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RELATIVISTIC GRAVITY AND THE ORIGIN OF INERTIA AND INERTIAL MASS

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Abstract. In physics there are two masses, the gravitational and inertial, and an open question: what is the relation between these two masses? The answer to that must pass through the explanation of inertia. In the experiments that have been made, it has been shown that the ratio of inertial mass to gravitational mass of an object is constant and equal for each material. This has led through the General Theory of Relativity in responses which led to dramatic predictions confirmed experimentally. In this paper we will follow a completely different path and will give different answers but our predictions will include earlier predictions that have been confirmed experimentally. Creating and using a theory in a radically different way of thinking, which affects the whole of physics, we will explain how the inertial forces are generated. In this process we will define the physical quantity of inertial mass. The nature of inertial mass, the way it changes and the effect on the measurement of time, will show that it is an extremely useful tool for the explanation of dark matter and dark energy.

1. INTRODUCTION

Inertial forces appear in a non-inertial reference system. But what determines an inertial reference system? According to Newton, an inertial reference system is a system that moves at a constant velocity with respect to the absolute space. According to Mach's principle, inertial reference system is a system that moves with constant velocity with respect to the distant stars. If the rest of the Universe determines the inertial frame, it follows that inertia is not an intrinsic property of the matter, but arises as a result of the interaction of matter with the rest of the matter in the Universe.

Alfred Einstein was inspired by Mach's principle. The General Theory of Relativity attempted to interpret inertia, considering that it is the gravitational effect of the whole Universe, but as pointed out by Einstein, it failed to do so [1]. Einstein showed that the field equation of General Relativity implies that an object in an empty Universe has inertial properties.

Following Mach's principle, Dennis Sciama in a paper in 1952 attempted to interpret inertia using a model where gravitational forces are described by equations similar to Maxwell equations [2]. In this paper, the inertia of an accelerating object is an inductive effect of all other objects of the Universe and the principle of equivalence is a result of the theory and need not be introduced axiomatically. But no answer was given to the question of how inertial forces appear exactly at the time when a body is accelerating.

In Electrodynamics inertia has the name radiation reaction. In an attempt to explain the radiation reaction in 1945 Wheeler and Feynman created a new theory, the Absorber theory of radiation [3][4]. According to this theory, the radiation reaction in an accelerated charge comes from all other charges in the Universe and travels with advanced waves, waves that propagate from the future to the past, to reach the charge exactly the moment when it starts accelerating. Paul Dirac also used advanced waves in a paper in 1938 [5], in order to interpret the radiation reaction. The Wheeler-Feynman theory revived one action at a distance formulation of Electrodynamics as derived by Schwarzschild, Tetrode

and Fokker [6][7][8][9]. It is known as the action at a distance theory because charges can interact with one another only over finite distance without a mediating entity (a field) that carries this action from one particle to the other. It is possible to formulate classical electrodynamics as an action at a distance theory and dispense with the field concept altogether [10][11][12].

Here we will follow Mach's principle in order to find a satisfactory explanation of the origin of inertia and inertial mass. If the inertia of an object is derived from the interaction of the object with all the other objects in the Universe, then, the forces with infinite range that can generate inertial forces are the gravitational and electrical forces. The force between two gravitational masses at rest is described by Newton's law of gravitation. The electric force between two point charges at rest in vacuum is described by Coulomb's law. Both these laws are of the same format. But while the two forces began with great similarity, they ended up described by two completely different theories, the General Theory of Relativity and Electrodynamics. But as emphasized by Poincare [13][14], if equilibrium is to be a frame-independent condition, it is necessary for all forces of non-electromagnetic origin to have precisely the same transformation law as of the Lorentz-force

2. GRAVITY

Let's suppose that we have two objects with gravitational masses and positive electric charges, at a point where the external gravitational and electric fields are zero. The two objects are placed at a distance from each other so that the attractive gravitational force is equal in magnitude and opposite in direction to the repulsive electric force exerted between them. There the objects are placed at rest, in respect to us, and they will remain at rest as they are in equilibrium. But what is observed by another observer moving in relation to the two objects? Let's imagine that if they collide, they will explode. It is impossible for one observer to see an explosion and for another not. The Special Theory of Relativity requires that the two objects must be in equilibrium also for an observer moving with a constant velocity in relation to them.

- If we want the system of two objects to be in equilibrium for each inertial observer, the gravitational force should be transformed in exactly the same way as the electric force is transformed in different inertial reference systems.

In order for this to happen, both forces must be described by similar equations. Since the equations of electromagnetism are relativistic invariants and verified by numerous experiments, it is logical to think that gravity should also be described by similar equations. Moreover, as we will prove in section 6, the gravitational and electric forces acting on an object, affect the spacetime metric perceived by this object.

- So, the idea is that the gravitational and electric forces are described by similar equations and both forces curve the spacetime with similar way.

It is known and well documented that we can produce all the equations of electromagnetic theory in Minkowski spacetime using the Special Theory of Relativity, on condition that [15]:

1. Coulomb's law describes the force exerted between the two electric point charges at rest in vacuum.
2. The electric charge is invariant. It has the same value in all inertial reference systems.
3. An electric charge produces a scalar potential and a vector potential, which together form a four-vector. The potentials produced by a charge moving in any way depend only upon the

velocity and position at the retarded time. Changes in these potentials are propagating in vacuum at the speed of light.

The methodology for doing this is described in many of the classic books on electromagnetism [16][17][18].

To apply the same methodology in Gravity and to obtain equations similar to those of electromagnetic theory, the same prerequisites must be met. But all we have is Newton's law of gravitation that is similar to Coulomb's law for electric point charges. If we want to apply the same methodology, we need the other two as well. We will accept them postulated:

Principle 1

The gravitational mass is invariant. It has the same value in all inertial reference systems.

Principle 2

A gravitational mass produces a gravitational scalar potential and a gravitational vector potential that together form a four-vector. The potentials produced by a gravitational mass moving in any way depend only upon the velocity and position at the retarded time. Changes in these potentials are propagating in vacuum at the speed of light.

So we arrive at the following equations, similar to Maxwell's equations, which we will call the equations of the gravitomagnetic theory in Minkowski spacetime and in SI units.

$$\vec{\nabla} \cdot \vec{E}_g = -\frac{\rho_g}{\epsilon_{0g}} \quad (2.1) \quad \vec{\nabla} \cdot \vec{B}_g = 0 \quad (2.2) \quad \epsilon_{0g} = \frac{1}{4\pi G}$$

$$\vec{\nabla} \times \vec{E}_g = -\frac{\partial \vec{B}_g}{\partial t} \quad (2.3) \quad \vec{\nabla} \times \vec{B}_g = -\mu_{0g} \vec{j}_g + \frac{1}{c^2} \frac{\partial \vec{E}_g}{\partial t} \quad (2.4) \quad \mu_{0g} = \frac{4\pi G}{c^2}$$

$$\vec{F} = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (2.5)$$

ρ_g : gravitational mass density \vec{j}_g : gravitational mass current density

\vec{E}_g : the gravitational field \vec{B}_g : the gravitational magnetic field

G : Newton's universal gravitational constant c : the speed of light in vacuum.

m : gravitational mass

The potentials produced by a gravitational mass moving with velocity \vec{v} at a distance r , is as the Lienard-Wiechert potential:

$$\varphi_g = -\frac{1}{4\pi\epsilon_{0g}} \left[\frac{m}{r - \vec{r}\vec{v}/c} \right] \quad (2.6)$$

$$\vec{A}_g = -\frac{1}{c^2} \frac{1}{4\pi\epsilon_{0g}} \left[\frac{m\vec{v}}{r - \vec{r}\vec{v}/c} \right] = \frac{1}{c^2} [\varphi_g \vec{v}] \quad (2.7)$$

In the above equations, what is in square brackets refers to the retarded position of the gravitational mass. Starting from the potentials, in order to find fields, we have the equations:

$$\vec{E}_g = -\nabla\varphi_g - \frac{\partial \vec{A}_g}{\partial t} \quad (2.8) \quad \text{and} \quad \vec{B}_g = \nabla \times \vec{A}_g \quad (2.9)$$

In this way, we easily obtain a relativistic invariant theory for Gravity, the Gravitomagnetic theory.

3. PRINCIPLE OF GENERAL RELATIVITY

The inertia appears in objects when they are accelerated. So the next step on the road to interpret inertia is to accept the extension of the Principle of Relativity for all reference systems, not only for inertial ones. There can be no privileged reference systems. We will do this with the next principle.

Principle 3

The laws of physics are the same in all frames of reference.

The principle of relativity is extended to all movements. In order to use Lorentz transformations to any reference system we should use the instantaneous speed. There is no absolute acceleration just as there is no absolute speed. With any experiments carried out in a closed laboratory, an observer accelerating cannot establish whether:

1. He is moving with acceleration $\vec{\alpha}$ and all other objects in the Universe are at rest.
2. He is at rest, while all the other objects in the Universe are moving with acceleration $(-\vec{\alpha})$.

An electric charge emits electromagnetic radiation when accelerating. This follows from the equations of electromagnetism. Since the same equations apply to gravitomagnetism, a gravitational mass when accelerating will emit gravitomagnetic radiation which will propagate in vacuum at the speed of light and is described by the same equation as that of electromagnetic radiation.

According to the principle of general relativity we accepted, acceleration is relative, so the radiation of a gravitational mass or an electric charge when accelerating is also relative. This means that when a mass or charge is accelerated in relation to an observer A, it emits radiation perceived by the observer A. If, for another observer B, there is no relative acceleration, there will be no radiation. That is, the radiation emitted by an object is always taken from another object. No radiation is emitted just in empty space. The radiation has necessarily a sender and a recipient.

Because acceleration is relative, the inertial force cannot be derived from the same object which is accelerating. It must be due to those objects in relation to which it is accelerated.

Suppose there are only two objects in the Universe, A and B. If B sees object A accelerate, then B receives radiation emitted by A. Because acceleration is relative, A sees B accelerate. So A receives radiation emitted by B. Both objects should receive radiation if acceleration is relative. Otherwise, we can discern which object is accelerated. But at the same time an inertial force is exerted on both objects if acceleration is relative. In order for all this to happen, one way is to accept the following procedure:

- Object A located at a time t_1 at point 1, with position vector \vec{r}_1 , is accelerating for B. So A emits radiation at time t_1 , which will reach B at time t_2 , when B is located at position 2, with position vector \vec{r}_2 . Radiation from A will exert a force \vec{F} on object B.
- Object B at a time t_2 at point 2, with position vector \vec{r}_2 , is accelerating for A. So B emits radiation at time t_2 , which will reach A at time t_1 when A is located at position 1, with position vector \vec{r}_1 . Radiation from B will exert a force $(-\vec{F})$ on object A.

Radiation from B to A is propagated opposite in space and time, in relation to the radiation from A to B. The time t_1 related to time t_2 with the equation:

$$t_2 = t_1 \pm \frac{|\vec{r}_2 - \vec{r}_1|}{c} \quad (3.1)$$

The sign in the equation (3.1) is determined by the principle of causality. The cause must precede the effect.

- If A accelerates first, A perceives as inertial force the radiation of B.
- If B accelerates first, B perceives as inertial force the radiation of A.

If A is accelerating first, the radiation is propagated to the object B with retarded waves. Object A perceives as inertial force the radiation of B which is propagated from object B on the object A with advanced waves. Both waves, retarded and advanced, are solutions of the Maxwell equations.

4. ACTION-REACTION

An object exerts a force on another object with radiation. But there is no reason to treat the force of radiation differently from the other forces. So we accept that the procedure described above applies to all forces. The Principle of General Relativity leads us to the relativistic generalization of the third law of Newton:

Principle 4

When an object A moves in any way and is located at time t_1 at a point with position vector \vec{r}_1 , it exerts a force-action \vec{F} at time t_2 , on an object B, located at a point with position vector \vec{r}_2 , moving in any way, then B exerts a force-reaction ($-\vec{F}$) on A at time t_1 with $t_1 = t_2 - \frac{|\vec{r}_2 - \vec{r}_1|}{c}$.

Comments on Principle 4.

- What is transmitted and has the effect of exerting force on one object is the potential of another object.
- The action is the cause and the reaction is the effect. Without action there is no reaction. Thus, the principle of causality is not violated.
- The above procedure always happens as a whole, i.e. action is always accompanied by reaction. So the concept of the field is auxiliary and not essential. It is only useful for calculations.
- At this point, we must make it clear that if we deal with phenomena occurring on our planet, the reaction comes from some thousandths or millionths of a second in the future.
- In classical physics we accept that the forces are transmitted instantaneously, and prove the principle of conservation of momentum from the third law of Newton. Now, with the relativistic generalization of the third law of Newton, we can prove the principle of conservation of momentum also for forces that are transmitted with finite speed.
- Strange as it may seem, if we want to interpret phenomena such as quantum entanglement, the possibility for a particle to receive the reaction from all other particles at exactly the moment when some force exerted on it, appears necessary [19].

4. INERTIA

Having created all the tools we need, we can now move on and consider an accelerated object.

5.1 Study of the motion of an object with only gravitational mass

Suppose we have a Universe where all objects contained are at rest. Suppose we have a point K where $\nabla\varphi_g = 0$, φ_g is the gravitational scalar potential at point K from all the Universe. We place an object S, with gravitational mass m , at rest at point K. The resultant gravitational force exerted on the object S is zero. The gravitational vector potential is zero, at the rest frame of S, since all objects are at rest in relation to S.

A force \vec{F} begins to be exerted on object S and it starts to move. Suppose at time t_1 object S is moving with velocity \vec{v} and acceleration $\vec{\alpha}$ in relation to the other objects in the Universe. Changes of the potentials of object S are propagated at the speed of light to the other objects, where they arrive at various later times t_i , different for every object, with:

$$t_1 = t_i - \frac{|\vec{r}_i - \vec{r}_1|}{c}$$

where r_i is the distance traveled to reach every object.

Since velocity and acceleration are relative, at the rest frame of S, the other objects move with velocity $(-\vec{v})$ and acceleration $(-\vec{\alpha})$ at the specific times t_i . The changes of the potentials of the other objects propagate backward in space and time and arrive at the time t_1 at object S. These changes cause at time t_1 , a force $(-\vec{F})$ to object S that we call gravitational inertial force.

Now the advanced gravitational vector potential \vec{A}_g at the rest frame of S at time t_1 is not zero but it will be from equation (2.7):

$$\vec{A}_g = \frac{1}{c^2} [\varphi_g(-\vec{v})]_{advanced}$$

The gravitational scalar potential changes but continues to be: $\nabla[\varphi_g]_{adv} = 0$, because if an object is in equilibrium to point K because $\nabla\varphi_g = 0$, it is in equilibrium for any observer. What is in square brackets refers to a later time than time t_1 . With this in mind, we can then remove the square brackets:

$$\vec{A}_g = \frac{1}{c^2} \varphi_g(-\vec{v}) \Rightarrow \vec{A}_g = -\frac{1}{c^2} \varphi_g \vec{v} \quad (5.1)$$

The gravitational force exerted on an object with gravitational mass m moving at speed \vec{v} , when it is in a gravitational field \vec{E}_g and a gravitational magnetic field \vec{B}_g is from eq. (2.5):

$$\vec{F}_g = m(\vec{E}_g + \vec{v} \times \vec{B}_g) \quad (5.2)$$

Where
$$\vec{E}_g = -\nabla\varphi_g - \frac{\partial\vec{A}_g}{\partial t} \quad \text{and} \quad \vec{B}_g = \nabla \times \vec{A}_g$$

In the rest frame of S, because $\vec{v}=0$ the gravitational magnetic force is: $m\vec{v}\times\vec{B}_g=0$. Moreover, as mentioned earlier, $\nabla\varphi_g=0$ ($\nabla[\varphi_g]_{adv}=0$ with the square brackets).

So at t_1 , the force exerted on S is:

$$\vec{F}_g = m\vec{E}_g \Rightarrow \vec{F}_g = -m\frac{\partial\vec{A}_g}{\partial t} \quad (5.3)$$

Equation (5.3) together with the (5.1) becomes:

$$\vec{F}_g = m\frac{1}{c^2}\frac{\partial(\varphi_g\vec{v})}{\partial t} \quad (5.4)$$

If we define as gravitational inertial mass m_{in-g} the inertial mass due to gravity:

$$m_{in-g} = -\frac{1}{c^2}m\varphi_g \quad (5.5)$$

The equation (5.4) becomes:

$$\vec{F}_g = -\frac{\partial(m_{in-g}\vec{v})}{\partial t} \quad (5.6)$$

We define as gravitational momentum \vec{p}_g :

$$\vec{p}_g = m_{in-g}\vec{v} \quad (5.7)$$

Because m_{in-g} is positive, speed and momentum have the same direction as required.

So if you replace (5.7) to (5.6) we have:

$$\vec{F}_g = -\frac{\partial\vec{p}_g}{\partial t} \quad (5.8)$$

$$\vec{F}_g = -\frac{\partial m_{in-g}}{\partial t}\vec{v} - m_{in-g}\frac{\partial\vec{v}}{\partial t} \quad (5.8a)$$

This is known to us as inertia force of an object that has only gravitational mass. We end up to the conclusion that the inertial force received by an object S with gravitational mass when accelerating is a gravitational force and is due to the acceleration of all the other objects in the Universe in relation to the object S. So we can express the second law of Newton as follows:

- When an object is accelerated, the force exerted on the object to accelerate it, needs to be equal in magnitude and opposite in direction to the inertial force.

It is obvious that the second law of Newton is now a consequence of the theory that we follow, and there is no need to introduce it axiomatically.

We accept that the gravitational inertial mass of an object S depends on the advanced gravitational scalar potential of all other objects in relation to which it is accelerated. The advanced gravitational scalar potential depends only upon the velocity and position at an advanced time.

Using (5.5) and (5.1) the gravitational vector potential \vec{A}_g becomes:

$$\vec{A}_g = -\frac{1}{c^2} \varphi_g \vec{v} \Rightarrow \vec{A}_g = \frac{m_{in-g} \vec{v}}{m} \Rightarrow m \vec{A}_g = m_{in-g} \vec{v}$$

So the gravitational momentum \vec{p}_g of an object S and the advanced gravitational vector potential \vec{A}_g from all the other objects at the rest frame of S, are connected with the equation:

$$\vec{p}_g = m \vec{A}_g \quad (5.9)$$

It is worth repeating that to get here we have accepted that the gravitational inertial mass of an object S is:

$$m_{in-g} = -\frac{1}{c^2} m \varphi_g$$

The total inertial mass m_{in} will simply be called inertial mass. For an object that has no electric charge, the gravitational inertial mass m_{in-g} is equal to the inertial mass m_{in} .

$$m_{in} = m_{in-g}$$

If we take the ratio of the inertial mass to gravitational mass of an object, we have:

$$\frac{m_{in-g}}{m} = -\frac{1}{c^2} \varphi_g \quad (5.10)$$

We see that the ratio of the inertial mass of an uncharged object to the gravitational mass is independent of the object. It depends only on the advanced gravitational scalar potentials due to all other gravitational masses in the Universe in respect to which it accelerates. So this ratio is equal for each object when the inertial mass is measured at the same point in space. This explains why all objects fall with the same acceleration in a gravitational field. The above results are confirmed experimentally very precisely. At another point in space, while the gravitational mass remains the same, the inertial mass changes so the ratio has another value.

Consider now an object that makes free fall in the gravitational field of another much bigger object K. The acceleration the object obtains is such, that the inertial force to be exerted on it is equal in magnitude and opposite in direction to the gravitational force it accepts from K. So the resultant gravitational force on it is zero. Therefore:

- The rest system of an object with gravitational mass which makes free fall in a gravitational field is equivalent to an inertial reference system.

An object S with gravitational mass which is accelerated in relation to the other objects receives an inertial force that is nothing else than a gravitational force. So:

- A reference system consisting only of gravitational masses which makes an accelerated motion in a space where there is no gravitational field is equivalent to an inertial reference system at rest in a gravitational field.

In the General Theory of Relativity the principle of equivalence for the gravitational forces is accepted as an initial axiom. Here we have proved that the principle of equivalence for the gravitational forces is a consequence of the theory.

In the General Theory of Relativity we accept that the inertial mass is equivalent to the gravitational mass. Here we have proved that the two masses can be completely different physical quantities but their ratio is equal for each object at the same point, but not the same everywhere.

Let us now examine how the gravitational inertial mass of an object S is transformed when S moves with velocity \vec{v} in relation to the other objects. In equation (5.5):

$$m_{in-g} = -\frac{1}{c^2} m \varphi_g$$

the gravitational mass m and the speed of light c , are invariant quantities. So the gravitational inertial mass will be transformed in the same way as the gravitational scalar potential φ_g .

We define gravitational scalar potential resting as φ_{g0} . The φ_{g0} is the gravitational scalar potential from all other objects, in the rest frame of S, when all other objects are at rest in relation to S. If the Universe consists of N discrete gravitational masses and each is located at a different distance from the object S, then:

$$\varphi_{g0} = -\frac{1}{4\pi g_0} \sum_{i=1}^N \frac{m_i}{r_i} \quad (5.11)$$

The distance r_i is measured in the reference system where all objects are at rest.

When S moves with velocity \vec{v} in relation to the other objects, the gravitational scalar potential of all other objects, in the rest frame of S is transformed in the same way as the electric scalar potential [15],[18][20]:

$$\varphi_g = \frac{\varphi_{g0}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.12)$$

Equation (5.5) shows that when S moves with velocity \vec{v} in relation to the other objects the gravitational inertial mass will vary in the same way:

$$m_{in-g} = \frac{m_{in-g0}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.13)$$

where m_{in-g0} is the gravitational rest mass. It is the gravitational inertial mass in the rest frame of object S when all other objects are at rest in respect to S.

We now see from equation (5.13) that the equation for the relativistic mass in the Special Theory of Relativity is proved directly and not indirectly, but with two important differences.

- The rest mass of an object S, is the inertial mass in the reference system where all the objects are at rest, including S. In Special Theory of Relativity the rest mass of an object S, is the inertial mass in the reference system where only the object S is at rest.
- The gravitational inertial mass of an object S varies only when S moves in relation to the other objects and not when S moves in relation to an observer.

If the Universe consists of N discrete gravitational masses and each mass is located at a different distance from an object, the gravitational rest mass of that object is:

$$m_{in-g0} = \frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=1}^N \frac{mm_i}{r_i} \quad (5.14)$$

The distance r_i is measured in the reference system where all the objects are at rest.

5.1.1 Dark matter

From equation (5.14) we observe that the gravitational rest mass of an object S depends on the distance of the other objects in respect to which it is accelerated. If we have an object only with gravitational mass, the total rest mass m_{in-0} is the gravitational rest mass m_{in-g0} .

So then (5.14) becomes:

$$m_{in-0} = \frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=1}^N \frac{mm_i}{r_i} \quad (5.15)$$

The distribution of matter in the Universe is not continuous. If it was continuous, the equation (5.15) would be dominated by the very distant gravitational masses, so much that the gravitational rest mass would be considered practically constant and independent of the object position in relation to other objects with very large gravitational mass such as the Milky Way, our galaxy. Let us think that our galaxy with a diameter of about one hundred thousand light years is far from the closest large galaxy, the Andromeda galaxy, which is almost identical to our own, one and a half million light years. The inertial mass of a star located at an average distance from the center of the Milky Way is affected almost fifty times more from the Milky Way than the Andromeda galaxy. In order to find fifty galaxies like Andromeda, we have to include galaxies that are almost twenty million light years far. So it seems that the position where a star is situated in a galaxy, affects significantly the inertial mass of the star. In places with higher density of stars the inertial mass of a star will be greater.

Two stars with the same gravitational mass have different inertial masses if they are in a different position in a galaxy. The star that is far from the center has a lower inertial mass than the star that is located near the center. To balance the gravitational attraction the star that is far away from the center will move with greater speed than the speed that it would have if the inertial mass was constant. This higher speed is observed. The inability to explain why speed is higher has driven to the concept of dark matter. This conclusion arises from a very simple approach we have taken. Certainly we need to do more precise calculations.

5.2. Study of the motion of an object with an electric charge

Let us now study the case that in the Universe there are electric charges at rest. Suppose we have a point K where $\nabla\varphi=0$, φ is the electric scalar potential from all the electric charges. We place an object S with electric charge q at rest in point K. The resultant electric force exerted on the object S is zero. Since all the other objects with electric charge are at rest, in relation to S, the electric vector potential due to them in the rest frame of S is zero. The process followed is identical to that which we followed in the previous section 5.1.

A force begins to be exerted on the object S and it starts to move. Suppose at time t_1 the object S is moving with velocity \vec{v} and acceleration \vec{a} in relation to the other electric charges. Changes of the electric potentials of object S are propagated at the speed of light to the other electric charges, where they arrive at various later times t_i , different for every electric charge, with:

$$t_1 = t_i - \frac{|\vec{r}_i - \vec{r}_1|}{c}$$

where r_i is the distance traveled to reach every electric charge.

Since velocity and acceleration are relative, at the rest frame of S, the other electric charges move with velocity $(-\vec{v})$ and acceleration $(-\vec{a})$ at different time moments in the future. The changes of the electric potentials of the other electric charges propagate backward in space and time and arrive at time t_1 at object S. These changes cause at time t_1 , a force $(-\vec{F})$ to object S that we call electric inertial force.

Now the advanced vector electric potential \vec{A} in the rest frame of S at time t_1 is not zero but it will be:

$$\vec{A} = \frac{1}{c^2} [\varphi(-\vec{v})]_{advanced}$$

The scalar electric potential, in the rest frame of S, changes but will continue to be $\nabla[\varphi]_{adv} = 0$ because if a charge is in equilibrium to point K, it is in equilibrium for any observer. What is in square brackets refers to a later time than t_1 . With this in mind, we can then remove the square brackets:

$$\vec{A} = \frac{1}{c^2} \varphi(-\vec{v}) \Rightarrow \vec{A} = -\frac{1}{c^2} \varphi \vec{v} \quad (5.16)$$

The electric force exerted on an object with electric charge q moving at speed \vec{v} , when it is in an electric field \vec{E} and a magnetic field \vec{B} is the Lorentz force:

$$\vec{F}_e = q(\vec{E} + \vec{v} \times \vec{B}) \quad (5.17)$$

$$\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t} \quad \text{and} \quad \vec{B} = \nabla \times \vec{A}$$

In the rest frame of S, because $\vec{v}=0$, no magnetic force is exerted on S. Moreover, we have said that $\nabla\varphi=0$ ($\nabla[\varphi_g]_{adv} = 0$ with the square brackets).

So at t_1 , the electric force exerted on S is:

$$\vec{F}_e = q\vec{E} \Rightarrow \vec{F}_e = -q \frac{\partial \vec{A}}{\partial t} \quad (5.18)$$

If we now define as electric inertia mass m_{in-e} the inertial mass caused by electric forces:

$$m_{in-e} = -\frac{1}{c^2} q\varphi \quad (5.19)$$

If we define as electric momentum \vec{p}_e the momentum due to electric forces:

$$\vec{p}_e = m_{in-e}\vec{v} \Rightarrow \vec{p}_e = -\frac{1}{c^2} q\varphi\vec{v} \Rightarrow \vec{p}_e = q\vec{A} \quad (5.20)$$

Then (5.16) becomes:

$$\vec{F}_e = -\frac{\partial(q\vec{A})}{\partial t} \Rightarrow \vec{F}_e = -\frac{\partial \vec{p}_e}{\partial t} \quad (5.21)$$

$$\vec{F}_e = -\frac{\partial m_{in-e}}{\partial t} \vec{v} - m_{in-e} \frac{\partial \vec{v}}{\partial t} \quad (5.21a)$$

At this point we must emphasize that the electric inertia mass according to equation (5.19) is not always positive, as is the gravitational inertia mass. If all charges are either positive or negative, the electric inertial mass is negative.

We conclude from equation (5.19) that the ratio of the electric inertial mass of a charged object to the charge is independent of the object:

$$\frac{m_{in-e}}{q} = -\frac{1}{c^2} \varphi$$

This ratio depends only on the advanced scalar potentials due to all other electric charges in the Universe in relation to which the object accelerates. So this ratio is equal for each charged object when the electric inertial mass is measured at the same point in space.

Let us now examine how the electric inertial mass of an object S is transformed when S moves with velocity \vec{v} in relation to the other charges. In equation (5.19):

$$m_{in-e} = -\frac{1}{c^2} q\varphi$$

the charge q and the speed of light c are invariant quantities. So the electric inertial mass will be transformed in the same way as the electric scalar potential φ .

We define electric scalar potential resting as φ_{e0} . The φ_{e0} is the electric scalar potential from all other charges, in the rest frame of S, when all other charges are at rest in relation to S. If the Universe consists of N discrete electric charges and each is located at a different distance from the object S, then:

$$\varphi_{e0} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{r_i} \quad (5.22)$$

The distance r_i is measured in the reference system where all charges are at rest.

When S moves with velocity \vec{v} in relation to the other charges, the electric scalar potential of all other charges, in the rest frame of S is known that is [15],[18][20]:

$$\varphi = \frac{\varphi_{e0}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.23)$$

Equation (5.19) shows that when S moves with velocity \vec{v} in relation to the other charges the electric inertial mass will vary in the same way:

$$m_{in-e} = \frac{m_{in-e0}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.24)$$

where m_{in-e0} is the electric rest mass. It is the electric inertial mass in the rest frame of object S when all other charges are at rest in respect to S.

If the Universe consists of N discrete electric charges and each charge is located at a different distance from an object, the electric rest mass of that object is:

$$m_{in-e0} = -\frac{1}{c^2} \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{qq_i}{r_i} \quad (5.25)$$

The distance r_i is measured in the reference system where all the objects are at rest.

5.2.1 A Universe only with electric charges

If there were only electric charges in the Universe, the total inertial mass m_{in} would be equal to the electric inertial mass m_{in-e} . Then from equation (5.19) we get:

$$\frac{m_{in}}{q} = -\frac{1}{c^2} \varphi$$

Thus, in a Universe with only electric charges all charges in an electric field would have the same acceleration. A charged object S which is accelerated in relation to the rest of the Universe receives an inertial force that is nothing else than an electric force. So in a Universe with only electric charges apply a principle of equivalence:

- A reference system consisting only of electric charges which makes an accelerated motion in a space where there is no electric field is equivalent to an inertial reference system at rest in an electric field.
- The rest system of an object with only electric charges which makes a free fall in an electric field is equivalent to an inertial reference system.

5.3 Study of the motion of an object with both gravitational mass and electric charge

In order to estimate the total inertia force exerted on the object S, we add the equations (5.8) and (5.21):

$$\vec{F}_{inertia} = -\frac{\partial \vec{p}_g}{\partial t} - \frac{\partial \vec{p}_e}{\partial t} \Rightarrow \vec{F}_{inertia} = -\frac{\partial \vec{p}}{\partial t} \quad (5.26)$$

$$\vec{F}_{inertia} = -\frac{\partial m_{in}}{\partial t} \vec{v} - m_{in} \frac{\partial \vec{v}}{\partial t} \quad (5.26a)$$

The total momentum \vec{p} is the sum of gravitational momentum \vec{p}_g and electric momentum \vec{p}_e :

$$\vec{p} = \vec{p}_g + \vec{p}_e \Rightarrow \vec{p} = (m_{in-g} + m_{in-e})\vec{v} \quad (5.27)$$

The total inertial mass m_{in} is the sum of gravitational mass and electrical inertial mass:

$$m_{in} = m_{in-g} + m_{in-e} \Rightarrow m_{in} = -\frac{1}{c^2}(m\phi_g + q\phi) \quad (5.28)$$

The transformation of total inertial mass when the object S moves with velocity \vec{v} in relation to the other objects is:

$$m_{in} = \frac{m_{in-g0} + m_{in-e0}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m_{in} = \frac{m_{in-0}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (5.29)$$

where m_{in-0} is the total rest mass. It is the inertial mass in the rest frame of object S when all other objects are at rest in respect to S.

In case we have N objects in the Universe with gravitational mass and M objects with electric charge, the total rest mass m_{in-0} is:

$$m_{in-0} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=1}^N \frac{mm_i}{r_i} - \frac{1}{4\pi \epsilon_0} \sum_{i=1}^M \frac{qq_i}{r_i} \right) \quad (5.30)$$

The momentum \vec{p} of an object S and the advanced gravitational and electric vector potential from all the other objects at the rest frame of S, are connected with the equation:

$$\vec{p} = m\vec{A}_g + q\vec{A} \quad (5.31)$$

In case we consider the motion of an electron in a semiconductor crystal, we have to take into account the change in the inertial mass of the electron because of the existence of other electrical charges in the space that it moves.

5.4. Zero inertial mass

Suppose in our Universe, dominated by the gravitational masses, we have two particles 1 and 2 with gravitational masses m_1 , m_2 and electric charges q_1 , q_2 at a distance r_2 . All other charges are far away. Equation (5.30) where we have two electric charges is applied.

$$\text{For particle 1} \quad m_{m-01} = \frac{1}{c^2} \left(\frac{1}{4\pi g_0} \sum_{i=2}^N \frac{m_1 m_i}{r_i} - \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_2} \right) \quad (5.32)$$

Suppose the two particles have like charges. In this case, the electric inertial masses of the particles are negative. We see from equation (5.32) that when they approach each other, the absolute value of the electric inertial mass increases. When the two particles approach each other the total inertial mass of particles decreases. When they approach each other at a certain distance, the absolute value of the electric inertial mass is equal to the gravitational inertial mass and so the total inertial mass is zero. At a shorter distance the inertial mass of particles becomes negative.

Let us now calculate this distance. Particle 1 with gravitational mass m_1 has gravitational rest mass:

$$m_{in-g01} = \frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=2}^N \frac{m_1 m_i}{r_i} \quad (5.33)$$

The equation (5.33) gives us the particle's rest mass of the contribution of gravitational forces. When the charged particle 1 approaching at a distance r_2 from particle 2, the rest mass decreases and becomes zero when:

$$\frac{1}{c^2} \frac{1}{4\pi g_0} \sum_{i=2}^N \frac{m_1 m_i}{r_i} = \frac{1}{c^2} \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_2} \quad (5.34)$$

If we combine (5.33) with (5.34) we get:

$$m_{in-g01} = \frac{1}{c^2} \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_2} \quad (5.35)$$

We consider that the gravitational rest mass remains constant as the two particles approach. From (5.35) we can calculate the distance where the rest mass becomes zero:

$$r_2 = \frac{1}{c^2} \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{m_{in-g01}} \quad (5.36)$$

In a weak electric field the rest mass is approximately equal to the gravitational rest mass:

$$m_{in-g01} = m_{in-01}$$

When measuring the rest mass of a particle in a weak electric field, gravitational rest mass is basically measured. So equation (5.36) becomes:

$$r_2 = \frac{1}{c^2} \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{m_{in-01}} \quad (5.37)$$

Equation (5.37) for two protons gives us: $r_2 = 1,53 \times 10^{-18} m$

Equation (5.37) for two electrons gives us: $r_2 = 2,81 \times 10^{-15} m$

With values:

$$q_p = +1,602 \times 10^{-19} C, \quad m_p = 1,672 \times 10^{-27} Kg, \quad q_e = -1,602 \times 10^{-19} C, \quad m_e = 9,109 \times 10^{-31} Kg$$

When the particles arrive at a distance less than the given value, the inertial masses of particles are negative. What negative inertial mass means is an open question.

6. THE SPACE TIME METRIC PERCEIVED BY AN OBJECT

We want to examine if spacetime metric perceived by an object with gravitational mass and electric charge depends on its position in a gravitational and electrical field. Here we will show that gravitational and electric forces change spacetime metric perceived by an object moving in it and not the metric of spacetime itself. The metric of spacetime itself is flat. When two different objects with different gravitational mass and electric charge are at the same point of a gravitational and electric field, each one will perceive a spacetime with different metric.

An object moves freely when no force is exerted on it or the resultant force exerted on it is zero. We restate Newton's First Law of Motion for curved spacetime as follows:

Principle 5

Every object that moves freely is an inertial observer and moves in a straight line in the spacetime that it perceives.

In an inertial observer time passes at a constant rate and the length of a rod is constant.

Three crucial classic experiments verify the General Theory of Relativity [21]: the red shift of spectral lines emitted by atoms in a region of strong gravitational potential, the deflection of light rays that pass close to the Sun and the precession of the perihelion of the planet Mercury. All the three above derive from Schwarzschild metric. Many efforts have been made in the past to produce Schwarzschild metric by using only the principle of equivalence, Newton's law of gravitation and the Special Theory of Relativity [21][22][23][24][25][26][27]. These efforts divided the scientific community in the past.

Today it is accepted that there is one way to produce the equations that determine the change of the rate of time and the change of length in a gravitational field using only Newton's law of gravitation, the Special Theory of Relativity and the principle of equivalence [28][29]. Since the principle of equivalence is a consequence of the new theory and Newton's law of gravitation is valid in this theory, this particular way will be followed here. Having a new theory of gravity different from the General Theory of Relativity we should get the same results in the three crucial experiments that we mentioned. For this reason we will move on to derive the spacetime metric that perceives an object with gravitational mass and electric charge. This spacetime metric will include Schwarzschild metric

6.1. Free fall in a gravitostatic field

We have an object S of gravitational mass M . The S is static and has spherically symmetric mass distribution. We use as coordinate system a spherical coordinate system with zero position at the center of the spherical object S .

A. Free fall of a clock

We have three identical clocks: A , B and C . Each clock has gravitational mass m . We let clock A at rest at a distance r from the center of S in position A . We let clock B at rest at infinity in position B and in the same radial direction as clock A . We let clock C make a free fall in the gravitational field of object S with zero initial speed starting from the position B . Clock C will move in a straight line that connects A and B (figure 1). When we say infinite, we mean far enough to assume that the gravitational field is zero. We want to compare the rate of clock A with the rate of clock B at infinity.

Because clock C makes a free fall it is an inertial observer, so time at clock C passes at a constant rate throughout the whole free fall. So, if let clock C make a free fall starting from position B with zero velocity, the clock C will work at the same rate with clock B that is at rest at infinity.

When clock C is at position B , if dt is the time interval that clock C measures between two events occurring at clock B and dt_B is the proper time interval that clock B measures between the same two events, then:

$$dt = dt_B$$

Therefore when clock C passes from point A , with velocity \vec{v}_A and compares its own rate with the rate of clock A , it is like comparing the proper rate of clock A with the proper rate of clock B .

When clock C passes from point A , clock C and clock A receive the same force of the gravitational field because they have the same gravitational mass. Thus we do not need the effect of gravity to compare the rates of clocks A and C . To compare the rate of clock C with the rate of clock A we only need the Lorentz transformations because of the relative speed they have.

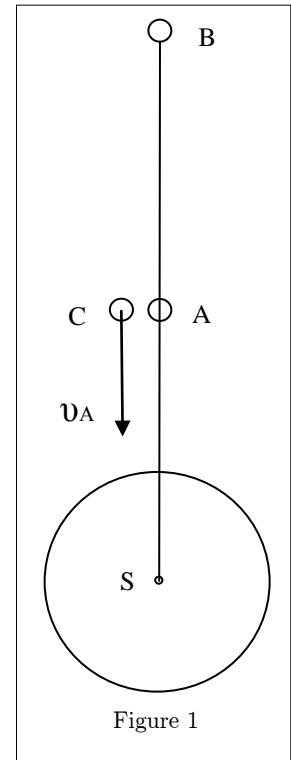
- When clock C passes from point A , it measures the time interval dt between two events occurring at clock A . Clock A measures the time interval dt_A for the same two events that is the proper time.

According to Lorentz transformations:

$$dt = \gamma dt_A \quad (6.1) \quad \text{Where: } \gamma = \frac{1}{\sqrt{1 - \frac{v_A^2}{c^2}}} \quad (6.2)$$

According to what we said before dt is also the proper time for the clock B , so equation (6.1) relates the proper time of clock B at infinity with the proper time of clock A at a distance r . So dt is the coordinate time and dt_A is the local time.

In order for clock C to measure an infinitesimal time interval on clock A , clock C needs two time instants, so we accept that the speed of clock C is constant at infinitesimal distances from position A .



B. Free fall of a rod

We have three identical infinitesimal rods for measuring length: A, B and C. Each rod has a gravitational mass m . We let rod A at rest at a distance r from the center of the object S in position A. We let rod B at rest to infinity in position B and in the same radial direction as rod A. We let rod C make a free fall in the gravitational field of object S with zero initial speed starting from the position B. All the rods are parallel to the straight line that connects A with B (figure 2). We want to compare the length of rod A with the length of rod B.

Because rod C makes a free fall it is an inertial observer, so the length of rod C is constant throughout the whole free fall. So if let rod C makes a free fall starting from infinity with zero velocity, the rod C will have the same length with rod B that is at rest at infinity.

When rod C is at position B, if dr is the length of rod B as measured by rod C and dr_B is the proper length of rod B, then:

$$dr = dr_B$$

Therefore when rod C passes from point A, with velocity \vec{v}_A and it compares its own length with the length of rod A, it is like comparing the proper length of rod A with the proper length of rod B.

When rod C passes from point A, rod C and rod A receive the same force of the gravitational field because they have the same gravitational mass. Thus we do not need the effect of gravity to compare the lengths of rods A and C. To compare the length of rod C with the length of rod A we only need the Lorentz transformations because of the relative speed they have.

- When rod C passes from point A, it measures that the length of rod A is dr . Rod A measures its own length, which is dr_A , that is the proper radial distance. According to Lorentz transformations:

$$dr = \frac{dr_A}{\gamma} \quad (6.3)$$

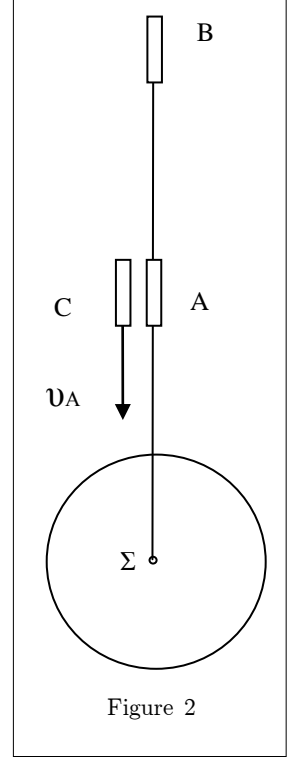
According to what we said before dr is also the proper radial distance for the rod B, so equation (6.3) relates the proper radial distance of rod B at infinity with the proper radial distance of rod A at a distance r . So dr is the radial coordinate distance and dr_A is the local proper radial distance.

Since length measurement needs two points at a certain distance and between the two points space time metric change for rod A, we can approximately accept that in infinitesimal distances from point A spacetime is flat.

We can calculate the magnitude of velocity \vec{v}_A by applying the conservation of energy when observer C makes a free fall for non-relativistic velocities. At the beginning observer C is at infinity, where the potential energy is zero. Thus the energy of the system is initially zero and the equation for conservation of energy is:

$$\frac{1}{2}m_{in-g}v_A^2 - G\frac{Mm}{r} = 0 \Rightarrow v_A^2 = 2G\frac{Mm}{rm_{in-g}} \quad (6.4)$$

Therefore:
$$\sqrt{1 - \frac{v_A^2}{c^2}} = \sqrt{1 - \frac{2GMm}{c^2 rm_{in-g}}} \quad (6.5)$$



By replacing (6.5) at (6.1) we have:

$$dt_A = \sqrt{1 - \frac{2GMm}{c^2 r m_{in-g}}} dt \quad (6.6)$$

By replacing (6.5) at (6.3) we have:

$$dr_A = \frac{dr}{\sqrt{1 - \frac{2GMm}{c^2 r m_{in-g}}}} \quad (6.7)$$

From the equations (6.6) and (6.7) we draw conclusions that apply to the gravitational field. These are:

1. The clock A at position A goes slower than clock B at infinity. Therefore, we conclude that when the clocks are in a gravitational field, they are slowed.
2. The rod A in position A is longer than the parallel rod B at infinity. Therefore, we conclude that a rod located in the gravitational field outside a static and spherically symmetric distribution of mass expands in the radial direction.

It is known and well documented that outside a static and spherically symmetric distribution of mass, the spacetime metric for a point that is at a distance r from the center of the object can be written in spherical coordinates [28]:

$$ds^2 = g_{00}(r)c^2 dt^2 + g_{rr}(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.8)$$

This metric should give us to infinity the Minkowski metric in spherical coordinates:

$$ds^2 = c^2 dt^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.9)$$

So we must have the boundary conditions:

$$\lim_{r \rightarrow \infty} g_{00}(r.t) \rightarrow 1 \quad \text{καί} \quad \lim_{r \rightarrow \infty} g_{rr}(r.t) \rightarrow -1 \quad (6.10)$$

- The spacetime distance between two events happening at the same point in space, $dr = d\theta = d\varphi = 0$, is:

$$ds^2 = g_{00}(r)c^2 dt^2 \quad (6.11)$$

But $\frac{ds^2}{c^2} = dt_A$ is the proper time at position r .

So the proper time at position r is:

$$dt_A = \sqrt{g_{00}} dt \quad (6.12)$$

From the equations (6.12) and (6.6) and from the boundary conditions (6.10) we have:

$$g_{00}(r) = \left(1 - \frac{2GMm}{rc^2 m_{in-g}} \right) \quad (6.13)$$

- The spacetime distance ds^2 between two events happening at the same time at different points in infinitesimal distance from the position A but on the same radial direction, $dt = d\theta = d\varphi = 0$ is the proper radial distance dr_A .

From the equation (6.8) when $dt = d\theta = d\varphi = 0$ we get:

$$ds^2 = g_{rr} dr^2 \quad (6.14)$$

But the proper radial distance at position r is:

$$dr_A = \sqrt{g_{rr}} dr \quad (6.15)$$

From the equations (6.15) and (6.7) and from the boundary conditions (6.10) we have:

$$g_{rr} = - \frac{1}{\left(1 - \frac{2GMm}{c^2 r m_{in-g}} \right)} \quad (6.16)$$

If we replace the equations (6.13) and (6.16) in the (6.8) we get:

$$ds^2 = \left(1 - \frac{2GMm}{c^2 r m_{in-g}} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GMm}{c^2 r m_{in-g}}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (6.17)$$

Equation (6.17) describes the spacetime metric perceived by an object with gravitational mass m and inertial mass m_{in-g} , who is at a distance r outside a static object S with gravitational mass M and spherically symmetric distribution mass.

We refer to spacetime metric perceived by an object and not to the spacetime metric that exists independent of the object. Certainly, as we have said, the ratio of gravitational mass to inertial mass at the same point is the same for every object so the spacetime metric perceived by a gravitational mass is the same for every gravitational mass. So it could be considered that this metric is the metric of spacetime itself, but this is wrong, and this will be apparent when we have both electric charges and gravitational masses.

The ratio of gravitational mass to inertial mass is the same for every object at the same point. In our position in the galaxy we have accepted that this ratio is equal to one.

$$\frac{m}{m_{in-g}} = 1 \quad (6.18)$$

Thus, when the ratio is equal to one, equation (6.17) gives us the metric:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (6.19)$$

Equation (6.19) is the well known Schwarzschild metric. Thus, all the phenomena that emerge from Schwarzschild metric also emerge from equation (6.17). These are:

- The red shift of spectral lines emitted by atoms in a region of strong gravitational potential.
- The deflection of light rays that pass close to the Sun if we accept that light moves in a straight line in spacetime perceived by the object that receives the light.
- The precession of the perihelion of the orbit of the planet Mercury.

At another point in our Galaxy, the ratio of gravitational to inertial mass is not equal to one. The phenomena are the same qualitatively but not quantitatively.

6.2 Dark Energy

From equation (6.17) arises that the equation relating the proper time at a distance r , from the center of S in position A (figure 1), with proper time at infinity is:

$$dt = \frac{dt_A}{\sqrt{1 - \frac{2GMm}{c^2 r m_{in-g}}}} \quad (6.20)$$

The equation (6.20) describes the red shift of spectral lines which is emitted by an atom in a gravitational field and is received by an object which is out of the gravitational field.

From the Schwarzschild metric (6.19) arises that the equation relating the proper time at a distance r , from the center of S in position A (figure 1), with proper time at infinity is:

$$dt = \frac{dt_A}{\sqrt{1 - \frac{2GM}{c^2 r}}} \quad (6.21)$$

- As it emerges from equation (6.21), red shift of spectral lines emitted by an atom in a star is the same for every star with the same gravitational mass.
- However as it emerges from equation (6.20), the red shift of spectral lines emitted by an atom in a star depends on the ratio of the gravitational mass to the inertial mass of the atoms emitting the light.

As the Universe expands, inertial mass of all objects decreases as time goes by as it emerges from equations (5.5) and (5.14). In this way, the inertial mass of an atom that emits light decreases as time goes by. As it emerges from equation (6.20) the light emitted by two identical supernovas Ia at different moments in the history of Universe, they will have different red shifts. When we see a supernova Ia at a distance l_1 it appears as it was at t_1 . When we see a supernova Ia at a distance l_2 it appears as it was in t_2 . When $l_1 < l_2$ then $t_1 > t_2$.

Thus the inertial mass of the atoms of a closer supernova Ia is smaller than the inertial mass of the atoms of a further identical supernova Ia. So, as it emerges from equation (6.20) closer supernovas have greater red shift. This phenomenon has been observed but the inability to explain why the red shift of spectral lines is greater has led to the theory that the Universe expands in an accelerating way because of dark energy. Certainly we need to do more precise calculations but the changes of gravitational inertial mass due to the expansion of the Universe are very useful for the explanation of the dark energy theory.

6.3 Free fall in a Universe that contains only electric charges

As we said in section 5.2.1, in a Universe with only electric charges all charges in an electric field would have the same acceleration. So the appropriate principle of equivalence is valid here.

We repeat exactly the same thought experiment we made in section 6.1 with the only difference that the clocks and the infinitesimal rods have electric charge q and the object S that is static and has spherically symmetric distribution of electric charge, has electric charge Q . For this reason it is not necessary to repeat all the procedure but to refer only to the points in which they are different.

We want to compare the rate of the clock and the length of the infinitesimal rod located in position A with a clock and a parallel infinitesimal rod at infinity in position B where the electric field is zero (figure 1, 2).

We choose the charges q and Q to be unlike in order to be attracted. When clock-rod C that make free fall, with zero initial speed starting from the position B, passes from point A, clock-rod C and clock-rod A receive the same force of the electric field because they have the same electric charge. Thus we do not need the effect of electric force to compare the rates of clocks A and C and the length of rods A and C. To compare the physical quantities we only need the Lorentz transformations because of the relative speed they have.

From the conservation of energy for non-relativistic velocities we have:

$$\frac{1}{2} m_{in-e} v_A^2 + K \frac{Qq}{r} = 0 \Rightarrow v_A^2 = -\frac{2KQq}{r m_{in-e}} \quad (6.22)$$

Thus:

$$\sqrt{1 - \frac{v_A^2}{c^2}} = \sqrt{1 + \frac{2KQq}{r_A m_{in-e}}} \quad (6.23)$$

And we end up to the equations:

$$dt_A = dt \sqrt{1 + \frac{2KQq}{r m_{in-e}}} \quad (6.24) \quad \text{and} \quad dr_A = \frac{dr}{\sqrt{1 + \frac{2KQq}{r m_{in-e}}}} \quad (6.25)$$

Following the same procedure as the one we followed in section 6.1 we come to the equation for the spacetime metric perceived by an object with electric charge q and inertial mass m_{in-e} , who is at a distance r from a spherically symmetric and stationary object with electric charge Q .

$$ds^2 = \left(1 + \frac{2KQq}{c^2 r m_{in-e}}\right) c^2 dt^2 - \frac{dr^2}{1 + \frac{2KQq}{c^2 r m_{in-e}}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (6.26)$$

In section 6.1 and section 6.3 we found the following phenomena:

- When we have clocks and rods which consist only of gravitational mass, the rate of the clocks and the length of the rods in the direction of the field, change when they are in a gravitational field
- When we have clocks and rods which consist only of electric charge, the rate of the clocks and the length of the rods in the direction of the field, change when they are in an electric field.

If we have an object C with both a gravitational mass m and an electric charge q in the field of an object S, with gravitational mass M and electric charge Q , both phenomena happen at the same time.

6.4 Free fall in a gravitostatic and an electrostatic field

We repeat exactly the same thought experiment we made in section 6.1 with the only difference:

- the clocks and the infinitesimal rods have gravitational mass m and electric charge q
- the object S has gravitational mass M electric charge Q and is static and has spherically symmetric distribution of gravitational mass and electric charge.

Because we repeat exactly the same thought experiment, it is not necessary to repeat all the procedure but to refer only to the points in which they are different. We choose the charges q and Q to be unlike in order to be attracted.

We want to compare the rate of the clock and the length of the infinitesimal rod located in position A with a clock and a parallel infinitesimal rod at infinity in position B where the gravitational and electric fields are zero (figure 1, 2).

According to principle 5, an infinitesimal object C (clock or rod) with gravitational mass and electric charge, which moves freely in the gravitational and electric field of the object S, is an inertial observer. When object C passes from point A, object C and object A receive the same force of the gravitational and electric field because they have the same gravitational mass and electric charge. Thus we do not need the effect of gravitational and electric force to compare the rates of clocks A and C and the length of rods A and C. To compare the physical quantities we only need the Lorentz transformations because of the relative speed they have.

We can estimate the magnitude of velocity by applying the conservation of energy when object C makes a free fall from infinity for non-relativistic velocities. At the beginning object C is at rest at infinity, where the potential energy is zero. Thus the energy of the system is initially zero and the equation for conservation of energy is:

$$\frac{1}{2} m_{in} v_A^2 - G \frac{Mm}{r} + K \frac{Qq}{r} = 0 \Rightarrow v_A^2 = \frac{2GMm}{r m_{in}} - \frac{2KQq}{r m_{in}} \quad (6.27)$$

Thus:
$$\sqrt{1 - \frac{v_A^2}{c^2}} = \sqrt{1 - \frac{2GMm}{r m_{in}} + \frac{2KQq}{r_A m_{in}}} \quad (6.28)$$

And we end up to the equations:

$$dt_A = dt \sqrt{1 - \frac{2GMm}{rm_{in}} + \frac{2KQq}{rm_{in}}} \quad (6.29) \quad \text{and} \quad dr_A = \frac{dr}{\sqrt{1 - \frac{2GMm}{rm_{in}} + \frac{2KQq}{rm_{in}}}} \quad (6.30)$$

Following the same process we did in 6.1 we arrive at the equation that describes the spacetime metric perceived by an object with gravitational mass m , electric charge q and inertial mass m_{in} , which is at rest at a distance r from an object S which is static and has spherically symmetric distribution of gravitational mass and electric charge with gravitational mass M and electric charge Q :

$$ds^2 = \left(1 - \frac{2GMm}{c^2 rm_{in}} + \frac{2KQq}{c^2 rm_{in}}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GMm}{c^2 rm_{in}} + \frac{2KQq}{c^2 rm_{in}}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (6.31)$$

We draw the following conclusions from equation (6.31) if the gravitational field is too weak:

1. Time goes more slowly for a negative electric charge which is located in the electric field of a positive charge while time goes faster for a positive electric charge which is located at the same point in the same electric field.
2. The space outside of a static and spherically symmetric distribution of a positive charge is expanded in its radial direction for a negative electric charge while the space is contracted in its radial direction for a positive electric charge which is located at the same point in the same electric field.

It is possible that we can measure:

- the red-blue shift of spectral lines emitted by atoms in a region of strong electric potential.
- the frequency shift of a laser beam emitted by a laser in a region of strong electric potential.

If two objects move at the same velocity at the same point and have the same gravitational mass but unlike charges, they will make completely different movements. So, we cannot support that spacetime is formed by the other objects and that a small testing object will simply move on it, independent of its properties. What we can say is:

- Spacetime itself is flat. What changes is the way each object perceives the spacetime.

CONCLUSIONS

In this paper we have proposed a new theory in order to explain the origin of inertia. Following a different way of thinking which is based on new postulates, we showed that the gravitational and electrical forces are described by the same equations and thus create similar effects. The inertial forces perceived by an accelerating object are gravitational and electrical forces created by induction because the acceleration is relative and thus at the rest frame of an accelerating object, it is all the other objects in the Universe which are accelerating. This gave us the ability to reveal the nature of the inertial mass and the way it changes. So now we have the answer to the question what is the relation between gravitational and inertial mass. The conclusions from the new theory are in agreement with all the previous experimental results. But the nature of the inertial mass seems very useful for the explanation of dark matter, dark energy and will certainly have many other applications. The properties of gravitational forces and the properties of the inertial mass open new horizons in physics.

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