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Relativistic Gravity and the Origin of Inertia and Inertial Mass

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Abstract

In order for the equilibrium of two bodies, with gravitational masses and positive electric charges, to be a frame-independent condition, the gravitational force should be transformed in exactly the same way as the Lorentz-force is transformed in different inertial frames. Thus, gravity should be described by a gravitomagnetic theory with equations which have the same mathematical form as those of the electromagnetic theory, with the gravitational mass as a Lorentz invariant. Using the gravitomagnetic theory, the principles of covariance and the equivalence principle we ensure the relativity of all kinds of translatory motion and we prove that,

1. The external inertial forces, perceived by an accelerating body, are real gravitational and electric forces, due to induction effects from the entire Universe.
2. The internal inertial force, perceived by an accelerating body, depends on the body's structure, but in the free fall is canceled, because of the equivalence principle. Thus, all bodies fall with the same acceleration.
3. The inertial rest mass of a body depends on the gravitational and electric scalar potentials, from the entire Universe, and can take positive or negative value.
4. The dependence of the inertial mass of a body, on the surrounding bodies, seems to be very important for the explanation of dark matter, dark energy and nuclear forces.
5. The gravitational and electric forces curve the spacetime perceived by a body and every freely moving body follows geodesic of it.
6. We derive a general spacetime metric where the Schwarzschild metric is a special case.
7. Finally, we extend the general spacetime metric to electric fields so that the similarity between the two forces is complete.

Keywords: relativistic gravity, gravitomagnetism, origin of inertia, origin of inertial mass, explanation of dark matter, explanation of dark energy, negative inertial mass.

1 Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. Inertial forces appear in a non-inertial reference frame. But what determines an inertial reference frame?

The first answer comes from Descartes and Newton, according to which, an inertial reference frame is a frame that moves at a constant velocity, with respect to the absolute space and the motion is absolute.

The inertial forces, such as the centrifugal force, must arise from acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter.

The second answer comes from Leibniz, Berkeley, Mach and is known as Mach' principle, according to which, an inertial reference frame is a frame that moves with constant velocity, with respect to the rest of the matter in the Universe, and the motion is relative. The inertial forces, such as the centrifugal force, are more likely caused by acceleration, with respect to the mass of the celestial bodies. This idea implies that the properties of space arise from the matter contained therein and are meaningless in an empty space.

The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important when we consider subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle is motion relative to other matter in the Universe has never found its complete expression in a physical theory. The Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity attempted to continue this relativization and interpret inertia, considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2].

The principle of equivalence is an essential part of General relativity. But although the principle of equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3]. Almost all of the results that have been the subject of experimental investigation can be described by the linear approximation of the field equations. The linear approximations of the field equations give us equations analogous to the equations of electromagnetism [4].

Finally, as pointed out by Henri Poincare, if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force [5][6]. But this does not happen with gravity as it is described by General relativity.

2 Relativistic Gravity

In an inertial reference frame K , let's have a system of two non spinning bodies with gravitational masses and positive electric charges, in a region of free space where there are no external forces. We suppose that the two bodies are at rest in the inertial frame K , under equilibrium conditions, i.e. the force of gravitational attraction balances that of electrostatic repulsion. But what is observed by another inertial frame of reference K' , moving with constant velocity relative to the frame K ? Let's imagine that if the bodies collide, they will explode. It is impossible for one observer to see an explosion and for another to not see it. So, the equilibrium must be a frame-independent condition. **Therefore, the gravitational force should be transformed in exactly the same way as the Lorentz-force is transformed in different inertial frames.**

- **In order for this to happen, gravity should be described by equations with the same mathematical form as those of electromagnetism.**

Moreover, as we will show later,

- If we want to ensure the relativity of all kinds of translatory motion, the gravitational and electric forces should affect the spacetime metric perceived by a body, in the same way.
- Different charged bodies perceive different spacetime metric, and every freely moving test-body follows geodesic of the metric that it perceives.

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformation, and the following series of remarks [7][8]:

1. The Coulomb potential for a stationary charge in vacuum is, $\varphi_e = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric charge produces a scalar potential and a vector potential, which together form a four-vector, $A_e = \left(\frac{\varphi_e}{c}, \mathbf{A} \right)$
3. The potentials produced by a charge moving in any way, depend only upon the velocity and position at the retarded time.

Therefore, if we want to obtain a gravitomagnetic theory, where its equations have the same mathematical form, as those of the electromagnetic theory, the same series of remarks must be met for gravity. We have only the first remark. The gravitational potential for a stationary gravitational mass m in vacuum is $\varphi_g = -\frac{1}{4\pi g_0} \frac{m}{r}$ where $g_0 = \frac{1}{4\pi G}$ and G is the Newton's universal gravitational constant. This is only the one remark. Therefore, we need the other two, as well. We will obtain them by the next two principles:

Principle 1

A gravitational mass produces a scalar potential and a vector potential, which together form a four-vector, $A_g = \left(\frac{\varphi_g}{c}, \mathbf{A}_g \right)$

From principle 1, follows that the gravitational mass is an invariant quantity. It has the same value in all inertial frames of reference. As we will show later, the inertial mass and the gravitational mass are different quantities. However, we will show that from the same point in a gravitational field, all the freely moving bodies have the same acceleration for non relativistic velocities.

Principle 2

The potentials produced by a gravitational mass moving in any way, depend only upon the velocity and position at the retarded time.

The potentials produced by a gravitational point-mass moving with velocity \mathbf{v} at a distance r from the point-mass, have the same mathematical form with the Lienard-Wiechert potentials for an electric point-charge moving with velocity \mathbf{v} at a distance r from the point-charge. The only difference is the minus sign,

$$\varphi_g = -\frac{1}{4\pi g_0} \left[\frac{m}{r - \mathbf{r}\mathbf{v}/c} \right] \quad (2.1)$$

$$\mathbf{A}_g = -\frac{1}{c^2} \frac{1}{4\pi g_0} \left[\frac{m\mathbf{v}}{r - \mathbf{r}\mathbf{v}/c} \right] = \frac{1}{c^2} [\varphi_g \mathbf{v}] \quad (2.2)$$

In the above equations, what is in square brackets refers to the retarded position of the gravitational mass and c is the speed of light in vacuum. Starting from the potentials, in order to find the fields, we have the equations

$$\mathbf{E}_g = -\nabla\varphi_g - \frac{\partial\mathbf{A}_g}{\partial t} \quad (2.3)$$

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g \quad (2.4)$$

The force, that a gravitational mass m experiences, when it moves with velocity \mathbf{v} in the above fields is,

$$\mathbf{F}_g = m(\mathbf{E}_g + \mathbf{v} \times \mathbf{B}_g) \quad (2.5)$$

where \mathbf{E}_g is the gravitational field and \mathbf{B}_g the gravitomagnetic field.

So, we have now a Gravitomagnetic theory, with equations that have the same mathematical form as those of the Electromagnetic theory. Therefore, we expect that there are gravitomagnetic waves that propagate, in vacuum with the speed of light, and that they are described by equations which have the same mathematical form as the corresponding equations for electromagnetic waves, but with one important difference. An isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment. However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity. The reason is simple. The electric dipole moment can move around with respect to the center of mass but the gravitational dipole moment is identical in location with the center of mass, and due to the law of conservation of momentum, cannot accelerate or radiate [9].

3 General relativity of motion

We will follow now, the fundamental idea of relativity of all kinds of translatory motion. In accordance with this idea we can detect and measure the translatory motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion.

The rotational motion of a rigid body, as a whole, cannot be relative. Different parts of the rotating body perceive different motion of the other bodies. Nevertheless, we can consider that a rotational rigid body consists of an infinite number of point particles and the velocity and acceleration of every point particle separately, is relative.

In order for all kinds of translatory motion to be relative, the laws of physics should have the same mathematical form and the same numerical values of quantities, when referred to different reference frames, which are in relative translatory motion, since otherwise the difference in form and in numerical values of quantities could provide a criterion for judging the absolute motion. So, first we accept the next principle,

Principle 3 - The principle of general relativity - The principle of covariance

The laws of physics which are valid in an inertial reference frame, i.e. the laws of Special theory of relativity, have the same mathematical form in all reference frames which are in relative translatory motion.

However, it does not end with the above principles. We will show that we need one more principle to ensure the relativity of all kinds of translatory motion. This arises from the fact that the expression of the equations of physics in a form which is independent of the reference frame does not in general prevent a change in their numerical content when we change from one reference frame to another and it is only

by relating such changes in numerical content to conceivable changes in gravitational and electric field, as we will show, that we are able to eliminate criteria for absolute motion and to preserve the idea of the relativity of all kinds of translatory motion [10].

We will consider now an accelerated reference frame by making a thought experiment, **the lab frame experiment**. We restrict our consideration, for the moment, only in gravitational fields. We will extend our consideration in electric fields, later. So, for the moment we assume that matter appears electrically neutral in the Universe. We will call this Universe, gravitational Universe.

Let's suppose that we have a space station, far from any massive body, that we use it as a laboratory. We will call the local reference frame of the space station, lab frame. An observer L is at rest in the lab frame. We assume that the distribution of matter in the Universe is such, that the gravitational field in the lab frame is zero. This means that the gravitational scalar potential φ_g , from the entire Universe, has the same value everywhere in the lab frame, and so,

$$\nabla\varphi_g = 0 \quad (3.1)$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitomagnetic vector potential due to one part of the mass-current, is canceled out by the vector potential due to another part of the mass-current, owing to its symmetry. This means that the gravitomagnetic vector potential \mathbf{A}_g from the entire Universe is zero, everywhere in the lab frame,

$$\mathbf{A}_g = 0 \quad (3.2)$$

This would also happen if all the bodies of the Universe were at rest, relative to the laboratory. So, we can say that the lab frame is at rest relative to the Universe, or at rest relative to the fixed stars.

In the lab frame, a small rocket starts to accelerate making translatory motion, with uniform proper acceleration, i.e. feels a constant force in its instantaneous rest frame. Let's have an observer R, at rest in the rocket. We will call the local rest frame of the rocket, rocket frame. Let's apply the covariant laws of gravitomagnetism in the rocket frame, following the principle of general relativity. Because we accept that acceleration is relative, the fixed stars are accelerating relative to the rocket frame, in the same way for all points of the rocket frame. Therefore the covariant laws of gravitomagnetism will give us an induced uniform gravitational field in the rocket frame, according to the Faraday's law for gravitomagnetism. We will consider this in detail, later.

So, the observer R is at rest in a local reference frame with a uniform gravitational field, while the observer L is at rest in a local inertial reference frame without gravity. We use an accelerating **local** reference frame because it is impossible to have a uniform gravitational field, **over a finite distance**, by using an accelerating frame. We assume that the gravitational forces between the rocket and the laboratory are negligible.

The observer L is at rest in a local inertial reference frame and so he can apply the laws of Special theory of relativity. The observer L, using the Special relativity, is able to describe what physical effects are observed by the observer R in the rocket frame. This is the well known study of a uniformly accelerated rigid reference frame, in Special relativity. So, from the viewpoint of the observer L, the well known physical effects, which are observed by an observer at rest in the rocket frame, are [11] [12]:

1. Redshift or blueshift of a light ray which moves parallel to the direction of the acceleration.
2. Varying coordinate speed of light; fixed local relative speed of light.
3. Spacetime is endowed with a metric.
4. Maximum proper time as the law of motion of freely moving bodies.

5. Horizons.

Let's consider now the description from the viewpoint of the observer R, in the rocket frame. From the viewpoint of the observer R, it is the lab frame that makes accelerating motion, relative to the rocket frame. So, if the induced gravitational field did not exist, by symmetry, the rocket frame would be equivalent to the lab frame and the above physical effects would have to occur in the lab frame and not in the rocket frame. Nevertheless, the uniform gravitational field exists and the above physical effects occur in the rocket frame and not in the lab frame. In order for this to happen, from the viewpoint of the observer R, the only way is to think that the induced uniform gravitational field, in the rocket frame, should be capable of causing all the above physical effects, with the same numerical values. For this reason we accept the next principle,

Principle 4 - The new principle of equivalence in the gravitational Universe

Physics in an accelerating local reference frame with proper acceleration $a = -g$, in a region without gravity, is equivalent to physics in a non accelerating local reference frame with a gravitational field, where all the released bodies fall with proper acceleration g .

From the viewpoint of the observer L, the lab frame is a local inertial reference frame without gravity and none of the above physical effects happen in the lab frame. **Nevertheless, from the viewpoint of the observer R, the lab frame makes free fall in the gravitational field that he perceives.** From the viewpoint of the observer R, none of the above physical effects happen in the lab frame, although it is accelerating, making free fall in the gravitational field. In order for this to happen, the only way is to think that the gravitational field must exactly cancel the acceleration of the lab frame so that, no sign of either acceleration or gravitation can be found by any physical means in the lab frame. So, we have an alternative expression of the new equivalence principle:

The new principle of equivalence (alternative expression)

Physics in a local reference frame, freely falling in a gravitational field, is equivalent to physics in an inertial reference frame without gravity.

From the viewpoint of the observer R, each of the fixed stars makes a free fall in the gravitational field created by the other accelerating fixed stars. Therefore, according to the equivalence principle, no sign of either acceleration or gravitation can be found by any physical means in the fixed stars and the lab frame that make free fall. So, the observer R, doesn't observe any radiation field of the fixed stars and the lab frame because radiation is a sign of acceleration or gravitation. In order for this to happen, the fields of the free falling stars are carried along convectively with the stars.

- **So, we come to the conclusion that, the instantaneous potentials of the free falling stars for the rocket frame, are the same with the potentials that they would have if they were moving with uniform velocity, equal to the instantaneous relative velocity.**

Therefore, we can find the instantaneous potentials of the free falling stars in the rocket frame, from the potentials of them in the lab frame, using just the Lorentz transformation with the instantaneous relative velocity.

Hence, when the rocket accelerates relative to the fixed stars it is equivalent to say either that

1. the rocket accelerates and thus radiates because of the acceleration, while the fixed stars are stationary and thus they do not radiate, or that

2. the rocket is stationary in a universal gravitational field where all the fixed stars make free fall, and thus they do not radiate, whereas the rocket which is at rest in the gravitational field radiates because of the gravitational field.

So, with the principle of general relativity and the new principle of equivalence we have ensured the relativity of all kinds of translatory motion.

- **Acceleration as well as velocity is relative.**

Moreover, the new equivalence principle shows us that,

- **the spacetime is endowed with a metric and the gravitational field affects the spacetime metric so that, the maximum proper time is the law of motion of a freely moving body in a gravitational field.**

The two above physical effects are so important that we will elevate them to physical principles. So, we will accept the next two principles:

Principle 5 - The principle of spacetime metric

Spacetime is endowed with a metric. The spacetime interval between two events is:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where $g_{\mu\nu}$ is the metric tensor.

Principle 6 - The principle of geodesic motion or of maximum proper time

Freely moving test bodies follow geodesic of the metric:

$$\delta \int ds = 0$$

The gravitational field affects the spacetime metric and so, we can say that gravity curves the spacetime. The spacetime of Special theory of relativity is the Minkowski spacetime which is a flat spacetime. While it is clear that flat and curved spaces are different entities, they are closely related. We are familiar from our experience with smooth curved surfaces that any smooth curved space can be approximated locally by a flat plane. This is the content of the local-flatness theorem. According to the local-flatness theorem, the metric in the immediate neighborhood of a point P is, to a close approximation, the Minkowski spacetime metric and the laws of Special relativity are valid there [13][14]. Therefore,

- **everywhere locally the laws of special relativity are valid.**

Let's follow now the principle of general relativity by applying the first Newton's law, which is a law of Special relativity, in the rest frame of an accelerating body. We assume that the body is small enough so that we can use the local-flatness theorem. Newton's first law of motion states that a body, subject to no forces, remains at rest or continues to move in a straight line with constant speed. In its rest frame the body is at rest and so, according to the first Newton's law the net force on the body is zero. So, we come to the conclusion that,

- **in the rest frame of any-body the total force on the body is zero.**

Now, having created all the tools we need, we can move on, and consider what are the inertial forces.

4 Inertia

4.1 Gravitational inertial forces

Let's continue now, the lab frame experiment. We assume that the lab frame and the rocket frame have the three sets of axes parallel and common the x, x' axis. The rocket starts to accelerate, along the x axis. We assume that the rocket frame is sufficiently small, so that, according to the local-flatness theorem, we can consider that the spacetime is flat in the rocket frame, and so, we can apply the laws of Special relativity. We have shown, in the previous chapter that we can find the potentials of the fixed stars in the accelerated rocket frame, from the potentials of them in the lab frame, using the Lorentz transformations with the instantaneous relative velocity.

The transformation laws which give the gravitational scalar potential φ'_g and the vector potential \mathbf{A}'_g in a moving frame S' , in terms of φ_g and \mathbf{A}_g in a stationary frame S , as measured at the same point in spacetime by people in the two frames, are

$$\begin{aligned} \varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}) & A'_{g-y} &= 0 \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g) & A'_{g-z} &= 0 \end{aligned} \quad \gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (4.1)$$

This assumes that the primed reference frame is moving with speed v in the positive x-direction, as measured in the unprimed reference frame [15][16]. We consider now that the rocket frame is the moving frame and the lab frame is the stationary frame. When the instantaneous speed of the rocket frame is v in the positive x-direction, as measured in the lab frame, it is straightforward to find the instantaneous potentials φ'_g and \mathbf{A}'_g in the accelerated rocket frame, in terms of φ_g and \mathbf{A}_g in the stationary lab frame, as measured at a point P common for both frames, by observers in the two frames at the same time. Substituting for A_{g-x} from equation (3.2) into equation (4.1), the potentials of the fixed stars, in the rocket frame, are

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.2)$$

$$\mathbf{A}'_g = -\frac{1}{c^2}\varphi'_g\mathbf{v} = -\frac{1}{c^2}\gamma(v)\varphi_g\mathbf{v} \quad (4.3)$$

The gravitational field in the rocket frame now, is given from the equation (2.3),

$$\mathbf{E}'_g = -\nabla'\varphi'_g - \frac{\partial\mathbf{A}'_g}{\partial\tau'} \quad (4.4)$$

where $\partial\tau'$ is the proper time interval, in the rocket frame.

The gravitomagnetic field, in the rocket frame, is zero because all the fixed stars make translatory motion in respect to the rocket frame and so, $\nabla' \times \mathbf{A}'_g = 0$.

The $\gamma(v)$ factor is the same everywhere in the rocket frame. Hence, from the equation (4.2), φ'_g is the same everywhere in the rocket frame and therefore,

$$\nabla'\varphi'_g = 0 \quad (4.5)$$

So, the gravitational field in the rocket frame becomes

$$\mathbf{E}'_g = -\frac{\partial\mathbf{A}'_g}{\partial\tau'} \quad (4.6)$$

Therefore, an induced gravitational field appears in the accelerated rocket frame, whereas in the lab frame there is no gravitational field. If a test-body K with gravitational mass m , is at rest in the rocket frame, will experience a gravitational force,

$$\mathbf{F}'_g = m\mathbf{E}'_g = -\frac{\partial (m\mathbf{A}'_g)}{\partial \tau'} \quad (4.7)$$

Substituting for \mathbf{A}'_g from equation (4.3) into equation (4.7), we have

$$\mathbf{F}'_g = \frac{m}{c^2} \frac{\partial(\varphi'_g \mathbf{v})}{\partial \tau'} = \frac{m}{c^2} \left(\frac{\partial \varphi'_g}{\partial \tau'} \mathbf{v} + \varphi'_g \frac{\partial \mathbf{v}}{\partial \tau'} \right) \quad (4.8)$$

We can have now, some very important results for non relativistic velocities, in the gravitational Universe, before moving on and considering the subject in the relativistic domain. For the moment, we restrict our consideration for non relativistic velocities and in an area where the gravitational scalar potential is constant. In this case, the gravitational scalar potential in the accelerated rocket frame is equal to the gravitational scalar potential in the lab frame. So, the equation (4.8) becomes

$$\mathbf{F}'_g = \frac{m\varphi_g}{c^2} \frac{\partial \mathbf{v}}{\partial t} = \left(-\frac{1}{c^2} m\varphi_g \right) (-\mathbf{a}) \quad (4.9)$$

where \mathbf{a} the acceleration of the test-body K.

If we recall now that the gravitational scalar potential φ_g of a gravitational mass is negative, it is obvious from the equation (4.9) that, the induced gravitational force on the test-body K, is opposite in direction to the acceleration of the body and thus resists to any change of its velocity. It is an inertial force.

Therefore, we will call the inertial force on the test-body K, which is given from the equation (4.7), external gravitational inertial force because it is due to external causes. So, the external gravitational inertial force $\mathbf{F}'_{ext-in-g}$, on the test-body K, is given by the equation

$$\mathbf{F}'_{ext-in-g} = \mathbf{F}'_g = -m \frac{\partial \mathbf{A}'_g}{\partial \tau'} \quad (4.10)$$

In addition to the external inertial force, there is also an internal inertial force. Let's consider what happens when a body is accelerating, relative to the fixed stars. The well known picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. We will call this self-force, internal inertial force, because it depends on the internal structure of the body [17].

According to the new equivalence principle, when a body makes free fall in a gravitational field, the gravitational field exactly cancels the acceleration so that, no sign of either acceleration or gravitation can be found by any physical means on the body. So, when a body makes free fall, every particle of the body exerts a force on every other, but the forces all balance in pairs and so, the internal net force is zero. **So, we come to the conclusion that, in the free fall of a body, the internal structure of the body does not play any role in the inertial force that the body experiences.**

- **In the free fall of a body in a gravitational field, the body experiences only the external inertial force.**

We imagine now, that the test-body **K** is a body without internal structure, like the elementary particles and thus, when it is accelerated by a force \mathbf{F} , it does not experience any internal inertial force. The inertial force on the test-body **K** is only the external inertial force. According to the first Newton's law, in the rest frame of the test-body **K**, the total force on the body is zero. Therefore, the force \mathbf{F} that accelerates the test-body **K** is

$$\mathbf{F} = -\mathbf{F}'_{ext-in-g} = -\mathbf{F}'_g = \left(-\frac{1}{c^2}m\varphi_g\right)\mathbf{a} \quad (4.11)$$

The equation (4.11) is the second Newton's law for non relativistic velocities. So, for non relativistic velocities, the gravitational inertial mass m_{in-g} of a body without internal structure, with gravitational mass m , is

$$m_{in-g} = \left(-\frac{1}{c^2}m\varphi_g\right) \quad (4.12)$$

Hence, the gravitational inertial mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential of the entire Universe. It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. With this consideration, the distant matter is of predominant importance, while local matter has only a small effect on the gravitational scalar potential

It may seem strange that we consider a body without internal structure, but in the free fall of a body in a gravitational field, as we have shown, the internal structure of the body does not play any role and the body experiences only the external inertial force. Therefore, if a body **A** with gravitational mass m_A , makes free fall in the gravitational field of a body **B**, which is static with spherically symmetric gravitational mass M , much bigger than the gravitational mass of the body **A**, the Newton's law of gravitation and the second law of Newton gives for the magnitude of the acceleration of the body **A**,

$$G\frac{m_A M}{r^2} = \left(-\frac{1}{c^2}m_A\varphi_g\right)a \quad (4.13)$$

where r is the distance of the body **A**, from the centre of the body **B**. It is obvious that the gravitational mass m_A is canceled in equation (4.13). Therefore, the acceleration of a body in a free fall, for non relativistic velocities, is independent of the gravitational mass of the body.

- **So, for non relativistic velocities, all the bodies released from the same point in a gravitational field, fall with the same acceleration.**

This is a fundamental experimental result that has been tested with enormous precision. In Einstein's General relativity, the above result is interpreted by accepting the equality of the gravitational and inertial mass.

4.2 Gravitational momentum and inertial mass in the gravitational Universe

Let's continue the lab frame experiment to find out, what is the inertial mass and the momentum of a body. We will start with the inertial mass and the momentum of the test-body **K** without internal structure, with gravitational mass m . We will find later the inertial mass and the momentum of a body with internal structure, a composite body. We consider the gravitational Universe where matter appears electrically neutral and so, we shall call the momentum of the test-body **K** gravitational momentum \mathbf{p}_g and its inertial mass, gravitational inertial mass m_{in-g} .

We suppose that the test-body K is accelerating by a force in the lab frame. In the inertial frame where the test-body K is momentarily at rest, the force is measured \mathbf{F}' and in the lab frame the same force is measured \mathbf{F} . The force is parallel to the relative motion of the two frames (longitudinal force) and so, according to the Lorentz transformation, it has the same value in the lab frame as it does in the rest frame of the test-body K. So,

$$\mathbf{F} = \mathbf{F}' \quad (4.14)$$

From the viewpoint of the lab frame, the gravitational momentum of the test-body K changes and the second Newton's law for the motion of it in the lab frame, is

$$\mathbf{F} = \frac{\partial \mathbf{p}_g}{\partial \tau} \quad (4.15)$$

In equation (4.15) $\partial \tau$ is the proper time interval in the lab frame. We use the partial derivative, for the second Newton's law, for a reason that will soon become apparent.

From the viewpoint of the rest frame of the test-body K, the gravitomagnetic vector potential from the entire Universe changes and causes the external gravitational inertial force which is

$$\mathbf{F}'_{ext-in-g} = -m \frac{\partial \mathbf{A}'_g}{\partial \tau'} \quad (4.16)$$

In equation (4.16) $\partial \tau'$ is the proper time interval in the rest frame of the test-body K. Here, every reference frame observes the motion of the other reference frame for the same proper time interval and thus we don't need to use the Lorentz transformation. The proper time interval is invariant. Therefore,

$$\partial \tau = \partial \tau' \quad (4.17)$$

The test-body K experiences the external gravitational inertial force $\mathbf{F}'_{ext-in-g}$ and the force \mathbf{F}' . For the rest frame of the test-body K, the body K is at rest and so, according to the first Newton's law, the total force on it is zero. So,

$$\mathbf{F}' = -\mathbf{F}'_{ext-in-g} \quad (4.18)$$

Hence, from all the above equations we have

$$\frac{\partial \mathbf{p}_g}{\partial \tau} = \frac{\partial (m \mathbf{A}'_g)}{\partial \tau} \quad (4.19)$$

By integration we have for the gravitational momentum of the test-body K, in the lab frame

$$\mathbf{p}_g = m \mathbf{A}'_g \quad (4.20)$$

The constant of integration is set equal to zero because we define the momentum to be zero when the velocity of the test-body K, relative to the lab frame, is zero. When the velocity of the test-body K, relative to the lab frame is \mathbf{v} , substituting for \mathbf{A}'_g from equation (4.3) into equation (4.20), we have the gravitational momentum of the test-body K, in the lab frame

$$\mathbf{p}_g = \gamma(v) \left(-\frac{1}{c^2} m \varphi_g \right) \mathbf{v} \quad (4.21)$$

From the equation (4.21), it is obvious that the gravitational momentum of the test-body K is a function of the gravitational scalar potential which depends on position and of the velocity which depends on time.

So, we should use partial derivative for the rate of change of the momentum in the second Newton's law. The gravitational inertial mass of the test-body K, in the lab frame is

$$m_{in-g} = \gamma(v) \left(-\frac{1}{c^2} m \varphi_g \right) \quad (4.22)$$

When the velocity of the test-body K, relative to the lab frame, is zero, the gravitational inertial mass m_{in-g} is equal to the gravitational inertial rest mass m_{in-g0}

$$m_{in-g0} = -\frac{1}{c^2} m \varphi_g \quad (4.23)$$

Let's see now, what is, the gravitational inertial rest mass of the test-body K. The instantaneous sum of the four-vector gravitational potentials of all the bodies in the Universe is also a four-vector [18]. It is the total four-vector gravitational potential A_g

$$A_g = \left(\frac{\varphi_g}{c}, \mathbf{A}_g \right) \quad (4.24)$$

The four-velocity of the test-body K is,

$$U = \gamma(c, \mathbf{v}) \quad (4.25)$$

The product mUA_g is Lorentz invariant, because the gravitational mass m is a Lorentz scalar, and the product of two four-vectors is also a scalar. Evaluating the product mUA_g in the rest frame of the test-body K, when the test-body K is at rest in the lab frame, we obtain

$$mUA_g = m(c, 0) \left(\frac{\varphi_g}{c}, \mathbf{A}_g \right) = m \varphi_g \quad (4.26)$$

So, from the equations (4.23) and (4.26), we come to the conclusions that,

1. the gravitational inertial rest mass of a body, without internal structure, is an invariant quantity.
2. the gravitational inertial rest mass of a body, without internal structure, is not an intrinsic property of the body but, in the lab frame, is the binding energy of the body with the entire Universe, when the body is at rest in the lab frame, divided by c^2 .

Therefore, the $m_{in-g}c$ and \mathbf{p}_g are the components of a four vector, the four-momentum of the test-body K without internal structure

$$P = m_{in-g0}U = m_{in-g0}\gamma(v)(c, \mathbf{v}) = (m_{in-g}c, \mathbf{p}_g) = \left(\frac{E_g}{c}, \mathbf{p}_g \right) \quad (4.27)$$

When the test-body K is at rest in the lab frame, if we accept that its "gravitational potential rest energy" is $U_{g0} = m \varphi_g$ then, the total gravitational rest energy of the test-body K, is

$$E_{g0} + U_{g0} = -m \varphi_g + m \varphi_g = 0 \quad (4.28)$$

So, the total gravitational rest energy of the Universe is zero! It's noteworthy that Richard Feynman writes in the "Lectures on Gravitation" [19]:

“Another spectacular coincidence relating the gravitational constant to the size of the universe comes in considering the total energy. The total gravitational energy of all the particles of the universe is something like GMM/R , where $R=Tc$, and T is the Hubble’s time. [...] If now we compare this number to the total rest energy of the universe, Mc^2 , lo and behold, we get the amazing result that $GM^2/R = Mc^2$, so that the total energy of the universe is zero. [...] Why this should be so is one of the great mysteries and therefore one of the important question of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate?”

We will derive now the equation for the gravitational momentum, with a second way.

In the lab frame the rocket, which is initially at rest, starts to accelerate. We suppose that the rocket frame is so small that we can consider that the spacetime is flat on it and so, we can apply the laws of special relativity. Exactly when the rocket starts to accelerate, the observer R releases the test-body K. From the viewpoint of the observer R, the rocket is stationary in a gravitational field. So, the test-body K makes free fall in a gravitational field because it experiences a gravitational force which is equal to the external gravitational inertial force on the rocket frame

$$\mathbf{F}'_{ext-in-g} = -m \frac{\partial \mathbf{A}'_g}{\partial \tau'} \quad (4.29)$$

where $\partial \tau'$ is the proper time interval, in the rocket frame. The second Newton’s law for the motion of the test-body K in the rocket frame, is

$$\mathbf{F}'_{ext-in-g} = \frac{\partial \mathbf{p}'_g}{\partial \tau'} \quad (4.30)$$

From equation (4.29) and (4.30) we have

$$\frac{\partial \mathbf{p}'_g}{\partial \tau'} = -\frac{\partial (m \mathbf{A}'_g)}{\partial \tau'} \quad (4.31)$$

By integration we have

$$\mathbf{p}'_g = -m \mathbf{A}'_g \quad (4.32)$$

The constant of integration is set equal to zero because we define the momentum to be zero when the velocity of the test-body K relative to the rocket frame is zero. When the instantaneous velocity of the rocket frame relative to the lab frame is \mathbf{v} , the instantaneous velocity of the test-body K relative to the rocket frame is $(-\mathbf{v})$. Substituting for \mathbf{A}'_g from equation (4.3) into equation (4.32), we have the gravitational momentum of the test-body K, in the rocket frame

$$\mathbf{p}'_g = \gamma(v) \left(-\frac{1}{c^2} m \varphi_g \right) (-\mathbf{v}) \quad (4.33)$$

So, we come to the same conclusions as before.

Let’s consider now, if the gravitational inertial mass of a body depends on whether its acceleration is in direction towards the center of our galaxy or in some other direction. This problem has the name “anisotropy of inertia” and was the subject of experimental investigation with negative results [20].

The gravitational scalar potential at a point P, in the field of a point-gravitational mass m , moving at a constant velocity \mathbf{v} , is given from an equation similar to the well known corresponding equation of electromagnetism. The only difference is the minus sign [21],

$$\varphi_g = -\frac{1}{4\pi g_0} \frac{m}{r (1 - \beta^2 \sin^2 \theta)^{\frac{1}{2}}} \quad (4.34)$$

where r is the distance from the point-gravitational mass to the field point P, $\beta = \frac{v}{c}$ and θ is the angle between the position vector and the velocity vector, of the point-gravitational mass. From the equation (4.34) we can see that any anisotropy of inertia comes from relativistic velocities.

From astronomical observations we know that, the very distant galaxies, move at relativistic velocities. However, their distribution in space is homogenous, as seen from afar. Because of this symmetrical distribution, the gravitational mass of a body, which is due to the distant galaxies, is independent of the direction of its acceleration relative to them. The nearest matter is certainly moving with non relativistic velocities.

Therefore, if a body moves also with non relativistic velocity, its gravitational inertial mass is independent of the direction of its acceleration. The results are in agreement with past experiments.

4.3 Electric momentum and electric inertial mass in an electric Universe

Let's imagine now, a Universe where all the bodies have only electric charges and no gravitational masses. We will call this Universe, electric Universe. To ensure the relativity of all kinds of translatory motion in the electric Universe, as we have shown in chapter 3, it is required to accept the principle of general relativity and an appropriate equivalence principle.

The new Principle of equivalence in an electric Universe

Physics in an accelerating local reference frame with proper acceleration $a = -g$, in a region without electric field, is equivalent to physics in a non accelerating local reference frame with an electric field, where all the released charges fall with proper acceleration g .

So, the electric field affects the spacetime metric and all the freely moving charges follow geodesic of the metric, as we have shown in chapter 3. **But, how is it possible positive and negative charges to move the same in the same induced electric field?**

Let's make the lab frame experiment in the electric Universe. We suppose that the distribution of the electric charges is such that, the electric scalar potential is the same everywhere in the lab frame so that, $\nabla\varphi_g = 0$ and the electromagnetic vector potential in the lab frame, is zero. Because the equations of electromagnetism have the same mathematical form with the equations of gravitomagnetism and we have done already the lab frame experiment in the gravitational Universe, it is not necessary to describe all the details again, but we will only state the important results.

From the viewpoint of the observer in the rocket frame, everything outside of the rocket frame makes a free fall in the universal electric field that he perceives and so, according to the new equivalence principle does not radiate. The electric momentum of a test-charge M without internal structure, with electric charge q , in the lab frame is

$$\mathbf{p}_e = \gamma(v) \left(-\frac{1}{c^2} q\varphi_e \right) \mathbf{v} \quad (4.35)$$

The electric inertial mass of the test-charge M, is

$$m_{in-e} = \gamma(v) \left(-\frac{1}{c^2} q\varphi_e \right) \quad (4.36)$$

The electric inertial mass, as we can see from equation (4.36), can be negative while the gravitational inertial rest mass is always positive, as we can see from equation (4.22). What does that mean? According to the second Newton's law, negative inertial mass means opposite acceleration. Thus, if a charge with

positive electric inertial mass is attracted from another charge, the same charge with negative electric inertial mass will be repelled.

Let's have an electric Universe with positive charges. If we assume, that the test-charge M has a positive charge, its electric inertial mass is negative and so, it is attracted from the other positive charges. If we assume now, that the test-charge M has a negative charge, its electric inertial mass is positive, and thus it is also attracted from the other positive charges.

Let's have an electric Universe with negative charges. If we assume, that the test-charge M has a positive charge, its electric inertial mass is positive and so, it is attracted from the other negative charges. If we assume that the test-charge M has a negative charge, its electric inertial mass is negative, and thus it is also attracted from the other negative charges.

So, in an electric Universe, all the kinds of charges will be attracted by each other!!! Therefore, what determines if a body is attracted or repelled by a force is both the kind of its inertial rest mass and the force acting on it, and not the force alone.

4.4 The momentum and the inertial mass in the real Universe

Let's consider now, a Universe with gravitational masses and electric charges. We will call this Universe, real Universe. To ensure the relativity of all kinds of translatory motion it is required to accept the principle of general relativity and an appropriate equivalence principle.

The new Principle of equivalence in the real Universe

Physics in an accelerating local reference frame with proper acceleration $a = -g$, in a region without gravitational and electric field, is equivalent to physics in a non accelerating local reference frame with a gravitational and an electric field, where all the released bodies fall with proper acceleration g .

According to the new principle of equivalence in the real Universe, there are combinations of gravitational and electric fields where all the bodies, charged or not, will move in the same way, as it happens with the induced gravitational and electric field that appears in the accelerating reference frame, but this is not always the case. So, the gravitational and the electric field affect the spacetime metric but we cannot say now, in the real Universe, that all the freely moving bodies follow geodesic of the metric.

- **So, in the real Universe, in order for the principle of geodesic motion to hold for anybody, charged or not, the gravitational and electric forces should affect the spacetime metric perceived by a body and not the spacetime itself. The spacetime doesn't have properties of its own.**

When different electric charges, are at the same point in an electric field, each one will perceive a spacetime with different metric. So, we must restate the principles of spacetime metric and geodesic motion.

Principle 5 - The principle of spacetime metric

The spacetime perceived by a body, is endowed with a metric.

Principle 6 - The principle of geodesic motion or of maximum proper time

Every freely moving test-body follow geodesic of the metric that it perceives.

What about the motion of a ray of light in the real Universe? We suppose that in the lab frame a ray of light propagates in a straight line and the rocket is accelerating in direction perpendicular to this line. For the rocket frame the ray of light will be judged to fall, like everything else. For the rocket frame the path of the ray of light does not depend on the kind of electric charges and therefore from the direction of the electric field in the area where the rocket is accelerating. So, in the real Universe, according to the new equivalence principle, the path of a ray of light behaves as if it is affected only by the gravitational field. Of course the issue of the motion of a ray of light is open but in any case, if we ignore the electric field, we can say that,

- **The path of a light ray, in the real Universe, is the same as the imaginary path of a body with gravitational mass, which moves with the speed of light in vacuum.**

Let's make the lab frame experiment, in the real Universe, to find the momentum and the inertial mass of a body. We suppose that the distribution of the gravitational masses and the electric charges is such that, the gravitational and electric scalar potential are the same everywhere in the lab frame so that, $\nabla\varphi_g = 0$ and $\nabla\varphi_e = 0$ and the gravitomagnetic and electromagnetic vector potential \mathbf{A}_g and \mathbf{A}_e in the lab frame, are zero.

We suppose that a test-body B without internal structure, with gravitational mass m and electric charge q , is accelerating by a force in the lab frame. In the inertial frame where the test-body B is momentarily at rest, the force is measured \mathbf{F}' and in the lab frame the same force is measured \mathbf{F} . The force is parallel to the relative motion of the two frames (longitudinal force) and so, it has the same value in the lab frame as it does in the rest frame of the test-body B. So,

$$\mathbf{F} = \mathbf{F}' \quad (4.37)$$

From the viewpoint of the lab frame, the momentum \mathbf{p}_{ge} of the test-body B changes and the second Newton's law for the motion of it in the lab frame, is

$$\mathbf{F} = \frac{\partial \mathbf{p}_{ge}}{\partial \tau} \quad (4.38)$$

where $\partial\tau$ is the proper time interval in the lab frame.

From the viewpoint of the rest frame of the test-body B, the gravitomagnetic and the electromagnetic vector potential from the entire Universe changes and causes the total external inertial force which is

$$\mathbf{F}'_{ext-in} = -m \frac{\partial \mathbf{A}'_g}{\partial \tau'} - q \frac{\partial \mathbf{A}'_e}{\partial \tau'} \quad (4.39)$$

where $\partial\tau'$ is the proper time interval in the rest frame of the test-body B. Here, every reference frame observes the motion of the other reference frame for the same proper time interval and thus we don't need to use the Lorentz transformation. Therefore,

$$\partial\tau = \partial\tau' \quad (4.40)$$

The test-body B experiences the total external inertial force \mathbf{F}'_{ext-in} and the force \mathbf{F}' . For the rest frame of the test-body B, the body is at rest and so, according to the first Newton's law, the total force on it is zero. So,

$$\mathbf{F}' = -\mathbf{F}'_{ext-in} \quad (4.41)$$

Hence, from all the above equations we have

$$\frac{\partial \mathbf{p}_{ge}}{\partial \tau} = \frac{\partial (m\mathbf{A}'_g + q\mathbf{A}'_e)}{\partial \tau} \quad (4.42)$$

By integration we have

$$\mathbf{p}_{ge} = (m\mathbf{A}'_g + q\mathbf{A}'_e) \quad (4.43)$$

The constant of integration is set equal to zero because we define the momentum to be zero when the velocity of the test-body B, relative to the lab frame, is zero. When the velocity of the test-body K, relative to the lab frame is \mathbf{v} , the gravitomagnetic vector potential, in the rest frame of the test-body K, is given from equation (4.3) and the electromagnetic vector potential from a similar equation

$$\mathbf{A}'_e = -\frac{1}{c^2}\varphi'_e\mathbf{v} = -\frac{1}{c^2}\gamma(v)\varphi_e\mathbf{v} \quad (4.44)$$

Substituting for \mathbf{A}'_g and for \mathbf{A}'_e from equations (4.3) and (4.44) into equation (4.43), we have the momentum of the test-body B, in the lab frame

$$\mathbf{p}_{ge} = \gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \mathbf{v} \quad (4.45)$$

The inertial mass m_{in-ge} of the test-body B is

$$m_{in-ge} = \gamma(v) \left[-\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \quad (4.46)$$

We assume now, that the Universe consists of n discrete gravitational masses, and m discrete electric charges, each at a different distance from the the test-body B. We also assume that all the gravitational masses and electric charges of the Universe are moving with non relativistic velocities, relative to the test-body B. In this case, according to equations (4.46), the inertial rest mass of the the test-body B is

$$m_{in-ge0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \sum_{i=1}^m \frac{qq_i}{r_i} \right) \quad (4.47)$$

The distance r is measured in the lab frame. The above equation shows us that, the inertial rest mass of a body, can take any value, positive or negative. It depends on the position of the body in relation to the other gravitational masses and electric charges. This seems to be very important for the explanation of dark matter, dark energy and certainly for phenomena of nuclear physics, as we will show below.

Having finished with the inertial mass of a point-particle without internal structure, let's consider now the inertial rest mass of a composite body M, a body with internal structure. The special theory of relativity accepts the principle of conservation of four-momentum, i.e. the sum of the four-momentum of all the particles going into a collision, is the same as the sum of the four-momentum of all those coming out. If we apply the conservation of four-momentum in a collision where n point particles, without internal structure, collide and create a composite body M, the total inertial rest mass m_{in-0} of the composite body M is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + \sum T/c^2 + E_{field}/c^2 \quad (4.48)$$

where $m_{in-ge0i}$ is the inertial rest mass of each particle, without internal structure, that makes up the body M, T is the kinetic energy of the relative motion of the particles that make up the body M, and E_{field} is the potential energy of the interaction of the particles that make up the body M [22]. **The total inertial rest mass m_{in-0} is also invariant** as is well known from the Special theory of relativity [23].

- So, if we have the inertial rest mass of each point particle, without internal structure, using the principle of conservation of four-momentum, in a collision, we end up with the relation that gives us the total inertial rest mass of the body M with internal structure, which is Lorentz invariant.

So, the four-momentum P of the body M, is

$$P = m_{in-0}U = m_{in-0}\gamma(v)(c, \mathbf{v}) = (m_{in}c, \mathbf{p}) = \left(\frac{E}{c}, \mathbf{p}\right) \quad (4.49)$$

Let's apply now the equation (4.47) in two interesting cases.

4.5 Dark matter

From equation (4.47), for non relativistic velocities, it follows that the inertial rest mass of a star is proportional to the gravitational scalar potential of the entire Universe. From astronomical observation, we know that the distribution of matter in the Universe is highly inhomogeneous; there are planets, the sun, stars, galaxies, clusters of galaxies and so on. So, it seems that the position where a star is located, affects significantly the inertial mass of the star. In places with higher density of matter the inertial mass of a star will be greater than the inertial mass of the same star, in a place with lower density of matter. This phenomenon has been observed, but the inability to explain it has led to the theory that in the Universe most of the matter is dark matter. It is very likely that the equation (4.47) provides a solution to this problem.

4.6 Zero inertial mass

Let's consider now for two charged particles, at what distance the contribution of the electric scalar potential to their inertial masses can be significant.

The inertial rest mass of a particle B without internal structure, with gravitational mass m and electric charge q , close to an electric charge Q , according to the equation (4.47), is

$$m_{in-ge0} = m_{in-g0} + m_{in-e0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^N \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \right) = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad (4.50)$$

For like charges there is a critical distance $r_{critical}$, where the inertial rest mass of the particle B becomes zero. Let's calculate this critical distance. When the inertial rest mass of the particle B is zero, equation (4.50) becomes

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r_{critical}} \Rightarrow r_{critical} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{m_{in-g0}} \quad (4.51)$$

For two protons $r_{critical} \approx 1,53 \times 10^{-18}m$ and for two electrons $r_{critical} \approx 2,81 \times 10^{-15}m$. Let us remember that the size of nucleus is of an order of $10^{-14}m$ and the nuclear force is attractive between nucleus at distances of about $10^{-15}m$.

What if two protons approach at a shorter distance? What if the inertial rest mass becomes negative? Do we really need the nuclear forces to explain phenomena of nuclear physics? A door is opening and many questions arise.

5 Spacetime metric

5.1 Spacetime metric outside of a static and spherically symmetric gravitational mass and electric charge

We have shown that the gravitational and electric forces affect the spacetime metric perceived by a body. The spacetime doesn't have properties of its own and different electric charges, at the same point in an electric field, will perceive a spacetime with different metric.

We will find now the spacetime metric that perceives a test-body A with gravitational mass m and electric charge q , outside of a static body B with spherically symmetric distribution of gravitational mass M and electric charge Q . We suppose that $M \gg m$ and $Q \gg q$. We will consider the freely motion of the test-body A in the radial direction of the field of the body B. We will follow a method based on a paper of F. Tangherlini [24][25][26].

It is well documented that we can bring the spacetime interval

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (5.1)$$

outside of a static body with spherically symmetric distribution of gravitational mass, into the standard Schwarzschild form [27]

$$ds^2 = g_{00} c^2 dt^2 + g_{11} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.2)$$

We assume now that,

1. The space-time interval, perceived by the test body A, is spherically symmetric like the body B. So, we can bring the spacetime interval perceived by the test body A outside of the body B, also into the standard Schwarzschild form. Therefore, equation (5.2) also applies to our case.
2. The scalar functions g_{00} and g_{11} , of the metric tensor, are function only of the distance r of the body A from the centre of the body B.
3. The inertial rest mass m_{in-0} , of the test body A, is constant during the radial motion.

The metric (5.2) should give us to infinity the Minkowski metric in spherical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.3)$$

So we must have the boundary conditions

$$\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1 \quad \lim_{r \rightarrow \infty} g_{11}(r) \rightarrow -1 \quad \lim_{r \rightarrow \infty} g_{00}(r)g_{11}(r) \rightarrow -1 \quad (5.4)$$

For radial motion of the test-body A the spacetime interval becomes

$$ds^2 = g_{00}(r) c^2 dt^2 + g_{11}(r) dr^2 \quad (5.5)$$

From the principle of geodesic motion we have

$$\delta \int ds = 0 \Rightarrow \delta \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = 0 \Rightarrow \delta \int \sqrt{L} d\tau = 0 \quad (5.6)$$

The L may be termed a 'lagrangian'. Using the relations

$$\dot{x}^\mu = \frac{dx^\mu}{d\tau} \quad \dot{x}^\nu = \frac{dx^\nu}{d\tau} \quad (5.7)$$

the lagrangian L becomes

$$L = g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \quad (5.8)$$

For radial motion the lagrangian becomes

$$L = g_{00}c^2 \left(\frac{dt}{d\tau} \right)^2 + g_{11} \left(\frac{dr}{d\tau} \right)^2 \quad (5.9)$$

From the lagrangian, using the calculus of variation, we obtain the Euler-Lagrange system of equations

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}} \right) - \frac{\partial L}{\partial t} = 0 \quad (5.10)$$

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0 \quad (5.11)$$

where

$$\dot{t} = \frac{dt}{d\tau} \quad \text{and} \quad \dot{r} = \frac{dr}{d\tau}$$

Because the Lagrangian does not depend on time, we have $\frac{\partial L}{\partial t} = 0$. Hence, from equation (5.10), arise that $\frac{\partial L}{\partial \dot{t}} = \text{const.}$ Therefore, the term $\frac{\partial L}{\partial \dot{t}}$ is a conserved quantity. Performing the differentiation in the equation (5.9), using the fact that the metric function $g_{00}(r)$ doesn't depend on \dot{t} and the relation $ds^2 = c^2 d\tau^2$, we obtain

$$\frac{\partial L}{\partial \dot{t}} = 2g_{00}c^2 \frac{dt}{d\tau} = \text{const.} \Rightarrow g_{00} \frac{dt}{ds} = k_0 \quad (5.12)$$

Equation (5.12) is a first integral for the equation of motion of the test-body A, in the radial direction and states that the body's energy k_0 (in units of $m_{in-0}c^2$) is a constant of the motion.

We can have a second first integral for the equation of motion in the radial direction by dividing the equation (5.5) by the spacetime interval ds^2

$$1 = g_{00}c^2 \left(\frac{dt}{ds} \right)^2 + g_{11} \left(\frac{dr}{ds} \right)^2 \quad (5.13)$$

Equation (5.13) is the second first integral for the equation of motion of the test-body A, in the radial direction and states the invariant relation between energy and momentum (in units of $m_{in-0}c^2$).

Eliminating $\frac{dt}{ds}$ from the equation (5.13), using the first integral from equation (5.12), we have

$$1 = \frac{c^2 k_0^2}{g_{00}} + g_{11} \left(\frac{dr}{ds} \right)^2 \quad (5.14)$$

Define a matrix $g^{\nu\sigma}$ as the inverse of $g_{\nu\sigma}$, that is, $g^{\nu\sigma} g_{\kappa\nu} = \delta_\kappa^\sigma$. So,

$$g^{00} = \frac{1}{g_{00}} \quad \text{and} \quad g^{11} = \frac{1}{g_{11}} \quad (5.15)$$

Dividing the equation (5.14) by g_{11} and using the equations (5.15) we have

$$\frac{1}{g_{11}} = \frac{c^2 k_0^2}{g_{00}g_{11}} + \left(\frac{dr}{ds} \right)^2 \Rightarrow g^{11} = c^2 k_0^2 (g^{00} g^{11}) + \left(\frac{dr}{ds} \right)^2 \quad (5.16)$$

Because we assume that g_{00} and g_{11} are functions only of r

$$\frac{dg^{00}}{ds} = \frac{\partial g^{00}}{\partial r} \frac{dr}{ds} = g^{00},r \frac{dr}{ds} \quad \text{and} \quad \frac{dg^{11}}{ds} = \frac{\partial g^{11}}{\partial r} \frac{dr}{ds} = g^{11},r \frac{dr}{ds} \quad (5.17)$$

where a comma denotes ordinary differentiation. Using the relations (5.17) we differentiate the equation (5.16) in respect to s and we obtain the following equation

$$g^{11},r \frac{dr}{ds} = c^2 k_0^2 (g^{00} g^{11}),r \frac{dr}{ds} + 2 \frac{dr}{ds} \frac{d^2 r}{ds^2} \Rightarrow \frac{d^2 r}{ds^2} = -\frac{c^2 k_0^2}{2} (g^{00} g^{11}),r + \frac{1}{2} g^{11},r \quad (5.18)$$

Finally, using the relation $ds^2 = c^2 d\tau^2$, we obtain the radial geodesic equation in the form,

$$\frac{d^2 r}{d\tau^2} = -\frac{c^4 k_0^2}{2} (g^{00} g^{11}),r + \frac{c^2}{2} g^{11},r \quad (5.19)$$

In the above equation (5.19) we see that because of the k_0 term, the radial acceleration depends on the energy which the test-body A had initially, i.e. the radial velocity with which the test-body A was launched.

Let's consider now the radial motion of the test-body A, with inertial rest mass m_{in-0} , into the gravitational field \mathbf{E}_{gB} and the electric field \mathbf{E}_{eB} of the body B. Because the body B is static there is no gravitomagnetic and electromagnetic field. The well-known equation of motion in an electric field, which apply now also for the gravitational field [28], is

$$m_{in-0} \frac{d^2 \mathbf{r}}{d\tau^2} = \gamma(v) m \mathbf{E}_{gB} + \gamma(v) q \mathbf{E}_{eB} \quad (5.20)$$

However, it is well known that the electric force $q \mathbf{E}_{eB}$ on the test-body A is independent of its velocity. The same happens with the gravitational force $m \mathbf{E}_{gB}$ because both forces are described by equations with the same mathematical form [29]. **So, for non relativistic velocities, the radial acceleration of the test-body A is independent of its velocity,**

$$\frac{d^2 \mathbf{r}}{d\tau^2} = \frac{1}{m_{in-0}} (m \mathbf{E}_{gB} + q \mathbf{E}_{eB}) \quad (5.21)$$

Therefore, if we want the equation (5.19) to be valid for non-relativistic velocities, this must be valid,

$$(g^{00} g^{11}),r = 0 \Rightarrow g^{00} g^{11} = \text{const.} \quad (5.22)$$

From the equation (5.22), using the boundary conditions (5.3) we have

$$g^{00} g^{11} = -1 \Rightarrow g^{11} = -\frac{1}{g^{00}} \Rightarrow g^{11} = -g_{00} \quad (5.23)$$

So, the equation (5.19), using the relations (5.22) and (5.23), becomes

$$\frac{d^2 r}{d\tau^2} = -\frac{c^2}{2} g_{00},r \quad (5.24)$$

Because the fields are produced by the static body B, the equation (5.21), using the potentials, becomes

$$\frac{d^2 \mathbf{r}}{d\tau^2} = -\frac{1}{m_{in-0}} \nabla (m \varphi_{gB} + q \varphi_{eB}) \quad (5.25)$$

Comparing the equations (5.24) and (5.25) we have

$$g_{00,r} = \frac{2}{m_{in-0}c^2} \nabla (m\varphi_{gB} + q\varphi_{eB}) \quad (5.26)$$

Because we assume that the inertial rest mass m_{in-0} of the test body A, is constant during the radial motion, using the boundary conditions (5.3), we obtain for g_{00} and g_{11}

$$g_{00} = 1 + \frac{2}{m_{in-0}c^2} (m\varphi_{gB} + q\varphi_{eB}) \quad (5.27)$$

$$g_{11} = -\frac{1}{1 + \frac{2}{m_{in-0}c^2} (m\varphi_{gB} + q\varphi_{eB})} \quad (5.28)$$

So, the spacetime metric that perceives the test-body A, at a distance r from the centre of the body B, for non relativistic velocities, is

$$ds^2 = \left[1 - \frac{1}{c^2 m_{in-0}} \left(\frac{2GMm}{r} - \frac{2KQq}{r} \right) \right] c^2 dt^2 - \frac{dr^2}{\left[1 - \frac{1}{c^2 m_{in-0}} \left(\frac{2GMm}{r} - \frac{2KQq}{r} \right) \right]} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.29)$$

where $K = \frac{1}{4\pi\epsilon_0}$. In the International system of units the gravitational mass and the inertial rest mass of a body, are considered to have the same value. So, the ratio of the gravitational mass to the inertial rest mass of the test-body A, in our position in our galaxy, is equal to one. Therefore, if we restrict our consideration, in our position and only for gravitational masses, the equation (5.29) reduces to the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.30)$$

So, all the phenomena that emerge from the Schwarzschild metric also emerge from the general spacetime metric (5.29). At another point in our Galaxy or in the Universe, the ratio of gravitational to inertial mass is not equal to one. There, the phenomena will be the same qualitatively but not quantitatively.

5.2 Dark Energy

Let's consider now the light emitted by an atom, with gravitational mass m and inertial rest mass m_{in-0} , at a distance r_{em} from the centre of a star, which is static with spherically symmetric distribution of gravitational mass M . From equation (5.29) arises the equation relating the proper time $d\tau_{em}$ at the point of emission, with the proper time $d\tau_{\infty}$ at infinity where is the point of observation [30]

$$d\tau_{\infty} = \frac{d\tau_{em}}{\sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-0}}}} \quad (5.31)$$

So, the frequency of light observed by an observer, in the point of observation, will be

$$f_{\infty} = f_{em} \sqrt{1 - \frac{2GMm}{c^2 r_{em} m_{in-0}}} \quad (5.32)$$

The equation (5.32) describes the red shift of spectral lines, which is emitted by an atom in a gravitational field and is received by a body, which is out of the gravitational field. This phenomenon is known as gravitational red shift.

As the Universe expands, according to the equation (4.47), the inertial rest mass of a star decreases. Thus, the inertial rest mass of an atom, which emits light, decreases over time. As it emerges from equation (5.32) the light emitted by two identical supernovas Ia, at different moments in the history of the Universe, will have different red shift. Because the atoms in a younger supernova have smaller inertial rest mass than the inertial rest mass of the atoms in an older supernova, the light emitted by a younger supernova Ia has greater red shift than the light emitted by an older supernova Ia. This phenomenon has been observed, but the inability to explain why the red shift of spectral lines is greater, has led to the theory that the Universe expands in an accelerating way, because of dark energy. It is very likely that the equations (4.47) and (5.32) provide a solution to this problem.

Conclusions

Finally, the physics that comes from the attempt to explore the origin of inertia shows us that the motion of a tiny body is affected by the entire Universe. As Dennis Sciama ended an article on inertia in Scientific American [31]: “If atomic properties are in fact so determined, we shall again be faced with the dual situation: Distant matter influencing local phenomena and local phenomena giving us information about distant matter. The scientist would then be able to claim that his imagination had out-stripped the poet’s. For he would see the world not in a “grain of sand” but in an atom” The Universe never stops surprising us!!!

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