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# Relativistic Gravity and the Origin of Inertia and Inertial Mass

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**Abstract.** If equilibrium is a frame-independent condition, the gravitational force should be transformed in exactly the same way as the electric force does in different inertial frames. In order for this to happen, gravity should be described by a gravitomagnetic theory in which the equations have the same mathematical form as those of the electromagnetic theory, with the gravitational mass as a Lorentz invariant. The Faraday's law of electromagnetism, will therefore become a fundamental law of the gravitomagnetism too, and allow us to prove the relativity of all kinds of translatory motion, by accepting the principles of covariance for all reference frames in relative translatory motion, and restating the equivalence principle for both the gravitomagnetism and electromagnetism. Following the principles, we prove that,

- 1) The external inertial forces, perceived by an accelerating body, are real gravitational and electric forces, due to induction effects from the entire Universe, and not fictitious forces.
- 2) The internal inertial force, perceived by an accelerating body, depends on the body's structure, but in the free fall is canceled, because of the equivalence principle, and the body experiences only the external inertial force. Thus, all bodies fall with the same acceleration.
- 3) The inertial rest mass of a body depends on the gravitational and electric scalar potentials, from the entire Universe, and can take positive or negative value. Both this value and the force acting on the body, determine if the body is attracted or repelled by the force.
- 4) The dependence of the inertial mass of a body, on the surrounding bodies, seems to be very important for the explanation of dark matter, dark energy and for phenomena of nuclear physics. The greater the density of stars in a galaxy the greater the inertial mass of the stars. As the Universe expands, the inertial mass of the atoms in a supernova Ia decreases and they emit light that becomes more and more red shifted. The inertial mass of a proton, at a distance smaller than  $10^{-18}$  m from another proton, becomes negative.
- 5) The gravitational and electric forces curve the spacetime perceived by a body and every freely moving body follows geodesic of the metric that it perceives.
- 6) We can derive the general spacetime metric that perceives a body outside of a static and spherically symmetric gravitational mass. The Schwarzschild metric is a special case of this.
- 7) Finally, we extend the general spacetime metric to electric fields so that the similarity between the two forces is complete.

The new theory satisfies the three criteria for viability: self-consistency, completeness, and agreement with past experiments.

**Keywords:** relativistic gravity, gravitomagnetism, origin of inertia, origin of inertial mass, explanation of dark matter, explanation of dark energy, negative inertial mass.

## 1. Introduction

The origin of inertial forces is a problem which has been of great concern to many thinkers since the time of Newton, but which so far has escaped a satisfactory solution. So, there is space for a new attempt. Inertial forces appear in a non-inertial reference frame. But what determines an inertial reference frame?

The first answer comes from Descartes and Newton, according to which, an inertial reference frame is a frame that moves at a constant velocity, with respect to the absolute space and the motion is absolute. The inertial forces, such as the centrifugal force, must arise from

acceleration with respect to the absolute space. This idea implies that space is an absolute physical structure with properties of its own and the inertia is an intrinsic property of the matter.

The second answer comes from Leibniz, Berkeley, Mach and is known as Mach' principle, according to which, an inertial reference frame is a frame that moves with constant velocity, with respect to the rest of the matter in the Universe, and the motion is relative. The inertial forces, such as the centrifugal force, are more likely caused by acceleration, with respect to the mass of the celestial bodies. This idea implies that the properties of space arise from the matter contained therein and are meaningless in an empty space.

The distinction between Newton's and Mach's considerations, is not one of metaphysics but of physics, for if Mach were right then a large mass could produce small changes in the inertial forces observed in its vicinity, whereas if Newton were right then no such effect could occur [1]. This seems to be very important when we consider subjects such as dark matter and dark energy.

The idea that the only meaningful motion of a particle is motion relative to other matter in the Universe has never found its complete expression in a physical theory. The Special theory of relativity eliminated absolute rest from physics, but acceleration remains absolute in this theory. Alfred Einstein was inspired by Mach's principle. The General theory of relativity attempted to continue this relativization and interpret inertia, considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so. Einstein showed that the gravitational field equations of General relativity imply that a body, in an empty Universe, has inertial properties [2].

The principle of equivalence is an essential part of General relativity. But although the principle of equivalence has been confirmed experimentally to high precision, the gravitational field equations of General relativity have not as yet been tested so decisively. Thus, it is not a theory fully confirmed experimentally and competing theories cannot be ruled out [3]. Almost all of the results that have been the subject of experimental investigation can be described by the linear approximation of the field equations [4]. The linear approximations of the field equations give us equations analogous to the equations of electromagnetism.

Finally, as pointed out by Henri Poincare, if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as that of the Lorentz-force [5][6]. But this does not happen with gravity as it is described by General relativity.

## 2. Relativistic Gravity

In an inertial reference frame  $K$ , let's have a system of two non spinning bodies with gravitational masses and positive electric charges, in a region of free space where there are no external gravitational and electric fields. The two bodies are in such distance, from each other, that the force of gravitational attraction balances that of electrostatic repulsion. The bodies are at rest in the inertial frame  $K$ , under equilibrium conditions.

But what is observed by another inertial frame of reference  $K'$ , moving with constant velocity relative to the frame  $K$ ? Let's imagine that if the bodies collide, they will explode. It is impossible for one observer to see an explosion and for another to not see it.

The theory of relativity is based on the premise that there is only a single "reality", but different observers can describe it differently. Thus, it is clear that any physical phenomenon, observed by an observer at rest in a reference frame, should be observed by observers at rest in any reference frame. This is so important that we will elevate it to a physical principle.

## Principle 0

**The physical phenomena are frame-independent.**

So, the equilibrium is a frame-independent condition. Therefore, in order for the above system of two charged bodies to be in equilibrium in all inertial frames,

- **The gravitational force should be transformed in exactly the same way as the electric force is transformed in different inertial frames.**

In order for this to happen, both forces must be described by equations with the same mathematical form. The equations of electromagnetism are Lorentz covariant and verified by numerous experiments. Therefore, it is logical to think that gravity should be described by equations with the same mathematical form as those of electromagnetism. Moreover, as we will show later, if we want to ensure the relativity of all kinds of translatory motion, the gravitational and electric forces should affect the spacetime metric perceived by a body, in the same way. Different charged bodies perceive different spacetime metric, and every freely moving test body follows geodesic of the metric that it perceives. So, the general idea that we will describe in this paper, is

- **The gravitational and electric forces should be described by equations with the same mathematical form and both forces should curve the spacetime perceived by a body, in the same way.**

According to Richard Feynman, we can reconstruct the complete electrodynamics using the Lorentz transformation, and the following series of remarks [7][8]:

1. The Coulomb potential for a stationary charge in vacuum is,  $\varphi = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
2. An electric charge produces a scalar potential and a vector potential, which together form a four-vector,  $A = (\frac{\varphi}{c}, \vec{A})$
3. The potentials produced by a charge moving in any way, depend only upon the velocity and position at the retarded time.

With those three remarks and the Lorentz transformation, we can reconstruct the complete electrodynamics. Therefore, the same series of remarks must be met for gravity, if we want to obtain a gravitomagnetic theory, where its equations have the same mathematical form, as those of the electromagnetic theory. But all we have is Newton's law of gravitation, which is similar to Coulomb's law for a stationary electric point charge, and so the gravitational potential  $\varphi_g$  at a distance  $r$  from a stationary gravitational mass  $m$  is,

$$\varphi_g = -G \frac{m}{r}$$

where  $G$  is the Newton's universal gravitational constant. This is only the one remark. Therefore, we need the other two, as well. We will obtain them by the next two principles:

## Principle 1

**A gravitational mass produces a scalar potential and a vector potential, which together form a**

**four-vector:**  $A_g = (\frac{\varphi_g}{c}, \vec{A}_g)$

From principle 1, follows that the gravitational mass is an invariant quantity. It has the same value in all inertial frames of reference. As we will show later, the inertial mass and the gravitational mass are different quantities. However, we will show that from the same point in a gravitational field, all the freely moving bodies have the same acceleration.

### Principle 2

**The potentials produced by a gravitational mass moving in any way, depend only upon the velocity and position at the retarded time.**

The potentials produced by a gravitational point-mass moving with velocity  $\vec{v}$  at a distance  $r$  from the point-mass, have the same mathematical form with the Lienard-Wiechert potentials for an electric point-charge moving with velocity  $\vec{v}$  at a distance  $r$  from the point-charge.

$$\varphi_g = -\frac{1}{4\pi g_0} \left[ \frac{m}{r - \vec{r}\vec{v}/c} \right] \quad (2.1)$$

$$\vec{A}_g = -\frac{1}{c^2} \frac{1}{4\pi g_0} \left[ \frac{m\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_g \vec{v}] \quad (2.2)$$

where  $g_0 = \frac{1}{4\pi G}$ . In the above equations, what is in square brackets refers to the retarded position of the gravitational mass and  $c$  is the speed of light in vacuum. Starting from the potentials, in order to find the fields, we have the equations

$$\vec{E}_g = -\vec{\nabla}\varphi_g - \frac{\partial \vec{A}_g}{\partial t} \quad (2.3)$$

$$\vec{B}_g = \vec{\nabla} \times \vec{A}_g \quad (2.4)$$

The force, that a gravitational mass  $m$  experiences, when it moves with velocity  $\vec{v}$  in the above fields is,

$$\vec{F}_g = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (2.5)$$

where  $\vec{E}_g$  is the gravitational field and  $\vec{B}_g$  the gravitomagnetic field.

So, we have now a Gravitomagnetic theory, with equations that have the same mathematical form as those of the Electromagnetic theory. Therefore, we expect that there are gravitomagnetic waves that propagate, in vacuum with the speed of light, and that they are described by equations which have the same mathematical form as the corresponding equations for electromagnetic waves, but with one important difference. An isolated electric source can radiate electric dipole radiation, with power proportional to the square of the second time derivative of the electric dipole moment. However, an isolated gravitational source cannot radiate gravitational dipole radiation, but quadrupole and radiation of higher polarity. The reason is simple. The electric dipole moment can move around with respect to the center of mass but the gravitational dipole moment is identical in location with the center of mass, and due to the law of conservation of momentum, cannot accelerate or radiate [9].

### 3. General relativity of motion

We will follow now, the fundamental idea of relativity of all kinds of translatory motion. In accordance with this idea we can detect and measure the translatory motion of a given body, relative to other bodies, but cannot assign any meaning to its absolute motion. **Acceleration as well as velocity is relative.**

The rotational motion of a rigid body, as a whole, cannot be relative. Different parts of the rotating body perceive different motion of the fixed stars. Nevertheless, we can consider that a rotational rigid body consists of an infinite number of point particles and the velocity and acceleration of every point particle separately, is relative. Then, having the field, that each point particle perceives, as we will show later, we can compose the overall picture of the field which the rotating body perceives.

In order for all kinds of translatory motion to be relative, the laws of physics should have the same mathematical form and the same numerical values of quantities, when referred to different reference frames, which are in relative translatory motion, since otherwise the difference in form and in numerical values of quantities could provide a criterion for judging the absolute motion [10]. **We will need two principles to ensure the relativity of all kinds of translatory motion.** The first principle, is

**Principle 3 - The principle of general relativity - The principle of covariance**  
**The laws of physics which are valid in an inertial reference frame, i.e. the laws of Special theory of relativity, have the same mathematical form in all reference frames which are in relative translatory motion.**

From the principle 0 and the principle 3 follows that, between frames in relative translatory motion, **any observer using the laws of physics in his reference frame, where he is at rest, is able to describe correctly what physical phenomena are observed by him and by an observer at rest in any other reference frame, because the physical phenomena are frame independent, but different observers can describe them differently.** Physics should be independent of reference frame choices.

However, it does not end with the above principles. We will show that we need one more principle to ensure the relativity of all kinds of translatory motion.

So, the laws of Special theory of relativity including the laws of gravitomagnetism, in a covariant form, i.e. the same mathematical form in all reference frames in relative translatory motion, are applied in all reference frames which make all kinds of translatory motion relative to an inertial frame.

Let's apply the first Newton's law, in the rest frame of an accelerating body. Newton's first law of motion states that a body, subject to no forces, remains at rest or continues to move in a straight line with constant speed. In its rest frame the body is at rest and so, according to the first Newton's law the net force on the body is zero. So, we come to the conclusion that,

- **In the rest frame of any-body the total force on the body is zero.**

We will consider now an accelerated reference frame by making a thought experiment and we will give it a name, because we will repeat it for a few times. We will call it, the lab frame experiment. We restrict our consideration, for the moment, only in gravitational fields. We will extend our consideration in electric fields, later. So, for the moment we assume that matter appears electrically neutral in the Universe. We will call this Universe, gravitational Universe.

Let's make now the lab frame experiment. We suppose that we have a space station, in the gravitational Universe, far from any massive object, that we use it as a laboratory. We will call the local reference frame, which we use to describe the events only in the area of the laboratory,

lab frame. An observer L is at rest in the lab frame. We assume that the distribution of matter in the Universe is such, that the gravitational field in the lab frame is zero. This means that the gravitational scalar potential  $\varphi_g$ , from the entire Universe, has the same value everywhere in the lab frame, and so,

$$\vec{\nabla}\varphi_g = 0 \quad (3.1)$$

We also suppose that the Universe expands symmetrically in all directions, with respect to the lab frame, so that the gravitomagnetic vector potential, due to one part of the mass-current is canceled out by the vector potential, due to another part of the mass-current owing to its symmetry. This means that the gravitomagnetic vector potential  $\vec{A}_g$  from the entire Universe is zero, everywhere in the lab frame.

$$\vec{A}_g = 0 \quad (3.2)$$

This would also happen if all the bodies of the Universe were at rest, relative to the laboratory. So, we can say that the lab frame is at rest relative to the Universe, or at rest relative to the fixed stars.

In the lab frame, a small rocket starts to accelerate, relative to the lab frame, with uniform proper acceleration, i.e. feels a constant force in its instantaneous rest frame. Let's have an observer R, at rest in the rocket. We will call the local rest frame of the rocket, which covers just the area of the rocket, rocket frame.

If we apply the covariant laws of gravitomagnetism in the rocket frame, because the fixed stars are accelerating relative to the rocket frame, they will give us an induced gravitational field according to the Faraday's law for gravitomagnetism. We will consider this in detail, later. **Therefore, the observer R is at rest in a local reference frame with gravity, while the observer L is at rest in a local inertial reference frame without gravity.** We assume that the gravitational forces between the rocket and the laboratory are negligible.

The observer L is at rest in a local inertial reference frame and so he can apply the laws of Special theory of relativity. The observer L, using the Special relativity, is able to describe what physical phenomena are observed by the observer R in the rocket frame, as we said. This is the well known study of a uniformly accelerated rigid reference frame, in Special relativity. So, from the viewpoint of the observer L, the well known physical phenomena that observed in the rocket frame, are [11] [12]:

- Redshift or blueshift of a light ray which moves parallel to the direction of the acceleration.
- Varying coordinate speed of light; fixed local relative speed of light.
- Spacetime is endowed with a metric.
- Maximum proper time as the law of motion of freely moving bodies.
- Horizons.

Let's consider now, what happens from the viewpoint of the observer R, in the rocket frame. If the induced gravitational field did not exist, by symmetry, the rocket frame would be equivalent to the lab frame and the above phenomena would have to occur in the lab frame and not in the rocket frame, because acceleration is relative. Nevertheless, the gravitational field exists and the above phenomena must occur in the rocket frame, **because the physical phenomena are frame-independent.** In order for this to happen, the only way is to think that the induced gravitational field should be capable of causing all the above phenomena, with the same numerical values. For this reason, in a Universe where matter appears electrically neutral we accept the next principle,

#### Principle 4 - The new principle of equivalence in the gravitational Universe

From the viewpoint of an observer at rest relative to the Universe and in a region of free space without gravity, physics in a local reference frame without gravity but accelerating with proper acceleration  $\vec{a} = -\vec{g}$ , is equivalent to physics in a non accelerating local reference frame with a gravitational field, where all the released bodies fall with proper acceleration  $\vec{g}$ .

From the viewpoint of the observer L, the lab frame is a local inertial reference frame without gravity and none of the above physical phenomena happen in the lab frame. Nevertheless, from the viewpoint of the observer R, the lab frame and the fixed stars, everything except of the rocket, make free fall in the universal gravitational field that he perceives. Because **physical phenomena are frame-independent**, from the viewpoint of the observer R, none of the above phenomena happen in the lab frame, although it is accelerating, making free fall in the universal gravitational field. In order for this to happen, the only way is to think that the gravitational field must exactly cancel the acceleration of the lab frame so that, no sign of either acceleration or gravitation can be found by any physical means in the lab frame. So, we have an alternative expression of the new equivalence principle:

#### The new principle of equivalence (alternative expression)

From the viewpoint of an observer at rest in a gravitational field, physics in a local reference frame, freely falling in that gravitational field, is equivalent to physics in an inertial reference frame without gravity.

From the viewpoint of the observer R, everything, except of the rocket, makes free fall in the universal gravitational field that he perceives. Therefore, according to the equivalence principle, no sign of either acceleration or gravitation can be found by any physical means in the fixed stars and the lab frame that make free fall. So, the observer R, doesn't observe any radiation field of the fixed stars and the lab frame, because radiation is a sign of acceleration. In order for this to happen, the fields of the free falling stars are carried along convectively with the stars, and the same happens with the field of the lab frame and so,

- **The instantaneous potentials of the accelerated lab frame, for the rocket frame, are the same with the potentials that the lab frame would have if it was moving with uniform velocity, equal to the instantaneous relative velocity.**

So, when the rocket accelerates relative to the fixed stars we can either say that the rocket accelerates and thus radiates because of the acceleration, while the fixed stars are stationary and thus they do not radiate, or that the rocket is stationary in a universal gravitational field where all the fixed stars make free fall, and thus do not radiate, whereas the rocket which is at rest in the gravitational field radiates because of the gravitational field according to the new equivalence principle. Therefore, with the principle of general relativity and the new principle of equivalence we have ensured the relativity of all kinds of translatory motion.

- **Acceleration as well as velocity is relative.**

Now, emerges a question. What would happen if there wasn't any other matter in the Universe, except of the lab frame and the rocket? In this case, the above phenomena would happen in both frames? The answer is yes and the reason is that in this case we cannot ignore the gravitational forces between the rocket frame and the lab frame for a reason that will become apparent in section 4.4.



The new equivalence principle shows us that, spacetime is endowed with a metric, and the gravitational field affects the spacetime metric so that the maximum proper time is the law of motion of a freely moving body, in a gravitational field, i.e. the motion of a body in a gravitational field is such that if the boundary conditions are specified by giving fixed start and finished events, then the world line of the body is the one having the maximum proper time. The two above phenomena are so important that we will elevate them to physical principles. So, we will accept the next two principles:

**Principle 5 - The principle of spacetime metric**

**Spacetime is endowed with a metric. The spacetime interval  $ds$  between two events is:**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where  $g_{\mu\nu}$  is the metric tensor.

**Principle 6 - The principle of geodesic motion or of maximum proper time**

**Freely moving test bodies follow geodesic of the metric:**

$$\delta \int ds = 0$$

The spacetime of Special theory of relativity is the Minkowski spacetime which is a flat spacetime. While it is clear that flat and curved spaces are different entities, they are closely related. We are familiar from our experience with smooth curved surfaces that any smooth curved space can be approximated locally by a flat plane. This is the content of the local-flatness theorem. According to the local-flatness theorem, the metric in the immediate neighborhood of a point P is, to a close approximation, the Minkowski spacetime metric, and the laws of Special relativity are valid there. [13][14][15]. Therefore, according to the local-flatness theorem,

- **Everywhere locally the laws of special relativity are valid.**

We will consider the effect of gravity in the spacetime metric in detail, later. Now, having created all the tools we need, we can move on, and consider what happens when a body is accelerating relative to the fixed stars. Let's find out what are the inertial forces.

## 4. Inertia

### §4.1 Gravitational inertial forces

Let's continue now, the lab frame experiment which we started in the previous chapter. Let's recall the equations (3.1) and (3.2) for the gravitational potentials from the entire Universe in the lab frame,

$$\vec{\nabla} \varphi_g = 0 \quad (3.1) \qquad \vec{\bar{A}}_g = 0 \quad (3.2)$$

We assume that the lab frame and the rocket frame have the three sets of axes parallel and common the  $x, x'$  axis. The rocket starts to accelerate, along the  $x$  axis. We have shown, in the previous chapter that, the instantaneous potentials of the accelerated lab frame, for the rocket frame, are the same with the potentials that the lab frame would have if it was moving with uniform velocity, equal to the instantaneous relative velocity.

The transformation laws which give the gravitational scalar potential  $\varphi'_g$  and the vector potential  $\vec{A}'_g$  in a moving frame  $S'$ , in terms of  $\varphi_g$  and  $\vec{A}_g$  in a stationary frame  $S$ , as measured at the same point in spacetime by people in the two frames, are

$$\begin{aligned}\varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}) & A'_{g-y} &= 0 & \gamma(v) &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g) & A'_{g-z} &= 0\end{aligned}\quad (4.1)$$

This assumes that the primed reference frame is moving with speed  $v$  in the positive x-direction, as measured in the unprimed reference frame.

We assume that the rocket frame is sufficiently small, so that, according to the local-flatness theorem, we can consider that the spacetime is flat in the rocket frame, and so, we can apply there the laws of Special relativity.

We consider now that the rocket frame is the moving frame and the lab frame is the stationary frame. When the instantaneous speed of the rocket frame is  $v$  in the positive x-direction, as measured in the lab frame, it is straightforward to find the instantaneous potentials  $\varphi'_g$  and  $A'_g$ , in the moving rocket frame, in terms of  $\varphi_g$  and  $\vec{A}_g$  in the stationary lab frame, as measured at a point P, common for both frames, by observers in the two frames, at the same time [16][17],

$$\begin{aligned}\varphi'_g &= \gamma(v)(\varphi_g - vA_{g-x}) & A'_{g-y} &= 0 \\ A'_{g-x} &= \gamma(v)(A_{g-x} - \frac{v}{c^2}\varphi_g) & A'_{g-z} &= 0\end{aligned}\quad (4.2)$$

Substituting for  $A_{g-x}$  from equation (3.2) into equation (4.2),

$$\varphi'_g = \gamma(v)\varphi_g \quad (4.3)$$

$$\vec{A}'_g = -\frac{1}{c^2}\varphi'_g\vec{v} = -\frac{1}{c^2}\gamma(v)\varphi_g\vec{v} \quad (4.4)$$

The gravitational field in the rocket frame now, is given from the equation (2.3)

$$\vec{E}'_g = -\vec{\nabla}'\varphi'_g - \frac{\partial\vec{A}'_g}{\partial\tau} \quad (4.5)$$

where  $\partial\tau$  is the proper time interval, in the rocket frame.

It is obvious, that the lab frame's velocity is measured the same from every location in the rocket frame. Therefore, the  $\gamma(v)$  factor is the same everywhere in the rocket frame. Hence, from the equation (4.3),  $\varphi'_g$  is the same everywhere in the rocket frame and therefore,

$$\vec{\nabla}' \varphi'_g = 0 \quad (4.6)$$

So, the equation (4.5) for the gravitational field in the rocket frame becomes

$$\vec{E}'_g = -\frac{\partial \vec{A}'_g}{\partial \tau} \quad (4.7)$$

Because the gravitational vector potential is the same everywhere in the rocket frame, we can replace the partial derivative in equation (4.7) with the derivative of a function of a single variable. But we choose to keep the partial derivative because in general, far from the lab frame, the gravitational vector potential changes, depending on position.

Because of the rocket's accelerating motion, relative to the lab frame, the rate of change of the gravitational vector potential, from the entire Universe, in the rocket frame is not zero. Therefore, an induced gravitational field appears in the rocket frame, whereas in the lab frame there is no gravitational field.

Let's have a test-body K, at rest in the rocket frame, with gravitational mass  $m$ . The test-body K will experience a gravitational force

$$\vec{F}'_g = m\vec{E}'_g = -\frac{\partial(m\vec{A}'_g)}{\partial \tau} \quad (4.8)$$

In the rocket frame the velocity of the body is zero, and so the gravitomagnetic force on the test-body K is zero in any case. Substituting for  $\vec{A}'_g$  from equation (4.4) into equation (4.8)

$$\vec{F}'_g = \frac{m}{c^2} \frac{\partial(\varphi'_g \vec{v})}{\partial \tau} = \frac{m}{c^2} \left( \frac{\partial \varphi'_g}{\partial \tau} \vec{v} + \varphi'_g \frac{\partial \vec{v}}{\partial \tau} \right) \quad (4.9)$$

**We can have now, some very important results for non relativistic velocities, in the gravitational Universe,** before moving on and considering the subject in the relativistic domain.

For the moment, we restrict our consideration for non relativistic velocities and in an area where the gravitational scalar potential is constant. In this case, the gravitational scalar potential in the accelerated rocket frame is equal to the gravitational scalar potential in the lab frame. So, the equation (4.9) becomes

$$\vec{F}'_g = \frac{m\varphi_g}{c^2} \frac{\partial \vec{v}}{\partial \tau} \quad (4.10)$$

If the acceleration of the rocket frame, relative to the lab frame is,

$$\vec{a} = \frac{\partial \vec{v}}{\partial \tau} \quad (4.11)$$

the equation (4.10) becomes

$$\vec{F}'_g = \frac{m\varphi_g}{c^2} \frac{\partial \vec{v}}{\partial \tau} = \left( -\frac{1}{c^2} m\varphi_g \right) (-\vec{a}) \quad (4.12)$$

If we recall now that the gravitational scalar potential  $\varphi_g$  of a gravitational mass is negative, it is obvious, from the equation (4.12), that the induced gravitational force on the test-body K resists to any change of its velocity. **It is an inertial force.**

Therefore, we will call the force on the test-body K, which is given from the equation (4.8), external gravitational inertial force because it is due to external causes. So, the external gravitational inertial force  $\vec{F}'_{ext-in-g}$ , on the test-body K, is given by the equation

$$\vec{F}'_{ext-in-g} = \vec{F}'_g = -m \frac{\partial \vec{A}'_g}{\partial \tau} \quad (4.13)$$

In addition to the external inertial force, there is also an internal inertial force. Let's consider what happens when a body is accelerating, relative to the fixed stars. The well known picture is something like this: We can think that a body consists of many particles. When the body is at rest or it's moving at uniform velocity, every particle exerts a force on every other, but the forces all balance in pairs, so that there is no net force. However, when the body is being accelerated, the internal forces will no longer be in balance, because of the fact that the influences take time to go from one particle to another. With acceleration, if we look at the forces between the various particles of the body, action and reaction are not exactly equal, and the body exerts a force on itself that tries to hold back the acceleration. We will call this self-force, internal inertial force, because it depends on the internal structure of the body [18].

According to the new equivalence principle, when a body makes free fall, the gravitational field exactly cancels the acceleration so that, no sign of either acceleration or gravitation can be found by any physical means on the body. So, when a body makes free fall, every particle of the body exerts a force on every other, but the forces all balance in pairs and so, the internal net force is zero. So, we come to the conclusion that,

- **In the free fall of a body, the internal structure of the body does not play any role in the inertial force that the body experiences. So, in the free fall, the body experiences only the external inertial force.**

We imagine now, that the test-body K is a body without internal structure, like the elementary particles, and thus, when it is accelerated by a force  $\vec{F}$ , it does not experience any internal inertial force. The inertial force on the test-body K is only the external gravitational inertial force. According to the first Newton's law, in the rest frame of the test-body K, the total force is zero. Therefore, the force  $\vec{F}$  that accelerates the test-body K is

$$\vec{F} = -\vec{F}'_{ext-in-g} \quad (4.14)$$

So, for non relativistic velocities, the force  $\vec{F}$  that accelerates the test-body K is

$$\vec{F} = -\vec{F}'_{ext-in-g} = -\vec{F}'_g = \left( -\frac{1}{c^2} m \varphi_g \right) \vec{a} \quad (4.15)$$

The equation (4.15) is the second Newton's law for non relativistic velocities. So, for non relativistic velocities, the gravitational inertial mass  $m_{in-g}$  of a body, with gravitational mass m, without internal structure, is

$$m_{m-g} = \left( -\frac{1}{c^2} m \varphi_g \right) \quad (4.16)$$

Hence, the gravitational inertial mass of a body, without internal structure, is not an intrinsic property of the body but is proportional to the gravitational scalar potential of the entire Universe. It's noteworthy that if we consider that the density of matter is roughly uniform throughout space, then the most distant matter dominates the gravitational scalar potential. This is because, although the influence of matter decreases with the distance, the amount of matter goes up as the square of the distance. With this consideration, the distant matter is of predominant importance, while local matter has only a small effect on the gravitational scalar potential.

It may seem strange that we consider a body without internal structure, but in the free fall of a body in a gravitational field, as we have shown, the internal structure of the body does not play any role and the body experiences only the external inertial force. Therefore, if a body A, with gravitational mass  $m_A$  makes free fall in the gravitational field of a body B, which is static with spherically symmetric gravitational mass  $M$ , much bigger than the gravitational mass  $m_A$  of the body A, the Newton's law of gravitation and the second law of Newton gives for the magnitude of the acceleration of the body A,

$$G \frac{m_A M}{r^2} = \left( -\frac{1}{c^2} m_A \varphi_g \right) a \quad (4.17)$$

where  $r$  is the distance of the body A, from the centre of the body B. It is obvious that the gravitational mass  $m_A$  is canceled in equation (4.17). Therefore, the acceleration of a body in a free fall, for non relativistic velocities, is independent of the gravitational mass of the body. So, we come to the conclusion that,

- **For non relativistic velocities, all bodies released from the same point in a gravitational field, fall with the same acceleration.**

This is a fundamental experimental result that has been tested with enormous precision. In Einstein's General relativity, the above result is interpreted by accepting the equality of the gravitational mass and the inertial mass.

## §4.2 Gravitational momentum and gravitational inertial mass

Let's find out now, what are the inertial mass and the momentum of a body. We will start with the inertial mass and the momentum of a point-particle K, without internal structure, like the elementary particles, and then, **in section 4.4**, we will find the inertial mass and the momentum of a composite body.

Let's continue the lab frame experiment. We suppose that the point-particle K, with gravitational mass  $m$  and without internal structure, is at rest in the lab frame. The rocket makes uniformly accelerated motion, relative to the lab frame towards the point-particle K.

We suppose that when the point-particle K, is in the immediate neighborhood of the rocket, the instantaneous velocity of the rocket frame, relative to the lab frame is  $\vec{v}$  and the instantaneous velocity of the lab frame, relative to the rocket, is  $(-\vec{v})$ .

The rocket frame is sufficiently small, so that we can consider that the spacetime is flat in the rocket frame, and the laws of Special relativity are applied, according to the local-flatness

theorem. Therefore, when the point-particle K is in the immediate neighborhood of the rocket, the observer R can study the motion of it, using the laws of Special relativity.

From the viewpoint of the observer R, an induced gravitational field appears when the rocket accelerates and so for the observer R, the fixed stars and the point-particle K make free fall in this universal gravitational field. So, for the observer R, the point-particle K makes free fall because it experiences a gravitational force  $\vec{F}$  which is equal to the external gravitational inertial force on the rocket frame

$$\vec{F} = \vec{F}'_{ext-in-g} = -m \frac{\partial \vec{A}'_g}{\partial \tau} \quad (4.18)$$

We consider a Universe where matter appears electrically neutral, and so we shall call the momentum of the point-particle K, gravitational momentum  $\vec{p}_g$ , and its inertial mass gravitational inertial mass  $m_{in-g}$ . So, from the viewpoint of the observer R, the second Newton's law for the motion of the point-particle K, is

$$\vec{F} = \frac{\partial \vec{p}_g}{\partial \tau} \quad (4.19)$$

We used the partial derivative, for the second Newton's law, for a reason that will soon become apparent. Substituting for  $\vec{F}$  from equation (4.18) into equation (4.19),

$$\frac{\partial \vec{p}_g}{\partial \tau} = -\frac{\partial (m \vec{A}'_g)}{\partial \tau} \quad (4.20)$$

When the velocity of the lab frame, and therefore the velocity of the point-particle K as well, relative to the rocket is  $(-\vec{v})$ , the equation (4.4) for the gravitational vector potential is valid.

**If we define for simplicity** a velocity  $\vec{u}$ , so that

$$\vec{u} = -\vec{v}$$

the gravitational momentum of the point-particle K, in the rocket frame, is given from the well known equation,

$$\vec{p}_g = m_{in-g} \vec{u} \quad (4.21)$$

and the equation (4.4) for the gravitational vector potential becomes

$$\vec{A}'_g = \frac{1}{c^2} \phi'_g \vec{u} = \frac{1}{c^2} \gamma(u) \phi_g \vec{u} \quad (4.22)$$

Substituting for  $\vec{A}'_g$  from equation (4.22) into equation (4.20),

$$\frac{\partial \vec{p}_g}{\partial \tau} = -\frac{\partial \left[ \frac{1}{c^2} \gamma(u) m \phi_g \vec{u} \right]}{\partial \tau} = -\frac{\partial \left[ \gamma(u) \left( -\frac{1}{c^2} m \phi_g \right) \vec{u} \right]}{\partial \tau} \quad (4.23)$$

By integration we have for the gravitational momentum of the point-particle K, in the rocket frame

$$\vec{p}_g = \gamma(u) \left( -\frac{1}{c^2} m \varphi_g \right) \vec{u} \quad (4.24)$$

where the constant of integration is set equal to zero because we define the momentum to be zero when the relative velocity  $\vec{u}$  is zero.

From the equation (4.24), it is obvious that the momentum of the point-particle K is a function of the gravitational scalar potential in the lab frame, and of the velocity of the point-particle K, relative to the lab frame. The gravitational scalar potential, although it is constant in the lab frame, in general it changes depending on position. The velocity of the accelerated point-particle depends on time. Therefore, recognizing that the momentum of the point-particle K may depend on position as well as time, we used partial derivative for the rate of change of the momentum in the second Newton's law.

Comparing the equations (4.21) and (4.24), we have the equation that gives us the gravitational inertial mass  $m_{in-g}$  of the point-particle K, in the rocket frame

$$m_{in-g} = \gamma(u) \left( -\frac{1}{c^2} m \varphi_g \right) \quad (4.25)$$

For a reason that will soon become apparent, we can call, what is expressed by the next equation, gravitational inertial rest mass

$$m_{in-g0} = -\frac{1}{c^2} m \varphi_g \quad (4.26)$$

Let's see now, what is, the gravitational inertial rest mass of the point-particle K. The instantaneous sum of the four-vector gravitational potentials of all the bodies in the Universe is also a four-vector [19]. It is the total four-vector gravitational potential  $A_g$

$$A_g = \left( \frac{\varphi_g}{c}, \vec{A}_g \right) \quad (4.27)$$

The four-velocity of the point-particle K is,

$$U = \gamma(c, \vec{u}) \quad (4.28)$$

The product  $mUA_g$  is Lorentz invariant, because the gravitational mass  $m$  is a Lorentz scalar, and the product of two four-vectors is also a scalar. Evaluating the product  $mUA_g$  in the rocket frame, where the point-particle K moves with instantaneous velocity  $\vec{u}$ , we obtain

$$mUA_g = \gamma(u) m(c, \vec{u}) \left( \frac{\varphi'_g}{c}, \vec{A}'_g \right) = \gamma(u) m \left( \varphi'_g - \vec{u} \vec{A}'_g \right) \quad (4.29)$$

Evaluating the product  $mUA_g$  in the rest frame of the point-particle K, the lab frame, we obtain

$$mUA_g = m(c, 0) \left( \frac{\varphi_g}{c}, \vec{A}_g \right) = m \varphi_g \quad (4.30)$$

Hence, when the point-particle K is at rest relative to the lab frame, the product  $mUA_g$  is equal to the product of its gravitational mass and the gravitational scalar potential of the entire Universe, as measured in the lab frame. Substituting for  $mUA_g$  from equation (4.30) into equation (4.26), we have

$$m_{in-g0} = -\frac{1}{c^2}m\varphi_g = -\frac{1}{c^2}mUA_g \quad (4.31)$$

So,

- **The gravitational inertial rest mass of a body, without internal structure, is an invariant quantity and is not an intrinsic property of the body but is the binding energy of the body with the entire Universe, when the body is at rest in the lab frame, divided by  $c^2$ .**

Therefore, the  $m_{in-g}c$  and  $\vec{p}_g$  are the components of a four vector, the four-momentum of the point-particle K without internal structure

$$P = m_{in-g0}U = m_{in-g0}\gamma(u)(c, \vec{u}) = (m_{in-g}c, \vec{p}_g) = \left(\frac{E_g}{c}, \vec{p}_g\right) \quad (4.32)$$

When the point-particle K is at rest in the rocket frame, its gravitational rest energy is

$$E_{g0} = m_{in-g0}c^2 = \left(-\frac{1}{c^2}m\varphi_g\right)c^2 = -m\varphi_g \quad (4.33)$$

When the point-particle K moves with velocity  $\vec{u}$  in the rocket frame, its gravitational energy is

$$E_g = m_{in-g}c^2 = \gamma(u)\left(-\frac{1}{c^2}m\varphi_g\right)c^2 = -\gamma(u)m\varphi_g \quad (4.34)$$

**Finally, in general, the gravitational inertial mass and the gravitational momentum of a point-particle with gravitational mass m and without internal structure, which moves with velocity  $\vec{v}$  relative to a reference frame S, for the reference frame S, are**

$$m_{in-g} = \gamma(v)m_{in-g0} = \gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right) \quad (4.35)$$

$$\vec{p}_g = \gamma(v)m_{in-g0}\vec{v} = \gamma(v)\left(-\frac{1}{c^2}m\varphi_g\right)\vec{v} \quad (4.36)$$

When the point-particle K is at rest in the lab frame, if we accept that its “gravitational potential rest energy” is

$$U_{g0} = m\varphi_g \quad (4.37)$$

then, the total gravitational rest energy of the point-particle K, from equation (4.33), is

$$E_{g0} + U_{g0} = -m\varphi_g + m\varphi_g = 0 \quad (4.38)$$

So, the total gravitational rest energy of the Universe is zero!



It's noteworthy that Richard Feynman writes in the "Lectures on Gravitation" [20]:

"Another spectacular coincidence relating the gravitational constant to the size of the universe comes in considering the total energy. The total gravitational energy of all the particles of the universe is something like  $GMM/R$ , where  $R=Tc$ , and  $T$  is the Hubble's time. Actually, if the universe were a sphere of constant density, there would be a factor of  $3/5$ , but let us neglect this, since our cosmological model is not all well known [...] If now we compare this number to the total rest energy of the universe,  $Mc^2$ , lo and behold, we get the amazing result that  $GM^2/R=Mc^2$ , so that the total energy of the universe is zero. Actually, we don't know the density nor that radius well enough to claim equality, but the fact that these two numbers should be of the same magnitude is a truly amazing coincidence. It is exciting to think that it costs nothing to create a new particle, since we can create it at the center of the universe where it will have a negative gravitational energy equal to  $Mc^2$  [...] Why this should be so is one of the great mysteries and therefore one of the important question of physics. After all, what would be the use of studying physics if the mysteries were not the most important things to investigate?"

Let's consider now, if the gravitational inertial mass of a body  $M$  depends on the angle between the vector position of the other gravitational masses, and the direction of acceleration of the body  $M$ , i.e. the gravitational inertial mass of the body  $M$  depends on whether its acceleration is in the direction towards the center of our galaxy or in a direction perpendicular to it. This problem has the name "anisotropy of inertia" and was the subject of experimental investigation with negative results [21].

The gravitational scalar potential at a point  $P$ , in the field of a point-gravitational mass  $m$ , moving at a constant velocity  $\vec{v}$ , is given from an equation similar to the well known corresponding equation of electromagnetism. The only difference is the minus sign [22].

$$\varphi_g = -\frac{1}{4\pi g_0} \frac{m}{r(1-\beta^2 \sin^2 \theta)^{\frac{1}{2}}} \quad (4.39)$$

where  $r$  is the distance from the point-gravitational mass to the field point  $P$ ,  $\beta = v/c$  and  $\theta$  is the angle between the position vector and the velocity vector, of the point-gravitational mass. From the eq. (4.39) we can see that any anisotropy of inertia comes from relativistic velocities.

As we said before, the most distant matter dominates the gravitational scalar potential and hence the gravitational inertial mass of a body. Thus, the very distant matter is of predominant importance, while local matter has only a small effect on the potential. From astronomical observations we know that, the very distant matter, i.e. galaxies, move at relativistic velocities. However, their distribution in space is homogenous, as seen from afar. Therefore, because of the symmetrical distribution, all the directions of acceleration of a particle are equivalent. Therefore, the gravitational inertial mass of the body  $M$  is independent of the angle between the vector position of the very distant matter and the direction of acceleration of the body  $M$ .

The nearest matter is certainly moving with non relativistic velocities. Therefore, if the body  $M$  moves also with non relativistic velocity, its gravitational inertial mass is independent of the angle between the vector position of any of the nearest matter and the direction of its acceleration. So,

- **The inertial mass of a body, for non relativistic velocities, does not depend on whether its acceleration is in the direction towards the center of our galaxy, or in a direction perpendicular to it.**

The results are in agreement with past experiments.

### §4.3 Electric momentum and electric inertial mass

The electric scalar potential and the electromagnetic vector potential produced by an electric point-charge  $q$ , moving with velocity  $\vec{v}$ , at a distance  $r$  from the point-charge, are the well known Lienard -Wiechert potentials

$$\varphi_e = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r - \vec{r}\vec{v}/c} \right] \quad (4.40)$$

$$\vec{A}_e = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \left[ \frac{q\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_e \vec{v}] \quad (4.41)$$

In the above equations, what is in square brackets refers to the retarded position of the point-charge  $q$ . Having the potentials, the well known equations of electromagnetism are

$$\vec{E}_e = -\vec{\nabla}\varphi_e - \frac{\partial\vec{A}_e}{\partial t} \quad (4.42)$$

$$\vec{B}_e = \vec{\nabla} \times \vec{A}_e \quad (4.43)$$

$$\vec{F}_e = q(\vec{E}_e + \vec{v} \times \vec{B}_e) \quad (4.44)$$

Let's imagine now, a Universe where there is only one kind of electric charges, positive or negative, and no gravitational masses. We will call this Universe, electric Universe. Let's assume now that we superimpose in the electric Universe two test-charges, the one positive and the other negative, without internal structure.

To ensure the relativity of all kinds of translatory motion in the electric Universe, as we have shown in chapter 3, it is required to accept the principle of general relativity and an appropriate equivalence principle.

#### The new Principle of equivalence in an electric Universe

**From the viewpoint of an observer at rest relative to the Universe and in a region of free space without electric field, physics in a local reference frame without electric field but accelerating with proper acceleration  $\vec{a} = -\vec{g}$ , is equivalent to physics in a non accelerating local reference frame with an electric field, where all the released charges move with proper acceleration  $\vec{g}$ .**

Moreover, as we have shown in chapter 3, from the new equivalence principle follow two phenomena that we elevated them to physical principles, the principle of spacetime metric and the principle of geodesic motion. We will do the same in the electric Universe. So, the electric field affects the spacetime metric and all the freely moving charges follow geodesic of the metric.

**But, how is it possible for the positive test-charge to move in the same way as the negative test-charge, in the same electric field, according to the equivalence principle in an electric Universe?**

Let's remake the lab frame experiment in the electric Universe. Because the equations of electromagnetism have the same mathematical form with the equations of gravitomagnetism, it is not necessary to describe all the details of the lab frame experiment again, but we will only state the important results.

In the lab frame of the electric Universe, we have for the electric scalar potential and the electromagnetic vector potential the relations

$$\vec{\nabla}\phi_e = 0 \qquad \vec{A}_e = 0 \qquad (4.45)$$

We suppose now that one of the superimposed test-charges, the point-charge M, with electric charge  $q$  and without internal structure, is at rest in the lab frame. The rocket makes uniformly accelerated motion, relative to the lab frame towards the point-charge M. From the viewpoint of the observer R, the observer in the rocket, an induced electric field appears when the rocket accelerates. The external electric inertial field in the rocket frame, is

$$\vec{E}_{in-e} = -\frac{\partial \vec{A}'_e}{\partial \tau} \qquad (4.46)$$

where  $\partial \tau$  is the proper time interval, in the rocket frame.

From the viewpoint of the observer R, the entire Universe make free fall in the induced electric field that he perceives, without any sign about acceleration, i.e. **a freely moving charge doesn't radiate and its internal structure does not play any role and thus the charge experiences only the external inertial force.**

We suppose that when the point-charge M, is in the immediate neighborhood of the rocket, the instantaneous velocity of the lab frame, relative to the rocket, is  $\vec{v}$  (in the gravitational Universe the instantaneous velocity of the lab frame relative to the rocket was  $-\vec{v}$ ). The electromagnetic vector potential, from the entire Universe, in the rocket frame, when the lab frame moves at instantaneous velocity  $\vec{v}$  relative to the rocket, is

$$\vec{A}'_e = \frac{1}{c^2} \phi'_e \vec{v} = \frac{1}{c^2} \gamma(v) \phi_e \vec{v} \qquad (4.47)$$

The rocket frame is sufficiently small, so that we can consider that the spacetime is flat in the rocket frame, and the laws of Special relativity are applied, according to the local-flatness theorem. Therefore, when the point-charge M, is in the immediate neighborhood of the rocket, the observer R can study the motion of it, using the laws of Special relativity.

From the viewpoint of the observer R, there is an electric field where the point-charge M makes free fall. So, from the viewpoint of the observer R, the point-charge M, makes free fall because it experiences an electric force  $\vec{F}$  which is the external electric inertial force

$$\vec{F} = \vec{F}'_{ext-in-e} = -\frac{\partial (q\vec{A}'_e)}{\partial \tau} \qquad (4.48)$$

The second Newton's law for the motion of the point-charge M, is

$$\frac{\partial \vec{p}_e}{\partial \tau} = -\frac{\partial (q\vec{A}'_e)}{\partial \tau} \qquad (4.49)$$

where  $\vec{p}_e$  the electric momentum of the point-charge M in the rocket frame. Substituting for  $\vec{A}'_e$  from equation (4.47) into equation (4.49),

$$\frac{\partial \vec{p}_e}{\partial \tau} = \frac{\partial \left[ \gamma(v) \left( -\frac{1}{c^2} q \phi_e \right) \vec{v} \right]}{\partial \tau} \qquad (4.50)$$

By integration we have for the electric momentum of the point-charge M, in the rocket frame

$$\vec{p}_e = \gamma(v) \left( -\frac{1}{c^2} q\varphi_e \right) \vec{v} \quad (4.51)$$

where the constant of integration is set equal to zero because we define the electric momentum to be zero when the relative velocity  $\vec{v}$  is zero. The electric inertial mass  $m_{in-e}$  of the point-charge M, is

$$m_{in-e} = \gamma(v) \left( -\frac{1}{c^2} q\varphi_e \right) \quad (4.52)$$

The electric inertial rest mass  $m_{in-e0}$  of the point-charge M, is

$$m_{in-e0} = -\frac{1}{c^2} q\varphi_e \quad (4.53)$$

Using the arguments of chapter 3, the inertial rest mass is an invariant quantity.

The equations for the electric inertial mass and the electric momentum have the same mathematical form with the equations for the gravitational inertial mass and the gravitational momentum. Nevertheless, in an electric Universe, we have something different and interesting. The electric inertial rest mass, as we can see from equation (4.53), can be negative while the gravitational inertial rest mass is always positive, as we can see from equation (4.26). What does that mean? According to the second Newton's law, negative inertial mass means opposite acceleration. Thus, if a charge with positive electric inertial mass is attracted to another charge, a charge with negative electric inertial mass will be repelled.

Let's have an electric Universe with positive charges. If we assume, that the point-charge M has a positive charge, its electric inertial mass is negative and so, it is attracted from the other positive charges. If we assume now, that the point-charge M has a negative charge, its electric inertial mass is positive, and thus it is also attracted from the other positive charges.

Let's have an electric Universe with negative charges. If we assume, that the point-charge M has a positive charge, its electric inertial mass is positive and so, it is attracted from the other negative charges. If we assume that the point-charge M has a negative charge, its electric inertial mass is negative, and thus it is also attracted from the other negative charges.

So, we come to the conclusions,

- **In an electric Universe, all the kinds of charges will be attracted by each other.**
- **What determines if a charge is attracted or repelled by an electric force is both the kind of its inertial rest mass and the force acting on it, and not the force alone.**

#### §4.4 The momentum and the inertial mass

Let's consider now, a Universe where there are both gravitational masses and electric charges. We will call this Universe, real Universe.

To ensure the relativity of all kinds of translatory motion, as we have shown in chapter 3, it is required to accept the principle of general relativity and an appropriate equivalence principle.

### The new Principle of equivalence in the real Universe

From the viewpoint of an observer at rest relative to the Universe and in a region of free space without gravitational and electric field, physics in a local reference frame without gravitational and electric field, but accelerating with proper acceleration  $\vec{a} = -\vec{g}$ , is equivalent to physics in a nonaccelerating local reference frame with a gravitational field, where all the released bodies fall with proper acceleration  $\vec{g}$ .

We will prove later that this is the appropriate equivalence principle. Moreover, as we have shown in chapter 3, from the equivalence principle follow two phenomena that we elevated them to physical principles, the principle of spacetime metric and the principle of geodesic motion. We want to do the same in the real Universe, but as we have shown in section 4.3 the electric fields affect the spacetime metric. So, in the real Universe we expect that both the gravitational and the electric fields affect the spacetime metric, but we know that a positive charge does not move the same with a negative charge in the same electric field. So,

- In order for the principle of geodesic motion to hold for any-body, charged or not, the gravitational and electric forces should affect the spacetime metric perceived by a body and not the gravitational and electric fields affect the spacetime itself.

**The spacetime doesn't have properties of its own.** When different electric charges, are at the same point in an electric field, each one will perceive a spacetime with different metric. So, we must restate the principle of geodesic motion.

**Principle 6 - The principle of geodesic motion or of maximum proper time**  
Every freely moving test bodies follow geodesic of the metric that it perceives.

We will remake the lab frame experiment in the real Universe. We have used this already for gravitational and electric fields separately and so, it is not necessary to describe all the details of the lab frame experiment again, but we will only state the important results.

In the lab frame, in the real Universe, we have for the gravitational scalar potential and the gravitomagnetic vector potential the relations

$$\vec{\nabla} \varphi_g = 0 \qquad \vec{A}_g = 0 \qquad (4.54)$$

and for the electric scalar potential and the electromagnetic vector potential the relations

$$\vec{\nabla} \varphi_e = 0 \qquad \vec{A}_e = 0 \qquad (4.55)$$

We can imagine a distribution of the charges in order to have the above results.

We suppose that a point-particle B, with gravitational mass  $m$ , electric charge  $q$  and without internal structure, is at rest in the lab frame. The rocket makes uniformly accelerated motion, relative to the lab frame, towards the point-particle B. From the viewpoint of the observer R, the observer in the rocket, an induced gravitational and electric field appears when the rocket accelerates. The total external inertial field in the rocket frame, is

$$\vec{E}_{in} = -\frac{\partial \vec{A}'_g}{\partial \tau} - \frac{\partial \vec{A}'_e}{\partial \tau} \qquad (4.56)$$

where  $\partial\tau$  is the proper time interval, in the rocket frame.

From the viewpoint of the observer R, the entire Universe makes free fall in the induced gravitational and electric field that he perceives, without any sign of acceleration, i.e. **a freely moving body doesn't radiate and its internal structure does not play any role and thus the body experiences only the external inertial force.**

We suppose that when the point-particle B is in the immediate neighborhood of the rocket, the instantaneous velocity of the lab frame, relative to the rocket, is  $\vec{v}$ . The gravitomagnetic vector potential, from the entire Universe, in the rocket frame, when the lab frame moves at instantaneous velocity  $\vec{v}$  relative to the rocket, is

$$\vec{A}'_g = \frac{1}{c^2} \varphi'_g \vec{v} = \frac{1}{c^2} \gamma(v) \varphi_g \vec{v} \quad (4.57)$$

The electromagnetic vector potential, from the entire Universe, in the rocket frame, when the lab frame moves at instantaneous velocity  $\vec{v}$  relative to the rocket frame, is

$$\vec{A}'_e = \frac{1}{c^2} \varphi'_e \vec{v} = \frac{1}{c^2} \gamma(v) \varphi_e \vec{v} \quad (4.58)$$

The rocket frame is sufficiently small, so that we can consider that the spacetime is flat in it, and the laws of Special relativity are applied, according to the local-flatness theorem. Therefore, when the point-particle B, is in the immediate neighborhood of the rocket, the observer R can study the motion of it, using the laws of Special relativity.

From the viewpoint of the observer R, there is a gravitational and electric field where the point-particle B is accelerating. So, from the viewpoint of the observer R, the point-particle B is accelerating because it experiences a gravitational and electric force  $\vec{F}$  which is the total external inertial force

$$\vec{F} = \vec{F}'_{ext-in} = -m \frac{\partial \vec{A}'_g}{\partial \tau} - q \frac{\partial \vec{A}'_e}{\partial \tau} \quad (4.59)$$

The second Newton's law for the motion of the point-particle B, is

$$\frac{\partial \vec{p}_{ge}}{\partial \tau} = - \frac{\partial (m \vec{A}'_g + q \vec{A}'_e)}{\partial \tau} \quad (4.60)$$

where  $\vec{p}_{ge}$  is the momentum, due to the external gravitational and electric inertial forces, of the point-particle B in the rocket frame. Substituting for  $\vec{A}'_g$  and for  $\vec{A}'_e$  from equations (4.57) and (4.58) into equation (4.60),

$$\frac{\partial \vec{p}_{ge}}{\partial \tau} = \frac{\partial \left[ \gamma(v) \left( -\frac{1}{c^2} (m \varphi_g + q \varphi_e) \right) \vec{v} \right]}{\partial \tau} \quad (4.61)$$

By integration we have for the momentum of the point-particle B, in the rocket frame

$$\vec{p}_{ge} = \gamma(v) \left[ -\frac{1}{c^2} (m \varphi_g + q \varphi_e) \right] \vec{v} \quad (4.62)$$

where the constant of integration is set equal to zero because we define the momentum to be zero when the relative velocity  $\vec{v}$  is zero.

The inertial mass  $m_{in-ge}$ , due to the external inertial forces, of the point-particle B is

$$m_{in-ge} = \gamma(v) \left[ -\frac{1}{c^2} (m\varphi_g + q\varphi_e) \right] \quad (4.63)$$

The inertial rest mass  $m_{in-ge0}$ , due to the external inertial forces, of the point-particle B is

$$m_{in-ge0} = -\frac{1}{c^2} (m\varphi_g + q\varphi_e) \quad (4.64)$$

Using the arguments of chapter 3, the inertial rest mass  $m_{in-ge0}$  is an invariant quantity.

**Having the equations, let's apply them to prove that the equivalence principle is the appropriate.**

We suppose now that the observer R releases a test-body K, with gravitational mass  $m$  and electric charge  $q$ , to move inside the rocket. **According to the equivalence principle, in the free fall of a body in the gravitational and electric field of the rocket frame, the internal structure of the body does not play any role and the body experiences only the total external inertial force.** From the viewpoint of the observer R, in the rocket frame, there is an induced gravitational and electric field and so the test-body K will experience a force

$$\vec{F} = \vec{F}'_{ext-in} = -m \frac{\partial \vec{A}'_g}{\partial \tau} - q \frac{\partial \vec{A}'_e}{\partial \tau} \quad (4.65)$$

where  $\partial \tau$  is the proper time interval in the rocket frame. Substituting for  $\vec{A}'_g$  and for  $\vec{A}'_e$  equations (4.57) and (4.58) into equation (4.65), with  $\varphi_g$  and  $\varphi_e$  constant in the rocket frame

$$\vec{F}_{in} = \left[ -\frac{1}{c^2} (m\varphi_g + q\varphi_e) \right] \frac{\partial [\gamma(v)\vec{v}]}{\partial \tau} \quad (4.66)$$

**We suppose that the test-body K moves with instantaneous velocity  $\vec{u}$  relative to the rocket frame, when the lab frame moves with velocity  $\vec{v}$  relative to the rocket frame.** Thus, the momentum  $\vec{p}_{ge}$  of the test-body K in the rocket frame is

$$\vec{p}_{ge} = \gamma(u) \left[ -\frac{1}{c^2} (m\varphi_g + q\varphi_e) \right] \vec{u} \quad (4.67)$$

The second Newton's law, in the rocket frame, for the test-body K is

$$\vec{F} = \frac{\partial \vec{p}_{ge}}{\partial \tau} \quad (4.68)$$

Substituting for  $\vec{F}$  and for  $\vec{p}_{ge}$  from equations (4.66) and (4.67) into equation (4.68),

$$\left[ -\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \frac{\partial[\gamma(v)\vec{v}]}{\partial\tau} = \left[ -\frac{1}{c^2}(m\varphi_g + q\varphi_e) \right] \frac{\partial[\gamma(u)\vec{u}]}{\partial\tau} \quad (4.69)$$

After canceling

$$\frac{\partial[\gamma(u)\vec{u}]}{\partial\tau} = \frac{\partial[\gamma(\vec{v})\vec{v}]}{\partial\tau} \Rightarrow \gamma^3(u) \frac{d\vec{u}}{d\tau} = \gamma^3(v) \frac{d\vec{v}}{d\tau} \quad (4.70)$$

We remind that the proper acceleration  $\vec{a}$  of a body, is its acceleration in the instantaneous reference frame where the body is at rest. If the instantaneous velocity of the lab frame, relative to the rocket frame is  $\vec{v}$ , then the proper acceleration of the lab frame [23], is

$$\vec{a} = \gamma^3(v) \frac{d\vec{v}}{d\tau} \quad (4.71)$$

where  $d\tau$  is the proper time interval in the rocket frame.

Therefore, from equation (4.70) and (4.71), we can see that all the released bodies have proper acceleration equal with the proper acceleration of the lab frame. So, in the induced gravitational and electric field of the rocket frame, all the released bodies, charged or not, are moving with the same proper acceleration. **Therefore, we can say that the observer R is at rest in a gravitational field with no evidence that an induced electric field exists.** So, the equivalence principle is the appropriate. Thus, with the principle of general relativity and the new principle of equivalence we have ensured the relativity of all kinds of translatory motion in the real Universe. Acceleration as well as velocity is relative.

### What about the motion of a light ray?

For the lab frame the light propagate in a straight line. For the rocket frame the light will be judged to fall, just like everything else. So, in the real Universe the path of a light ray behaves as if it is affected by the gravitational field. Therefore,

- **The path of a light ray in a gravitational field is the same as the imaginary path of a body with gravitational mass, which moves with the speed of light in vacuum.**

This result is in agreement with past experiments [24].

Let's have a point-particle M, which has gravitational mass  $m$ , electric charge  $q$  and no internal structure. We assume now, that the Universe consists of  $n$  discrete gravitational masses, and  $m$  discrete electric charges, each at a different distance from the point-particle M. We also assume that all the discrete gravitational masses and discrete electric charges of the Universe are moving with non relativistic velocities, relative to the point-particle M. In this case, according to equations (4.64), the gravitational rest mass of the point-particle M is

$$m_{in-ge0} = \frac{1}{c^2} \left( \frac{1}{4\pi g_0} \sum_{i=1}^n \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \sum_{i=1}^m \frac{qq_i}{r_i} \right) \quad (4.72)$$

The distance  $r_i$  is measured in the lab frame.



- **The above equation shows us that, the inertial rest mass of a body, can take any value, positive or negative. It depends on the position of the body in relation to the other gravitational masses and electric charges.**

This seems to be very important for the explanation of dark matter, dark energy and certainly for phenomena of nuclear physics, as we will show below.

Having finished with the inertial mass of a point-particle without internal structure, let's consider now the inertial rest mass of a composite body M. The special theory of relativity accepts the principle of conservation of four-momentum, i.e. that the sum of the four-momentum of all the particles going into a collision, is the same as the sum of the four-momentum of all those coming out. If we apply the conservation of four-momentum in an inelastic collision where n point particles, without internal structure, collide and create a composite body M, the total inertial rest mass  $m_{in-0}$  of the composite body M is

$$m_{in-0} = \sum_{i=1}^{i=n} m_{in-ge0i} + \sum T/c^2 + E_{field}/c^2 \quad (4.73)$$

where  $m_{in-ge0i}$  is the inertial rest mass of each particle, without internal structure, that makes up the body M, T is the kinetic energy of the relative motion of the particles that make up the body M, and  $E_{field}$  is the potential energy of the interaction of the particles that make up the body M [25]. **The total inertial rest mass  $m_{in-0}$  is also invariant** as is well known from the Special theory of relativity [26].

- **So, if we have the inertial rest mass of each point particle, without internal structure, using the principle of conservation of four-momentum, in an inelastic collision, we end up with the relation that gives us the total inertial rest mass of the body M with internal structure which is Lorentz invariant.**

So, the total inertial mass  $m_{in}$  of the body M is

$$m_{in} = \gamma(v)m_{in-0} \quad (4.74)$$

The total momentum  $\vec{p}$  of the body M is

$$\vec{p} = \gamma(v)m_{in-0}\vec{v} \quad (4.75)$$

The total energy  $E$  of the body M is

$$E = m_{in}c^2 = \gamma(v)m_{in-0}c^2 \quad (4.76)$$

Therefore, the  $m_{in}c$  and  $\vec{p}$  are the components of a four vector, the four-momentum  $P$  of the body M, and having the four-momentum of the body we have everything we need.

$$P = m_{in-0}U = m_{in-0}\gamma(u)(c, \vec{u}) = (m_{in}c, \vec{p}) = \left(\frac{E}{c}, \vec{p}\right) \quad (4.77)$$

### §4.5 Radial acceleration

Let's consider now the motion of a test-body A, with gravitational mass  $m$  and electric charge  $q$ , in the gravitational field and electric field of a body B, which is static with spherically symmetric distribution of gravitational mass and electric charge. Let's imagine that we superimpose the body B, at rest in the area of the lab frame. Because the body B is static there is no gravitomagnetic and electromagnetic field.

We suppose that we release the test-body A to move freely in the radial direction in the gravitational field and electric field of the body B. Then the equation of its motion is

$$\frac{d\vec{p}}{dt} = m\vec{E}_{gB} + q\vec{E}_{eB} \quad (4.78)$$

where  $\vec{E}_{gB}$  the gravitational field,  $\vec{E}_{eB}$  the electric field of the body B at the position of the test-body A and  $dt$  the time interval in the lab frame. Let's recall the relation

$$dt = \gamma(v) d\tau \quad (4.79)$$

where  $d\tau$  the proper time interval in the rest frame of the test-body A. Using the relation (4.79) the equation (4.78) becomes

$$\frac{d\vec{p}}{d\tau} = \gamma(v) m\vec{E}_{gB} + \gamma(v) q\vec{E}_{eB} \quad (4.80)$$

The total inertial rest mass  $m_{in-0}$  of the test-body A is invariant and the equation (4.80) becomes

$$m_{in-0} \frac{d^2\vec{r}}{d\tau^2} = \gamma(v) m\vec{E}_{gB} + \gamma(v) q\vec{E}_{eB} \quad (4.81)$$

Let's recall the relations

$$\vec{E}_{gB} = -\vec{\nabla} \varphi_{gB} \quad \vec{E}_{eB} = -\vec{\nabla} \varphi_{eB} \quad (4.82)$$

Using the relations (4.82) the equation (4.81) becomes

$$\frac{d^2\vec{r}}{d\tau^2} = -\frac{\gamma(v)}{m_{in-0}} \vec{\nabla} (m\varphi_{gB} + q\varphi_{eB}) \quad (4.83)$$

Therefore,

- **The radial acceleration of a body in a gravitational and an electric field is not depended on its speed, for non relativistic velocities.**

### §4.6 Dark matter

From equation (4.72), for non relativistic velocities, it follows that the gravitational rest mass of a star is proportional to the gravitational scalar potential of the entire Universe. As we have said in section 4.1, if the distribution of matter in the Universe was homogeneous, the equation (4.72) would be dominated by the very distant matter, so much that the inertial rest mass of a body would be considered practically constant and independent of the position in relation to other bodies with very large gravitational masses, such as the Milky Way.

However, from astronomical observation, we know that the distribution of matter in the Universe is highly inhomogeneous; there are planets, the sun, stars, galaxies, clusters of galaxies and so on. So, it seems that the position where a star is located in a galaxy, affects significantly the inertial mass of the star.

In places with higher density of matter the inertial mass of a star will be greater than the inertial mass of the star, in a place with lower density of matter. The density of matter in a galaxy is higher, close to the centre of the galaxy compared with the density at the edge of the galaxy. Therefore, two stars with the same gravitational mass, but in different positions in a galaxy, have different inertial masses. The star that is far from the centre of the galaxy has lower inertial mass than the star near the centre of the galaxy. As the star moves around the centre of the galaxy, to balance the gravitational attraction the star that is far away from the centre, will move with greater speed than the speed that it would have if its inertial mass was constant. This higher speed is observed. The inability to explain why speed is higher has driven to the concept of dark matter.

#### §4.7 Zero inertial mass

Let's consider now, at what distance should two charged particles approach, so that the contribution of the electric scalar potential to their inertial masses can be important.

We assume that we superpose, in the lab frame and in a short distance, an electric charge  $Q$  and a point particle  $K$ , which has not internal structure, with gravitational mass  $m$  and electric charge  $q$ . We suppose that the point particle  $K$  and the electric charge  $Q$  are at rest in the lab frame and at a distance  $r$ . The inertial rest mass of the body  $K$ , according to the equation (4.72), is

$$m_{m-ge0} = m_{m-g0} + m_{in-e0} = \frac{1}{c^2} \left( \frac{1}{4\pi g_0} \sum_{i=2}^N \frac{mm_i}{r_i} - \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \right) = m_{in-g0} - \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} \quad (4.84)$$

We suppose that the charges  $Q$  and  $q$  are like charges. In this case, when the point particle  $K$  approaches the electric charge  $Q$ , the inertial rest mass of the point particle  $K$ , decreases. At a critical distance  $r_{critical}$ , the inertial rest mass of the point particle  $K$  becomes zero. Let's calculate this critical distance. When the inertial rest mass of the point particle  $K$  is zero, equation (4.84) becomes

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{r_{critical}} \Rightarrow r_{critical} = \frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \frac{qQ}{m_{in-g0}} \quad (4.85)$$

From (4.85) we can calculate the critical distance where the rest mass becomes zero. The gravitational rest mass of the electron and the proton are,

$$m_e = 9,109 \times 10^{-31} \text{ Kg} \quad m_p = 1,672 \times 10^{27} \text{ Kg} \quad (\text{S.I.})$$

The absolute value of the electron's charge and the proton's charge is

$$|q_e| = 1,602 \times 10^{-19} \text{ C} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = 8,988 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \quad (\text{S.I.})$$

Using the above values, the equation (4.77) gives us, for two protons  $r_{critical} \approx 1,53 \times 10^{-18} m$  and for two electrons  $r_{critical} \approx 2,81 \times 10^{-15} m$ . So,

- **At a distance about  $r \approx 10^{-15}m$  the contribution of the electrical forces to the inertial mass of a particle can be predominant.**

This conclusion is very important for nuclear physics. What if two protons approach at a shorter distance? What if the inertial rest mass becomes negative? Do we really need the nuclear forces to explain phenomena of nuclear physics? A door is opening and many questions arise.

## 5. Spacetime metric

### §5.1 Spacetime metric outside of a static and spherically symmetric gravitational mass and electric charge

We have shown in section 4.4, that the gravitational and electric forces affect the spacetime metric perceived by a body. The spacetime doesn't have properties of its own and different electric charges, at the same point in an electric field, will perceive a spacetime with different metric.

Four classic famous experiments verified the Schwarzschild metric, derived from the field equations of the General Theory of Relativity [27], the gravitational red shift of light, the deflection of light by the Sun, the perihelion precession of Mercury's orbit and the Shapiro time delay effect. So, if we can derive the Schwarzschild metric, without the field equations of General relativity, we will be in agreement with past experiments. We will do this by using a method based on a paper of F. Tangherlini in 1962 [28][29][30].

So, let's find the spacetime metric that perceives a test-body A with gravitational mass  $m$  and electric charge  $q$ , outside of a body B which is static (and therefore stationary) with spherically symmetric distribution of gravitational mass  $M$  and electric charge  $Q$ . We suppose that  $M \gg m$  and  $Q \gg q$ . It is well documented that we can bring the spacetime interval  $ds$  from the principle of spacetime metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (5.1)$$

outside of a static and spherically symmetric body B, into the standard Schwarzschild form [30]

$$ds^2 = g_{00}(r) c^2 dt^2 + g_{11}(r) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.2)$$

This metric should give us to infinity the Minkowski metric in spherical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.3)$$

So we must have the boundary conditions

$$\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1 \quad \lim_{r \rightarrow \infty} g_{11}(r) \rightarrow -1 \quad \lim_{r \rightarrow \infty} g_{00}(r) g_{11}(r) \rightarrow -1 \quad (5.4)$$

**For the radial motion of the test-body A the spacetime interval becomes**

$$ds^2 = g_{00}(r) c^2 dt^2 + g_{11}(r) dr^2 \quad (5.5)$$

Let's find now the lagrangian for the radial motion of the test-body A. From the principle 5 of spacetime metric and the principle 6 of geodesic motion we have

$$\delta \int ds = 0 \Rightarrow \delta \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau = 0 \Rightarrow \delta \int \sqrt{L} d\tau = 0 \quad (5.6)$$

The L may be termed a 'lagrangian'. Using the relations

$$\dot{x}^\mu = \frac{dx^\mu}{d\tau} \quad \dot{x}^\nu = \frac{dx^\nu}{d\tau} \quad (5.7)$$

the lagrangian L becomes

$$L = g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \quad (5.8)$$

For radial motion the lagrangian becomes

$$L = g_{00} c^2 \left(\frac{dt}{d\tau}\right)^2 - g_{11} \left(\frac{dr}{d\tau}\right)^2 \quad (5.9)$$

From the lagrangian, using the calculus of variation, we obtain the Euler-Lagrange system of equations

$$\frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}}\right) - \frac{\partial L}{\partial t} = 0 \quad (5.10) \quad \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \quad (5.11)$$

where

$$\dot{t} = \frac{dt}{d\tau} \quad \text{and} \quad \dot{r} = \frac{dr}{d\tau}$$

Let's find now the two first integral for the equation of the motion in the radial direction. Because the Lagrangian does not depend on time, we have

$$\frac{\partial L}{\partial t} = 0 \quad (5.12)$$

$$\text{So,} \quad \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}}\right) - \frac{\partial L}{\partial t} = 0 \Rightarrow \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{t}}\right) = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = \text{const.} \quad (5.13)$$

So, from equation (5.13), the term  $\frac{\partial L}{\partial \dot{t}}$  is a conserved quantity. Performing the differentiation in the equation (5.9) and using the relation  $ds^2 = c^2 d\tau^2$  and the fact that the metric function  $g_{00}(r)$  doesn't depend on  $t$ , we obtain

$$\frac{\partial L}{\partial \dot{t}} = 2g_{00} c^2 \frac{dt}{d\tau} = \text{const.} \Rightarrow g_{00} \frac{dt}{ds} = k_0 \quad (5.14)$$

Equation (5.14) is a first integral for the equation of motion in the radial direction and states that the body's energy  $k_0$  (in units of  $m_{in-0} c^2$ ) is a constant of the

**motion.** We can have a second first integral for the equation of motion in the radial direction by dividing the equation (5.5) by the spacetime interval  $ds^2$

$$1 = g_{00}c^2 \left( \frac{dt}{ds} \right)^2 + g_{11} \left( \frac{dr}{ds} \right)^2 \quad (5.15)$$

Equation (5.15) is the second first integral for the equation of motion in the radial direction and states the invariant relation between energy and momentum (in units of  $m_{in-0}c^2$ ). Eliminating  $\frac{dt}{ds}$  from the equation (5.15), using the first integral from equation (5.14), we have

$$1 = \frac{c^2 k_0^2}{g_{00}} + g_{11} \left( \frac{dr}{ds} \right)^2 \quad (5.16)$$

Let's recall that  $g^{\nu\sigma} g_{\kappa\nu} = \delta_{\kappa}^{\sigma}$ . So,

$$g^{00} = \frac{1}{g_{00}} \quad \text{and} \quad g^{11} = \frac{1}{g_{11}} \quad (5.17)$$

Dividing the equation (5.16) by  $g_{11}$  and using the equations (5.17) we have

$$\frac{1}{g_{11}} = \frac{c^2 k_0^2}{g_{11} g_{00}} + \left( \frac{dr}{ds} \right)^2 \Rightarrow g^{11} = c^2 k_0^2 (g^{00} g^{11}) + \left( \frac{dr}{ds} \right)^2 \quad (5.18)$$

Because  $g_{00}(r)$  is function only of  $r$

$$\frac{dg^{00}}{ds} = \frac{\partial g^{00}}{\partial r} \frac{dr}{ds} = g^{00},r \frac{dr}{ds} \quad \text{and} \quad \frac{dg^{11}}{ds} = \frac{\partial g^{11}}{\partial r} \frac{dr}{ds} = g^{11},r \frac{dr}{ds} \quad (5.19)$$

Using the relations (5.19) we differentiate the equation (5.18) in respect to  $s$  and we obtain the following equation where a comma denotes ordinary differentiation

$$g^{11},r \frac{dr}{ds} = c^2 k_0^2 (g^{00} g^{11}),r \frac{dr}{ds} + 2 \frac{dr}{ds} \frac{d^2 r}{ds^2} \Rightarrow \frac{d^2 r}{ds^2} = -\frac{c^2 k_0^2}{2} (g^{00} g^{11}),r + \frac{1}{2} g^{11},r \quad (5.20)$$

**Finally, using the relation  $ds^2 = c^2 d\tau^2$ , we obtain the radial geodesic equation in the form,**

$$\frac{d^2 r}{d\tau^2} = -\frac{c^4 k_0^2}{2} (g^{00} g^{11}),r + \frac{c^2}{2} g^{11},r \quad (5.21)$$

In the above equation (5.21) we see that because of the  $k_0$  term, the radial acceleration depends on the energy which the test-body had initially, i.e. the radial velocity with which the test-body was launched. **But as we have shown in section 4.5, the radial acceleration of a body in a gravitational and an electric field is not depended on its speed, for**

non relativistic velocities. So, for non relativistic velocities, the equation (5.21) must reduce to the equation

$$\frac{d^2 r}{d\tau^2} = +\frac{c^2}{2} g^{11},{}_r \quad (5.22)$$

In order for this to happen

$$(g^{00} g^{11}),{}_r = 0 \Rightarrow g^{00} g^{11} = \text{const.} \quad (5.23)$$

Using the boundary conditions (5.3) and the relations (5.17) we have

$$g^{00} g^{11} = -1 \Rightarrow g^{11} = -\frac{1}{g^{00}} \Rightarrow g^{11} = -g_{00} \quad (5.24)$$

So, the equation (5.22) using the relation (5.24), becomes

$$\frac{d^2 r}{d\tau^2} = -\frac{c^2}{2} g_{00},{}_r \quad (5.25)$$

Comparing the equations (5.25) and (4.83) we have

$$g_{00},{}_r = \frac{2\gamma(v)}{m_{in-0} c^2} \nabla (m\varphi_{gB} + q\varphi_{eB}) \quad (5.26)$$

Using the boundary condition  $\lim_{r \rightarrow \infty} g_{00}(r) \rightarrow 1$  we obtain for  $g_{00}(r)$

$$g_{00} = 1 + \frac{2\gamma(v)}{m_{in-0} c^2} (m\varphi_{gB} + q\varphi_{eB}) \quad (5.27)$$

Using the equation (5.17), the equation (5.24) becomes

$$g^{00} g^{11} = -1 \Rightarrow g_{00} g_{11} = -1 \Rightarrow g_{11} = -\frac{1}{g_{00}} \quad (5.28)$$

and so we obtain for  $g_{11}(r)$

$$g_{11} = -\frac{1}{1 + \frac{2\gamma(v)}{m_{in-0} c^2} (m\varphi_{gB} + q\varphi_{eB})} \quad (5.29)$$

Using the equations for the gravitational and electric scalar potential

$$\varphi_{gB} = -G \frac{M}{r} \quad \text{and} \quad \varphi_{eB} = K \frac{Q}{r} \quad (5.30)$$

we obtain the final equation that give us the spacetime metric that perceives a test-body A with gravitational mass  $m$  and electric charge  $q$ , outside of a body B which is static with spherically symmetric distribution of gravitational mass  $M$  and electric charge  $Q$ .

$$ds^2 = \left[ 1 - \frac{\gamma(v)}{c^2 m_{in-0}} \left( \frac{2GMm}{r} - \frac{2KQq}{r} \right) \right] c^2 dt^2 - \frac{dr^2}{\left[ 1 - \frac{\gamma(v)}{c^2 m_{in-0}} \left( \frac{2GMm}{r} - \frac{2KQq}{r} \right) \right]} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.31)$$

Let's recall that, in the International system of units the gravitational mass and the inertial rest mass of a body, are considered the same and so the ratio of gravitational mass to inertial mass, in our position in our galaxy, is equal to one. Therefore, if we restrict our consideration only for gravitational masses and for non relativistic velocities, the equation (5.29) reduces to the Schwarzschild metric

$$ds^2 = \left( 1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (5.32)$$

So, all the phenomena that emerge from the Schwarzschild metric also emerge from the general spacetime metric (5.30). At another point in our Galaxy or in the Universe, the ratio of gravitational to inertial mass is not equal to one. There, the phenomena will be the same qualitatively but not quantitatively.

## §5.2 Dark Energy

As the Universe expands, the inertial rest mass of all matter decreases. Thus, the inertial rest mass of an atom, which emits light, decreases over time. As it emerges from equation (5.31) the light emitted by two identical supernovas Ia, at different moments in the history of the Universe, will have different red shift. The inertial rest mass of the atoms of a closer supernova Ia is smaller than the inertial rest mass of the atoms of a further identical supernova Ia because the closer a star is to us, the more recent the light that we see from it. So, the closer supernovas Ia have greater red shift. This phenomenon has been observed but the inability to explain why the red shift of spectral lines is greater has led to the theory that the Universe expands in an accelerating way because of dark energy.

## Conclusions

Finally, the physics that comes from the attempt to explore the origin of inertia shows us that the motion of a tiny body is affected by the entire Universe. As Dennis Sciama ended an article on inertia in Scientific American [31]: "If atomic properties are in fact so determined, we shall again be faced with the dual situation: Distant matter influencing local phenomena and local phenomena giving us information about distant matter. The scientist would then be able to claim that his imagination had out-stripped the poet's. For he would see the world not in a "grain of sand" but in an atom" The Universe never stops surprising us!!!

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