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To cite this version:

João Peterson, Vinicius Lopes, Americo Cunha Jr. Maximization of the electrical power generated by a piezo-magneto-elastic energy harvesting device. XXXVI Congresso Nacional de Matemática Aplicada e Computacional (CNMAC 2016), Sep 2016, Gramado, Brazil. hal-01471301

HAL Id: hal-01471301

https://hal.science/hal-01471301v1

Submitted on 19 Feb 2017

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Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Maximization of the electrical power generated by a piezo-magneto-elastic energy harvesting device

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Abstract. This work deals with the maximization of the mean power in a piezoelectric energy harvesting device. The voltages are acquired by applying different force amplitudes and initial positions for the system. These signals are treated in order to identify and separate chaotic from regular (non chaotic) results, through 0-1 test for chaos. An optimization problem is numerically solved in order to identify an optimal configuration of parameters.

Key-words. nonlinear dynamics, energy harvesting, chaos, optimization, test 0-1 for chaos

1 Introduction

Energy harvesting devices are mechanisms able to collect energy from external sources, and store it as electric potential, for a possible use as electric energy [4]. There is a wide range of possible applications for such devices, that goes from electrical sources for equipment displays to indicators lights mounted on embarked dispositives, such as cars'. They can also be useful to support small equipment placed far from electrical distribution system or in dangerous (restricted access) areas, where the best option may be an independent supply of energy [5]. Furthermore, in general these equipment have an extremely rich non-linear behavior [4].

In order to fully understand the potential of energy harvesting, and to improve its efficiency, it is necessary to understand how their parameters' values on its nonlinear dynamic behavior. In this sense, this work aims to study the nonlinear dynamics of the energy harvesting device proposed by [1], which is composed by the classical Moon and Holmes magneto-elastic beam [3], coupled with laminae of piezoelectric material. The

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paper also aims to solve an optimization problem, in order to maximize the mean electrical power in the device. Therein, in section 2 presents the modeling of the harvesting device. Numerical experiments with this model are discussed in section 3. Finally, the section 4 summarizes the main conclusions of this work.

2 Modeling of the nonlinear dynamic system

2.1 Physical model

For the nonlinear dynamics study proposed here, the mathematical and theoretical development are focused on the harvesting device proposed by [1], which is sketched in Figure 1. The structure supports a slim beam made of a ferromagnetic material, and subjected to two magnetic fields, provided by magnets placed in the lower part of the structure. The excitation of an external force creates vibration and, once this movement is perceived by the piezoelectric laminae, placed on highest part of blade, the mechanical energy is converted into voltage.

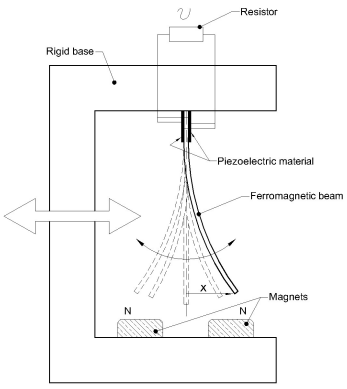


Figure 1: Schematic representation of the energy harvesting device proposed by [1].

2.2 Mathematical model

As shown in [1], the dynamics of the harvesting device of Figure 1 is described by the following set of ordinary differential equations

$$\ddot{x} + 2\xi\dot{x} - \frac{1}{2}x(1 - x^2) - \chi\upsilon = f\cos\Omega t, \tag{1}$$

$$\dot{\upsilon} + \lambda\upsilon + \kappa\dot{x} = 0, \tag{2}$$

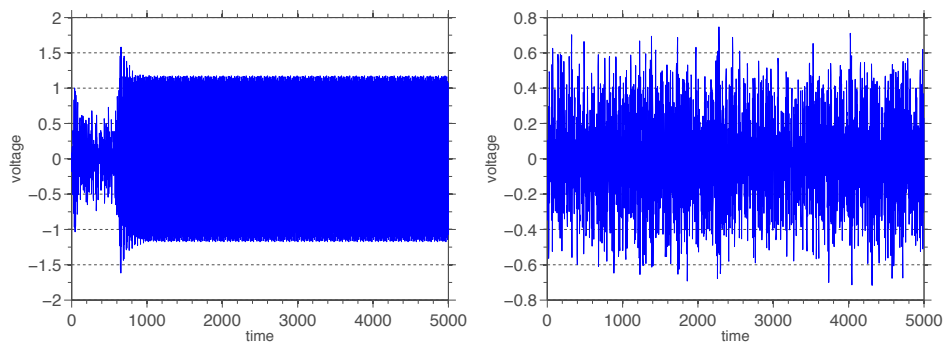
where x is the beam's extreme displacement, ξ is the mechanical damping ratio, χ represents a piezoelectric coupling term in the mechanical equation, f is the amplitude of excitation, Ω is the forced excitation frequency, υ means the voltage by the dispositive, κ is a piezoelectric coupling term in the electrical circuit equation and, finally, λ is a reciprocal time constant. These differential equations are supplemented by the initial conditions

x_0, $\dot{x}_0$, and v_0. All of these parameters are dimensionless, assuming the following nominal values $\Omega = 0.8$, $\xi = 0.01$, $\chi = 0.05$, $\kappa = 0.5$, $\lambda = 0.05$, $x_0 = 1$, $\dot{x}_0 = 0$, and $v_0 = 0$. The value of f will be specified soon.

3 Numerical experiments

3.1 Exploration of nonlinear dynamics

The voltage signal may have regular (non chaotic) or chaotic behavior [6], depending on the values of certain parameters of the harvesting device underlying dynamic system, such as f. This fact is illustrated in Figure 2, through the voltage time series, and in Figure 3 through Poincaré sections in $x\,\dot{x}$ plane and the corresponding phase space trajectory projection. Similar results are observed if x_0 is changed. Therefore, depending on the chosen value for the pair x_0 and f, it is possible to conclude that the dynamics presents different characteristics. This fact was observed for the first time by [1], which did a similar analysis using the pair $(\dot{x}_0, f)$.



(a) $f = 0.115$ (b) $f = 0.083$.

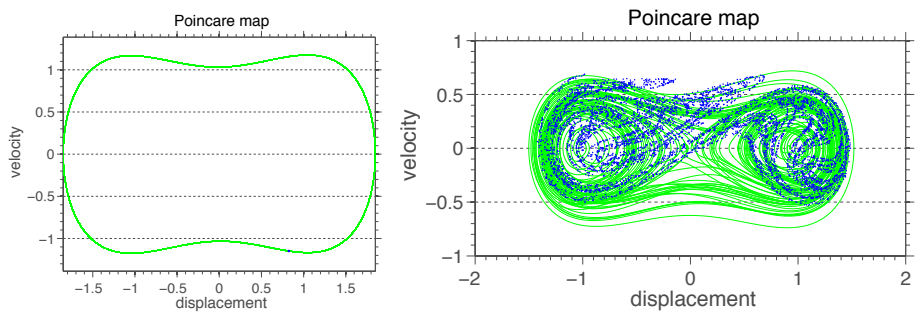
Figure 2: Time series of the voltage in the energy harvesting device, showing regular (left) and chaotic (right) behaviors.

3.2 Optimization of mean power in the harvesting device

Aiming to find a pair (x_0, f), in a given operating window $x_{0_{min}} \leq x_0 \leq x_{0_{max}}$ and $f_{min} \leq f \leq f_{max}$, that maximizes the mean power in the harvesting device, an optimization problem is solved in this work. The objective function is the mean power

$$\langle \mathcal{P} \rangle = \frac{1}{\tau} \int_t^{t+\tau} P(t')\, dt', \tag{3}$$

being $\mathcal{P}(t) = \upsilon^2(t)$ the instantaneous power in the device, and τ a representative interval of (dimensionless) time in the steady state regime. For purposes of energy utilization, only the voltage presenting regular (not chaotic) behavior is of interest. This condition is introduced in the optimization problem as a constraint.



(a) $f = 0.115$ (b) $f = 0.083$.

Figure 3: Poincaré sections in $x\,\dot{x}$ plane (blue dots) and the corresponding projection of the phase space trajectory (green line), showing regular (left) and chaotic (right) behaviors.

In what follows, the strategy to solve the optimization problem is described. First, the operating window is discretized using a rectangular grid of points, where each point corresponds to a pair (x_0, f). Then, for each grid point, the system dynamics is integrated. Now the problem turns into separate interesting (non chaotic) time series of voltages, to compute the maximum mean power. At this point, the test 0-1 for chaos [2] is applied. The pairs (x_0, f) associated with the non chaotic dynamics define the admissible region for the optimization problem. The mean power is computed in each point of the admissible region, and the maximum value is determined.

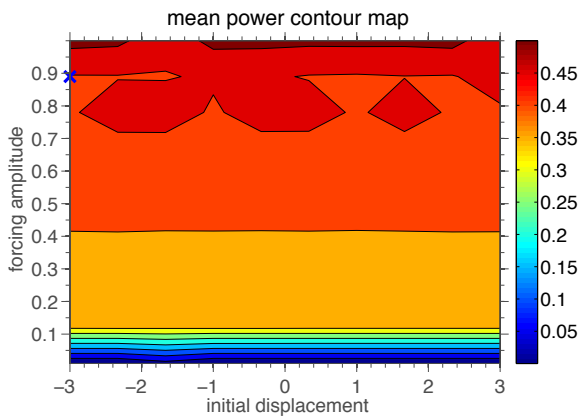


Figure 4: Contour map of the mean (dimensionless) power in the harvesting device. The maximum value is approximately 0.45, and occurs at $(x_0, f) = (-3, 0.89)$ (blue cross).

The contour map of $\langle \mathcal{P} \rangle$, defined over $-3 \leq x_0 \leq 3$ and $0.01 \leq f \leq 1$, is shown in Figure 4. For $f \lesssim 0.7$ the initial displacement interference is negligible. As the force amplitude increases, the contour map reveals variations in $\langle \mathcal{P} \rangle$ value across the x_0 direction. In fact, it is possible to note an asymmetry with respect to $x_0 = 0$. The highest (dimensionless) mean power is approximately 0.45, obtained at the pair $(x_0, f) = (-3, 0.89)$, which is indicated in Figure 4 with a blue cross, in the border of the domain.

4 Final remarks

This work analyzed the non-linear dynamics of a piezoelectric harvesting device, and addressed an optimization problem seeking to maximize the electrical power in this device. Results presented by time series of voltage demonstrated that system dynamics may be chaotic or regular, depending on its initial displacement and force amplitude. Such behavior was also observed through an analysis using Poincaré maps. In the optimization problem solution, test 0-1 for chaos was used to identify the admissible region of parameters, and an optimal configuration (x_0, f) was obtained numerically. The main contribution of this work is the solution of the optimization problem reported above, once, to the best of authors' knowledge, there is no work in the open literature doing this type of analysis. In a future work, the authors' want to analyze how other parameters' influence over the non-linear dynamic behavior and see which pair of parameters are the most effective to maximize the mean power in the device.

Acknowledgments

The authors are indebted to the Brazilian agencies CNPq, CAPES, and FAPERJ for the financial support given to this research. They also thank Dr. Jorge Gripp (ITA) for the enlightening comments.

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