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# **Water flow in soil at small water contents: A simple approach to estimate the relative hydraulic conductivity in sandy soil**

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*Running title: Soil hydraulic conductivity at small water content*

## **Summary**

The unsaturated soil hydraulic properties are key properties in the fields of soil science and civil engineering. Because of their strong dependence on water content, there are considerable experimental and numerical difficulties in their determination, specifically in the dry range. This situation is encountered regularly in arid and semi-arid regions. The models commonly used for predicting unsaturated hydraulic conductivity function rely on pore bundle concepts that account for capillary flow only and neglect film flow. Furthermore, the assumption of a local equilibrium between liquid water and its vapour is no longer valid at small water contents. Thus, with classical approaches, the experimental identification of hydraulic characteristics can fail at small water contents. To emphasize the weakness of capillary models, soil column experiments have been carried out with two sandy soils from Burkina Faso. Special care was taken to prevent any transport processes that are not directly related to liquid transport. Data from profiles of water content were introduced into an inverse numerical procedure to identify the coefficients of a new relative hydraulic conductivity function. Our results show that this simple approach is suitable for the analysis of flow processes at small water contents. It provides a simple, robust and inexpensive method to identify the properties of unsaturated conductivity function that account for capillary and film flows.

*Keywords: soil hydraulic property, soil water flow, soil water profile, modelling, capillary flow, film flow, relative hydraulic conductivity.*

## **Highlights**

- A new kind of soil column experiment is proposed to focus on the dry range.
- A new relative hydraulic conductivity function accounting for film flow is introduced.
- The weaknesses of classical relative hydraulic conductivity functions are emphasized.
- It provides a simple, cheap and robust method to identify the relative hydraulic conductivity.

## Introduction

The description of water flow processes in soil has been the focus of intensive research in the fields of agronomy, hydrology and geomechanics. If the situation associated with temperate regions where water content varies from intermediate to large seems to be well understood, the case for dry soil in arid and semi-arid zones still requires further investigation. The development of alternative modelling approaches for water resources management dedicated to arid soil is of prior importance for sub-Saharan countries such as Burkina Faso (Bénet *et al.*, 2009; Ouédraogo *et al.*, 2013).

As water content decreases, hygroscopic effects become predominant and adsorptive forces prevail over capillary ones (Or & Tuller, 1999; Tokunaga, 2009). Liquid water is present in the form of adsorbed layers around soil particles and its thermodynamic state can differ markedly from bulk water. Therefore, the modelling of transport phenomena at small water contents requires a specific description. In particular, it has been emphasized that vapour diffusion can contribute considerably to water transport (Saito *et al.*, 2006), which leads to reconsidering the classical assumption of a local equilibrium (Smits *et al.*, 2012; Ouédraogo *et al.*, 2013; Massman, 2015). In such situations, liquid filtration, vapour diffusion and phase change are simultaneous transport processes that must be taken into account to describe global water exchanges accurately in natural soil.

In particular, the physical processes that govern liquid water flow in the small water content range are still in question. In general, numerical modelling based on the Richards' equation requires the complete determination of its hydraulic properties, i.e. the hydraulic conductivity and water retention functions. Numerous attempts have been made to derive theoretically the relative hydraulic conductivity function directly from the water retention curve based mainly on capillary bundle models (Burdine, 1953; Mualem, 1976). Although fairly accurate at large water contents where water is held mainly by capillary forces, these descriptions generally underestimate the permeability at small water contents (Sakai *et al.*, 2009). The van Genuchten–Mualem permeability function is appropriate for intermediate water contents ( $w > 0.04 \text{ kg kg}^{-1}$ ), which highlights the shortcomings of capillary bundle models to predict the permeability function in this dry range of the soil (Kébré *et al.*, 2013a). Some theoretical extensions to the full range of water content have been proposed such as a segmented soil water retention function with a non-zero value of residual water content, (Zhang *et al.*, 2016). One challenge to derive closed form equations for the soil hydraulic conductivity function is

that it should be mathematically simple, continuously differentiable and physically consistent in the full range of soil water content (Peters, 2013; Peters, 2014; Iden *et al.*, 2015; Raneem *et al.*, 2016).

During the past decade, several authors have emphasized the major contribution of flow in thin liquid films, which are likely to become important at small water contents (Or & Tuller, 1999; Tuller & Or, 2001; Peters & Durner, 2008b; Tokunaga, 2009; Lebeau & Konrad, 2010). To address this issue, Tuller & Or (2001) proposed a mathematically complex model of unsaturated hydraulic conductivity resulting from film and corner flow. An empirical formulation has been proposed by Peters & Durner (2008b) to extend classical relative hydraulic conductivity functions to take film flow into account. Although easily integrated in the usual numerical models, this approach has been used successfully to fit previous experimental data better. Alternatively, some theoretical approaches have been developed for direct formulations of film flow dynamics (Tokunaga, 2009; Lebeau & Konrad, 2010; Wang *et al.*, 2013). The starting point is to partition water retention into capillary and adsorptive components to introduce the macroscopic contribution of thin film flow. Without any empirical coefficients, these models rely on physical properties that could be difficult to identify such as the Hamaker constant. More recently, correction factors have been proposed to reproduce experimental measures (Zhang, 2011).

In parallel to these theoretical descriptions, various experimental methods have been proposed to determine the relative hydraulic conductivity function. Multi-step outflow (MSO) (Eching *et al.*, 1994; Schneider *et al.*, 2006; Nasta *et al.*, 2011), multi-step inflow (MSI) (Fujimaki *et al.*, 2004), evaporation (Peters & Durner, 2008a; Schindler *et al.*, 2010) or condensation (Sakai *et al.*, 2009) methods provide experimental data to be used in the inverse numerical approach. In general, the use of tensiometers to measure the water potential at different depths sets a lower limit to the applicable water potential of about  $100 \text{ J kg}^{-1}$ . Indeed, if suction falls below the air-entry value of the tensiometer or below the water vapour pressure, discrepancies in the measurements are expected (Schneider *et al.*, 2006). These experimental difficulties would explain why most of the improved hydraulic functions proposed in the literature have been validated against the same set of experimental data published more than 20 years ago (Peters & Durner, 2008b; Lebeau & Konrad, 2010; Zhang, 2011).

In the above mentioned methods, the water flux is estimated globally through the continuous weighing of samples, and combining the contributions from liquid water and vapour. If the role of vapour diffusion is negligible in the intermediate moisture range, its relative

contribution to water transport becomes significant at small water contents. Thus, recent research has explicitly introduced an additional term to describe the unsaturated hydraulic conductivity,  $K$ , as the sum of capillary  $K^{\text{cap}}$ , film  $K^{\text{film}}$  and vapour parts  $K^{\text{vap}}$  (Peters *et al.*, 2015; Rudyanto *et al.*, 2015). This representation, however, assumes implicitly a local equilibrium between liquid and gaseous phases, whereas recent research questions the appropriateness of this hypothesis (Smits *et al.*, 2012; Ouédraogo *et al.*, 2013; Massman, 2015). In the case of dry soil, non-equilibrium between vapour and liquid water has been shown experimentally to require non-equilibrium modelling of water transport. Therefore, with standard methods such as the evaporation method, overestimation of liquid water flux might lead to an incorrect determination of relative hydraulic conductivity. Furthermore, the extrapolation of hydraulic functions towards the dry range can lead to numerical instabilities (Schelle *et al.*, 2013).

The aim of this paper is to propose a simple procedure to identify the hydraulic conductivity function in the dry range that accounts for both capillary and film flows. To be appropriate for non-equilibrium description, liquid flow processes must be identified separately from other transport mechanisms. We did a new kind of soil column experiment without the use of tensiometers on two kinds of sandy soil sampled in Burkina Faso, a sub-Saharan country. Profiles of the evolution of water content from the experiment were then used as data in a numerical inverse procedure to characterize the relative hydraulic conductivity of these sandy soils. This approach is a simple, robust and cheap characterization procedure. It provides data to overcome the shortcomings of the use of soil water potential in capillary bundle models to compute the hydraulic conductivity function in dry soil.

## **Materials and methods**

### *Basic soil properties*

Two upper layers of a natural Arenosol (IUSS Working Group WRB, 2006) from Nasso, Burkina Faso, (latitude 11.210733°, longitude -4.420386°) were investigated. The first soil sample, denoted NH1, was taken from the top layer of the soil (0–30-cm depth) and the second one, denoted NH2, comes from a greater depth (30–70-cm depth) in the same profile. The main morphological properties of the two samples are summarized in Table 1. According to the International Society of Soil Science (ISSS) classification, the textures of these samples

are sand for NH1 and loamy sand for NH2, which correspond globally to sandy soil. Figure 1 shows their particle-size distributions; the silty and clayey fractions are almost negligible.

Physical analysis was done by dry sieving after washing (France standard NF P94-056, 1996) for particles larger than 80  $\mu\text{m}$  and sedimentation analysis (France standard NF P94-057, 1996) for smaller particles. Bulk density,  $\rho_s$ , of each sample was measured by the core sampler method on undisturbed soil (France standard NF X31-501, 1992). If the soil particle density is assumed to be  $\rho_s^* = 2650 \text{ kg m}^{-3}$ , the saturated gravimetric water content,  $w_{\text{sat}}$ , can be determined. The saturated hydraulic conductivity  $K_{\text{sat}}$  was measured with a variable head permeameter (France standard NF X30-441, 2008).

### *Soil water retention curve*

Classical experimental devices were used to acquire data to characterize the soil water retention of the two sandy soil samples: (i) pressure plate apparatus at constant temperature (30°C) for large water contents that correspond to the funicular state and (ii) saturated salt solutions for small water contents in the hygroscopic state. For both methods, at equilibrium, gravimetric water content,  $w$ , was determined by weighing after oven-drying at 105°C for 48 hours. The results and discussion about water retention function modelling, especially at small water contents, are presented in Kébré *et al.* (2013b).

Various potentials have been used to characterize the thermodynamic state of water such as suction, matric potential, pressure head, relative humidity, water activity and so on. Following the ideas put forward by Bénet *et al.*, (2012) and Ouédraogo *et al.*, (2013), water content is plotted as a function of the absolute value of the water chemical potential rather than pressure head or suction. Indeed, a variety of micro-scale processes governs soil–water interactions and liquid pressure cannot be defined at small water contents. From its thermodynamic definition, the chemical potential can describe the state of water consistently over the whole range of water content, from saturation to dryness. It can be seen as a unifying concept that is well-defined within a thermodynamic framework.

The models used in our research are given below:

- VG80 (van Genuchten, 1980)

$$S_e = \frac{w - w_r}{w_{\text{sat}} - w_r} = \left[ 1 + (\alpha \mu_1)^n \right]^{-m} \quad (1)$$

- FX94 (Fredlund & Xing, 1994)

$$w = \frac{\left[ 1 - \frac{\ln \left( 1 + \frac{\mu_1}{\mu_r} \right)}{\ln \left( 1 + \frac{10^6}{\mu_r} \right)} \right] \frac{w_{\text{sat}}}{\left[ \ln \left( e + \left( \frac{\mu_1}{\mu_i} \right)^n \right) \right]^m}}{\quad} \quad (2)$$

- FS95 (Fayer & Simmons, 1995)

$$w = w_a \left( 1 - \frac{\ln \mu_1}{\ln \mu_0} \right) + \left[ w_{\text{sat}} - w_a \left( 1 - \frac{\ln \mu_1}{\ln \mu_0} \right) \right] \left[ 1 + (\alpha \mu_1)^n \right]^{-m} \quad (3)$$

In these equations,  $S_e$  is the effective saturation,  $w_r$  ( $\text{kg kg}^{-1}$ ) and  $w_{\text{sat}}$  ( $\text{kg kg}^{-1}$ ) are the residual and saturated water content, respectively,  $w_a$  ( $\text{kg kg}^{-1}$ ) is a curve-fitting parameter that represents the soil water content at  $\mu_1 = 10 \text{ J kg}^{-1}$ ,  $\mu_0$  ( $\text{J kg}^{-1}$ ) is the chemical potential at oven dryness, usually taken as  $10^6 \text{ J kg}^{-1}$ ,  $\mu_r$  ( $\text{J kg}^{-1}$ ) is a parameter used to adjust the lower portion of the curve,  $\mu_i$  ( $\text{J kg}^{-1}$ ) is a fitting parameter closely related to the air-entry value for the soil and  $\alpha$  ( $\text{kg J}^{-1}$ ),  $n$  and  $m$  are fitting parameters.

### *Laboratory column experiments*

As mentioned above, to obtain experimental data that enables us to determine the hydraulic conductivity at small water contents remains important, but measurements of water content profiles are a reliable alternative (Vereecken, 2008). Therefore, the objective was to determine the profiles of gravimetric water content in a homogeneous soil column subjected to a water potential gradient.

A robust and inexpensive experimental approach is proposed. Two portions of wet soil were prepared and stored for 24 hours to homogenize the water content prior to compaction. Each portion of soil was prepared at the desired initial water content to reach the field-measured dry density given in Table 1, i.e. water content in the upper half of the column was  $w = 0.06 \text{ kg kg}^{-1}$ , whereas it was  $w = 0.02 \text{ kg kg}^{-1}$  in the lower part. Several soil columns made of PVC tubes (20-cm high and 7.5-cm diameter) were prepared with the portions of soil and were compacted in with a standardized Proctor hammer (Controlab, Paris, France). Finally, both column ends were hermetically sealed to avoid dehydration so that the average water content remained constant.



During one experiment, six soil columns were prepared with identical initial conditions and placed in a thermo-regulated chamber (Figure 2). Because a destructive method was used to determine the water content profile, one soil column was removed from the thermo-regulated chamber at given time intervals. The soil was then extracted from the PVC tube and sliced into several 5-mm thick samples. The water content of each slice was measured by differential weighing between the wet and dry sample after 48 hours of oven-drying at 105°C.

The experiment resulted in a redistribution of water from the upper to the lower part of the column because of the moisture gradient. The water activity was almost one between water contents of 0.02 kg kg<sup>-1</sup> and 0.06 kg kg<sup>-1</sup>, (Figure 3). Thus, vapour pressure is uniform over the soil column and is equal to the saturated vapour pressure, which prevents phase change and vapour diffusion. Therefore, the only water transfer process that can occur is liquid flow governed by capillary and gravity effects.

#### *Relative hydraulic conductivity*

Several attempts have been made to predict the relative hydraulic conductivity function theoretically. Most statistical models conceptualize the pore space as either bundles of cylindrical tubes of uniform size or assemblies of cylindrical capillary tubes of various sizes (Lebeau & Konrad, 2010). The most widely used formulations are those of Burdine (1953) and Mualem (1976):

$$k_r(w) = S_e^q \left\{ \frac{\int_{w_r}^w \frac{dw}{\mu^\xi(w)}}{\int_{w_r}^{w_{sat}} \frac{dw}{\mu^\xi(w)}} \right\}^\varepsilon \quad (4)$$

where  $\mu(w)$  describes the soil-water retention curve,  $q$ ,  $\xi$  and  $\varepsilon$  are specific constants ( $q=2$ ,  $\xi=2$ ,  $\varepsilon=1$  in Burdine (1953) and  $q=0.5$ ,  $\xi=1$ ,  $\varepsilon=2$  in Mualem (1976)).

van Genuchten (1980) substituted his model of the soil-water characteristic curve (Equation (1)) in the above statistical model, and derived a closed-form expression for the relative hydraulic conductivity function. One of the most popular models used in soil science is the van Genuchten–Mualem model (denoted by VG80–M) given by:

$$k_r(S_e) = \sqrt{S_e} \left[ 1 - \left( 1 - S_e^{\frac{1}{m}} \right)^m \right]^2 \quad (5)$$

where  $m = 1 - 1/n$  is the shape parameter from Equation (1).

Similarly, models of the soil-water retention curve proposed by Fredlund & Xing (1994) and Fayer & Simmons (1995) (Equations (2) and (3), respectively) did not lead to a closed-form equation, and a numerical solution is required to determine the relative hydraulic conductivity function with Equation (4). We denote the relative hydraulic conductivity models computed from Mualem's (1976) integral formulation in conjunction with the Fredlund & Xing (1994) and Fayer & Simmons (1995) water retention models as FX94–M and FS95–M, respectively.

Theoretical and experimental investigations suggest that water transport processes that take place at large water contents (low chemical potential) are quite different from those that occur at small water contents (high chemical potential) (Or & Tuller, 1999; Peters & Durner, 2008b; Tokunaga, 2009; Lebeau & Konrad, 2010). In dry soil, the water phase is distributed in thin films that do not behave as bulk water.

The pioneering research of Or & Tuller (1999) and Tuller & Or (2001) provides a set of equations that are derived from hydrodynamic considerations based on the assumptions of a specific pore geometry to account for capillary and film flows. Their mathematically complex model is not easily associated with one of the classical descriptions of the soil-water retention curve. To simplify, the relative hydraulic conductivity function for unsaturated soil can be described as a linear superposition of the contributions of capillary flow,  $k_r^{\text{cap}}$ , and film flow,  $k_r^{\text{film}}$ , by the following (Peters & Durner, 2008b; Lebeau & Konrad, 2010; Zhang, 2011):

$$k_r = k_r^{\text{cap}} + k_r^{\text{film}}, \quad (6)$$

where  $k_r^{\text{cap}}$  can be described by a classical pore bundle model, whereas  $k_r^{\text{film}}$  has various formulations that depend on the authors.

To represent the relative contribution of film flow, Peters & Durner (2008b) introduced a weighting coefficient,  $\omega$ . The values of  $\omega$  identified from experimental data range from  $10^{-7}$  to  $10^{-2}$ , which suggests that this coefficient should be seen more appropriately as a physical characteristics rather than as a weighting coefficient.

Similarly, Lebeau & Konrad (2010) describe the relative effect of film flow based on a partitioning function that depends only on the water content. In contrast to using a weighting coefficient, their formulation combines the capillary and film flow models without requiring any additional fitting parameter and is based on a three-parameter water retention function derived from Campbell & Shiozawa (1992) and Kosugi (1999).

When power functions of the saturation are used to describe  $k_r^{\text{cap}}$  and  $k_r^{\text{film}}$  as proposed by Peters & Durner (2008b), a weighting function is not required because each term rapidly becomes negligible with respect to the other as water content varies. Therefore, we proposed to use Equation (6) directly without any weighting coefficient. With a simple power function for an empirical approach to describe relative film flow (Peters & Durner (2008b), the ‘unconstrained new function’ for relative hydraulic conductivity is the following:

$$k_r(S_e) = \sqrt{(S_e)} \left[ 1 - \left( 1 - S_e^{\frac{1}{m}} \right)^m \right]^2 + \beta S_e^\gamma, \quad (7)$$

which introduces two fitting coefficients,  $\beta$  and  $\gamma$ . This formulation has the advantage of two independent terms to describe capillary and film flow that are not coupled through a weighting factor. In general, it improves identification of the coefficients in inverse analysis.

#### *Computational approach*

Experimental conditions ensure that liquid flow occurs only in the soil columns. The numerical modelling is based, therefore, on the classical Richards equation. Following B enet *et al.*, (2012) and Ou edraogo *et al.*, (2013), the mass balance equation is written in term of gravimetric water content and the gradient of chemical potential is taken as the controlling force of the liquid flow:

$$\frac{\partial w}{\partial t} - K_{\text{sat}} \frac{\rho_1}{\rho_s g} \frac{\partial}{\partial z} \left[ k_r \left( \frac{\partial \mu_1}{\partial w} \frac{\partial w}{\partial z} + g \right) \right] = 0 \quad (8)$$

where  $g$  ( $\text{m s}^{-2}$ ) is the gravity acceleration,  $k_r$  is the relative hydraulic conductivity function,  $K_{\text{sat}}$  ( $\text{m s}^{-1}$ ) is the hydraulic conductivity at saturation. The mass density of water  $\rho_1$  is taken to be  $1000 \text{ kg m}^{-3}$ .

This equation is discretized based on a finite-volume formulation with a one-dimensional regular mesh where the unknown,  $w$ , is at the centre of the grid blocks. A first-order upstream scheme is used to describe the convective term. Temporal discretization is done by a fully implicit scheme to obtain unconditional numerical stability. To handle non-linearities, a Newton–Raphson method ensures accurate convergence with a moderate time step. This numerical modelling was validated with HYDRUS-1D software (Simunek *et al.*, 2008) by comparing the evolution of saturation profiles under various conditions.

To identify the coefficients of the relative hydraulic conductivity function (Equation (7)), the inverse analysis was based on numerical evolution of the water content profiles by Equation (8). The criterion to be minimized is based on the root mean square error (RMSE), with the difference between experimental and numerical data obtained from the water content profiles at four successive time steps measured from the soil column experiments:

$$\text{RMSE} = \sqrt{\frac{1}{nm} \sum_{i=1}^n \sum_{j=1}^m [w^*(z_i, t_j) - w(z_i, t_j)]^2}, \quad (9)$$

where  $w^*$  and  $w$  denote the experimental and predicted water contents, respectively, at depth  $z_i$  and time  $t_j$ ,  $n = 38$  is the number of observation depths and  $m = 4$  is the number of profiles.

The numerical minimization procedure was based on the Nelder–Mead downhill simplex method or Levenberg–Marquardt algorithm. Both approaches lead to identical coefficients, which emphasizes the robustness of this inverse analysis.

## Results and discussion

The sorption isotherm and water retention curves are given in Figures 3 and 4 for the two sandy soil samples under investigation. The values of the coefficients of Equations (1) to (3) are given in Table 2, and the corresponding retention curves are represented in Figure 4(a,b). Because we aim to predict the liquid water flow process at small water contents, the concept of residual water content is meaningless and we assume  $w_r = 0$  in Equations (1) and (7). Globally, a good agreement between experimental and predicted data is achieved with the three soil water retention models. However, at very small water contents, the model proposed by Fredlund & Xing (1994) leads to the best fit to experimental data.

With identical boundary and initial conditions, the water content profiles predicted with Equation (8) for sample NH1 are compared to experimental data in Figure 5. Kébré *et al.* (2013a), have already shown that the use of the van Genuchten–Mualem model (Equation (5)) for the relative hydraulic conductivity, VG80–M, in Equation (8) leads to good agreement between predicted and experimental water content profiles for intermediate water contents ( $w > 0.04 \text{ kg kg}^{-1}$ ), whereas there are marked discrepancies at small water contents. Even the flexibility introduced by taking independent values of  $m$  and  $n$  (van Genuchten & Nielsen, 1985) for Equation (4), the related hydraulic conductivity function, does not enable the

prediction of water content profiles that match experimental profiles at the small water content range. Although the retention curve models from Fredlund & Xing (1994), FX94, and Fayer & Simmons (1995), FS95, fit best to the experimental soil-water retention data at small water contents (Kébré *et al.*, 2013b), their related relative hydraulic conductivity, FX94–M and FS95–M, used to predict water content profiles from Equation (8) lead to unsatisfactory descriptions of flow processes (see Figure 5b) because the relative hydraulic conductivity is overestimated.

Even if the VG80–M model gives better results than FX94–M and FS95–M, the three approaches fail in the range of small water contents. The results show that statistical models based on the capillary bundle concept are not appropriate for predicting the hydraulic conductivity at very small water contents. The drawback is that these models have assumed that the liquid thermodynamic state is identical over the full range of water contents and that a single concept can be used to describe water transport from saturation to dryness (Lebeau & Konrad, 2010). This shortcoming can be attributed to the neglect of film and corner flows that are probably the dominant flow process in the dry range (Peters & Durner, 2008b).

Coefficients for the relative hydraulic conductivity (Equation (7)) identified from soil column experiments with the inverse process are given in Table 3 for the two sandy soil samples, NH1 and NH2. Comparisons between measured and predicted water content profiles are shown in Figure 6, whereas the unsaturated conductivity functions ( $K = K_{\text{sat}} k_r$ ) are plotted as a function of chemical potential in Figure 7. The results from VG80–M and the model proposed by Lebeau & Konrad (2010) are also given for comparison. It emphasizes the film correction for small water contents (high chemical potential) where it diverges from the VG80–M description, and the unsaturated conductivity is several orders of magnitude larger.

Figure 6(a,b) shows the strength of the experimental and numerical approaches for inverse identification of the relative hydraulic conductivity for the sandy soil under investigation. Profiles of predicted water content are close to those of measured data and cohere to the features of global shape. A comparison of Figures 5(a) and 6(a) shows that introduction of the film flow correction clearly improves the description of water transport at small water contents ( $w < 0.04 \text{ kg kg}^{-1}$ ) without inducing marked modifications at intermediate water content ( $w > 0.04 \text{ kg kg}^{-1}$ ). The simple power function introduced in Equation (7) is adequate to account for film flow accurately.

There are some discrepancies, however. First, at the interface between the upper and lower parts of the soil columns, the sharp change in water content is not described correctly by numerical predictions. Because of the method of preparation of the soil columns, the bond between the two parts does not seem to be correct because water continuity was not ensured at the beginning of experiment. This might result in local variation in soil properties and introduce an additional hydraulic resistance. Clearly, numerical prediction tends to soften this water content front. Second, the error increases mainly with depth and time because the water has then crossed more heterogeneities (Lambot *et al.*, 2004). Indeed, an important cause of the discrepancies originates from the initial conditions because it is impossible to make the conditions in the six soil columns (replication scheme) exactly the same. This introduces a random bias that can impair the inverse analysis.

The evolution of unsaturated hydraulic conductivity as a function of chemical potential (Figure 7(a,b)) focuses on the two flow mechanisms, capillary flow for low chemical potential with a fast decreasing slope and thin film flow for large chemical potential with a slight decreasing slope. The transition between both states corresponds to the point where adsorptive forces predominate with respect to capillary forces (Lebeau & Konrad, 2010). This transition point for soil sample NH1 corresponds to a chemical potential of  $100 \text{ J kg}^{-1}$  and a water content of about  $0.04 \text{ kg kg}^{-1}$ . This corroborates previous investigations which have identified that film flow processes start to prevail when soil water potential becomes larger than  $100 \text{ J kg}^{-1}$  in various soil textures (sand, loam, silt) (Tuller & Or, 2001; Peters & Durner, 2008b; Zhang, 2011).

Figure 6(a,b) and Table 3 shows that the results are slightly better for the more sandy sample, NH1, than for the loamy sand, NH2, when the coefficient of determination  $R^2$  and the RMSE are compared. The two soil samples differ mainly in the amount of clay (Figure 1). This difference is emphasized through the sorption behaviour because NH2 is more hygroscopic than NH1 (Figure 3). The smaller water content ( $w = 0.02 \text{ kg kg}^{-1}$ ) imposed at the bottom of the soil columns is somewhat above the hygroscopic domain for the NH1 sample, but this is clearly not the case for the sample NH2.

## Conclusions

The term for film flow (a simple empirical relation) introduced in the relative hydraulic conductivity function clearly improves the prediction of water content in the sandy soil samples by increasing water transport in the dry range. We have shown a reasonable

agreement between experimental and predicted water content profiles, which underlines the strength of this approach. Therefore, the residual water concept becomes meaningless. Our approach provides a simple and robust experimental method to identify the relative hydraulic conductivity without the requirement of any sophisticated or expensive device. We still observed discrepancies, however, in particular when the clay fraction become non-negligible. Further investigations are required to address this issue with the experimental approach described above.

For developing nations, especially in regions that suffer drought, the implementation and the popularization of such a simple and cheap approach would contribute to efforts to manage land and water resources effectively for sustained agricultural production.

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### **References**

- Bénet, J.-C., Ramirez-Martinez, A., Ouedraogo, F. & Cherblanc, F. 2012. Measurement of the chemical potential of a liquid in porous media. *Journal of Porous Media*, **15**, 1019–1029.
- Burdine, N.T. 1953. Relative permeability calculation size distribution data. *Transactions of the American Institute of Mining and Metallurgical Engineers*, **198**, 71–78.
- Campbell, G.S. & Shiozawa, S. 1992. Prediction of hydraulic properties of soils using particle-size distribution and bulk density data. *Proceedings of International Workshop on Indirect Methods for Estimating Hydraulic Properties of Unsaturated Soils* (eds M. Th. van Genuchten, F. J. Leij & L. J. Lund), pp. 317–328, University of California, Riverside, CA.
- Eching, S.O., Hopmans, J.W. & Wendroth, O. 1994. Unsaturated hydraulic conductivity from transient multi-step outflow and soil water pressure data. *Soil Science Society of America Journal*, **58**, 687–695.
- Fayer, M.J. & Simmons, S.C. 1995. Modified soil water retention functions for all matric suctions. *Water Resources Research*, **31**, 1233–1238.
- Fredlund, D. G. & Xing, A. 1994. Equation for the soil-water characteristic curve. *Canadian Geotechnical Journal*, **31**, 521–532.

- Fujimaki, H., Inoue, M. & Konishi, K. 2004. A multi-step inflow method for estimating hydraulic conductivity at low pressure under the wetting process. *Geoderma*, **120**, 177–185.
- Iden, S. C., Peters, A. & Durner, W. 2015. Improving prediction of hydraulic conductivity by constraining capillary bundle models to a maximum pore size. *Advances in Water Resources*, **85**, 86–92.
- IUSS Working Group WRB, 2006. *World Reference Base for Soil Resources 2006*. World Soil Resources Reports No 103. FAO, Rome.
- Kébré, M. B., Cherblanc, F., Ouédraogo, F., Bénét, J.-C. & Zougmore, F. 2013a. Flow at low water contents: A simple approach for inverse estimation of van Genuchten–Mualem soil hydraulic parameters. *Proceedings of the 4th International Conference "HYDRUS Software Applications to Subsurface Flow and Contaminant Transport Problems"* (eds J. Simunek, M. Th. van Genuchten & R. Kodesova) pp. 203–212, Department of Soil Science and Geology, Czech University of Life Sciences, Prague.
- Kébré, M.B., Ouédraogo, F., Betaboalé, N., Zougmore, F. & Cherblanc, F. 2013b. Soil water characteristic curve modeling at low water content: Empirical and semi-empirical approaches. *International Journal of Engineering Research*, **2**, 432–437.
- Kosugi, K. 1999. General model for unsaturated hydraulic conductivity for soils with log-normal pore-size distribution. *Soil Science Society of America Journal*, **63**, 270–277.
- Lambot, S., Hupet, F., Javaux, M. & Vanclooster, M. 2004. Laboratory evaluation of a hydrodynamic inverse modeling method based on water content data. *Water Resources Research*, **40**, W03506.
- Lebeau, M. & Konrad, J.M. 2010. A new capillary and thin film flow model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research*, **46**, W12554.
- Massman, W.J. 2015. A non-equilibrium model for soil heating and moisture transport during extreme surface heating. *Geoscientific Model Development*, **8**, 3659–3680.
- Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research*, **12**, 513–521.
- Nasta, P., Huynh, S. & Hopmans, J.W. 2011. Simplified multistep outflow method to estimate unsaturated hydraulic functions for coarse-textured soils. *Soil Science Society of America Journal*, **75**, 418–425.



- Or, D. & Tuller, M. 1999. Liquid retention and interfacial area in variably saturated porous media: upscaling from single pore to sample scale model. *Water Resources Research*, **35**, 3591–3606.
- Ouédraogo, F., Cherblanc, F., Naon, B. & Bénet, J.-C. 2013. Water transfer in soil at low water content. Is the local equilibrium assumption still appropriate? *Journal of Hydrology*, **492**, 117–127.
- Peters, A., Iden, S.C. & Durner, W. 2015. Revisiting the simplified evaporation method: Identification of hydraulic functions considering vapor, film and corner flow. *Journal of Hydrology*, **527**, 531–542.
- Peters, A. 2013. Simple consistent models for water retention and hydraulic conductivity in the complete moisture range. *Water Resources Research*, **49**, 6765–6780.
- Peters, A. 2014. Reply to comment by S. Iden and W. Durner on "Simple consistent models for water retention and hydraulic conductivity in the complete moisture range". *Water Resources Research*, **50**, 7535–7539.
- Peters, A. & Durner, W. 2008a. Simplified evaporation method for determining soil hydraulic properties. *Journal of Hydrology*, **356**, 147–162.
- Peters, A. & Durner, W. 2008b. A simple model for describing hydraulic conductivity in unsaturated porous media accounting for film and capillary flow. *Water Resources Research*, **44**, W11417.
- Rudiyanto, Sakai, M., van Genuchten, M.T., Alazba, A.A., Setiawan, B.I. & Minasny, B. 2015. A complete soil hydraulic model accounting for capillary and adsorptive water retention, capillary and film conductivity, and hysteresis. *Water Resources Research*, **51**, WR017703.
- Sakai, M., Toride, N. & Šimůnek, J. 2009. Water and vapor movement with condensation and evaporation in a sandy column. *Soil Science Society of America Journal*, **73**, 707–717.
- Saito, H., Šimunek, J. & Mohanty, B.P. 2006. Numerical analysis of coupled water, vapor and heat transport in the vadose zone. *Vadose Zone Journal*, **5**, 784–800.
- Schelle, H., Heise, L., Janicke, K. & Durner, W. 2013. Water retention characteristics of soils over the whole moisture range: a comparison of laboratory methods. *European Journal of Soil Science*, **64**, 814–821.

- Schindler, U., Durner, W., von Unold, G. & Muller, L. 2010. Evaporation method for measuring unsaturated hydraulic properties of soils: Extending the measurement range. *Soil Science Society of America Journal*, **74**, 1071–1083.
- Schneider, K., Ippisch, O. & Roth, K. 2006. Novel evaporation experiment to determine soil hydraulic properties. *Hydrology and Earth System Sciences*, **10**, 817–827.
- Šimůnek, J., van Genuchten, M.T. & Šejna, M. 2008. Development and applications of the HYDRUS and STANMOD software packages and related codes. *Vadose Zone Journal*, **7**, 587–600.
- Smits, K.M., Ngo, V.V., Cihan, A., Sakaki, T. & Illangasekare, T.H. 2012. An evaluation of models of bare soil evaporation formulated with different land surface boundary conditions and assumptions. *Water Resources Research*, **48**, W12526.
- Tokunaga, T.K. 2009. Hydraulic properties of adsorbed water films in unsaturated porous media. *Water Resources Research*, **45**, W06415.
- Tuller, M. & Or, D. 2001. Hydraulic conductivity of variably saturated porous media – Film and corner flow in angular pore space. *Water Resources Research*, **37**, 1257–1276.
- van Genuchten, M.T. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, **44**, 892–898.
- van Genuchten, M.T. & Nielsen, D.R., 1985. On describing and predicting the hydraulic properties of unsaturated soils. *Annales Geophysicae*, **3**, 615–628.
- Vereecken, H., Huisman, J.A., Bogena H., Vanderborght, J., Vrugt, J.A. & Hopmans, J.W. 2008. On the value of soil moisture measurements in vadose zone hydrology: A review. *Water Resources Research*, **44**, W00D06.
- Wang, Y., Zhang, J., Ma, Y., Zhao, M. & Edmunds, W. M. 2013. A new theoretical model accounting for film flow in unsaturated porous media. *Water Resources Research*, **49**, 5021–5028.
- Zhang, Z.F., Oostrom, M. & White, M.D. 2016. Relative permeability for multiphase flow for oven-dry to full saturation conditions. *International Journal of Greenhouse Gas Control*, **49**, 259–266.
- Zhang, Z.F. 2011. Soil water retention and relative permeability for conditions from oven-dry to full saturation. *Vadose Zone Journal*, **10**, 1299–1308.

## TABLES

**Table 1** Textural characteristics, saturated gravimetric water content and saturated conductivity  $K_{\text{sat}}$ .

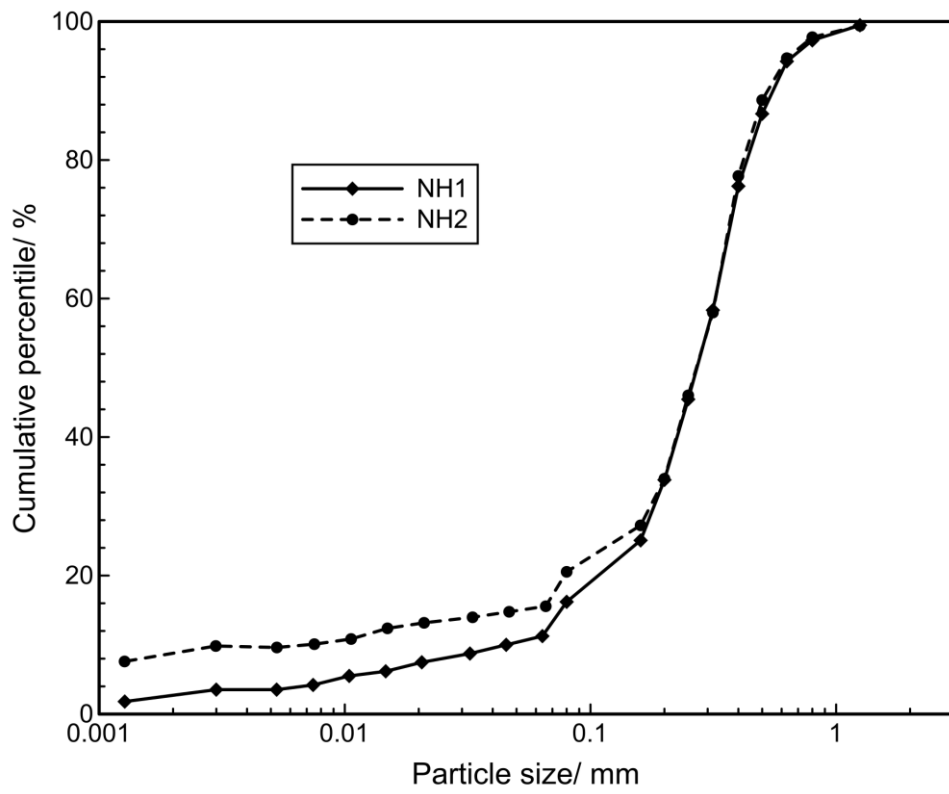
|     | Sand | Silt | Clay | $\rho_s$            | $w_{\text{sat}}$     | $K_{\text{sat}}$      |
|-----|------|------|------|---------------------|----------------------|-----------------------|
|     |      | /%   |      | /kg m <sup>-3</sup> | /kg kg <sup>-1</sup> | /m s <sup>-1</sup>    |
| NH1 | 90   | 7    | 3    | 1688                | 0.215                | $3.65 \times 10^{-5}$ |
| NH2 | 85   | 6.5  | 8.5  | 1652                | 0.228                | $9.78 \times 10^{-6}$ |

**Table 2** Evaluation of parameter values from Equations (1) to (3) for the water retention curve with the statistics  $R^2$  and RMSE for the accuracy of prediction.

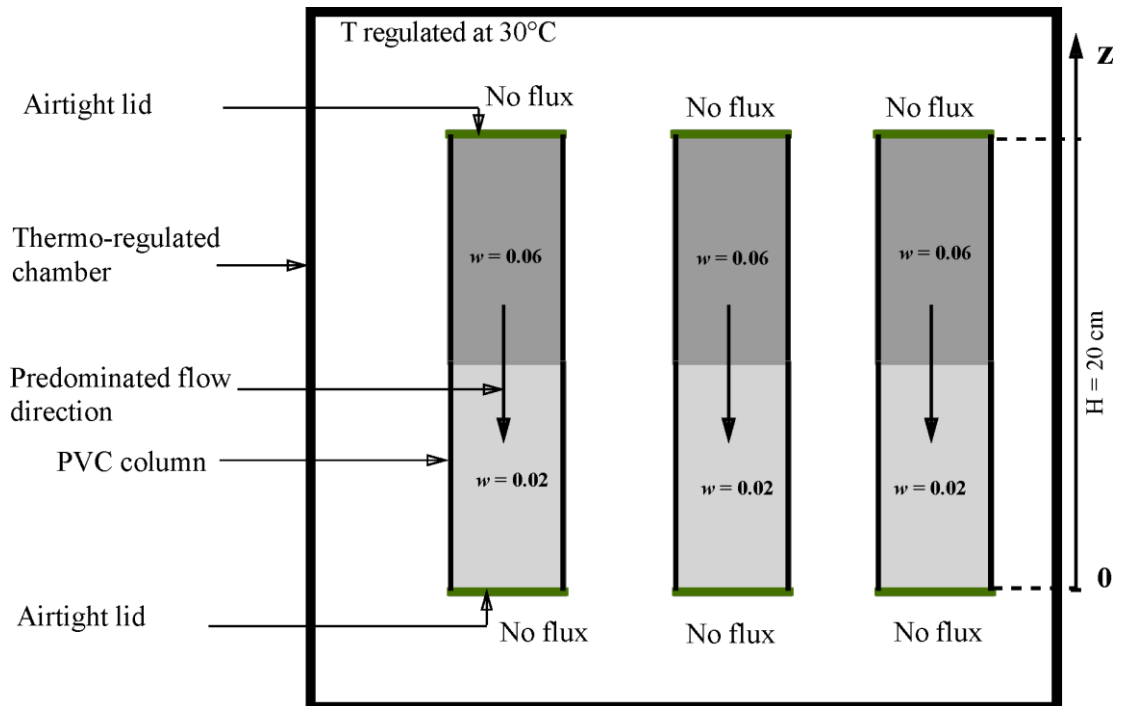
| Sample and model | $w_a$<br>/kg kg <sup>-1</sup> | $a'$<br>/kg J <sup>-1</sup> | $n$   | $m$       | $\mu_r$<br>/J kg <sup>-1</sup> | $\mu_i$<br>/J kg <sup>-1</sup> | $R^2$<br>/% | RMSE<br>/kg kg <sup>-1</sup> |
|------------------|-------------------------------|-----------------------------|-------|-----------|--------------------------------|--------------------------------|-------------|------------------------------|
| <b>NH1</b>       |                               |                             |       |           |                                |                                |             |                              |
| VG80             | –                             | 0.9127                      | 1.392 | $1 - 1/n$ | –                              | –                              | 99.30       | 0.004171                     |
| FX94             | –                             | –                           | 1.648 | 0.9318    | 50.03                          | 2.086                          | 99.84       | 0.003319                     |
| FS95             | 0.03377                       | 0.4375                      | 1.615 | 0.5069    | –                              | –                              | 99.79       | 0.003843                     |
| <b>NH2</b>       |                               |                             |       |           |                                |                                |             |                              |
| VG80             | –                             | 1.902                       | 1.297 | $1 - 1/n$ | –                              | –                              | 99.52       | 0.005225                     |
| FX94             | –                             | –                           | 1.431 | 0.7436    | 59.42                          | 1.067                          | 99.69       | 0.004231                     |
| FS95             | 0.04664                       | 1.544                       | 3.536 | 0.1271    | –                              | –                              | 99.69       | 0.004231                     |

**Table 3** Coefficients from Equation (7) for relative hydraulic conductivity and evaluation of the accuracy of prediction of gravimetric water content profiles with  $R^2$  and RMSE

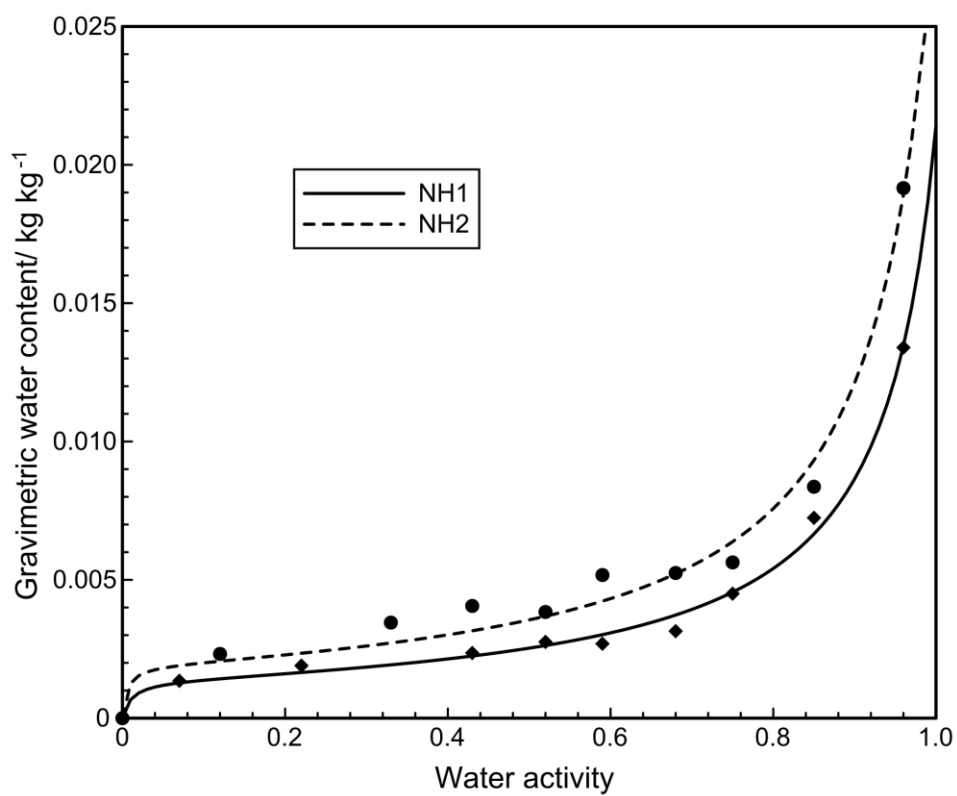
|     | $\beta$                | $\gamma$ | $R^2 /$<br>% | RMSE /<br>kg kg <sup>-1</sup> |
|-----|------------------------|----------|--------------|-------------------------------|
| NH1 | $2.405 \times 10^{-6}$ | 1.981    | 97.84        | 0.002141                      |
| NH2 | $5.490 \times 10^{-4}$ | 4.210    | 94.37        | 0.003215                      |



**Figure 1** Soil granulometric curves for soil samples NH1 and NH2.

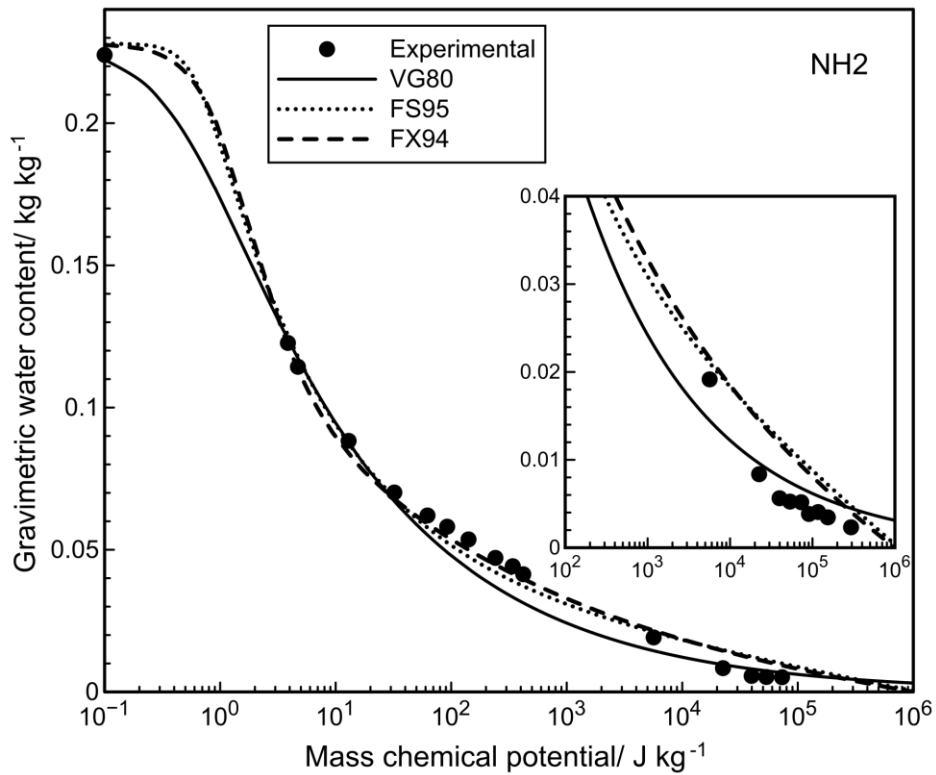
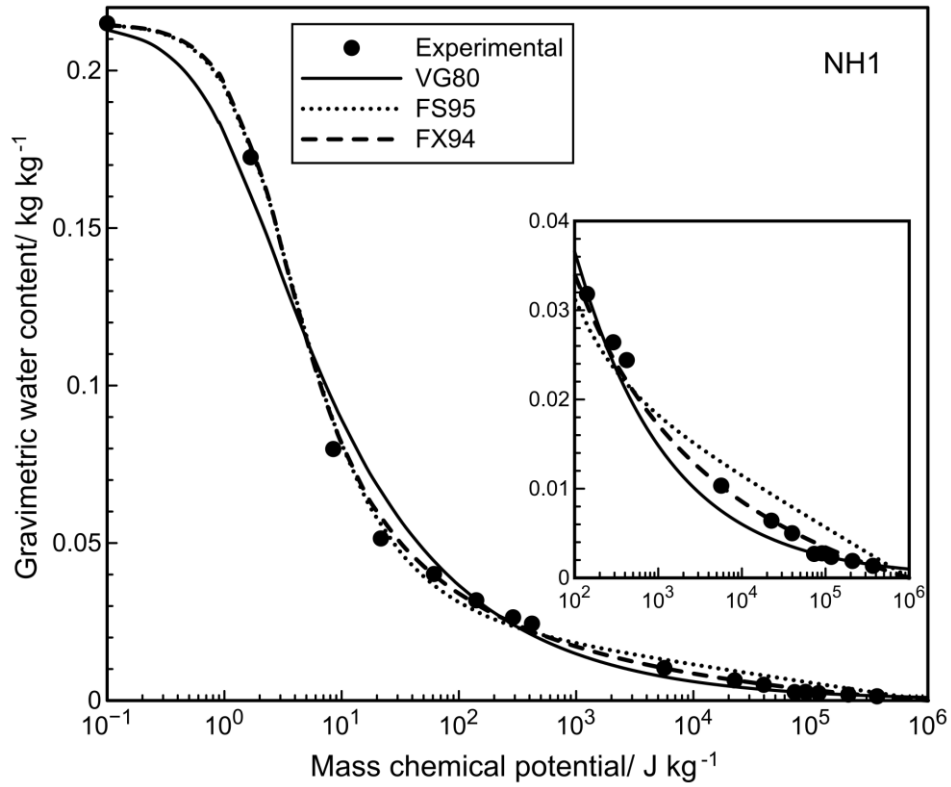


**Figure 2** Laboratory set-up of the soil column experiment. T is temperature.

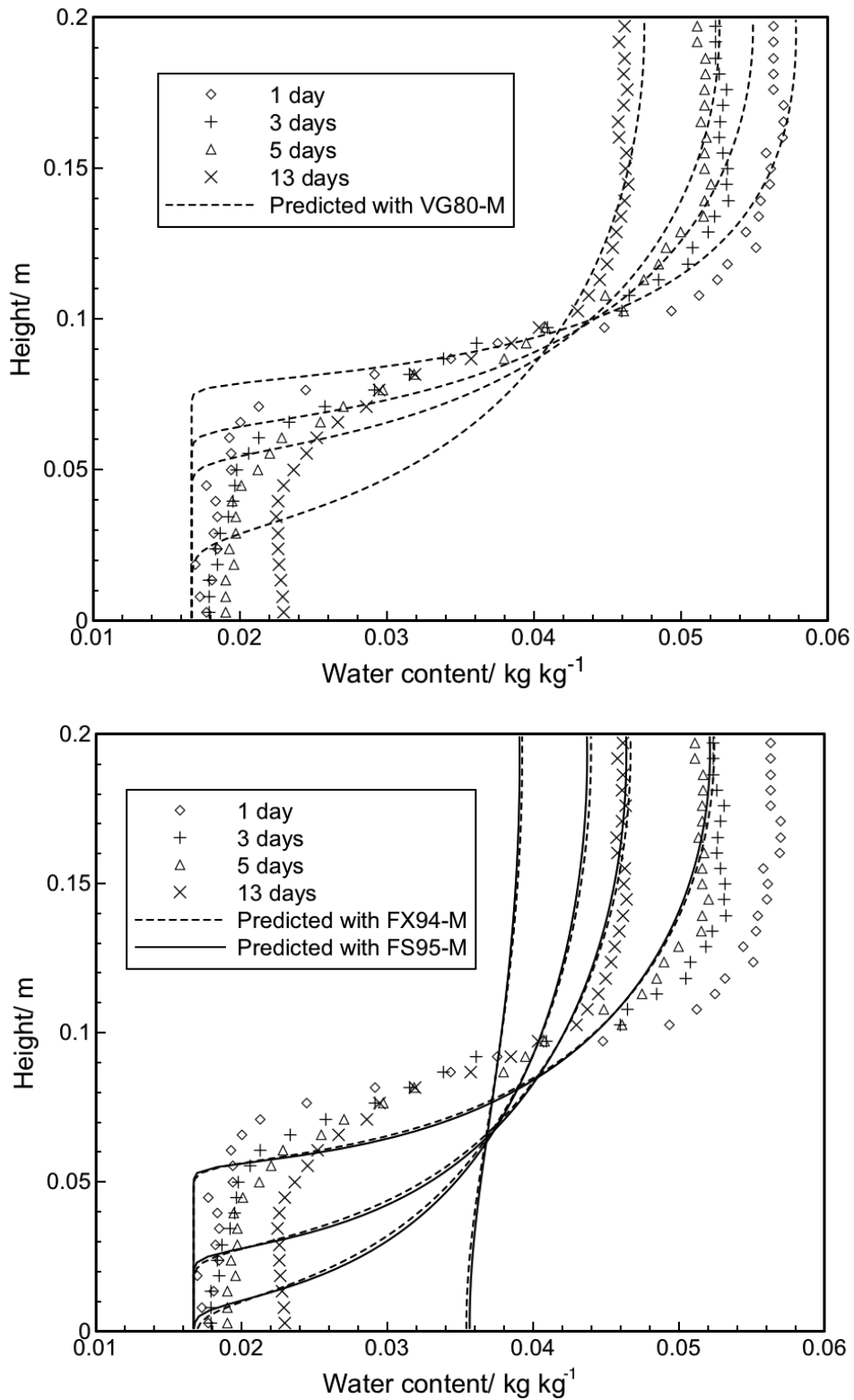


**Figure 3** Sorption isotherm curves for soil samples NH1 and NH2.

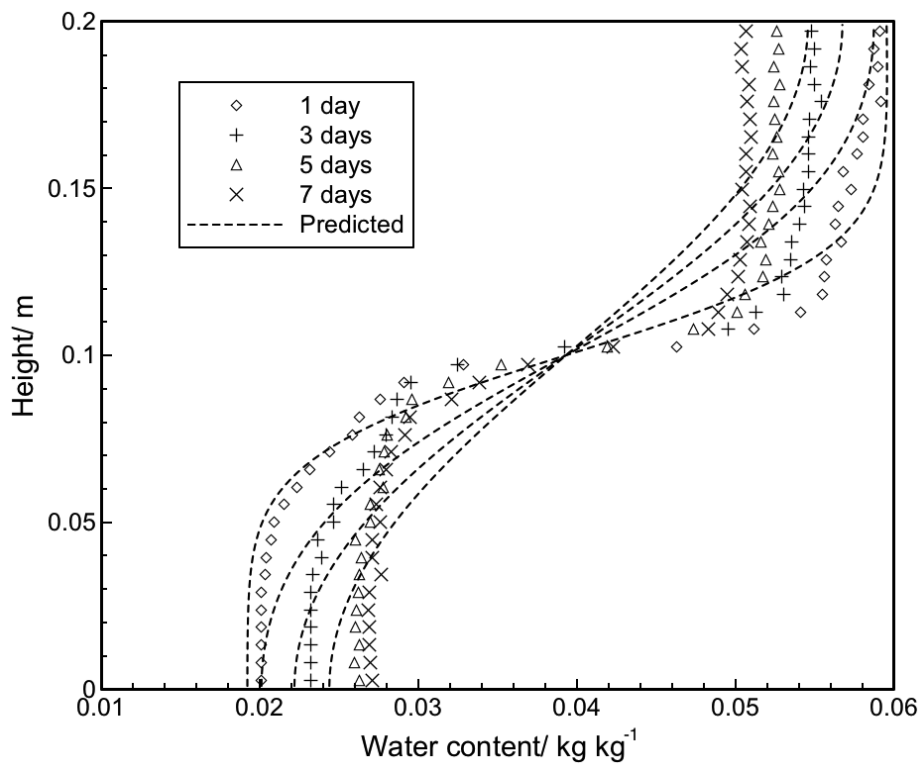
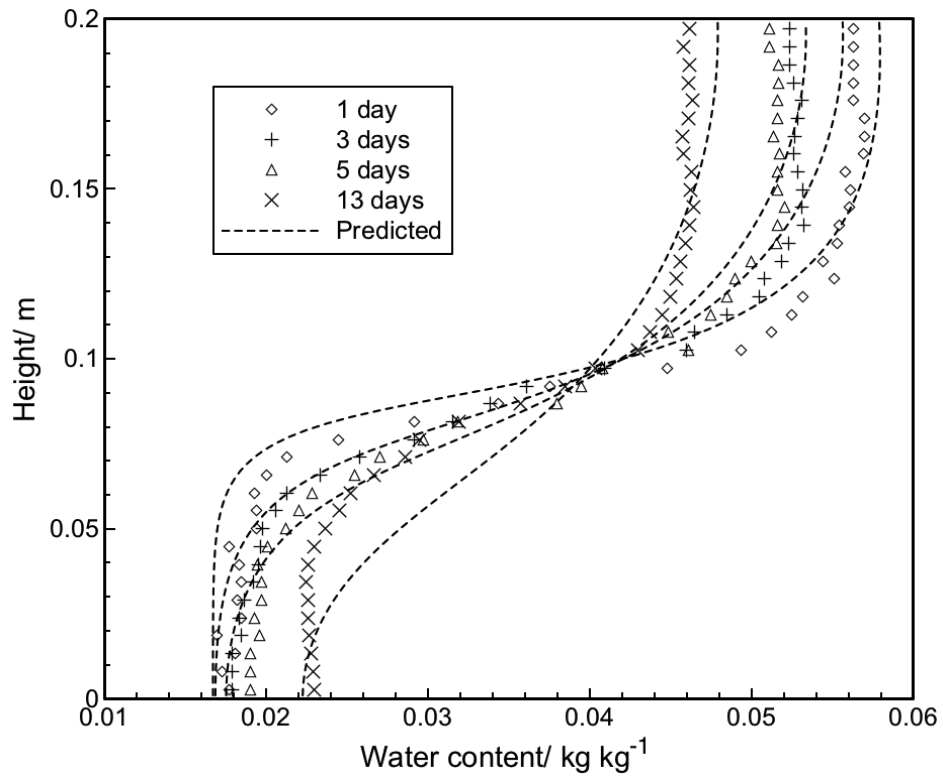




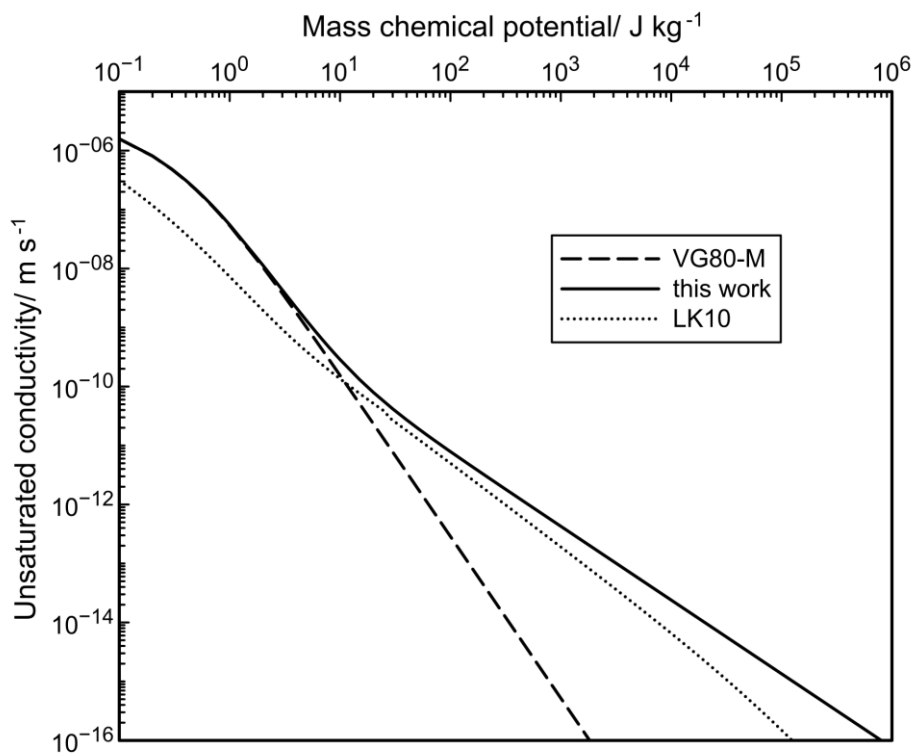
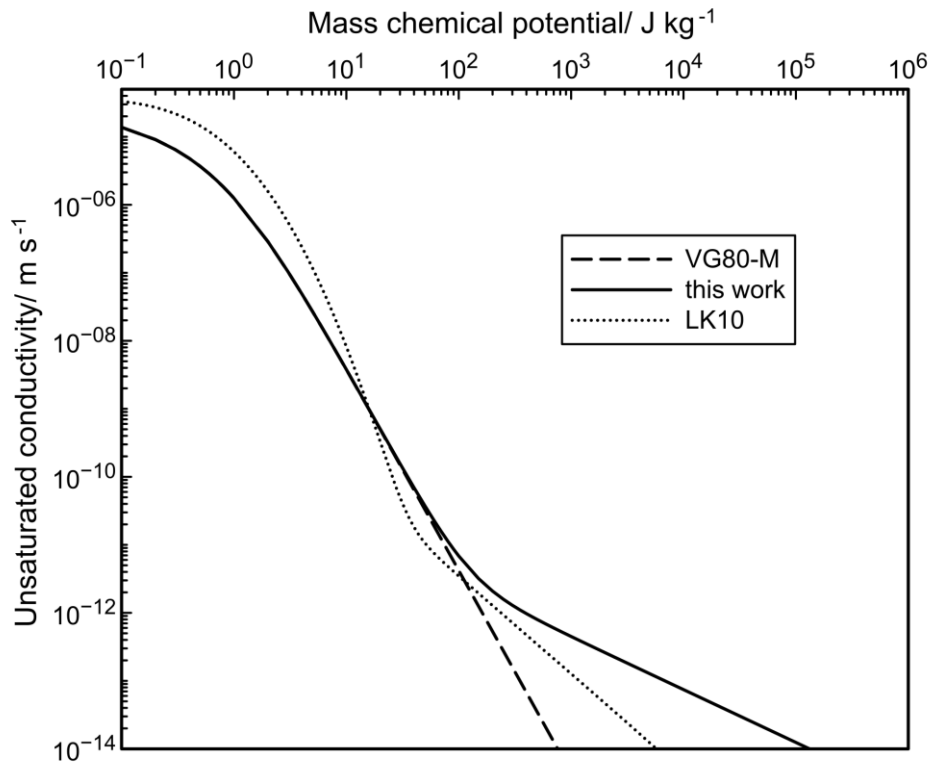
**Figure 4** Experimental and theoretical soil-water retention curves for samples (a) NH1 and (b) NH2.



**Figure 5** Profiles of gravimetric water content along soil columns: a comparison between experimental results and numerical prediction based on the models: (a) VG80-M, (b) FX94-M and FS95-M for soil sample NH1.



**Figure 6** Profiles of gravimetric water content along soil columns: a comparison between experimental results and numerical predictions based on the description proposed in Equation (7) for soil samples (a) NH1 and (b) NH2.



**Figure 7** Unsaturated hydraulic conductivities as a function of the chemical potential for soil samples (a) NH1 and (b) NH2: a comparison between the results from VG80–M, LK10 and this research (result of Equation (7)). Plotted on a logarithmic scale.