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Sinn/Bedeutung and Intension/Extension in Gödel's *Max Phil IX*

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Introduction

On 18th November 1942 P.A. Schilpp, the editor of the famous series *The library of living Philosophers*, asked Gödel to contribute to the volume in honour of Bertrand Russell. The volume was planned for the fall or early winter of 1943, and Schilpp suggested that Gödel's essay should take the title "Russell's mathematical logic".¹ Gödel sent a first manuscript on the 17th of May 1943, and the revised version on the 28th of September.

On the same day that he had been asked to contribute the essay, the 18th of November 1942, Gödel began a new philosophical notebook, the *Max Phil IX*. From a formal point of view, *Max Phil IX* introduces a break in the long series *Max Phil III-VIII*, written from around October 1940 to November 1942: *Max Phil III-VIII* are actually numbered continuously from page 1 to page 680, whereas *Max Phil IX* starts from page 1 again. *Max Phil IX* is followed by *Max Phil X*, which is dated by Gödel 12.3.1943- 27.1.1944. The two latter notebooks were therefore written mainly in the same period as the Russell paper.

As Wang suggests in his *Reflections on Kurt Gödel*,² the Russell paper marks the transition from Gödel's publications on definite logical and mathematical results to more philosophical and history-oriented papers. Actually Gödel himself considered it as a particularly appropriate statement of his philosophical point of view as he wrote in the *Grandjean Questionnaire* published in the same book by Wang.³ We know from the same source that Gödel himself qualifies this philosophical point of view as "a conceptual and mathematical realism".⁴

The general frame of the Russell Paper clearly comes from Gödel's intensive study on Leibniz, which began around 1943.⁵ It opens and ends with Leibniz and with what Gödel calls his project of the *characteristica universalis*, interpreted as a tool for both the *ars inveniendi* and the *ars demonstrandi*. Considering logic in the Leibnizian sense as the science prior to all others and containing the ideas and principles underlying them, Gödel proposes a sort of critical evaluation of how Russell, following Frege and

1 (Gödel 2003) p.217.

2 (Wang 1988) p.305.

3 *Ibid.*, p. 18.

4 *Ibid.*, p. 20, section 1.2.3.

5 *Ibid.*, p. 19.

Peano, has made progress towards Leibniz's project or, respectively, has turned away from it.⁶

The general thesis of the paper is clear and can be summed up as follows: Russell's main contribution concerns his analysis of the paradoxes, which he freed from all mathematical technicality, but his philosophical attitude with his gradually decreasing realism towards concepts and classes has prevented him from producing a definite and clear analysis of the primitive concepts of mathematical logic. From Gödel's point of view, this is exactly what is missing in order to realise Leibniz's project.

Despite this clear frame and its explicit aim, the Russell paper is not easy to analyse in its details, neither from the point of view of its structure nor from the point of view of its content.⁷ The critical appreciation of Russell's work (and especially of his vicious circle principle) contains, beyond the critical part, some positive allusions concerning the direction that should be taken to realise Leibniz's project. All of this positive part is very enigmatic and no answers have been given to many of the questions concerning it until now. Let's point out just some of these problems.

Gödel distinguishes in the paper a realistic attitude towards classes and concepts, from a constructivist or nominalist one; the latter reduces classes and concepts to linguistic entities or to creations or constructions of the mind. Concerning the former, Gödel says that, on the one hand, classes can be considered as pluralities of things or structures of pluralities of things and, on the other hand, concepts can be considered as properties and relations among things. If we take the assertion of the *Grandjean Questionnaire* seriously, it follows that not only classes (or sets) have to be taken as objective entities but also properties and relations. What does it mean exactly to recognise properties and relations as objective entities? What exactly are the consequences of Gödel's realistic attitudes towards concepts in his diagnosis of the paradoxes? In which way does this realistic attitude influence his conception of the relationship between logic and mathematics? How should we understand his program of constructing a strong type-free theory of concepts? Did he work on the project?

As *Max Phil* IX and X were written in exactly the same period as the Russell paper, it is natural to ask whether they contain the expected clues necessary in order to clarify these open questions. From our point of view, the answer is positive but the objective complexity of the *Max Phil* prevents us from affirming that this positive response implies immediate answers to all of our questions. *Max Phil* IX and X do indeed contain a large amount of logical reflections (*Bemerkungen*) directly connected to the content of the Russell paper: remarks on Leibniz, Frege, Peano, Russell, the problems of definition, judgement, definite descriptions, paradoxes, the nature of functions and concepts, the relation between logic and mathematics, and so on. Other theological, ontological, physical, political and psychological *Bemerkungen*, contained in the same notebooks and related to the logical ones, show how deep and how wide Gödel's philosophical concerns were in this period: he clearly put together different subjects, using analogy without restriction, in order to find a general solution to the

6 The lasting persistency of Gödel's faith in the Leibnizian project of a general method for the solution of all problems including mathematical ones is witnessed by many other passages in Gödel's works. For example, in his 1961 paper, (Gödel 1996), at the beginning of pp. 382-383.

7 See for example, Charles Parsons' introduction to the Russell paper in (Gödel 1996). An analysis of the structure of the Russell paper was proposed in (Crocco 2012), pp. 217-256.

theoretical and practical problems of philosophy. *Max Phil* Notebooks from IX on are probably the place in the Archives where it is most explicit that Gödel is really searching for the basis of a new metaphysical monadology, adapted to the science of his time. The *Bemerkung* that we commented on in the paper belonging to the first part of this volume clearly shows that although for Gödel the central problem of this part of the *Max Phil* is to find the list of the primitive concepts and the axioms and definitions of them, what Gödel actually does throughout these two notebooks is to give us his after-thoughts on philosophers and logicians, whom he often does not even quote explicitly. This situation pushes us to a cautious attitude: a definite answer to our logical problems concerning the Russell paper will be possible only when the work of interpreting and connecting together all these apparently scattered *Bemerkungen* is finished. Until then, some partial but important improvements can be accomplished on the basis of some specific remarks of the *Max Phil*.

Here, for the sake of analysis, I will partially neglect the kaleidoscopically amazing richness of Gödel's way of thinking, and concentrate essentially on our first question mentioned above, which I consider as taking precedence over the others and to be central to understanding the Russell paper: the peculiarity of Gödel's realism is that it concerns not only mathematical objects but also concepts. Now *Max Phil* IX and X provide us with precious material with which to clarify the essential question of what Gödel really means by concepts. We will see that, even from this restricted point of view, it is impossible to set apart logical from metaphysical investigations.

I will divide this paper into three main parts. In the first part I will present three conjectures about the Gödelian notion of concept that I formulated on the basis of the available published material some years ago, before the analysis of the *Max Phil*. In the second part, I will present a first group of remarks from *Max Phil* IX, showing that all the elements, which I conjecturally considered to be a part of the enigmatic Gödelian notion of concept are actually discussed in these manuscripts. In the third part I will partially comment upon two longer remarks from *Max Phil* IX, which seem to give very good clues to specify and partially corroborate one of my conjectures.

1. Some conjectures on Gödel's notion of concept

In a paper published in 2006,⁸ I formulated three conjectural theses concerning different aspects of the notion of concept. These three conjectural theses were based on clues which came from the analysis of four texts, namely: i) the Russell paper (and some Gödelian annotations on it written by Gödel himself and published as textual notes at the end of the second volume of the *Collected Works* (Gödel 1990); ii) the Gödelian conference of 1951 (Gödel 1951); iii) the drafts of "Is mathematics syntax of language?" (Gödel 1953-9); and finally iv) the transcription of Gödel's conversations during the seventies with his friend Hao Wang (Wang 1996). The necessity to either disprove or corroborate these conjectures was one of the main reasons for exploring Gödel's *Nachlaß* for me.

(a) My first conjectural thesis concerned the semantic interpretation of the notion of concept. It seemed to me that in order to understand the Gödelian notion of concept

⁸ (Crocco 2006).

we need four terms: two from the old traditional opposition *intension/extension* and two from the Fregean distinction *Sinn/Bedeutung*. There is a long tradition that comes essentially from Carnap, which identifies the intension of a conceptual term (a propositional function) with its *Sinn* and the extension with its *Bedeutung*. My thesis is that Gödel does not belong to this tradition and does not accept this identification. Actually the four terms seem necessary to him in order to reassess the distinction between what is real and what is possible together with the Leibnizian distinction between *ideas in mente homini* and *ideas in mente Dei*, i.e. between a concept of possibility linked to human conceivability and a metaphysical notion of possibility. From my point of view, assessing these distinctions is crucial for the elimination of many misunderstandings about Gödel's *Begriffsrealismus*, and also fundamental to understanding correctly the relationship between logic and mathematics in Gödel's thinking.

At that time I developed my conjecture in this way: Gödel considers *Sinn* of conceptual terms as ways to access their *Bedeutungen*, as rules for the determination of these *Bedeutungen*. Nevertheless, it seems that according to him the notion of *Bedeutung* can be split into an intensional and an extensional part. Extensionally, the *Bedeutungen* of predicates are classes (that is pluralities of n-tuples of things). Intensionally they are the concepts themselves (properties or relations) considered as objective entities.⁹

(b) My second conjectural thesis concerned the relationship between Frege's and Gödel's notions of concept. According to our first conjecture, both Frege and Gödel seem to agree on the fact that conceptual terms denote the concept themselves, but it is well known that for Frege, concepts as *Bedeutungen* are these unsaturated entities in search of completion, which are transformed into objects (and more exactly *Wertverläufen*) as soon as we try to speak about them. On the contrary, I affirmed that the notion of concept in an objective sense is incompatible for Gödel with the idea that concepts are unsaturated entities, extracted from propositions and in need of completion. Concepts in the objective sense are prior to the propositions in which they occur and the relation they have to their argument is an external one. As a consequence, in every propositional function $\varphi(x)$, we can make explicit the link between the concept and its argument, and we can make it explicit in both an extensional and in an intensional way. Therefore $\varphi(x)$ means intensionally that x *esti* φ , and extensionally that x belongs to the class of φ . Hence, the problem of the intensional paradoxes is the problem of a clear analysis of the *esti* relation.

(c) The third conjecture related concepts to paradoxes and concerned Gödel's attitude towards the strategies which have been proposed in order to solve the intensional paradoxes. It can be formulated as follows. The paradox of concepts¹⁰ (concerning for example the concept of not applying to itself when applied to itself) is, according to Gödel, the real open problem of modern logic, upon which depends the fate of the Leibnizian project of the *characteristica universalis* as an *ars inveniendi* and an *ars demonstrandi*. The analysis of the strategies that are able to solve this problem is central

⁹ I proposed the same thesis, with a different terminology, in (Crocco 2012) p. 243.

¹⁰ This paradox must be accurately distinguished from the linguistic paradoxes as suggested by Gödel in (Wang 1996) pp. 269-273.

to Gödel's concern. Gödel considers that the diagnosis of the paradox must not be contrary to either the idea that to each predicate corresponds a concept (as *Sinn*, that is something that can be understood), nor to the fact that a concept (in the objective sense) is ontologically previous to all its possible arguments. The intensional paradoxes come from the idea that a concept can be meaningfully applied to every argument whatsoever. Gödel thinks that universal applicability of a concept must generally be the case, because of the very nature of the notion of concept, but he is convinced that there exist limited cases, which have not yet been completely identified, in which the concept cannot be applied meaningfully to its argument. Gödel calls this strategy to avoid paradoxes the strategy of the limited ranges of significance. The paradoxical concept of not applying to itself is therefore a perfectly acceptable concept applicable to many arguments. Nevertheless it cannot result in a significant proposition when applied to itself: its range of significance is limited by very specific exceptions. The intensional theory of simple types which Gödel speaks about in the last part of his Russell paper is, according to Gödel, a living and non-artificial example of intensional logic based on the idea of limited ranges of significance. Therefore it is based on an assumption completely different from that of the theory of order (the name that Gödel gives to the ramified theory of types) and it is logically independent from the vicious circle principle. This last point is hardly accepted by the current literature on the subject. Owing to a tradition coming from Ramsey, it is generally affirmed that the theory of simple types comes directly from the ramified theory, by a simplification and an extensionalization of the latter.

These three conjectures are clearly interconnected. If the analysis of the *esti* is the central task for solving the intensional paradox (b), this is exactly because only one general relation in the objective sense exists, which links objective concepts to their arguments (a), without distinctions of types or of orders (c). Affirming, as Gödel does in the Russell paper, that the limited ranges of significance strategy is the right one for the intensional paradoxes (c) is to say that concepts in the objective sense are previous to their arguments and from the propositional contexts in which they occur (b), in the same way as structures in *mente Dei* (a) are independent from their realisations in the spatio-temporal world and are even previous to them.

Because of this interconnection, a completely satisfactory analysis of Gödel's notion of concept and of his *Begriffsrealismus* should take into account the three aspects. Nevertheless, for reasons of space, this present paper is essentially devoted to the analysis of conjecture (a) in light of the evidence of *Max Phil IX*.¹¹ Actually the two long *Bemerkungen* that we analyse in the third part of this paper mention problems connected with conjectures (b) and (c), but I shall leave these questions to further research.¹²

11 We limit ourselves here to *Max Phil IX*, essentially because it is contemporary to the Russell paper. Nevertheless, there is much more material on the subject of *Sinn* and *Bedeutung* in the rest of *Max Phil*. To mention only the last notebook *Max Phil XV*, there is a very explicit assertion connecting the distinction between *Sinn* and *Bedeutung* to the problem of analyticity: *Phil. Zum Sinn jedes Begriffs gehört es, dass er eine objektive Bedeutung hat. Daber ist die Objektivität der Begriffswelt ein analytischer Satz. Phil.* To the *Sinn* of each concept appertains that it has a *Bedeutung*. Hence the objectivity of the world of concepts is an analytic proposition.

12 See Gabriella Crocco, Julien Bernard, "Gödel and the paradox in *Max Phil X*", in this book.

2. Intension/extension; *Sinn/ Bedeutung*

The observations that we present in this section can be divided into three groups. In the first one Gödel explicitly uses the pair of notions intensional/extensional to qualify two different aspects in the learning of natural languages. In the second group Gödel explicitly mentions and comments upon Frege's pair of notions *Sinn* and *Bedeutung* and Carnap's interpretation of them. In the third group Gödel mentions the same Fregean pair *Sinn/Bedeutung* in a more metaphysical context, together with other pairs of opposite terms, which belong to the history of philosophy. The two first groups of *Bemerkungen* simply show that Gödel makes use of both pairs (extension/intension, and *Sinn/Bedeutung*) in the *Max Phil* notebooks as he does in the published corpus.¹³ They also show that he seems perfectly aware on the one hand of Frege's use of *Sinn/Bedeutung* and on the other hand of Carnap's misleading interpretation of them.

The aim of this section is to indicate the different role that Gödel assigns to *Sinn/Bedeutung* on one side and *intension/extension* on the other, setting the stage for the two long *Bemerkungen* of section 3, which give some concrete and valuable hints on why the four terms are all needed by Gödel in his attempt to detail his *Begriffsrealismus*.

2.1. The intensional and extensional learning of a language

There are many observations in *Max Phil* where Gödel discusses the differences between the extensional and intensional point of view. The observation that I will comment upon here comes from page 21 of *Max Phil* IX. It is particularly interesting because the notion of *Bedeutung* is mentioned there and offers a privileged access to understanding its connection with the pair extension/intension.

Observation (Philosophy): Are the extensional aspects acquired in parallel to the intensional ones in the learning of a language? That is: 1. the knowledge of the rare words, and 2. the exact knowledge of the common words and the principles according to which we can construct both the *Bedeutung* (or *Bedeutungen*) of the combinations of these words and the precise *Bedeutung* of their roots.

Bemerkung (Philosophie): Geht das *extensionale* und *intensionale* Lernen einer Sprache parallel? Das heißt: 1.) Die Kenntnis auch der seltenen Worte, und 2.) die genaue Kenntnis der häufigen <Worte> und der Prinzipien, nach denen die Bedeutung [die Bedeutungen] der Zusammensetzung konstruiert wird und die genaue Bedeutung der Stämme.

The opening question fixes very clearly the context of the use of the two adjectives intensional/extensional in this *Bemerkung*, i.e. the process of learning a language (which could concern a child learning his native language, or an adult learning a foreign language). Learning a language implies the learning of its words and the question addressed by Gödel presupposes clearly the distinction of two aspects in the mastering of a language: the intensional one and the extensional one. But how can we interpret the second part of the *Bemerkung*?

It must be stressed that, although Gödel does not answer here the question, which opens the *Bemerkung*, an indirect answer to it seems to be given in the 1961 paper in a

¹³ The distinction intension/extension is crucial throughout the Russell paper. The *Sinn/Bedeutung* pair of notions is mentioned twice in the three first volumes of the *Collected Works*: in the Russell paper (Gödel 1990), note 7, p. 122, and in version III of the Carnap paper, (Gödel 1996), p. 350, note 40.

different context. In a famous passage of it Gödel affirms that the better understanding of language is one of the essential components in the general development of a child. He adds that as soon as the child goes beyond the most primitive use of language (the designation of objects through words) this better understanding of language is realized through the ability to grasp the basic concepts on which language rests.¹⁴ Gödel says that this understanding implies knowledge of the use of the words and of their logical connections.¹⁵ Therefore, if we identify extensional learning with the learning of the language through designation and ostension of objects, and if we identify intensional learning with the learning of the concepts designated by the terms, then, on the basis of the 1961 paper, it seems that the answer to the question in our *Bemerkung* should be negative. The extensional learning of language is not parallel to the intensional learning. The first is more primitive in respect to the second and it occurs before it. First we learn to designate objects of the external world. Then we learn how to name their properties and their relations, which in return allows us to recognize and identify their singularities and differences.

Now, if we accept this answer, and the identification, which it implies (i.e. extensional learning = learning by ostension of objects, intensional learning = learning of the concepts designated by the terms of the language) do we understand better the last part of the *Bemerkung*? The answer seems to be positive.

Gödel does not specify what extensional learning is, because it seems quite obvious for him that extensional learning of a word is just learning by ostension: it is learning of a word by indication of the objects exemplifying it. Being able to point out examples in the real world is a way of showing extensional competence in the use of the terms. It seems very plausible that the second part of Gödel's observation specifies on the contrary what intensional learning is supposed to be. This explanation is given enumerating possible kinds of intensional learning. Firstly Gödel mentions the acquisition of rare words. This is easily explainable: if a word is not frequently used, the objects exemplifying it cannot be easily pointed out, and it is probable that this word will be acquired not by ostension but through the relations which it has with other words. Secondly concerning common words he mentions the acquisition of the exact knowledge of them, the acquisition of the principles according to which they can be composed to indicate complex entities and the knowledge of the roots of the words.

These three aspects can also be easily explained. Concerning the first two, the acquisition of the exact knowledge of a common word belongs to the intensional mastering of a language because, in the common use of a language, in general we do not signify precise concepts but rather a neighborhood of concepts.¹⁶ Actually we are not generally interested in an analysis of borderline cases, unless we encounter one of them in our experience. In the 1961 paper Gödel seems to suggest that a precise knowledge of concepts can be acquired nevertheless by reflecting on the relations between words and on their use. The first *Bemerkung* of *Max Phil X* insists on the same point: that the mastering of logical terms implies the mastering of their ways of composition, and therefore the mastering of their possible applications.

14 (Gödel 1996), p. 383.

15 *Ibid.* p. 385.

16 Cf. *Bemerkung* p. 89b, *Max Phil IX*.

Finally Gödel mentions the knowledge of the roots of the words for which no exemplification in terms of concrete objects can be given because roots of words are not generally used to name things and therefore their knowledge cannot be acquired by ostension. They only indicate what different words have in common.

Note that in the latter two points Gödel uses the term *Bedeutung*. Intensional competence in the learning of a language is knowledge of the *Bedeutung* of the roots of the words and of the principle for forming complex *Bedeutungen* signified by complex terms. What is important here is that knowledge of some aspects of the *Bedeutung* of a term are qualified as intensional or more precisely their learning belongs to the intensional mastering of the learning of a language. If we were right in identifying intensional learning with learning of the concepts designated by the terms of the language, it should follow that the *Bedeutung* of a term, i.e. what this term signifies is, intensionally, a concept and, extensionally, the class of object falling under it. This conception seems quite compatible with the passage of the 1951 paper (Gödel 1990) on page 320 mentioned in my paper of 2006, where Gödel says that concepts are what is denoted by linguistic terms. It is also compatible with the observation in *Max Phil XI* page 16, where Gödel says that every concept, even the logical ones are components of the world as they are components of facts.¹⁷ Therefore concepts in intension, as components of the real, are kinds of *Bedeutungen*.

2.2. Frege's *Sinn/Bedeutung* and Carnap's interpretation of them in *Max Phil IX*

A possible objection to our conclusion of the previous section would be that Gödel does not use the term *Bedeutung* in a technical sense as opposed to *Sinn*. Nothing proves that Gödel does not use *Bedeutung* as a synonym of *Sinn*, as is the case in current German. But this objection is poor considering other material from the same *Max Phil IX*. Many *Bemerkungen* prove that Gödel knows Frege's distinction of *Sinn* and *Bedeutung* and the modern interpretation of it from Carnap onward. They also indicate that he is interested in the use of the notion of *Sinn* as opposed to that of *Bedeutung*, to express the human contribution in the act of knowledge.

Carnap was one of the first logicians to explicitly discuss Frege's distinction and to connect it with the traditional intension/extension distinction. We can find such an interpretation in all of Carnap's main books from *Der Logische Aufbau der Welt* of 1928 (sections 43–45) to the *Die logische Syntax der Sprache* (sections 38, 65 and ff.) and right up to *Introduction to semantics*, (sections 10 and 31). The latter was published at the beginning of 1942 and in the same year Alonso Church published a review of it¹⁸ where Frege's distinction is also assimilated to the intension/extension one, and where the question which occupies Gödel at the beginning of the Russell paper is extensively discussed. In his review of Carnap's *Introduction to semantics*, Church interprets intension as *Sinn* and extension as *Bedeutung*, but we know that this is not acceptable from Frege's point of view, because a saturated entity as a class (or more exactly a *Wertverlauf*) cannot be the *Bedeutung* of an unsaturated expression. This fact

¹⁷ “Even the ‘or’ appears in the world (as an element of some facts)”: see the paper of Amélie Mertens in this book for a further comment on this observation in *Max Phil XI*.

¹⁸ (Church 1942).

is known only from the publication of the *Ausführungen über Sinn und Bedeutung*, in Frege's *Nachgelassene Schriften*, (Frege 1969), but it is also understandable from the published texts of 1891/2. It is possible that Gödel was aware that for Frege the *Bedeutung* of predicates are the concepts themselves. What is certain is that Gödel, who was already familiar with Carnap's *Logische Syntax*, was reading about this subject during the redaction of the Russell paper and was becoming interested in Frege's distinction, probably also through the references in Russell's *Inquiry into meaning and truth*, published in 1940 and mentioned in the Russell paper.¹⁹

The first remark presented in this section 2.2 has the form of a question.²⁰ It directly concerns Frege's pair of notions *Sinn* and *Bedeutung*. Gödel seems to give an attentive interpretation of Frege's distinction, although he doubts its conformity to Frege's analysis.

Remark (Philosophy): *Sinn* and *Bedeutung* come from the double relation of the language to the thing and to the listener (or speaker)? p. 40)

Bemerkung (Philosophie): *Sinn* und *Bedeutung* kommen von der doppelten Relation der Sprache auf die Sache und den Zuhörer (oder Sprechenden)? (p. 40)

Gödel asks himself about the nature of the Fregean distinction. He hypothesizes that it concerns the use of language and seems to suggest that for Frege language has a double relation: on one side with what Gödel calls the things (and that could be interpreted as what the language points to, i.e. the entities of the outer world) on the other side to the actors of the linguistic exchange: the speaker and the listener. The hypothesis is repeated in an affirmative assertion in the next *Bemerkung*, in a way that seems to evoke an implicit dialogue with the Carnapian interpretation.

Remark Gr: Frege's theory of *Sinn* and *Bedeutung* cannot be easily interpreted by supposing a systematic equivocation in common language, because he supposes that in the use of language both *Sinn* and *Bedeutung* play an equal role, i.e. the *Bedeutung* in relation to the object and the *Sinn* in relation to the subject. (p. 63)

Bemerkung (Grammatik): Die *Frege'sche* Theorie von *Sinn* und *Bedeutung* kann nicht einfach als Behauptung einer systematischen Äquivokation in den gewöhnlichen Sprachen interpretiert werden, denn er behauptet, dass in der Verwendung der Sprache *Sinn* und *Bedeutung* beide gleichzeitig <eine> Rolle spielen, nämlich die *Bedeutung* im Objekt und der *Sinn* im Subjekt.

The systematic equivocation mentioned by Gödel was generally attributed to the opposite pairing of intension/extension. A general term, a common name, can be considered to be in relation both with a property (its intension) and with the class of objects satisfying it (its extension), therefore it is possible to say that, in natural language, general terms are "equivocally" in relation to both intension and extension.

According to my conjecture, Gödel considers that the *Bedeutungen* indicated by linguistic conceptual terms can be concepts or classes, and therefore that there is a possible double indication (equivocation) in the way language points to the world as *Bedeutung*. Now if we identify intension with *Sinn* and extension with *Bedeutung*, as

19 Gödel's *Bemerkung* in *Max Phil IX* p. 26, confirms this interpretation.

20 To be exact, the *Bemerkung* contains a single phrase, ending with a question mark, but it presents no subject-predicate inversion, which it would need to be grammatically correct. This indicates more the expression of a general doubt, than the formulation of an explicit question.

Carnap and Church do, we should say that the systematic equivocation in the use of the terms of natural language concerns *Sinn* and *Bedeutung*. This is exactly what this *Bemerkung* seems to put in doubt.

Gödel seems to refuse such an interpretation on the basis of his previous interpretation of Frege's pair of notions. A linguistic expression is related on one side towards the subject employing it (the speaker or the listener understanding the language) and on the other side toward the objects²¹ of the world to which it points. Both these dimensions are essential in the use of the language. They are semantic components of it, both playing a role in its use. Therefore the neglect of one of them provokes a fundamental misunderstanding in the interpretation of the role and the nature of language.

The next *Bemerkung* is also relative to an argument from Carnap that we find explicitly in chapter 3 of *Meaning and Necessity* published in 1947, but that is implicit in *Introduction to Semantics*. The extensionalization of intension and, therefore of *Sinn*, if we accept Carnap's identification, is at the heart of the Carnapian semantic analysis. It cannot satisfy Gödel because it results in an elimination of all possible references to the subject in the semantic analysis of language. This is unacceptable because it causes the neglect of what makes language useful for knowledge:

Page 26 Remark (Grammar): Carnap asserts that the choice of an extensional language has nothing to do with the rejection of '*Sinn*'. We can speak extensionally about *Sinn*, but the *Sinn* of objects such as 'proposition', and 'concept' is just that they make possible the understanding of language [its perception] and therefore a language is 'understandable' inasmuch as its Symbols [and its combinations of symbols] are conceived as designating such objects and inasmuch as the assemblage of the symbols denotes the production of objects of this kind. Otherwise a language would be sterile and not understandable.

Page 26 Bemerkung (Grammatik): Carnap behauptet, dass <die> Wahl einer *extensionalen* Sprache nichts mit Verwerfung des „Sinns“ zu tun hat. Man kann auch *extensional* über den Sinn sprechen, aber der Sinn derartiger Objekte wie „Satz“, „Begriff“ ist gerade, dass sie das Verständnis der Sprache ermöglichen [ihre Wahrnehmung] und daher eine Sprache nur „verstanden“ werden kann, wenn ihre Symbole [und Symbolkombinationen] als solche Objekte bezeichnend aufgefasst werden und die Zusammenfassung von Symbolen die Erzeugung neuer derartiger Objekte bedeutet; [26] andernfalls <ist> eine Sprache unfruchtbar und unverständlich.

Gödel here seems to object to the Carnapian elimination of the subject in the semantic analysis, because this elimination cuts out the link between logical analysis and theory of knowledge, and it is exactly this connection that Gödel is searching for in order to develop his logic both as an *ars inveniendi* and an *ars demonstrandi*. The extensionalization of the '*Sinn*' corresponds to the neglect of the analysis of the principles of compositions of symbols expressing complex concepts, and the learning of this principle of composition is essential in order to learn their complex *Bedeutungen* as Gödel affirms in the previous *Bemerkung*. We cannot eliminate the analysis of the *Sinn* of 'proposition' or 'concept' without neglecting what makes the use of language fruitful for knowledge. It is important for our purpose to stress the beginning of the second

21 Note that Gödel uses the term '*Objekt*' and not '*Gegenstand*', '*Ding*' or '*Sache*'. '*Objekt*' seems in general to convey an intentional accent.

phrase of the *Bemerkung*. Gödel says that it is possible to speak extensionally about *Sinn*. That seems to imply that even for the level of *Sinn*, it is possible to distinguish an intensional mode of speaking from an extensional one. But this extensionalization of *Sinn* destroys the possibility to understand language.

Sinn/Bedeutung and possibility/reality

The last *Bemerkung* of this section 2, mentions the Fregean distinction *Sinn/Bedeutung* in a complex of philosophical oppositions, which from the Gödelian point of view can be considered analogous. It suggests how the notion of *Sinn* appears as linking together, through the semantic analysis of language, epistemological and metaphysical aspects essential to the analysis of the relationship between logic and mathematics.

Remark (Philosophy): The following oppositions come out to be the same:

1. Description (or concept, symbol) and thing
2. Phenomenon and thing in itself
3. *Sinn* and *Bedeutung*
4. Possibility and reality
5. [Plurality and unity?]

The light and what is illuminated; what is in us and what is in front of us, in the act of knowledge. This distinction appears in different specifications (levels): 1. logic and mathematics, 2. mathematics and reality (are the phenomena related to the things something merely possible?) (pp. 23-24)

Bem<er>kung> (*Phil<osophie>*): Folgende Gegensätze kommen auf dasselbe hinaus:

1. Beschreibung (oder Begriff, Symbol) und Ding. 2. Erscheinung und Ding an sich.
3. Sinn und Bedeutung
4. Möglichkeit und Wirklichkeit
- [5. Viel und Eines?]

Licht und Beleuchtetes, was im Erkenntnisakt in uns ist und was vor uns. [24]
Dieser Unterschied tritt, in verschiedenen Ausprägungen (Stufen) auf: 1. Logik und Mat<hematik> > 2. Mat<hematik> und Wirklichkeit (die Erscheinungen sind im Verhältnis zum Ding etwas bloß Mögliches?) (pp. 23-24)

Gödel proposes a list of 5 pairs of opposite notions that he says, come out to be the same. This means that, from his point of view, each first element of the five listed oppositions is analogous with one of the other elements appearing in the first position and each second element of the above mentioned list is analogous to the other elements appearing in the second position. We have therefore on one side description (or concept, symbol), phenomenon, *Sinn*, possibility and plurality, and on the other side thing, thing in itself, *Bedeutung*, reality and unity.

Two pairings evoked in the list are related to a philosophical tradition, which is easy to identify. Among them the Fregean one (3) and the opposition between phenomenon and thing in itself (2), which reminds us explicitly of Kant's distinction between the external and unknowable cause of our knowledge and the constitution of knowable objects by the subject through the faculties of intuition and understanding. Possibility and reality seem to evoke the Leibnizian analysis of the metaphysical distinction of what is conceived by God, through the combination of the ideas, the articulations of His mind, and what is realised by Him as the actual real world because of His magnanimity. The first and the last items of the list are more difficult to identify. Because of the possible assimilation that (1) leaves open between description,

concept and symbol, the first pairing seems to belong to the modern logical analysis of Russell or Carnap. The last item mentions a distinction from the old Pythagorean speculations that runs through the whole history of philosophy. Many interesting conjectures can be formulated about the interrogative mark following it and about the parentheses that seem to reinforce the expression of Gödel's doubt on the legitimacy of the opposition to appear in the list. As the analysis of them will exceed the limit of our present task, I will not enter into detail here, but would like to point out that in Wang's book whole and unities, universals and particulars are considered by Gödel as fundamental aspects of the world, see (Wang 1996) pp. 295-296, n° 9.124, n° 9.125.

It is worthwhile to stress that the first two items refer more to language or knowledge, whereas the last two have a very metaphysical flavour. *Sinn/ Bedeutung* occupies the middle place. The last parentheses, with the question about the relationship between phenomena and possibilities, indicate that there is indeed a gap between these two levels of analysis. A hypothesis that will be explored in the next section is that the notion of *Sinn* is the place of articulation between the conceptual element in the act of knowledge (what comes from us) and the metaphysical possibility.

In the first two lines after the fifth item, Gödel indicates what is common to all the oppositions and therefore what grounds the analogy, which justifies the assertion in the first line of the *Bemerkung*. On the one hand, the first term of each item belongs to what is conceptual and produced by a mind, "the light" of reason, says Gödel. The second items belong to what is actual and part of reality, that is, what can be illuminated. It is possible to recognise a difference from the first way of expressing the heart of the analogy (*Licht und Beleuchtetes*) and the second one (*was im Erkenntnisakt in uns ist und was vor uns*). There is no reference to human mind in the first couple (*Licht und Beleuchtetes*), while this reference is very explicit in the second. We will come back later on to this point.

The distinction outlined by Gödel between what comes from the mind and what is in front of it is important to understanding the relations between logic and mathematics and their respective contribution to our understanding of reality. Logic is to mathematics as what is conceptual (and therefore concerns possibility) is to what is actual (and therefore concerns reality). Gödel is very explicit about that in Wang's book (Wang 1996 8.6.1 and ff.). At the same time mathematics is to reality as what is conceptual is to what is actual and "factual", because mathematics does not directly concern the objects of the outer world and their properties and relations, but the properties and relations of the concepts that can be applied to our experience. This is very clearly stated in version V of the Carnap paper ((Gödel 1996) p. 360) with the example of the description of the reactions of electronic tubes. Mathematics adds to our knowledge in this very simple case the "general laws as to how systems of tubes connected in a certain way will react". Therefore, representing the general aspects of reality it can be seen in this respect as the light that allows us to understand what is in front of us. Nevertheless, mathematics is involved in the conceptual analysis of reality only from the extensional point of view and therefore it is only marginally concerned by the analysis of the fundamental concepts structuring it.²²

22 Gödel affirms this clearly in (Wang 1996): n°8.2.4 and n°8.6.2. In the latter section the opposition between logic and mathematics is clearly stated in terms of that between intensions (concepts) and

3. The double role of concepts: joining metaphysics to theory of knowledge

The content of the observations analysed in section 2 allows us clearly to affirm that there is no possibility to assimilate *Bedeutungen* of conceptual terms to extensions because intensional aspects of language concern also *Bedeutungen* (2.1). They allow us also to understand how Gödel seems to reinterpret Frege's notion of *Sinn* as the cognitive component in the use of language (2.2-2.3). Considering Leibniz's distinction between *ideas in mente homini* and *ideas in mente Dei*, it seems that concepts as *Sinn* are for Gödel *ideas in mente homini*, that is, conditions of possibility for the use of language and for knowledge. But our conjecture (a) postulated that the four terms extension/intension, Sinn/Bedeutung were necessary to him in order to reassess the distinction between what is real and what is possible together with the Leibnizian distinction between *ideas in mente homini* and *ideas in mente Dei*, i.e. between a concept of possibility linked to human conceivability and a metaphysical notion of possibility. We have until now not yet presented any evidence of a possible use by Gödel of concepts as *ideas in mente Dei*, that is objective structures of the possible and of the real.

The two last *Bemerkungen* presented give evidence in this sense and corroborate our conjecture (a) in at least two points:

- I. the first one indicates clearly that concepts have to be considered as objective entities (ideas, universals) which are structures of reality, i.e. they hold the world together and we perceive them although only in an incomplete way; and
- II. the second *Bemerkung* confirms that concepts as objective entities—independent of our constructions or conventions—are explicitly called *Bedeutungen* in opposition to *Sinn*. It explains the notion of *Sinn* as a procedure, performed by a mind applying concepts to reality.

They also give us an insight about the wideness of the metaphysical and epistemological and logical problems related to the Gödelian notion of concept and therefore leave us with other open questions. Both the problem of the *esti*, and the intensional paradoxes are evoked here in a context, which is mainly metaphysical and epistemological.

3.1 Concepts as God's servants and structures of the real

Bem<erku>ng (*Phil<osophie>*): Wodurch unterscheiden sich die Begriffe als Gegenstände betrachtet von anderen Gegenständen? Das heißt, was ist das Wesen des Begriffes (der Ideen, *Universale*), insofern sie <=die Begriffe> nicht Konstruktionen des menschlichen Geistes [Regeln zur Erzeugung einer scharfen Einteilung aller Dinge] sind? Man meint mit „Begriff“ meistens das letztere, denn Ideen haben ursprünglich eine rein *psychol<ogische>* Bedeutung. Auch betrachtet man „wohldefinierter Begriff“ als eine *Species* der Begriffe, nicht aber „wohl definierter Mensch“ als eine *Species* der Menschen. Die Frage involviert auch die Analyse des „ε“ (oder der „Anwendung“ eines Begriffes auf ein Argument).

extensions (sets). We will come back to this subject in our conclusion. *Max Phil IX* contains another important *Bemerkung* on the difference between logic and mathematics on page 62.

Jeder Gegenstand, von den leblosen bis zu den Engeln ist seinem Wesen nach etwas, das „reagiert“, einen „Charakter“ hat (d. h. es folgt ein bestimmter „*actus*“ auf eine bestimmte *passio*), auch Begriffe <sind> vielleicht in ihren<?> Weise<n> aufzufassen. Aber Unterschiede gegen<über> Gegenständen:

1. Ihre geringere Zahl,
2. dass sie mit einem anderen Organ wahrgenommen werden (und jederzeit zur Wahrnehmung bereitstehen),
3. dass die Gegenstände nur „durch Begriffe“ wahrgenommen werden. [53] Das ist die einzige Art und Weise wie wir sie von anderen unterscheiden und „wiedererkennen“. Daraus braucht man aber nicht zu schließen, dass die Begriffe nur dazu da sind, um uns eine Wahrnehmung der Gegenstände und eine Orientierung in der Welt zu gestatten (das ist die *psychol<ogische>* Auffassung: Sie sind „Organe“ des Geistes), **sondern umgekehrt, weil sie das sind „was die Welt zusammenhält“**. **So ist nur durch ihre Wahrnehmung** eine Erkenntnis²³ möglich und deswegen haben wir eine unmittelbare „Kenntnis“ von ihren Inhalten. Gott befiehlt seinen Geistern, dass sie nicht nur seine, sondern auch unsere Diener sind.
4. Vielleicht sind die Begriffe „Geister“ und dass einem Ding ein Begriff zukommt, ist bloß ein Zeichen dafür, dass dieses Ding von diesem Begriff „gewählt“ wurde. In dieser Wahl [d. h. der *ε*-Relation] <ist> das Ding das *Pass<ive>* und der Begriff das Aktive.²⁴ Daher, wenn sich die *ε*-Relation ändert, so sagt man nicht, dass der Begriff sich geändert <hat>, sondern das Ding. Die Gesetze, nach denen diese Wahl erfolgt, sind bestimmt durch den Charakter des Begriffes und seinen Gehorsam gegen Gott [das sind die Naturgesetze]. [54] Insofern die Dinge „aktiv“ sind, sind sie das, was Begriffe sind (sie zeigen dadurch ein bestimmtes „Wesen“), aber **das Aktive in den nicht-begrifflichen Dingen hat unmittelbar keine Veränderung in einem von ihnen verschiedenen Ding zur Folge**, sondern das geschieht erst dadurch, dass „Gott seine Diener schickt, dies zu bewirken“. Was der Mensch tun kann, ist nur ein „Wollen“, für die Wirkung desselben ist er aber nicht die Ursache. Die beiden Wahrheitswerte repräsentieren die beiden möglichen „Handlungen“ eines Begriffes gegenüber einem Ding. Jede Tatsache ist also letzten Endes eine „Handlung“ *actus* irgend jemandes.²⁵ Der Unterschied gegen<über> unseren Handlungen besteht darin, dass die Begriffe als Motivation gewissermaßen das ganze Universum vor sich haben und zwar die wirklichen Dinge, wir nur einen kleinen Teil und zwar im Bild dargestellt durch die „Organe“ der Erkenntnis. Ebenso ist unsere Wirkung nur „mittelbar“. Das Merkwürdige ist, dass in diesem Sinn die *primitiveren* Dinge [z. B. Atome] der Wirkungsweise der [55] Begriffe ähnlich<er> wären als die höheren (wie der Mensch). Aber es ist denkbar, dass das eben nur für einfache Begriffe gilt (welche tatsächlich etwas „Physisches“ sind), dass dagegen die komplizierten <Begriffe> (wie sich schon an ihrer Struktur zeigt) ein Heer von „Dienern“—die untergeordnete Begriffe—haben, welche unserem Gehirn und unseren Handlungsorganen entsprechen (oder ist die Wirkung beim

23 Und Orientierung in der Welt. *Gödel's footnote.*

24 Das „Beherrschende“. *Gödel's footnote.*

25 Oder die beiden möglichen Handlungen des Dinges gegen sich [mit der Folge der Unterordnung unter den Begriff] bei Begriff<en>, welche einen *actus* des Dinges ausdrücken. Die einzige Aktivität der Dinge besteht also darin, dass sie sich gewiss<en> Begriffen unterordnen. *Gödel's footnote.*

Begriff immer „unmittelbar“ und gerade das das Charakteristische? Mittelbar ist nur die in *Def<initionen>* ausgedrückte Bedingung, unter der er wirkt, d. h. „wählt“). Das Ganze ist ein „**Begriffs-Animismus**“ oder logischer *Animismus*. Der Unterschied zwischen Begriff und Ding ist also, dass die Dinge das „rein Leidende“ sind,²⁶ d. h. sie haben keine andere Wirkung als auf sich selbst. Das Einzige, was sie tun können, ist „zu wollen zu sein“. Der *Solips<ist>* in diesem Sinn scheint mehr berechtigt <zu sein>, als der, welcher sagt, dass wir nur Dinge, die in uns sind, wahrnehmen können. Der zweite Unterschied²⁷ zwischen Ding und Begriff ist, dass nur der Letztere die Verknüpfungen liefert, welche die Welt zu einer „Struktur“ machen, d. h. < das ist > insofern richtig, dass die Beziehungen zweier Dinge nur durch Vermittlung der Begriffe, zu denen sie in Beziehung stehen, zustande kommen. Sogar: Auch die Wahrnehmung anderer Dinge <kommt> nur durch Vermittlung der Begriffe <zustande>. Der Zustand, in dem wir sind, wenn wir einen Begriff wahrnehmen, ist die zweite Art wie Begriffe „auswählen“ [nach *Aristot<eles>* letzten Endes dasselbe, da die Wahrnehmung bedeutet, dass die Form „in uns“ ist]. (pp. 52-55).

The question opening the first remark, concerns what Gödel calls *Gegenstände*, that is objective entities which have an essence, a capacity of acting and reacting in the real world and are therefore real in the plain sense of the word.²⁸

It is important to stress the steps through which Gödel articulates his question. Every entity in the scale of the beings is characterized from its essence, which determines its ability to react. If concepts are not considered as creations of the human mind, but as entities of the world that can be perceived, then they can also be considered as entities having a character. Is there something that differentiates them from the other entities in such a perspective—that is to say from non-conceptual things?

Gödel proposes his own analysis in four points, whereas the last is the longest one and contains the definite answer to the initial question. This conception can be summed up as a sort of logical animism, as he calls it, and represents one of the most amazing expressions of his *Begriffsrealismus*. The first three points in the *Bemerkung* prepare the explication of the latter assertion. Considering concepts not as constructions of the human mind, that is as rules to differentiate objects, but as ideas, or universals, i.e. properties and relations of the other *Gegenstände*, (i.e. entities being instantiated by different concrete objects) the first difference obviously concerns their numbers: one idea can be instantiated and exemplified by many entities, in consequence ideas are fewer than any other kind of entities. The second concerns the way we learn about them: not through our senses but by a special organ that Gödel postulates.²⁹ The third difference concerns the process of this learning. It is direct, and the perception of things (our ability to identify and recognise them) can only be achieved through ideas. The fourth point presents a non-Leibnizian doctrine according to which ideas are themselves minds and not only articulations of God's mind.

26 Begriffe sind „auch Leidende“. *Gödel's footnote*.

27 Der aber aus dem Ersten folgt. *Gödel's footnote*.

28 *Max Phil IX* contains a *Bemerkung* on p. 66, where Gödel speaks about what he calls the „*oberste Einteilung der Gegenstände*“. ‘*Gegenstände*’, he says later on, can be ‘*Einheiten*’ or ‘*Vielheiten*’, where the latter are identified with concepts, and the former with things (*Dinge*).

29 See Russell paper (Gödel 1990), p. 128 and (Wang 1974), p. 85.

There is no mentioning in this remark of the Fregean opposition between *Sinn* and *Bedeutung*, nor of the logical one between intension/extension. The exposition here is mainly a metaphysical one but it is said from the beginning that this metaphysical analysis has consequences on the logical analysis of the *esti* (relation of the application of a concept to its argument).

There are two passages of this *Bemerkung*, which are fundamental to our purpose. We find the first at point 3 and the second at the end of the *Bemerkung*. Gödel stresses in point 3 the fact that concepts are for us necessary to recognize things and to orient ourselves in the world. But he adds that this is not evidence of what he calls “*die psychologische Auffassung*”. Concepts considered as ideas or universals are not the wheels or the organs of the human mind. On the contrary they are independent from it and are what hold the world together, as Gödel says quoting from Goethe’s *Faust*. The two last lines of point 3 are especially important for us because they offer a Gödelian interpretation or correction of Leibniz’s doctrine of the relationship between God and his ideas. In Gödel’s interpretation, God gives orders to his servants (the ideas) to be also our servant. Ideas are not only God’s servants but they also act in human minds allowing them to perceive reality.

One of the interesting consequences of this assertion that Gödel explores in point 4, concerns the interpretation of the truth-values. If the kind of logical animism proposed by Gödel is correct then the *esti* relation is the mark of the choice accomplished by the concept. Here the things are passive, the concepts are active and the truth-values are the two opposite behaviours of the concepts vis-à-vis the other *Gegenstände*. The choice accomplished by the ideas is determined: 1. by the character of the concepts (their ability of *actio* and *passio* in relation to their essence); 2. by the laws of nature which act on concepts as constraints generated by God’s will to which concepts are submitted; and 3. in the case of human minds, by their will or wish to become different from what they are (their “*wollen*”). It is this last point that illustrates the relationship between the human mind and God’s mind through the mediation of ideas. The wish of the human mind causes the reaction to the ideas.

The Leibnizian idea of metaphysical possibility, as conceivability in God’s mind, is at work here, but in a wider context where ideas are also considered to be active entities. Therefore God’s thinking seems not to be related to a deterministic calculation but to a dynamic activity to which ideas but also human minds, through their wishes, participate. It is possible that Gödel’s aphorism in Wang’s conversation (“the meaning of the world is the separation of wish and fact”) is the later expression of this idea.³⁰

The last part of the *Bemerkung* contains a long comparative analysis of actions of concepts in respect to actions of the other beings. I will not comment on this but just stress that Gödel says that concepts are called, in this sense, structures of reality. They are what hold the world together because relations between things are possible only through the relations between the concepts under which they fall.

³⁰ (Wang 1996), 9.4.1, p. 309.

3.2 Concepts as *Sinn* distinct from their *Bedeutungen*, and our extensional talking about them

The last *Bemerkung* concerns the relationship between concepts as objective entities, (objective ideas) and concepts as procedures of the human mind (subjective or psychological). Gödel presents differences and similarities between these two ways of considering concepts in eleven points. From point 1 to point 6 Gödel considers mainly concepts as procedures applied by the human mind to orient itself in the world and he makes explicit reference to the possibility of referring extensionally to *Sinn*. Point 7 concerns concepts as universals and presents two ways of considering them: the Aristotelian one and the Platonic one, each of them requiring a specific interpretation of the relation *esti*. Point 8 comes back to the differences between procedures and universals in respect of the notion of application. Points 9–11 are inspired by Russell's analysis in chapter II of the *Principia*, where the vicious circle principles and the axiom of reducibility are introduced. We will come back to them in the article *Gödel and the paradox*, later in this book.

Bem<erku>ng (Phil<osophie>): Über das Verhältnis der Begriffe als idealisierte Verfahren und als Ideen [d. h. *subj<ektiv>* oder *psych<isch>* oder *obj<ektiv>*]

1. Im ersten Sinn ist ein Begriff eine Aussagefunktion [oft *sentent<ial> funct<ion>*], das heißt der Sinn eines Ausdrucks $\phi(x)$.
2. Besteht aber der Sinn einer Aussage in dem Verfahren ihrer Verifikation? Das würde heißen, jede Aussage hat die Form: Wenn ich mich so und so verhalte, geschieht das und das [auf eine solche Handlung folgt ein solches Leiden] und dann würde Sinn einer Aussagefunktion = Verfahren.
[Aber vielleicht kann der Sinn einer Aussage auch das Verhalten von etwas Anderem betreffen und dann wäre der Sinn einer Aussagefunktion nicht ein Verfahren, sondern vielleicht ein Begriff im objektiven Sinn?]
3. Unter *Def<inieren>* einer Sache versteht man die Kombination der einfachen Begriffe, welche die Sache ergibt [entweder in dem Sinn, dass es sie ist oder dass es sie beschreibt].
4. Der Geist erschafft Begriffe insofern als er die Elemente des Denkens kombiniert. Diese [59] {Am Oberen Rand der Seite 59 durchgestrichen: Alle die Beschreibungen zweiter Stufe, *Frege* Sinn und Bedeutung, beide anwesend, Begriff der Möglichkeit nach schon vorher vorhanden, logische Frage, die die Grundbegriffe betrifft—*psych<isch>* oder *met<aphysisch>*} Kombinationen haben zwar der Möglichkeit nach schon vorher bestanden, aber nur in dem Sinn, dass eben der Geist sie durch Kombination bilden kann. Kommt in einer solchen Kombination ein *ext<ensionales>* „alle“ vor, so handelt es sich um eine Kombination aller dieser Dinge, welche, da sie nicht wirklich ausgeführt werden kann, „beschrieben“ wird durch das Wort „alle“ [das „alle“ wäre also ein Sinn zweiter Stufe, welcher sich zum gewöhnlichen Sinn verhält wie dieser zu den Dingen, dass sie nämlich durch ihn „erfasst“, dem Denken zugänglich gemacht werden].
5. Daher kann man nicht *ext<ensional>* von allem Sinn³¹ sprechen, denn die so beschriebene Kombination ist *prin<zipiell>* undurchführbar, weil nach ihrer Durchführung nichts mehr möglich wäre [das wäre der Tod], dagegen <ist>

31 Das heißt „alle Begriffe“. *Gödel's footnote*.

objektiv vielleicht ein zeitloses und vollendetes Leben denkbar. Unser Leben ist nie vollendet, weil wir immerfort irren [d. h. falsch konstruieren, kombinieren]. Daher streben wir beständig und wird immer wieder Neues erschaffen. Dies Neuerschaffen wird ein Ende haben mit dem Ende der Welt, das heißt Gottes Geduld ist dann zu Ende.³²

6. **Daher kann man auch nicht von alle<n> a-Begriffen sprechen und involviert die impräd<ikative> Def<inition> von Begriff einen Zirkel, denn<?>: sie wäre eine Kombination von Dingen, unter denen das durch Kombination zu erzeugende Ding selbst schon vorkommt oder ein Verfahren dessen Durchführung zu einem Schritt führt, der in der Anwendung eben dieses Verfahrens besteht.**
7. Universalien sind entweder Teile [Splitter] der Dinge [*Arist<oteles>*], die unter sie fallen, dann gilt die *Abtrakt<ions>*-Theorie und *a b* bedeutet: *b* ist Teil von *a*, dann sind sie *Akz<idenzien>* und haben keine selbstständige Existenz, oder sie existieren abgesondert von den Dingen [sind *Subst<anzen>*] und die ε -Relation bedeutet: Der Begriff *b* hat das Ding *a* erwähnt.³³ Was zur Folge hat, dass in dem Ding jetzt eine Marke³⁴ dieser Wahl [das *Akz<idens>*] sich befindet. Im letzten Fall ist das „ ε “ eine *ext<erne>* Relation [welche nicht auf einer Qualität der Dinge beruht, sondern umgekehrt Qualitäten der Dinge zur Folge hat]. Im ersten Fall ist es eine *int<erne>* Relation, welche im Vorhandensein der Qualität in dem Ding besteht.
8. Im Falle der Begriffe ist das „ ε “ die Operation der „Anwendung“, d. h. vielleicht das versuchsweise Zusammenstellen, wobei die Aufmerksamkeit auf „Richtigkeit“ oder „Gleichheit“ gerichtet wird und das Resultat entweder „richtig“ [gleich] oder „unrichtig“ [ungleich] im Sinne einer „Evidenz“ ist. Die Kombination von der unter 4. die Rede ist, besteht in einer in bestimmter Reihenfolge wiederholten Anwendung dieser Operation, ausgehend von den Grundbegriffen, nämlich dem ihnen entsprechenden „Sinn“ [nicht ihrer Bedeutung, d. h. ihnen selbst].
9. Das Reduzibilitäts*sax<iom>* besagt, dass es zu jedem Begriff eine umfangreiche Idee (*Universale*) gibt; was man damit begründen kann, dass die Möglichkeit, die betreffende Klasse von Dingen auszusondern,³⁵ ihren Grund in einer objektiv gemeinsamen Beschaffenheit der Dinge dieser Klasse haben muss. Man könnte sagen, diese gemeinsame Beschaffenheit bestehe eben in dem, was die *Def<inition>* [besser das *Definiens*] sagt. Aber eine objektive Beschaffenheit kann nur im Vorhandensein gewisser Merkmale in den betrachteten Dingen bestehen, während durch *Def<inition>* diese Dinge gewissermaßen „von außen“ beschrieben sind. Eine *Universale* ist etwas Einfaches und die Beschaffenheit könnte höchstens im Vorhandensein mehrerer Universalien bestehen, aber nicht in einer Struktur von „Sinnen“, wie es das Definierende ist. Das *Def<iniens>* sagt, dass gewisse Operationen an dem Ding ausgeführt, ein gewisses Resultat haben.

32 Wollen wir überhaupt dieses vollendete Sein? Oder ist es ein Zug unseres Wesens, dass wir immer etwas Neues wollen und alles andere uns starr / tot vorkommt? Auch wenn alles zusammen da wäre → = Unersättlichkeit. *Gödel's footnote.*

33 Plato sagt „teilhaben“, was vielleicht heißt „ähnlich werden“. *Gödel's footnote.*

34 Ein Abbild des Begriffes. *Gödel's footnote.*

35 Mit Hilfe des Begriffes „alle“ und den logischen Operationen. *Gödel's footnote.*

Das ist nicht eine Beschaffenheit (insofern diese Teil des Dinges ist), sondern höchstens Kriterium für eine solche. Die Aussage, dass es zu jedem Begriff eine umfanggleiche Idee gibt, ist allerdings sicher falsch, wenn in voller Allgemeinheit formuliert.

10. Man kann durch Kombinationen von Sinnen einen neuen Sinn erzeugen, aber nicht durch Kombination von Ideen eine neue Idee.
11. Das {*ext<nsionale>*} Reduzibilitäts*sax<iom>* für Begriffe ist sinnlos für alle *a*-Begriffe und falsch für jede Ordnung. (pp. 58-62).

There are two assertions in this *Bemerkung*, which are particularly relevant to our problem. The first assertion is to be found in point number eight. Speaking about the possible combinations of concepts performed by the human mind, Gödel stresses that they are the results of the reiterated application of the *esti* relation starting from the primitive (fundamental) concepts considered as *Sinn* and he adds in parentheses that *Sinn* must be intended here as opposite to *Bedeutung*, whereas the *Bedeutung* of a primitive concept is nothing more than the concept itself. The second assertion is found in point number 5 (and also 6), where Gödel says that it is impossible to speak extensionally about all *Sinn*. In order to clarify these two assertions, I will comment upon the *Bemerkung* starting from point 7 and then coming back to points 1-4. This will give us some clues in order to better understand points 8, 5 and 6.

In the case of concepts considered as universals (7) we have two ways of looking at them, both having consequences on the interpretation of the *esti* relation. The first way, inspired by Aristotle, considers the universals as a part (fragments) of the primary substances (the things), which fall under them. In this case the universal is grasped by the process of abstraction and has no existence outside the thing itself. The relation of *esti*, the logical application of the universal to the thing, has to be seen as an internal relation, grounded in the thing and expressing the fact that the universal is part of the thing or, using the Aristotelian terminology, it is an "accident" of it, a quality of the thing.

The second way to consider universals is the Platonic, where they are independent from things and where the possible realisations of them are the phenomenal qualities that can be perceived by us. Gödel says that in this case the *esti* relation is an external one. Here again we encounter the terminology used in the previous remark to explain the *esti* as an external relation. Gödel says that the universals choose the things and that the qualities of the latter, perceived by human minds, are marks of these choices. These marks could be interpreted as a sort of phenomenal manifestation of the universals, through which we can grasp them.

How can we consider concepts now if we analyse them with respect to the notion of an idealised procedure performed by a subject (the second way to consider concepts that Gödel evokes in the beginning of the *Bemerkung*)?

It is possible to read 1-4 as giving four (not exclusive) aspects of the notion of a concept as an idealised procedure. In a first sense (1), they can be considered in a Fregean terminology as *Sinn* of unsaturated expressions as $\phi(x)$, or as sentential functions in the Russellian terminology, that is, the conceptual counterpart of propositions. In which sense can the *Sinn* of an unsaturated expression or of a sentential function be considered as procedures? One way of understanding this is to consider them as ways to associate objects with propositions. This fits with the interpretation

given in the previous *Bemerkung*, where subjective concepts are called rules for the repartition of things through which we recognise how an object can be differentiated from others through its properties and through the relations it has with them.

In a second sense (2), subjective concepts as *Sinn* can be considered as procedures to verify sentences. A concept as *Sinn* then takes the form of a sequence of instructions that have to be accomplished in order to assess if such and such object has such and such property or is in such and such relation to the others. The notion of *Sinn* viewed from this perspective has an operational content that can be made explicit in the form indicated by Gödel with a clear behaviouristic flavor: when I act in such and such a way such and such consequences follow. The parentheses in the second part of point (2) in Gödel's *Bemerkung* shows an alternative interpretation on which Gödel hesitates. If we assume the logical *animisms* of the previous remark than the behaviour expressed by the *Sinn* can belong not to human beings but to God, the master of concepts considered as his servants. In this perspective the *Sinn* does not express a human procedure but something objective related to God and to metaphysical conceivability, that is, God's ability to think up possible worlds.

In a third sense (3), subjective concepts can be considered as procedures to define entities, to produce or present them. In the latter sense, definitions are structures of *Sinn* (9), which constitute paths towards the *definienda*.

In a fourth sense (4) concepts are the results of combinatorial procedures. If we consider concepts as *Sinn*, relative to the human mind, we can conclude that complex concepts are obtained (even created) through combinations, starting from the fundamental ones, considered as the fundamental elements of thought. (Later in 10, Gödel says that we can create only a new *Sinn* from combinations of pre-existing ones but we cannot create ideas, as objective entities, through combinations of other ideas).

Now, in light of Gödel's contrast between objective ideas (7) and *Sinn* as subjective procedures (1-4) we can understand the content of 8. In the case of concepts considered as human procedures (*Sinn*), the *esti* relation is always an attempted association of a procedure to a thing, in order to verify if the application of the procedure to the thing is correct. Complex procedures, says Gödel, are obtained by applying the fundamental concepts one to the other through the *esti*. But these fundamental concepts that the human mind combines are not *Bedeutungen*, they are not the objective entities, which hold the world together. They are *Sinn*, that is, they are what correspond in our mind to the objective concepts. They are the subjective counterparts of the objective entities.

The same idea is expressed in the lines written by Gödel at the top of pages 59 and crossed out. Gödel explicitly mentions here the Fregean pair of notions *Sinn* and *Bedeutung* as already being present in philosophy before Frege and stresses the problem of the double interpretation of the primitive concepts in respect to human (psychical) or metaphysical conceivability. He calls it a logical problem, perhaps because the gap between the combination of the concepts as *Sinn* in the human mind, and the combination of concepts as *Bedeutungen*, in God's mind, expresses itself in the crucial problem of the intensional paradoxes.

What we are doing, by applying a concept (as a *Sinn*) to an argument through *esti* is an attempted recognition of the correctness of the combination on the base of the evidence available to us. In (5) Gödel stresses how we are often at fault in our striving to create, that is, to produce new combinations again and again. He also suggests

an analogy between our own life, which is always incomplete and our impossibility to speak extensionally of all *Sinn*, that is of all human concepts. This analogy gives us an interesting clue to understanding point 6. The constraint of predicativity applies to concepts as *Sinn* as they are produced by composition and therefore are not independent from the human mind and cannot be considered as actually given to the mind. They are only potentially there. For this reason the interpretation of generality in the extensional way, as referring to the class of all constructions that can be accomplished by the human mind, is impossible and similarly the concept of all concepts applying to *a* (the *a-Begriffe*)³⁶ is circular. It has to be stressed here that Gödel is talking of an extensional *all*. This implies that he is considering that the universal quantifier refers (in such an extensional interpretation) to the totality of the procedures, which can potentially be obtained through composition by the human mind.

4. Conclusion and perspectives

We conjectured that the four notions intension/extension, *Sinn/Bedeutung* were necessary to understand Gödel's *Begriffsrealismus*. The *Bemerkungen* analysed in sections 2 and 3 partially confirm this conjecture introducing new elements for our inquiry.

The first *Bemerkung* presented in section 2.1 asserts that there is an intensional learning of language and an extensional one. The former concerns properties and relations insofar as we are interested in their possible reciprocal connections. The latter concerns concepts as classes insofar as we are interested in the plurality of objects, which fall under concepts. This seems compatible with the fact that in his conversations with Wang, Gödel speaks about intensions and extensions as respectively the subject matter of logic and of mathematics, opposing properties and relation (intensions) to sets (extensions) [Wang 1996](8.6.1). The *Bemerkung* presented in 3.1 develops a similar line of thought from a metaphysical point of view and with another terminology. It opposes non-conceptual entities to conceptual ones (ideas and universals), the latter being considered as elements of reality as being what hold the world together, being components of facts.

Taking that for granted, it seems that Gödel is also interested in the difference between subjective concepts, that is concepts in the human mind, and objective concepts, that is structures of the possible and of the real, considered as articulations of God's mind. Gödel identifies the latter with the *Grundbegriffe*, the primitive concepts from which all the others can be obtained. In the *Bemerkung* presented in 3.2, point 8, Gödel qualifies these primitive concepts as *Bedeutungen*, opposing them to their subjective counterparts, called *Sinn*, and he analyzes the possibility of an extensional mode of speech concerning the latter. We interpret this as follows: it is possible to refer to *Sinn* (subjective properties and relations) in an extensional way, that is, considering classes of them, but such an extensional speech has an intrinsic limitation, owing to the nature of these entities.

36 See (Russell Whitehead 27), vol. 1, chapter II, section VI, p. 55.

As a consequence, we have two pairs, one concerning the level of being, the other applying to the conceptual level of mind (finite or infinite ones). On the one hand intensions are opposed to extensions because abstract structures (properties and relations) are opposed to classes (multiplicity of things). On the other hand conceptual structures in the human mind (*Sinn*) are opposed to conceptual structures in God's mind (*Bedeutungen*) structuring reality.

There are nevertheless at least two fundamental open questions related to such an interpretation. The first (I) concerns Gödel's interpretation of the Fregean notion of *Sinn* as something subjective. It seems incompatible both with Frege's doctrine of *Sinn* and with Gödel's own interpretation of the Fregean doctrine as it appears in the published Gödelian corpus mentioned. Actually in both the Russell and the Carnap paper, Gödel seems to consider the Fregean *Sinn* as something objective.³⁷

A possible clarification of this contradiction is given by the last *Bemerkung* presented in section 3.2:

- a) *Sinn* considered as idealised procedures of the human mind are inter-subjectively expressible through language. Gödel indicates four ways to consider *Sinn* (1-4) and none of them are related to the private states of the subject. This is enough to fit the Fregean doctrine according to which the notion of *Sinn* is open to everyone able to master a language. There is no doubt that Gödel's analysis, contrary to Frege's, is essentially concerned with the ways of acquisition of concepts and by the acts that human minds have to accomplish in order to form and use them. Nevertheless this difference of approach does not prevent Gödel from considering Frege's semantic distinction as a valuable one.
- b) The parentheses in point 2 express Gödel's fundamental terminological hesitation, which can conceal the apparent contradiction between the idea of *Sinn* as subjective procedures with the assertions in the Russell and the Carnap papers. Gödel says in parentheses at point 2 that the sense of a sentence can concern the behaviour of someone other than a human mind, and it seems quite natural, considering the content of the remark in section 3.1, to ascribe such behaviour to God. Concepts are therefore those objective articulations of God's mind (His servants) constituting for us an objective *Begriffswelt*, although they cannot exist independently of God's mind.

This conclusion leads up to our second question (II) concerning the relationship between *Sinn/Bedeutung* on the one side and possibility/reality on the other. Gödel says that objective *Grundbegriffe* as ideas, are *Bedeutungen* and opposes them to *Sinn* considered as human concepts. How is it possible to reconcile such an assertion with the hypothesis that he makes in the same *Bemerkung* at point 2, where objective concepts are considered as *Sinn*? How can we reconcile such an assertion with the content of the *Bemerkung* presented in 2.3 where we have on one side *Sinn* and possibility, opposed on the other side to *Bedeutung* and reality?

The solution to this problem is deeply related to the way in which Gödel re-actualises Leibniz's doctrine to the science of the 20th century. Its consequences spread to a large range of logical, ontological, and epistemological problems on

³⁷ They are considered respectively as the conceptual counterparts of existing facts and as something independent from human constructions and conventions.

which Gödel works throughout the *Max Phil*: the problem of the individual concepts and their real counterparts, the definition of time, the relationship between essence and accidents, the status of relations and so on. Only the analysis of all of Gödel's notebooks will (hopefully) give a solution to it. In the meantime, two little schemas can help to visualize the notions at stake justifying Gödel's hesitation.

The first schema has four entries and takes the content of the parentheses of point 2 in the *Bemerkung* presented in section 3.2 seriously together with the content of 2.3 where the difference between *Sinn* and *Bedeutung* is assimilated to that between the possible and the real. This distinction might correspond to the Leibnizian distinction between conceivability in God's mind and conceivability in the human mind. God conceives metaphysical possibilities, combining the fundamental ideas, which are articulations of His mind. In so doing He conceives the possible and the real. The latter is chosen by God as actual (*das Beleuchtete*) because in His omnipotent knowledge (*das Licht*) He recognizes it as the best of the possible worlds and gives it reality. The act of knowledge, accomplished by human minds is also an act of conceiving, i.e. an operation of the mind, which constitutes objects, throwing light on the sensory data coming to the subject from the outer world. Here *Sinn* and *Bedeutung* are opposed as possibility and reality. But it should be stressed that *Bedeutungen* are in space and time and *Sinne* are in the conceptual or logical space and might be objective or subjective. The difference between God's mind and human minds is the difference between the infinite and the finite, the objective and the subjective where the latter can be thought to approximate the first.

	God's Mind	Human Minds
<i>Sinn</i> = possibility = conceivability	Concepts as ideas = Objective <i>Sinn</i>	Concepts as idealised procedures = Subjective <i>Sinn</i>
<i>Bedeutungen</i> = reality	Things and Properties and relations of real things as components of facts = Space-time realisations of <i>Grundbegriffe</i> ,	

The second schema takes the assertion of point 8 in the *Bemerkung* presented in 3.2 seriously where the objective *Grundbegriffe* are called *Bedeutungen*. In light of the content of the *Bemerkung* presented in 2.3, it tries to reconcile the discrepancy in the following way: the notions of *Sinn* and *Bedeutung* should be interpreted as in the *Bemerkungen* of section 2.2, where they essentially play a role in relation to language. As a consequence, if we place ourselves from the human point of view, linguistic terms express subjective concepts (*Sinn* considered as idealised procedures) and refer to (*bedeuten*) two kinds of entities: the intensional ones (*ideas in mente Dei*) and the extensional ones (objects and facts of reality). Certainly concepts are also components of facts as Gödel affirms explicitly, but in the sense that their realizations in the space-time are components of reality.

Human concepts = Subjective concepts= <i>Sinn</i> as idealised procedures	
<i>ideas in mente Dei</i> Objective concepts (<i>Grundbegriffe</i>) (intensional <i>Bedeutung</i>)	Realisations of the <i>ideas in mente Dei</i> in time and space Reality, extensional <i>Bedeutungen</i> (real objects and facts)

According to this schema, facts and objects that are part of reality are the starting point from which our perceptions of the *Grundbegriffe* (as *idea in mente Dei*) can be realized. Reality is the “material” that we have to take into consideration to elevate ourselves in order to access the *Begriffswelt*, and this is of great importance for knowledge (the *Bedeutung*).

Whatever Gödel’s preferred solution is, with the help of our two diagrams, we can sketch the three kinds of problem that Gödel has to solve. They are clearly interconnected.

From an epistemological point of view the problem is to describe the way in which objective concepts are related to subjective *Sinn* and the way in which our perception of reality (containing objects and facts) is the result of such an interaction. Gödel affirms that we perceive concepts immediately, and that we perceive objects (clearly and distinctly) only through concepts. Is that so because objective concepts (as *ideas in mente Dei*) act directly on our mind? Or does the perception of the realisations of God’s ideas in space and time open to us an access to the *Begriffswelt* through a sort of detour *via materia*?

From a logical point of view the open question is the relationship between the application of ideas, and more exactly of the *Grundbegriffe*, through the objective *esti* in order to form complex concepts and the application of subjective concepts through their subjective counterpart in our mind. The gap between them is the source of the intensional paradoxes.³⁸

From a metaphysical point of view the problem is to describe the relationship between space-time reality and God’s ideas. This problem clearly implies the analysis of the concepts of possibility and necessity, and the question of time. The realisation of the *Grundbegriffe* (the *Bedeutungen* of point 8 in the *Bemerkung* presented in 3.2) in space and time implies their “localisation” and their “contextualisation”. Should this process be explained with the help of the Platonic notion of participation (point

38 Gödel says in the same *Max Phil IX* at page 48:

Bem<erku> (*Gr<ammantik>*): Letzter Grund für *Antin<omien>* ist, dass wir nicht sehen, was die ε -Relation eigentlich ist [im Reich der Begriffe], sondern wir sehen einen Ersatz in dem, was wir konstruiert haben. Ebensowenig sehen wir, was der Begriff „Begriff“ ist.

Remark (Grammar): The ultimate reason for the antinomies, is that we do not see what the *esti*-relation really is (in the realm of concepts), rather we see its substitute in what we have constructed. Just as we do not see what the concept “concept” is.

7 of the same *Bemerkung*)? And in which way is that compatible with the Leibnizian notion of substance and of individual concepts?

In the current state of our reflection, we have no definite answer to these questions, but at least we have a better understanding of the problems which constitute the background of the Russell paper. At the beginning of the paper, the discussion about the descriptive phrases, involves the whole problem.³⁹ It is not by chance, therefore, that Gödel closes his paper by confessing his trouble: “there is something behind it” he says “which is not yet completely understood”.⁴⁰

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³⁹ See (Crocco 2012) where in a long analysis of section 4, pp. 230-241, I discuss the relationship between the problem of descriptive phrases, the Gödelian interpretation of *Sinn* and *Bedeutung* and finally Gödel's use of the Leibnizian notion of individual concept.

⁴⁰ (Gödel 1990), p. 123.

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