



HAL
open science

A Theoretical Derivation of the Milgrom MOND Equation

Shlomo Barak

► **To cite this version:**

| Shlomo Barak. A Theoretical Derivation of the Milgrom MOND Equation. 2017. hal-01471151v4

HAL Id: hal-01471151

<https://hal.science/hal-01471151v4>

Preprint submitted on 23 Jul 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A Theoretical Derivation of the Milgrom MOND Equation

Shlomo Barak

Taga Innovations 16 Beit Hillel St. Tel Aviv 670017 Israel

Corresponding author: shlomo@tagapro.com

Abstract

Based on General Relativity, we derive theoretically, the Milgrom phenomenological MOND equation that explains Rotation Curves in galaxies. This is made possible by taking into account the inhomogeneous space expansion, in and around, galaxies. Our derivation supports the effort to dispel the need for Dark Matter, or the need to modify known and accepted physics.

Key Words: Gravitation; MOND; Dark Matter;

1 Introduction

The issue of Rotation Curves in galaxies led, in 1933, to the Zwicky hypothesis of Dark Matter [1]. Alternatively it led, in 1983, to the suggestions by Milgrom [2] to modify Newtonian Gravity (MOND) and by Bekenstein [3], in 2004, to modify General Relativity (TeVes).

Distance and time are considered fundamental attributes of the reality and the result of their division is velocity. Based on the Rindler discussion [4] on clock synchronization, we suggest considering distance and velocity as fundamental and time as merely a practicality. This has implications for how we interpret the first term in the Schwarzschild metric. It leads us to conclude that the Newtonian gravitational central acceleration, g_N , can be looked upon as a gradient in light velocity, created by space curving (contraction) around a mass.

Light velocity is **a constant of nature**, since Local Observers measuring light velocity in their own zones of space arrive at the same result. But **it is not a constant** for a Faraway Observer examining other zones of space rather than its own.

Space in the universe expands, but hardly within galaxies. We show that the inhomogeneous expansion of space in and around galaxies creates a similar gradient in light velocity in and around them. This causes an additional, overlooked, universal central acceleration, g_0 . The geometric mean of these two central accelerations g_N and g_0 yields the Milgrom MOND equation.

2 Remarks on Distance Time and Light Velocity

2.1 Space as a Lattice

By attributing a cellular structure to space, we can explain its expansion, its elasticity and can introduce a cut-off in the wavelength of the vacuum state spectrum of its vibrations. Without this limitation on the wavelength, infinite energy densities arise. The need for a cut-off is

addressed by Sakharov [5], Misner et al [6], and by Zeldovich [7]. In addition, the Bekenstein Bound sets a limit to the information available about the other side of the horizon of a black hole [8]. Smolin [9] argues that:

There is no way to reconcile this with the view that space is continuous for that implies that each finite volume can contain an infinite amount of information.

We relate to space as a 3D elastic, deformable lattice, rather than a bent manifold. We show that Riemannian geometry is applicable also for a 3D elastic, deformable lattice [10].

2.2 Space Density

Space density ρ is defined as the number of space cells per unit volume. Space density in a zone of space without deformations (far away from masses) is denoted ρ_0 .

2.3 Distance

Consider the linear dimension of a space cell as the local observer's miniature yardstick. Hence, in a space lattice the **distance** between two points is simply the number of space cells on the straight line between the two points for a Euclidian space and on the geodesic for a Riemannian (deformed) space.

We consider a **standard unit of length** as a yardstick with **the same number** of space cells along its length, anywhere in space. A one centimeter yardstick, for example, contains the same number of space cells anywhere, but the linear dimensions of these cells might vary in different zones of space. Hence, the length of standard yardsticks of local observers, in different zones of space, as seen by faraway observers, may also vary.

2.4 Time

We do not know what time is, we only know what motion is.

We can build devices called clocks; whose hand movement (or whatever is analogous to the hand movement) describes to us the "passing of time". The time of an event would be the

position of the hands at the moment, and at the place of the event. The hands of the clock must move in a rhythmic circular motion so that each cycle is exact. In the case of a non-circular, but rhythmic motion the physical conditions must be preserved so that every motion is like the one preceding it. Thus we can talk about the clock's rhythm, or frequency of its motions, where each cycle represents a unit of time.

We consider a **standard unit of time** as the time that it takes for a light beam to cross a standard unit of length. If the yardstick moves with respect to space we relate as a unit of time to the time it takes for the beam to move back and forth. This device is our **standard clock**.

2.5 Light Velocity

The discussion here and in Section 3 will clarify the fact that observers in all zones of space, regardless of their space densities, will claim to get the same result measuring light velocity with their standard yardsticks and clocks. Hence we relate to Light Velocity as a **constant of nature**. However, each and every faraway observer finds that according to his measurements and understanding light velocity elsewhere, where local observers reside, might vary according to space density in their locality. This is the result of light velocity dependence on the permittivity and permeability of space. But the permittivity and permeability of space depend on the density of space [11]. Hence, we should not consider light velocity as a constant and relate to the **coordinate speed of light** of GR [12] as a real speed.

3 The Metric and Light Velocity

Schwarzschild, in 1916, was the first to find a solution to Einstein's field equation - a general spacetime metric - for the exterior of a spherically-symmetric star of radius R , i.e., for $r > R$:

$$ds^2 = -g_{00}c^2 dt^2 + 2g_{0r} dr dt + g_{rr} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

The metric's elements g_{00} , g_{0r} and g_{rr} are functions of r and t .

According to [13] the line element ds^2 is:

$$ds^2 = -e^{-\frac{2GM}{c^2 r}} c^2 dt^2 + e^{\frac{2GM}{c^2 r}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (2)$$

We denote a **gravitational scale factor**, a :

$$a = e^{-\frac{GM}{c^2 r}} \quad (3)$$

For the surface of the sun or the edge of our galaxy: $GM/rc^2 \sim 10^{-6}$ and thus $GM/rc^2 \ll 1$.

For $GM/rc^2 \ll 1$ equation (3) is approximated as:

$$a = (1 - GM/rc^2) \quad a < 1 \quad r \rightarrow \infty \quad a \rightarrow 1 \quad (4)$$

We rewrite equation (2) to become:

$$ds^2 = -a^2 c^2 dt^2 + a^{-2} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (5)$$

The metric in equation (5) is derived by a faraway observer OB1 – far away from the center of a mass, M , that serves as the origin of his co-ordinates.

For OB1, a radial distance interval, dl , close to M , contains a smaller number of his yardstick units, dr , than dr_0 , the number of the local observer OB2 yardstick units that dl contains. This is the result of the OB2 yardstick contraction (curving), which is the contraction of his local space. Hence:

$$dr_0 = a^{-1} dr \quad a < 1 \quad (6)$$

From the **synchronization of clocks**, [4] Rindler arrives (p. 184) at:

$$dt_0 = a dt \quad a < 1 \quad (7)$$

Thus, for OB1, a time interval, dt , contains a larger number of time units, dt_0 , than the number of time units, dt_0 , for OB2.

The 4D **spacetime interval** between two events [4]; the “emission” of a short pulse of light at point A and the “arrival” of this pulse at point B is:

$$ds^2 = 0.$$

Hence, using equation (5):

$$-a^2c^2dt^2 + a^{-2}dr^2 = 0 \quad (8)$$

$$acdt = a^{-1}dr \quad (9)$$

$$dr/dt = a^2c \quad (10)$$

This, $dr/dt = c'$, for OB1, is the light velocity close to a mass M. Light velocity, for OB1, far away from M, is c (standard light velocity), whereas $dr/dt = c' < c$.

This, $dr/dt = c'$, is a local, real and slower, light velocity since, according to equation (4), $a < 1$.

In the literature dr/dt in equation (10) is called **coordinate speed of light**, [12]. This is a misleading name, since dr/dt should be considered a **real speed** [14].

Substituting dr from equation (6) and dt from equation (7) in equation (8) gives:

$$dr/dt = adr_0/a^{-1}dt_0 = a^2dr_0/dt_0 \quad (11)$$

Comparing equation (11) to equation (10), gives:

$$dr_0/dt_0 = c \quad (12)$$

And from (11) again:

$$c' = a^2c \quad (13)$$

The results here and the discussion in Section 2.5 lead us to conclude that OB1 and OB2 measuring light velocity locally in their own zones of space arrive at the same result.

Due to this invariance, light velocity is considered “**a constant of nature**”, despite the fact that in different zones of space it behaves differently.

In conclusion:

$$dr_0 = a^{-1} dr \quad (6)$$

$$dt_0 = a dt \quad (7)$$

$$c' = a^2 c \quad (13)$$

4 The Gravitational Field as a Gradient in Light Velocity

Substituting a , equation (4), in equation (10), gives for the case $GM/rc^2 \ll 1$:

$$dr/dt = a^2 c = (1 - GM/rc^2)^2 c \sim (1 - 2GM/rc^2) c = (1 + 2\phi/c^2) c \quad (14)$$

From equation (14) and $dr/dt = c'$ (Section 3) we get the gravitational potential ϕ :

$$\phi = \frac{1}{2} c (c' - c) \quad (15)$$

Note that $c' < c$, which complies with $\phi < 0$. The field strength (central acceleration g) is thus:

$$E_g = g = - d\phi/dr = - \frac{1}{2} c dc'/dr \quad (16)$$

$$\mathbf{E}_g = \mathbf{g} = - \frac{1}{2} c \nabla \mathbf{c}' \quad (17)$$

Thus, the gravitational field (central acceleration) can be considered a gradient in light velocity.

Note that c' is not a scalar, it is a vector \mathbf{c}' , and $\nabla \mathbf{c}'$ is a gradient of a vector. This gradient involves Christoffel symbols which are involved in the GR field equation.

To check our derivation we take (14) and $c' = dr/dt$ and get:

$$c' = (1 - 2GM/rc^2) c \quad (18)$$

$$dc'/dr = 2GM/r^2 c \quad (19)$$

Hence, according to equation (16) the **central acceleration** is:

$$g = \frac{1}{2} c dc'/dr = GM/r^2 \quad (20)$$

$$\mathbf{g} = - (GM/r^3) \mathbf{r} \quad (21)$$

Note that M is a **gravitational mass**, since it comes from Einstein's GR field equation.

Note also that equations (16) and (17) are **general equations**, not necessarily limited to the Newtonian gravitational field.

Our paper [15] shows that this understanding leads to the understanding of free fall and yields the proof of the equivalence of inertial mass and gravitational mass.

5 The Overlooked Central Acceleration

The **cosmological scale factor** (CSF), a , in the epoch of galaxies formation 500–700 Myr

($z = 8-11$) after the Big-Bang [16], is notated a_b . Taking $z = 9$ gives:

$a_b = 1/(z+1) = 0.1$, whereas the present CSF in the intergalactic space is $a_0 = 1$.

$$a_b = 0.1 \quad a_0 = 1 \quad (22)$$

Note that the CSF, a , in this section **is not** the gravitational scale factor, a , of Section 3, although both relate to space density.

Space in the universe expands, but space within galaxies does not. We, however, assume that at some point in the galaxy or on its skirt space starts to expand gradually to reach $a_0 = 1$.

A simple **toy function** for a variable CSF, in and around galaxies, is:

$$a = a_b + (a_0 - a_b)[1 - \exp(-r/(R/4))] \quad (23)$$

R is the Hubble sphere radius (which is not the R of Section 3). For $r = 0$, $a = a_b$ and for $r \rightarrow R$, $a = 0.98$, which is close to $a_0 = 1$.

Note that the radius of the universe is many times larger than the Hubble sphere radius R . Taking $R/4$ is **arbitrary**, but based on the size of “dark matter halos” it is **reasonable**; it **should**, however, be supported by observations.

Substituting the values $a_b = 0.1$ and $a_0 = 1$, of equation (22), in equation (23) gives:

$$a = 0.1 + 0.9[1 - \exp(-4r/R)] \quad (24)$$

For $r \ll R$ equation (24) becomes:

$$a = 0.1 + 3.6r/R \quad (25)$$

According to equations (13) and (25) and using the Hubble parameter $H = c/R$ (defined as $H = \dot{a}/a$) gives for $r \ll R$ the following value for dc'/dr :

$$dc'/dr = c \, d/dr (a^2) = c \, 2a \, da/dr = 2 \cdot a \cdot 3.6 \cdot c / R$$

Taking for a its average value $(a_b + a_0)/2 \sim 0.5$ gives:

$$dc'/dr = 0.36H \quad (26)$$

The H value as of today - the Hubble constant H_0 , [17], is:

$$H_0 = 2.26 \pm 0.25 \times 10^{-18} \text{ sec}^{-1}.$$

Substituting this value (without the error range, since we are using an artificial toy function) in equation (26) gives:

$$dc'/dr = 0.36 H_0 = 0.81 \times 10^{-18} \text{ sec}^{-1} \quad (27)$$

The value for the **central acceleration**, due to the inhomogeneous space density, as of today, is calculated using equations (16) and (27):

$$g = -\frac{1}{2} c \, dc'/dr = -1.22 \times 10^{-8} \text{ cm s}^{-2}. \quad (28)$$

This acceleration, notated g_0 , is:

$$g_0 = -1.22 \times 10^{-8} \text{ cm s}^{-2} \quad (29)$$

The **geometric average (mean)** of the Newtonian **central acceleration** $g_N = -GM/r^2$, and g_0 gives:

$$g = -\frac{\sqrt{g_0 GM}}{r} \quad (30)$$

Equation (30) **resembles** the Milgrom MOND **phenomenological** equation [2]. According to MOND, for central accelerations smaller than g_0 , the Newtonian central acceleration should be modified to become equation (30). Our g_0 , in contrast, is **a real** additional central acceleration. Thus, it is clear that our understanding is not related at all to MOND. We adhere to GR and its weak field Newtonian approximation and dispel the need to modify them.

Note the fit of our value for g_0 , in equation (29), to the observed [2] MOND g_0 , which is:

$$g_0 = -1.2 \pm 0.2 \times 10^{-8} \text{ cm s}^{-2}. \quad (31)$$

Note, that the MOND theory uses the notation a_0 rather than g_0 .

We take the **geometric average (mean)**, since central accelerations are related to radii of space curvatures [10]. This subject, however, is beyond the scope of this paper.

Gravitation is the contraction of space, whereas space expansion is the dilation of space. g_N is the result of gravitational space contraction (curving) whereas g_0 is the result of space dilation (curving). Let r_0 denote the distance from the center of a galaxy at which space contraction was balanced by space dilation, in the epoch of the galaxy's creation. This balance at r_0 , with the larger g_0 **of that time**, is expressed by the equality $g_N = g_0$, or:

$$GM/r_0^2 = g_0 \quad (32)$$

Thus:

$$r_0 = (GM/g_0)^{1/2} \quad (33)$$

Note the following:

With time H becomes smaller and so does the gradient in light velocity, see equation (26). Thus, the zone of balance, at r_0 , see (33), moves forward, away from the center of the galaxy, as if “Dark Matter Halos” grow with time.

Our central acceleration, equation (17), is based on a gradient in light velocity; hence we can explain lensing, including the lensing of empty zones of space with inhomogeneous space density.

6 Summary

The inhomogeneous expansion of space around galaxies creates a universal, so far overlooked, central acceleration, g_0 that explains Rotation Curves.

Acknowledgements

We would like to thank Mr. Roger M. Kaye for his linguistic contribution and technical assistance.

References

- [1] F, Zwicky: *Helv. Phys. Acta*, 6, 110 (1933)
- [2] M, Milgrom: A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal* 270: 365–370 (1983), and MOND Theory arXiv: 1404.7661 (2014)
- [3] J. D. Bekenstein: Relativistic gravitation theory for the modified Newtonian dynamics paradigm, *Physical Review D*, 70 (8): 083509, (2004), arXiv:astro-ph/0403694
- [4] W. Rindler: *Relativity* Oxford (2004) p. 184, p. 232, p. 236
- [5] A. D. Sakharov: *Soviet Physics-Doklady*, Vol. 12 No. 11, P.1040 (1968)
- [6] C. W. Misner, K. S. Thorne, J. A. Wheeler: *Gravitation*, (1970) P. 426 P. 1202

- [7] Y. B. Zeldovich and I. D. Novikov: Stars And Relativity (1971) P. 71
- [8] J.D. Bekenstein.: Phys. Rev. D 7, p. 2333 (1973)
- [9] L. Smolin: Three Roads to Quantum Gravity (2001)
- [10] S. Barak: On Bent Manifolds and Deformed Spaces hal-01498435
(2017) <https://hal.archives-ouvertes.fr/hal-01498435>
- [11] S. Barak: The Geometrodynamics of Space. hal-01435685 (2017)
<https://hal.archives-ouvertes.fr/hal-01435685>
- [12] B.W. Carroll and D. A. Ostlie: An Introduction to Modern Astrophysics (2007) p.1052,
p.635
- [13] B. F. Schutz: General Relativity, Chapter 10, Cambridge (2003)
- [14] S. Barak: Time is not Fundamental - unlike Distance and Velocity. hal-01502214
(2017) <https://hal.archives-ouvertes.fr/hal-01502214>
- [15] S. Barak: On the Essence of Gravitation and Inertia, Part 1: Inertia and Free Fall
of an Elementary Particle hal-01404143v5 (2016)
<https://hal.archives-ouvertes.fr/hal-01404143v5>
- [16] S. R. Bernard et al: Galaxy candidates at $z \sim 10$ in archival data from the brightest of
reionizing galaxies (BORG[z8]) survey The Astrophysical Journal, Volume 827,
Number 1 (2016)
- [17] C. L. Bennett et al: The 1% concordance Hubble constant, arXiv:1406.1718v2
[astro-ph.CO](2014)