Sensor Management using Expected Risk Reduction approach

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***Abstract*—This paper introduces an Expected Risk Reduction approach to Sensor Management and Multi-Target Tracking in a surveillance context that implements an IR-Radar sensor suite. Due to operational restrictions (for instance, electromagnetic emission constraints), it is assumed that there are more targets than given sensors are capable of tracking simultaneously when a radar emission control is applied. It is also presumed that an incorrect target classification entails a cost that is different for each target class. The Expected Risk Reduction is then applied to a simulated IR-Radar sensor management to preserve an acceptable level of kinematic accuracy on targets of high cost. Finally, empirical statistical tests show that a track on high priority targets is maintained better when the aforementioned approach is introduced than in the case of other conventional methods, such as the information gain approach or the round robin assignment.**

I. Introduction

The key to successful target tracking lies in the optimal extraction of useful information about the target's state from the observations in the presence of sensor's imperfections. This task is usually realized by maintaining an estimate of the target's state over time using algorithms such as a Kalman filter [1]. Nonetheless, in most cases, due to operational restrictions, sensors are not able to maintain a satisfactory estimate of all target states in a given area over a period of time. Consequently, the targets with poor estimates will be lost.

In many target tracking scenarios a sensor can be controlled by changing the position, orientation or motion of the sensor platform, which may have a significant impact on the quality of the estimation performance of the tracking system. At times, the control decisions are driven by a manual intervention, or by some deterministic control policy, which does not guarantee the optimality.

The field of view (FOV) restrictions in a pan tilt zoom (PTZ) camera that attempts to autonomously locate a target of interest provides a good example of operational limitations. The PTZ camera needs to zoom in on potential targets to classify them, at the sacrifice of collecting state measurements on other targets. Analogous sensor management problems exist for IR-Radar sensors. Common radar systems use a "pencil beam" mode and steering capabilities (Electronic Scanning Radar) with the adequate resolution to both classify and track targets of interest.

In scenarios embracing a large number of targets, an autonomous sensor management is typically employed to track as many targets as possible. This usually involves scheduling the sensor to measure the target track estimate, characterized by the highest level of uncertainty of the true target state. The uncertainty reduction is commonly obtained using the Kullback Leibler divergence, the Fisher information gain, or the Renyi divergence [2]–[4].

When only a subset of the total targets can be successfully tracked, the prioritization of target tracks is crucial and cannot be realized by means of information gain-based metrics. To overcome the limitations on existing metrics, a statistical risk model used to calculate an expected cost as a metric, has gained interest in recent research [4], [5].

This paper takes into consideration a surveillance context with electromagnetic emission constraints. In that case, there are too many maneuverable targets to be tracked by the IR-Radar multisensory system. A significant amount of research in this field has been conducted [6]–[10]. It is assumed in this paper that only a subset of all total targets needs to be tracked, and that initially all the target kinematic states are known (targets tracks may have been provided by an upper level tactical module). Besides, their classification states may be unknown. Finally, it is also supposed that there is a cost resulting from an incorrect decision on a target's true classification.

In this paper, the task of a sensor manager is to decide which targets the sensor should focus on in order to reduce the expected cost of an incorrect classification decision. Thus, the cost value and the event of losing a target track are strictly correlated. Therefore, the expected cost incorporates both the track kinematic and the classification estimate.

Section II presents the models used for maneuvering target tracking and the classification state. Section III highlights the calculation of the expected cost and the amount of its reduction according to new measurements. The results of experiments applying this specific metric are shown in details in Section IV.

II. Kinematic and Classification State Estimation

For each target, there is information about kinematic true state and its classification, and both are represented by X as follows:

$$X = [X_{kinematic} \quad X_{classification}]$$

The specific values of X are discussed in the following subsections.

A. Kinematic State Estimation

A Kalman filter is often employed in kinematic measurements to estimate the position, velocity, and acceleration of a target [11], [12]. The kinematic and observation models in target tracking can be formulated as follows:

$$\begin{aligned} x_{k+1} &= f_k(x_k) + v_k \\ z_k &= h_k(x_k) + w_k \end{aligned}$$

where x_k is the target state at the discrete time k, z_k is the observation, v_k and w_k are process and measurement noise respectively, while f_k and h_k are some time-varying functions. Usually, estimates of these true target states are updated by a standard extended Kalman filter (EKF) [12]. Given the latest estimate $x_{k|k}$ and latest state error covariance $P_{k|k}$, the prediction is such as:

$$x_{k+1|k} = f_k(x_{k|k}) \tag{1}$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + G_k Q_k G_k^T \tag{2}$$

The update to these estimates given the measurement z_{k+1} at time $k+1$ is:

$$K_{k+1} = P_{k+1|k} H_{k+1}^T (H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1})^{-1} \tag{3}$$

$$x_{k+1|k+1} = x_{k+1|k} + K_{k+1}(z_{k+1} - h_k(x_{k+1|k})) \tag{4}$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} \tag{5}$$

where F_k and H_k terms are the local linearization of the functions f_k and h_{k+1} as follows:

$$\begin{aligned} F_k &= \left.\frac{\partial f_k(x)}{\partial x}\right|_{x=x_{k|k}} \\ H_{k+1} &= \left.\frac{\partial h_k(x_{k+1})}{\partial x}\right|_{x=x_{k+1|k}} \end{aligned}$$

In this paper, both targets and sensors are defined in a three dimensional space. Consequently, the true kinematic state consists of a three dimensional position and velocity:

$$X_{kinematic} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

IR measurements basically provide angular information, that is to say an azimuth angle θ and an elevation angle ϕ, whereas radar measurements consist of an azimuth angle θ, an elevation angle ϕ, a range r and range rate $\dot{r}$ as shown hereafter.

$$\begin{aligned} Z_{IR} &= \begin{bmatrix} \theta \\ \phi \end{bmatrix} \\ Z_{Radar} &= \begin{bmatrix} \theta \\ \phi \\ r \\ \dot{r} \end{bmatrix} \end{aligned}$$

where:

$$\begin{aligned} \theta &= \arctan\left(\frac{y}{x}\right) \\ \phi &= \arctan\left(\frac{z}{\sqrt{x^2+y^2}}\right) \\ r &= \sqrt{x^2+y^2+z^2} \\ \dot{r} &= \frac{(x\dot{x} + y\dot{y} + z\dot{z})}{r} \end{aligned}$$

To avoid singularities in the linearization process, measurements are converted to Cartesian coordinates. The linearization on the noise is achieved as shown by Bar-Shalom and Li [13]. In this way, measurements are represented as:

$$\begin{aligned} Z_{IR}^{Cartesian} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\ Z_{Radar}^{Cartesian} &= \begin{bmatrix} x \\ y \\ z \\ \dot{r} \end{bmatrix} \end{aligned}$$

Where:

$$\begin{aligned} x &= r\cos(\phi)\cos(\theta) \\ y &= r\cos(\phi)\sin(\theta) \\ z &= r\sin(\phi) \end{aligned}$$

It is assumed that the initial position of targets is known. The matrix H is given as shown below. Note that it corresponds here to a linear filter.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The measurement error covariance matrix R is defined as follows:

$$R = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} & \Sigma_{xz} \\ \Sigma_{yx} & \Sigma_{yy} & \Sigma_{yz} \\ \Sigma_{zx} & \Sigma_{zy} & \Sigma_{zz} \end{bmatrix}$$

The subsequent paragraph demonstrates how to calculate the matrix R using linearization. Measures θ_m, ϕ_m and r_m are defined with respect to true θ, ϕ and r data as hereafter:

$$\begin{aligned} \theta_m &= \theta + \tilde{\theta} \\ \phi_m &= \phi + \tilde{\phi} \\ r_m &= r + \tilde{r} \end{aligned}$$

where errors $\tilde{\theta}, \tilde{\phi}$ and $\tilde{r}$ are assumed to be independent with zero mean and standard deviations $\sigma_\theta, \sigma_\phi, \sigma_r$ respectively.

Denoting (x, y, z) the true Cartesian position and using first order terms of the Taylor series expansion of the Cartesian measurements at (θ_m, ϕ_m, r_m), i.e., applying linearization, yields the Cartesian coordinate errors as hereafter:

$$\begin{aligned} x_m - x &\approx \tilde{r}\cos(\phi)\cos(\theta) - \tilde{\phi} r_m \sin(\theta)\cos(\theta) \\ &\quad - \tilde{\theta} r_m \cos(\phi)\sin(\theta) \\ y_m - y &\approx \tilde{r}\cos(\phi)\sin(\theta) - \tilde{\phi} r_m \sin(\phi)\sin(\theta) \\ &\quad + \tilde{\theta} r_m \cos(\phi)\cos(\theta) \\ z_m - z &\approx \tilde{r}\sin(\phi) + \tilde{\phi} r_m \cos(\phi) \end{aligned}$$

The mean of the errors, as given by the above equations is zero. Consequently, the elements of the corresponding covariance matrix R are:

$$\begin{aligned} \Sigma_{xx} &= \sigma_r^2 \cos^2(\phi)\cos^2(\theta) + \sigma_\phi^2 r^2 \sin(\phi)\cos(\theta) \\ &\quad + \sigma_\theta^2 r^2 \cos^2(\phi)\sin^2(\theta) \\ \Sigma_{yy} &= \sigma_r^2 \cos^2(\phi)\sin^2(\theta) + \sigma_\phi^2 r^2 \sin^2(\phi)\sin^2(\theta) \\ &\quad + \sigma_\theta^2 r^2 \cos^2(\phi)\cos^2(\theta) \\ \Sigma_{zz} &= \sigma_r^2 \sin^2(\phi) + \sigma_\phi^2 r^2 \cos^2(\phi) \\ \Sigma_{xy} &= \sigma_r^2 \cos^2(\phi)\cos(\theta)\sin(\theta) + \sigma_\phi^2 r^2 \sin^2(\phi)\cos(\theta)\sin(\theta) \\ &\quad - \sigma_\theta^2 r^2 \cos^2(\phi)\cos(\theta)\sin(\theta) \\ \Sigma_{xz} &= \sigma_r^2 \cos(\phi)\sin(\phi)\cos(\theta) - \sigma_\phi^2 r^2 \cos(\phi)\sin(\phi)\cos(\theta) \\ \Sigma_{yz} &= \sigma_r^2 \cos(\phi)\sin(\phi)\sin(\theta) - \sigma_\phi^2 r^2 \cos(\phi)\sin(\phi)\sin(\theta) \end{aligned}$$

In this paper, we set $\sigma_r^2 = 1$ m^2 and $\sigma_\theta^2 = \sigma_\phi^2 = 3.0462 \times 10^{-4}$ rad^2 (i.e., the standard deviation of 1 degree). The process noise covariance matrix Q is given by:

$$Q = \Phi_s \begin{bmatrix} T^4/4 & T^3/2 & 0 & 0 & 0 & 0 \\ T^3/2 & T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & T^4/4 & T^3/2 & 0 & 0 \\ 0 & 0 & T^3/2 & T^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & T^4/4 & T^3/2 \\ 0 & 0 & 0 & 0 & T^3/2 & T^2 \end{bmatrix}$$

In this paper, the scan rate of IR sensor is $T_{IR} = 1$ second, while for Radar sensor $T_{Radar} = 2$ seconds, and $\Phi_s = 5$ m/s^2. Finally the state transition model F is a basic constant velocity model:

$$F = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

B. Classification State Estimation

The classification state estimate is formulated hereafter. Assuming there are n possible classification states for each target, J is a random variable that stands for the true classification with support $\{j \,|\, j \in [1, n]\}$.

$$X_{classification} = \begin{bmatrix} P(J = 1) \\ \vdots \\ P(J = n) \end{bmatrix}$$

The classification probability is updated by applying the Bayes' theorem as shown below. The classification measurement is represented by a discrete random variable M with support $\{m \,|\, m \in [1, n]\}$.

$$\begin{aligned} P'(J = i) &\triangleq P(J = i \,|\, M = m) \\ &= \frac{P(M = m \,|\, J = i)P(J = i)}{P(M = m)} \\ &= \frac{P(M = m \,|\, J = i)P(J = i)}{\sum_{r=1}^{n} P(M = m \,|\, J = r)P(J = r)} \end{aligned} \tag{6}$$

where P' indicates the posterior probability. To simplify classification notations in this paper, measurement likelihoods $P(M = m \,|\, J = i)$ are represented by a normalized confusion matrix CC.

$$CC = \begin{array}{c} \\ 1 \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{ccc} 1 & \dots & n \end{array} \\ \begin{pmatrix} P(M = 1 \,|\, J = 1) & \dots & P(M = 1 \,|\, J = n) \\ \vdots & \ddots & \vdots \\ P(M = n \,|\, J = 1) & \dots & P(M = n \,|\, J = n) \end{pmatrix} \end{array}$$

III. Statistical Risk and KL Divergence

A. Type 1 Error Cost

There is a decision to be made on target classification. An incorrect decision results in a cost that can be interpreted as a cost of a lost target of interest, or a loss of sensor resources. In this paper, an incorrect decision is tagged as being a type 1 error during statistical hypothesis testing. A type 1 error corresponds to an incorrect rejection of a true null hypothesis, and occurs when H_0 is true yet rejected.

The matrix CM_1, as defined below, contains the cost of committing a type 1 error. Each column represents the true classification, and each row stands for the decision on classification. The diagonal is zero, since there is no cost when the correct decision takes place.

$$CM_1 = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \dots & n \end{array} \\ \begin{pmatrix} 0 & c_{1_1} & \dots & c_{1_n} \\ c_{1_1} & 0 & \dots & c_{1_n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1_1} & c_{1_2} & \dots & 0 \end{pmatrix} \end{array}$$

B. The Expected Cost for Making a Type 1 Error

The expected cost of a type 1 error on target classification is influenced by many factors, including the current classification, the probability of the actual target being lost (or not), and the decision about this classification. The above-mentioned factors are modeled by random variables. The expected cost is obtained by applying the law of total expectation, as detailed below.

Let C_1 be a discrete random variable representing the cost of the type 1 error. The cost matrix CM_1 contains entries $\{c_{1_{ij}}\}$ where each $c_{1_{ij}}$ entry occurs when a decision falsely rejects H_0, resulting in a type 1 error. J corresponds to a categorical random variable representing the current classification, $\{j|j \in [1, n]\}$. I is a categorical random variable denoting the decision on classification, $\{i|i \in [1, n]\}$. $\hat{I}$ is a discrete, uniformly distributed, random variable denoting the classification decision on a reacquired target after its lost, $\{i|i \in [1, n]\}$. L is a Bernoulli random variable representing whether or not the actual target is lost, where the event space is $\{0, 1\}$. Using the law of the iterated expectation for each random variable that determines the cost, we describe the expected cost by:

$$\begin{aligned} & E_{c_1}(C_1|I = i) \\ = \; & E_{c_1}(C_1|I = i,\, L = 1,\, J = i)P(L = 1)P(J = i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i)P(L = 1)P(J \neq i) \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J = i)P(L = 0)P(J = i) \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \\ = \; & E_{c_1}(C_1|I = i,\, L = 1,\, J = i)P(L = 1)P(J = i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i)P(L = 1)P(J \neq i) \\ & + 0 \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \end{aligned} \tag{7}$$

In the above summation, the third term becomes zero since the correct decision is made and the target is not lost. If, however, the situation was the opposite and the target was lost, at this point, it would be necessary to consider the case in which the target would be reacquired. Note that regardless of the classifier's accuracy, it is possible that the acquired target is not the original one. Thus (7) would be as follows:

$$\begin{aligned} & E_{c_1}(C_1|I = i) \\ = \; & E_{c_1}(C_1|I = i,\, L = 1,\, J = i,\, \hat{I} = i)P(L = 1)P(J = i)P(\hat{I} = i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J = i,\, \hat{I} \neq i)P(L = 1)P(J = i)P(\hat{I} \neq i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i,\, \hat{I} = i)P(L = 1)P(J \neq i)P(\hat{I} = i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i,\, \hat{I} \neq i)P(L = 1)P(J \neq i)P(\hat{I} \neq i) \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \\ = \; & 0 \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J = i,\, \hat{I} \neq i)P(L = 1)P(J = i)P(\hat{I} \neq i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i,\, \hat{I} = i)P(L = 1)P(J \neq i)P(\hat{I} = i) \\ & + 0 \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \end{aligned} \tag{8}$$

In the above summation, the first term becomes zero once the correct decision has been made, even though the track had been lost and later reacquired. Hence, there is no cost. In the second term, the cost is observed when the target is lost and reacquired and a wrong decision about its classification takes place. In the third term, the cost is present and the target classification decision is never correct, even after the target has been lost and reacquired. The fourth term of summation represents the case in which the classification decision is incorrect (i.e. $I = i$ and $J \neq i$), and the reacquired target is characterized by a different classification (i.e. $\hat{I} \neq i$ and $J \neq i$). Consequently, the cost is zero, because it does not exist with regard to the initial decision of $I = i$ before the track had been lost. The last term illustrates the cost stemming from a wrong target classification, since the target is never lost. These terms are related to specific rows and columns of the cost matrix CM_1 as shown below:

$$\begin{aligned} & E_{c_1}(C_1|I = i) \\ = \; & E_{c_1}(C_1|I = i,\, L = 1,\, J = i,\, \hat{I} \neq i)P(L = 1)P(J = i)P(\hat{I} \neq i) \\ & + E_{c_1}(C_1|I = i,\, L = 1,\, J \neq i,\, \hat{I} = i)P(L = 1)P(J \neq i)P(\hat{I} = i) \\ & + E_{c_1}(C_1|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \\ = \; & \sum_{c_{1_{ij}}} c_{1_{ij}} \Big(P(C_1 = c_{1_{ij}}|I = i,\, L = 1,\, J = i,\, \hat{I} \neq i) \\ & \qquad P(L = 1)P(J = i)P(\hat{I} \neq i) \Big) \\ & + \sum_{c_{1_{ij}}} c_{1_{ij}} \Big(P(C_1 = c_{1_{ij}}|I = i,\, L = 1,\, J \neq i,\, \hat{I} = i) \\ & \qquad P(L = 1)P(J \neq i)P(\hat{I} = i) \Big) \\ & + \sum_{c_{1_{ij}}} c_{1_{ij}} P(C_1 = c_{1_{ij}}|I = i,\, L = 0,\, J \neq i)P(L = 0)P(J \neq i) \\ = \; & \sum_{r \in I} c_{1_{ri}} P(L = 1)P(J = i)P(\hat{I} \neq i) \quad \forall r \neq i \\ & + \sum_{r \in J} c_{1_{ir}} P(L = 1)P(J = r)P(\hat{I} = i) \quad \forall r \neq i \\ & + \sum_{r \in J} c_{1_{ir}} P(L = 0)P(J = r) \quad \forall r \neq i \end{aligned} \tag{9}$$

Note that the first term in (9) is a function of the rows of the cost matrix over column $J = i$. This implies an incorrect decision after the target was reacquired. Finally, assuming $\hat{I}$ being uniformly distributed, and P_{lost} the probability of the actual target to be lost, Equation (9) can be rewritten as below:

$$\begin{aligned} E_{c_1}(C_1|I=i) &= \sum_{r\in I} c_{ri}\, P(J=i) P_{lost} \frac{n-1}{n}\, \forall r \neq i \\ &+ \sum_{r\in J} c_{ir}\, P(J=r) P_{lost} \frac{1}{n}\, \forall r \neq i \\ &+ \sum_{r\in J} c_{ir}\, P(J=r)(1-P_{lost})\, \forall r \neq i \end{aligned} \tag{10}$$

The probability P_{lost} is assumed to be the portion of a multivariate normal distribution $\mathcal{N}(\hat{x}_k, \hat{P}_k)$ not contained in the sensor's field of view when the sensor's aim-point is centered on kinematic state of the target ($\hat{x}$ is the mean state estimate and $\hat{P}$ is the state estimate covariance).

C. The Expected Risk Reduction

When a decision on a target classification is made, the goal is to minimize the risk. In this way, the minimum expected cost is chosen among all possible decisions for each track classification. The risk always decreases with new measurements and reduces the probability of the target being misclassified or lost [4], [5], [14].

The expected risk reduction (ERR) is achieved using the minimum expected cost presented in (10). Note that probabilities in this equation change as measurements are accumulated by a sensor. It is assumed that probabilities change as a Bayesian update. Denoting R as the minimum cost before a measurement update, we can calculate the ERR as:

$$R_i \triangleq E_{c_1}(C_1|I=i) \tag{11}$$

$$R = \min_i \{R_i\} \tag{12}$$

Assuming that the posterior probabilities are denoted by P'_{lost} and $P'(J=i)$, the risk using these updated probabilities is:

$$\begin{aligned} R' &= \min_i \{R'_i\} \\ &= \min_i \left\{ \begin{array}{l} \sum_{r\in I} c_{ri}\, P'(J=i) P'_{lost} \frac{n-1}{n} \\ + \sum_{r\in J} c_{ir}\, P'(J=r) P'_{lost} \frac{1}{n} \\ + \sum_{r\in J} c_{ir}\, P'(J=r)(1-P'_{lost}) \end{array} \right\} \forall r \neq i \end{aligned} \tag{13}$$

When the classification probability is updated through the direct application of Bayes' theorem, as detailed in (6), then (13) can be rewritten as follows:

$$\begin{aligned} R' &= \min_i \{R'_i\} \\ &= \min_i \left\{ \begin{array}{l} \sum_{r\in I} c_{ri}\, \frac{P(M=m|J=i)P(J=i)}{P(M=m)} P'_{lost} \frac{n-1}{n} \\ + \sum_{r\in J} c_{ir}\, \frac{P(M=m|J=r)P(J=r)}{P(M=m)} P'_{lost} \frac{1}{n} \\ + \sum_{r\in J} c_{ir}\, \frac{P(M=m|J=r)P(J=r)}{P(M=m)} (1-P'_{lost}) \end{array} \right\} \end{aligned} \tag{14}$$

Since any classification measurement M is possible, it is necessary to calculate an additional expectation over all possible measurements. The expectation over all possible measurements $\langle R' \rangle$ is calculated as:

$$\begin{aligned} \langle R' \rangle &= \sum_{m\in M} R'\, P(M=m) \\ &= \sum_{m\in M} \min_i \left\{ \begin{array}{l} \sum_{r\in I} c_{ri}\, (P(M=m|J=i)P(J=i) \\ \quad P'_{lost} \frac{n-1}{n}) \quad \forall r \neq i \\ + \sum_{r\in J} c_{ir}\, (P(M=m|J=r)P(J=r) \\ \quad P'_{lost} \frac{1}{n}) \quad \forall r \neq i \\ + \sum_{r\in J} c_{ir}\, (P(M=m|J=r)P(J=r) \\ \quad (1-P'_{lost})) \quad \forall r \neq i \end{array} \right\} \end{aligned} \tag{15}$$

Taking into consideration that the expected cost decreases in value with new measures [5], [14], ERR is given by:

$$ERR = R - \langle R' \rangle \tag{16}$$

D. Kullback-Leibler Divergence

The Kullback-Leibler Divergence (KL divergence) is a measure of the similarity between two probability distributions. It is conventionally defined for two probability distributions P and Q.

Specifically, the KL divergence of Q from P, denoted $D_{KL}(P||Q)$, is the amount of information lost when Q is used to approximate P. When P and Q are discrete probability distributions, KL divergence is defined as in [2]:

$$D_{KL}(P||Q) = \sum_i P_i \log \frac{P_i}{Q_i} \tag{17}$$

In this paper, to calculate the KL divergence we use two multivariate normal distributions, with means μ_1, μ_2 and covariance matrices Σ_1, Σ_2. The two distributions have the same dimension, k. In this case, the KL divergence corresponds to:

$$\begin{aligned} D_{KL}(\mathcal{N}_1||\mathcal{N}_2) &= \frac{1}{2} \Big((\mu_2-\mu_1)^T \Sigma_2^{-1} (\mu_2-\mu_1) - k \\ &\quad + trace(\Sigma_2^{-1}\Sigma_1) + \log\left(\frac{\det \Sigma_2}{\det \Sigma_1}\right) \Big) \end{aligned} \tag{18}$$

IV. Experiments

In order to evaluate the performance of the sensor management processing ERR metric, we run a set of two scenarios. Each of them concerns a 300-second duration. The first scenario involves 10 maneuvering targets, 4 of which are targets of interest (targets 2, 3, 8, and 10), and consequently should be tracked. The second scenario embraces 15 maneuvering targets, 4 of which are targets of interest (targets 3, 5, 12, and 15).

The sensor suite combines an Infrared Search and Track (IRST) and Radar. Both of them are located at the reference position ($x = y = z = 0$). Every second the IR sensor obtains measurements of all targets while Radar sensor takes a measurement of only one target every 2 seconds (using electromagnetics emission constraints). Thus, every 2 seconds a sensor management algorithm based on the risk metrics decides which target track is to be estimated using the Radar sensor report. The sensor management used in this paper is performed through contact reports combining IRST and Radar sensors.

Radar sensor field of view (FOV) is a 500 m^2 region centered on the estimated track position. The target is considered lost if the ground truth position is outside of this FOV. In such a case, a state estimate is very poor and no further measurements are made on targets.

Several simulations using the Monte Carlo method are performed. As for classification of targets, two types of tests were run: one where all target classifications are initially unknown and another one where all target classifications are initially known. For all tests, each target starts with a high accuracy kinematic track. Thus, tasks of the IR-Radar are to correctly classify, maintain a track on, and allocate measurements to the targets of interest. The ground truth over 300 seconds for scenario 1 and 2 is shown in Figure 1 and Figure 2.

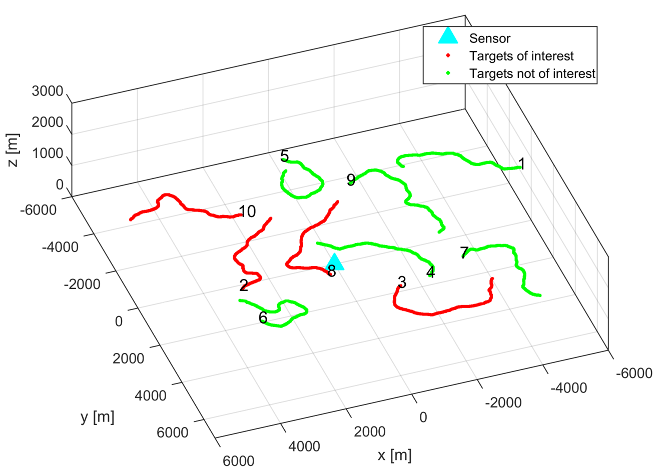

Fig. 1. Scenario 1: targets of interest (in red), targets not of interest (in green), and the sensor location (cyan triangle).

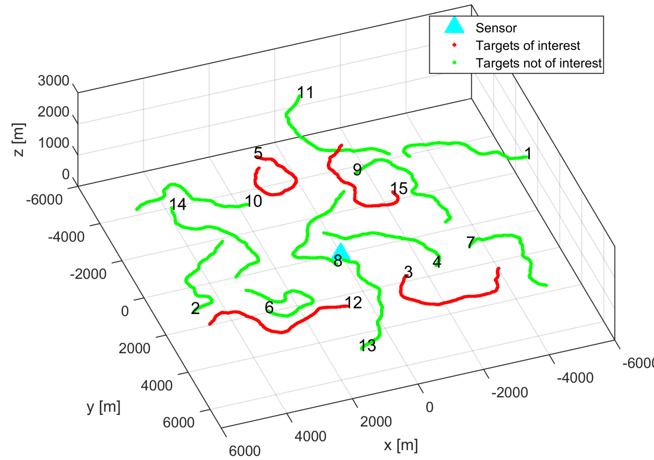

Fig. 2. Scenario 2: targets of interest (in red), targets not of interest (in green), and the sensor location (cyan triangle).

For comparison purposes, the expected risk reduction approach was contrasted with three different sensor management methods involving the Kullback-Leibler divergence, the random assignment, and the round robin assignment; in the latter, the targets are repeatedly selected in a specific order. One thousand Monte Carlo runs were conducted using each method. The track error was calculated between the ground truth position and the estimate for all tracks. For each analysis below, the 5% of the highest and the lowest error measurements were discarded to remove outliers.

A. *Two Classification*

For evaluating ERR metric, a binary classification state is considered where the target to be tracked is either a target of interest ($J = 1$) or a target of non interest ($J = 2$). The binary classification measurement M has support $\{m | m \in [1, 2]\}$. Cost matrix CM_1 and confusion matrix CC are:

$$CM_1 = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 \\ 30 & 0 \end{pmatrix} \end{array} \qquad CC = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix} \end{array}$$

1) Initial target classifications are known: In this case, all target classifications are initially known. Table I presents the resulting median error on each target for each sensor manager method in scenario 1.

For all targets of interest, the ERR approach maintains the value of an acceptable error that is lower when compared to others methods. The sensor manager using ERR maintains a track on targets 2, 3, 8, and 10 with the error's value lower than sensor's FOV radius (250m). The Kullback-Leibler divergence has poor performance for target 3 and 10. The random method performs very poorly and the round robin method has poor performance for target 3.

TABLE I
THE MEDIAN POSITION ERROR IN METERS FOR SCENARIO 1. TARGETS OF INTEREST ARE 2, 3, 8, AND 10.

Class	Target	ERR	KLdiv	Random	Round Robin
2	1	8949	8949	7943	107
1	2	27	122	1976	112
1	3	40	7191	5875	6215
2	4	4900	29	2365	46
2	5	7777	36	4605	62
2	6	6053	6053	3500	85
2	7	5778	5778	25001	88
1	8	31	40	3765	91
2	9	7848	7848	7600	120
1	10	41	6867	4218	105

When using ERR, more measurements can be assigned to targets 2, 3, 8, and 10, since they are targets of interest. Thus, tracking performance is improved yet the track quality for targets of no interest is diminished.

To examine more accurately the results presented by metrics, it is necessary to notice that the average track error is less than 250 meters and so, still within the sensor's FOV. If the average error on targets 2, 3, 8, and 10 is greater than 250 meters, it means that the sensor manager method was ineffective on average.

Table II focuses on p-values for a Student t-test (H_0: true mean error is $\leq$ 250 meters). We can observe that ERR approach is the only method that effectively maintains track on all of targets of interest.

TABLE II
P-VALUES FOR H_0: RESIDUAL ERROR ≤ 250 METERS

Target	ERR	KLdiv	Random	Round Robin
2	1	1	0	1
3	1	0	0	0
8	1	1	0	1
10	1	0	0	1

Table III represents the resulting median error on each target for each sensor manager method in scenario 2. For all targets of interest, the ERR approach maintains track on targets 3, 5, 12, and 15. The error does not exceed sensor's FOV radius (250m). The Kullback-Leibler divergence performs well only for target 15. The random method performs very poorly, and the round robin method performs well only for target 5.

Table IV provides p-values for a Student t-test (H_0: true mean error is $\leq$ 250 meters).

2) Initial target classifications are unknown: In this case, all target classifications are initially unknown. Table V provides the resulting median error on each target for each sensor manager method in scenario 1. For all targets of interest, the ERR approach and the Kullback-Leibler divergence perform well for targets 2 and 8. The random method performs very poorly, and the round robin method only fails to track target 3.

Table VI shows the resulting median error on each target for each sensor manager method in scenario 2 where all target classifications are initially unknown. For all targets of interest, the ERR approach maintains tracks on targets 5 and 15 with the error not exceeding the sensor's FOV radius (250m). The Kullback-Leibler divergence performs well only for target 15. The random method performs very poorly, and the round robin method performs well only for target 5.

TABLE III
THE MEDIAN POSITION ERROR IN METERS FOR SCENARIO 2. TARGETS OF INTEREST ARE 3, 5, 12, AND 15.

Class	Target	ERR	KLdiv	Random	Round Robin
2	1	8949	8949	7880	8949
2	2	4965	4965	2826	132
1	3	226	7191	6336	6220
2	4	4900	44	3165	56
1	5	33	7777	5571	72
2	6	6053	6053	4274	104
2	7	5778	5778	6617	101
2	8	5497	68	4361	102
2	9	7848	7848	7416	115
2	10	6867	6867	5154	114
2	11	4841	4841	3539	105
1	12	39	7102	5280	7102
2	13	7657	32	5694	7657
2	14	6913	6913	4733	124
1	15	34	35	4992	6690

TABLE IV
P-VALUES FOR H_0: RESIDUAL ERROR ≤ 250 METERS

Target	ERR	KLdiv	Random	Round Robin
3	1	0	0	0
5	1	0	0	1
12	1	0	0	0
15	1	1	0	0

TABLE V
THE MEDIAN POSITION ERROR IN METERS FOR SCENARIO 1. TARGETS OF INTEREST: 2, 3, 8, AND 10. ALL TARGET CLASSIFICATIONS ARE INITIALLY UNKNOWN.

Class	Target	ERR	KLdiv	Random	Round Robin
2	1	8487	8949	8029	107
1	2	30	122	1980	112
1	3	7191	7191	6082	6216
2	4	1578	29	2440	46
2	5	1748	36	4369	62
2	6	1405	6053	3382	85
2	7	2539	5778	22672	88
1	8	34	40	3703	91
2	9	2749	7848	7981	120
1	10	3269	6867	4468	105

B. Three Classifications

In order to evaluate the ERR metric, we also consider a tertiary classification state where the target being tracked is either a target of high interest ($J = 1$), of medium interest ($J = 2$), or of low interest ($J = 3$). The tertiary classification measurement M has support $\{m|m \in [1, 2, 3]\}$. Cost matrix CM_1 and confusion matrix CC are:

$$CM_1 = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 20 & 1 \\ 30 & 0 & 1 \\ 30 & 20 & 0 \end{pmatrix} \end{array} \qquad CC = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{pmatrix} \end{array}$$

TABLE VI

MEDIAN POSITION ERROR IN METERS FOR SCENARIO 2. TARGETS OF INTEREST ARE 3, 5, 12, AND 15. INITIALLY, ALL TARGET CLASSIFICATIONS ARE UNKNOWN.

Class	Target	ERR	KLdiv	Random	Round Robin
2	1	8597	8949	7880	8949
2	2	637	4965	2826	132
1	3	7191	7191	6336	6220
2	4	571	44	3165	56
1	5	54	7777	5571	72
2	6	5676	6053	4274	104
2	7	3825	5778	6617	101
2	8	1057	68	4361	102
2	9	7848	7848	7416	115
2	10	6867	6867	5154	114
2	11	1516	4841	3539	105
1	12	7102	7102	5280	7102
2	13	2539	32	5694	7657
2	14	3832	6913	4733	124
1	15	138	35	4992	6690

Initially, all target classifications are unknown. Table VII shows the resulting median error on each target for each sensor manager method for scenario 1. For all targets of interest, the ERR approach performs well only for target 2. The Kullback-Leibler divergence performs well for targets 2 and 8. The random method performs very poorly, and the round robin method only fails to track target 3. While comparing Table V and VII, we can observe that the ERR method improves results in the tertiary classification on target 10 (hight interest) despite the opposite impact on targets 2 and 8 (medium interest).

TABLE VII

THE MEDIAN POSITION ERROR IN METERS FOR SCENARIO 1. THERE ARE THE FOLLOWING TARGETS OF INTEREST: 2, 8, AND 10. INITIALLY, ALL TARGET CLASSIFICATIONS ARE UNKNOWN.

Class	Target	ERR	KLdiv	Random	Round Robin
3	1	8704	8949	7909	107
2	2	47	122	2182	112
3	3	7191	7191	5543	6220
3	4	2112	29	2228	46
3	5	1994	36	3600	62
3	6	4193	6053	3346	85
3	7	5660	5778	10838	88
2	8	293	40	4290	91
3	9	2716	7848	7616	120
1	10	1825	6867	3587	105

Table VIII illustrates the resulting median error on each target for each sensor manager method in scenario 2. For all targets of interest, the ERR approach performs well only in the case of target 5, and shows that the value of the average error on targets 12 and 15 is lower than when other methods are employed, but higher than the sensor's FOV. The Kullback-Leibler divergence performs well only for target 8. The random method performs very poorly, and the round robin method can only track target 5.

If we compare Table VI and VIII, we can notice improved results for the tertiary classification of tracks on target 12 when the ERR method is applied. As for tracks on targets 5 and 15 (medium interest), the outcomes are less satisfactory. However, the value of an average error on target 12 turns out to be higher than sensor's FOV.

TABLE VIII

THE MEDIAN POSITION ERROR IN METERS FOR SCENARIO 2. THERE ARE THE FOLLOWING TARGETS OF INTEREST: 5, 12, AND 15. INITIALLY, ALL TARGET CLASSIFICATIONS ARE UNKNOWN.

Class	Target	ERR	KLdiv	Random	Round Robin
3	1	8251	8949	7823	8949
3	2	2174	4965	2495	132
3	3	7191	7191	6205	6211
3	4	1890	44	2765	56
2	5	157	7777	5230	72
3	6	6053	6053	4324	104
3	7	5778	5778	22830	101
3	8	2438	67	4193	102
3	9	7848	7848	7209	115
3	10	6867	6867	5104	114
3	11	3727	4841	3656	105
1	12	2095	7102	5119	7102
3	13	7657	32	5950	7657
3	14	6913	6913	4822	124
2	15	308	35	5208	6690

V. CONCLUSION

This paper introduces the Expected Risk Reduction approach to sensor management in the case of an IR-Radar sensor suite. The ERR is based on the expected cost of an incorrect decision on a target's classification. This cost was then conditioned on the event of losing a target track which allowed for achieving the combination of classification and kinematic uncertainty in the same metric. When all target classifications were initially known, it has been proved that the ERR approach can maintain a track on targets of interest when it is not possible for a single sensor to track all targets in the environment. In the case where all target classifications were initially unknown, the ERR approach did not perform very well. Nevertheless, it is clear that there was a significant reduction of the value of the average error for targets of interest, which, however, remained higher than sensor's FOV. The ERR turns out to be effective when it comes to the IR-radar case, specifically when the initial classification of targets are known. The round robin method has a good performance when the number of targets is not too high, however, it never prioritizes tracking targets of interest.

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