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# On a Singular Solution in Higgs Field (III) - Condensates and Representation of Certain $f_0$ Mesons' Masses<sup>\*</sup>

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**Abstract:** We have recently discussed the mass and the basic structure of SM Higgs boson ( $H^0$ ) by obtaining asymptotic solution for the equation of motion of nonlinear Klein-Gordon type partial differential equation. In this paper, we will treat with above in mind, - masses of glueball (GB) of ground state and of certain  $f_0$  mesons, ur- Higgs boson (ur- $H^0$ ) which will consist of a number of GBs and/or  $f_0$  above for respective fullerene structure, a representation of these  $f_0$  mesons' masses by masses of  $\pi$  octet and GB, and transformation of ur- $H^0$  into  $H^0$ .

**Key words:** Higgs boson mass, glueball, gluon,  $f_0$  meson, Bethe-Salpeter equation

## Nomenclature

G: gauge coupling constant defined by  $(g^2 + g'^2)^{1/2}$

$G_F$ : Fermi constant

$I(p, p', P_B)$ : irreversible part of the process

M: rest mass

$P$ : total momentum of bound state

T: operator of time ordered

$W_\mu$ : gauge field of W

$Z_\mu$ : gauge field of Z

a: constant

c: velocity of light

**e**: phenomenological parameter of strength

$g, g'$ : gauge coupling constant of SU(2), U(1), respectively.

m: relativistic mass

$q$ : relative momentum

s: relativistically invariant distance from origin

t; time

$x_i$ : coordinates of Minkowski space

## Greek letters

$\varphi$ : isospinor scalar Higgs field

$\phi$ : scalar Higgs field

$\phi_{B_i}(p, P_B)$ : BS amplitude

$\Delta'_{Fa}, \Delta'_{Fb}$ : modified Feynman propagators

$v$ : vacuum expectation value

$\alpha, \xi$ : gauge parameter

$\varepsilon_i$ : infinitesimal Grassmann number

$\varepsilon_{photon}$ : energy of emitted photon

$\sigma$ : string tension

## 1. Introduction

In preceding paper [1] the mass and the basic structure of  $H^0$  were discussed by obtaining asymptotic solution for the Euler-Lagrange equation of nonlinear Klein-Gordon type, in Higgs field with newly developed mass triangle method, and also the basic structure of  $H^0$  by referring to a tightly bound virtual top quark-pair ( $tt^*$ ). Though we saw an intimate relation between  $H^0$  and ( $tt^*$ ), in our calculation  $H^0$  had a smaller mass than the predicted one by the dynamical strong coupling theory of top quark condensation [2], and also seemed to have a truncated-Octahedron (tr-O) mass structure composed of heavy and light pseudo-scalar mesons' masses of all spin 0. There we considered that the mass deference would come from different phase state of each  $H^0$ . Therefore, to understand this in detail we shall hereafter start with investigating the phase transition of  $H^0$  via a relativistic energy equation with describing a phase transition diagram of  $H^0$ , after review of former result. Then we at first see that the ground state mass of GB is calculated at 502.55 MeV/c<sup>2</sup> which is expected as  $f_0(500)$  meson's mass. The GBs will attract mutually with neighbors among original their components of gluons in different colors, so that they could gradually form cluster. And we show that our computed masses of  $f_0(1370)$ ,  $f_0(1500)$  and  $f_0(1710)$  are within each  $f_0$  meson's mass from experiment while they will construct respective fullerene structure for ur- $H^0$  as well as  $f_0(500)$ , provided that the mass of ur- $H^0$  (120.611 GeV/c<sup>2</sup>) will

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consist of a number of masses of GB or  $f_0$  in which all (pure) GB-fullerene may have an icosahedral ( $I_h$ ) rotational symmetry. Finally we propose a representation by which  $f_0$  mesons masses above are reproduced respectively with masses of several light pseudo-scalar mesons such as  $\eta$ ,  $K^0$ ,  $K^0_{\text{bar}}$ ,  $K^\pm$ ,  $\pi^0$ ,  $\pi^\pm$  and GB, under the consideration of those junction networks. Where the mass of  $f_0(1500)$  is described only by the mass of GB. And also  $ur\text{-}H^0$  will transform into  $H^0$  under mass invariance through, for instance,  $\gamma f_0$  reaction to  $\eta_c$  as its component via radiative decay of  $J/\Psi$ . Along with these discussions, a massive gluon propagator for virtual top quark-pair decay is calculated by Bethe-Salpeter equation.

## 2. Review - formula of SM Higgs boson mass

### 2.1. SM Higgs boson mass formula

EOM of Higgs field [1] should have a solution at the point of vacuum expectation value ( $\phi = v$ ), or  $\phi = 0$ . When we choose an asymptotic form for  $s \rightarrow 0$  as,

$$\varphi(s) \sim avs^3 \left\{ 1 - \exp(-s_0/s) \right\} \Big|_{s \rightarrow 0}, \quad (1)$$

$$\text{where } s \equiv \sqrt{c^2 t^2 - x_i x^i}.$$

Then asymptotically.  $\varphi(0) \sim 0$ ,  $\varphi'(0) \sim 0$ . And expanding near  $s \rightarrow 0$ , we can take an asymptotic form near singular solution ( $\varphi=0$ ) as,

$$\varphi \sim \varepsilon^2 v, \quad (\varepsilon \rightarrow 0) \quad (2)$$

So we have a Higgs mass formula from EOM as

$$m_\varphi^2 = 2 \left\{ \left( \sqrt{W_\mu^+ W^{-\mu}} \right)^2 / (2\varepsilon_\lambda/g)^2 \right\} + \left\{ (Z_\mu)^2 / (2\varepsilon_\lambda/G)^2 \right\} \quad (3)$$

Finally we get the value of rest mass for  $H^0$

$$M_{H^0} = \frac{2M_W}{\sqrt{1 + \cos^2 \theta_W}} = \frac{2M_W M_Z}{\sqrt{M_W^2 + M_Z^2}} = 120.611_{-0.022}^{+0.023} \left[ \text{GeV}/c^2 \right] \quad (4)$$

### 2.2. Top quark mass formula

Extended EOM of Higgs scalar field [1] from Euler-Lagrange equation is

$$\begin{aligned} \lambda \varphi^3 + 3\lambda v \varphi^2 + \left[ \left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) + \left\{ m_\varphi^2 - \frac{1}{2} g^2 W_\mu^+ W^{-\mu} - \frac{1}{4} G^2 (Z_\mu)^2 \right\} \right] \varphi \\ - \left\{ g M_W W_\mu^+ W^{-\mu} + \frac{1}{2} G M_Z (Z_\mu)^2 \right\} + \frac{1}{v} \left( m_{b_i} \bar{b} b + m_{c_i} \bar{c} c + m_{t_i} \bar{t} t \right) \\ + \frac{1}{v} \left( m_{d_i} \bar{d} d + m_{u_i} \bar{u} u + m_{s_i} \bar{s} s \right) = 0 \end{aligned} \quad (5)$$

Then extended Higgs boson mass formula is

$$m_\varphi^2 = 2 \left\{ \left( \sqrt{W_\mu^+ W^{-\mu}} \right)^2 / (2\varepsilon_\lambda/g)^2 - m_{q_{i(bsd)}} \left( \sqrt{q_{bsd} q_{bsd}} \right)^2 / \left( 2\varepsilon_\lambda / \sqrt{2\sqrt{2}G_F} \right)^2 \right\}$$

$$+ \left\{ (Z_\mu)^2 / (2\varepsilon_\lambda/G)^2 - 2m_{q_{i(cu)}} \left( \sqrt{q_{cu} q_{cu}} \right)^2 / \left( 2\varepsilon_\lambda / \sqrt{2\sqrt{2}G_F} \right)^2 \right\} \\ - 2m_{t_i} \left( \sqrt{\bar{t} t} \right)^2 / \left( 2\varepsilon_\lambda / \sqrt{2\sqrt{2}G_F} \right)^2 \quad (6)$$

It will be understood that first and second terms in right-hand side of Eq.(6) are both related to decay of top quark to weak boson and to other quark. And we assume that the probability of  $m_t$ -decay process obeys binominal distribution of being  $k$ -times in  $n$ -trials (-particles) with  $r_0$  ( $0 < r_0 < 1$ ) as decay-mode parameter,

$$\begin{aligned} {}_n C_k \cdot r_0^k (1-r_0)^{n-k} m_t \\ \equiv {}_n C_k \cdot r_0^{k-1} (1-r_0)^{n-k} m_{W(bsd)} + {}_n C_k \cdot r_0^k (1-r_0)^{n-k-1} m_{Z(cu)} \\ = {}_n C_k \cdot r_0^{k-1} (1-r_0)^{n-k-1} \left\{ (1-r_0) m_{W(bsd)} + r_0 m_{Z(cu)} \right\}, \\ \therefore m_t = m_{W(bsd)} / r_0 + m_{Z(cu)} / (1-r_0) \end{aligned} \quad (7)$$

Thus the stationary mass value of top quark is

$$M_t = (1/2) \left\{ \left( M_W^2 - M_b^2 - M_s^2 - M_d^2 \right)^{\frac{1}{4}} + \left( M_Z^2 - M_c^2 - M_u^2 \right)^{\frac{1}{4}} \right\}^2 \\ \approx 171.26(6) \left[ \text{GeV}/c^2 \right], \text{ with } M_b = 4.68 \text{ GeV}/c^2 \text{ (1S Mass)}. \quad (8)$$

which is consistent with CDF/D0's experimental result [4].

Since  $H^0$  is expected to be a composite scalar meson,

$$M_{(\bar{t}t)^*} \equiv M_{H^0} = M_t / \sqrt{2} = 121.10(3) \left[ \text{GeV}/c^2 \right]. \quad (9)$$

$$\therefore \Delta M \equiv M_{(\bar{t}t)^*} - M_{H^0} = 0.49(2) \left[ \text{GeV}/c^2 \right].$$

which is little smaller than masses of  $K^{\pm,0}$  mesons, and is smaller than mass of  $\eta$  meson. Therefore it is expected that  $H^0$  is to be a scalar meson after emitting one photon from the virtual top quark pair:  $(t\bar{t})^*$ ,

$$(\bar{t}t)^* \rightarrow \gamma H^0. \quad (10)$$

### 2.3. Basic structure of SM Higgs boson mass

Let us consider that  $H^0$  mass is constructed basically by heavy mesons' masses of all spin 0, such as

$$\left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \quad (11)$$

It is expected that they will form a polyhedron composed of planes of hexagon in space. Because effective number of the planes turns out to be 4 (four), basic structure of  $H^0$  mass is represented as

$$\begin{aligned} M_{H^0} (3\eta_{c,\dots}(\text{exper. values})) \\ \equiv \sum_{M_i} \left[ 3\eta_c + 10\pi^+ \pi^- + 4 \left\{ \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right\} \right] \\ \left( = 120.612 \text{ GeV}/c^2 \right). \end{aligned} \quad (12)$$

## 3. Structure of ur-Higgs boson

## -Condensates and a representation of certain $f_0$ mesons' masses

### 3.1 A naïve relativistic energy equation for the system

Before we later apply Bethe-Salpeter equation on  $(tt)^*$ , we shall at first make a naïve relativistic energy equation for  $(tt)^*$  with considering Cornell potential  $V$  of quarkonium in lattice QCD of Wilson loop) [5] in Figs. 1 and 2.

$$V(r) = -e/r + \sigma r, \quad (13)$$

where  $r$ : distance from the centre of the string

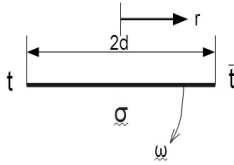


Fig.1 String Tension  $\sigma$

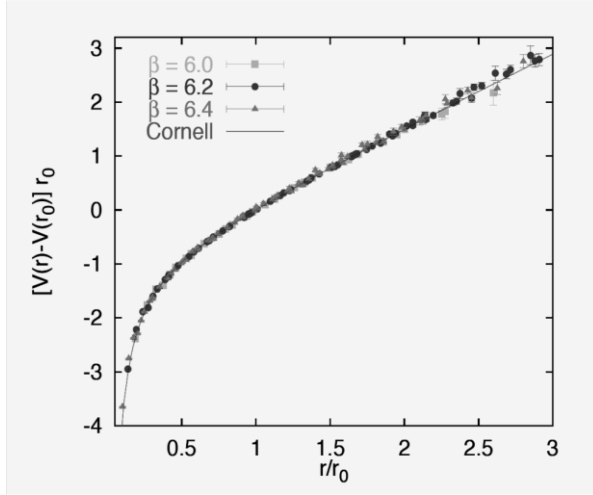


Fig. 2 Quenched Wilson action  $SU(3)$  potential [5], normalized to  $V(r_0) = 0$ .

Hence we shall write for a tightly bound energy as [1]

$$V_{t,bound} \equiv M_{H^0} c^2 = \left( M_t / \sqrt{2} \right) c^2 - \varepsilon_{photon} \quad (14)$$

because we will treat the situation in next subsection that total mass of the system is zero, we here set its tightly bound (total) energy to equalize to the mass of  $H^0$  which will be produced at second stage from massless vector particle (gluon). Then by

substituting Eq.(14) into Eq.(13), we have a naïve (approximate) relativistic energy equation for the system as

$$\left( M_t / \sqrt{2} \right) c^2 - \varepsilon_{photon} \approx -e_{Total} / r + \sigma r. \quad (15)$$

So if we take  $r = 0.4$  fm, then  $e_{Total} = -48.0$  GeVfm where

we adopt  $\sigma = 1.5$  GeV/fm [5]. Here we consider that there would be the latent heat between the molecular-like state:  $M = 2M_t$  of the dynamical strong coupling theory of top quark condensation and pure liquid state:  $M = M_t / \sqrt{2}$  of our calculation. It is expected that the former would not be in tightly bound state. Then we could describe the diagram of phase transition into  $H^0$  as shown in Fig.3. We will later return to this diagram. It is interesting that the rate of outgoing energy from the system to the space ( $= +Q$ ) by deficit of mass is fairly large during the condensation:

$$\eta_{+Q} \equiv \left( 2 - 1/\sqrt{2} \right) M_t c^2 / \left( 2M_t c^2 \right) \approx 0.646 \quad (16)$$

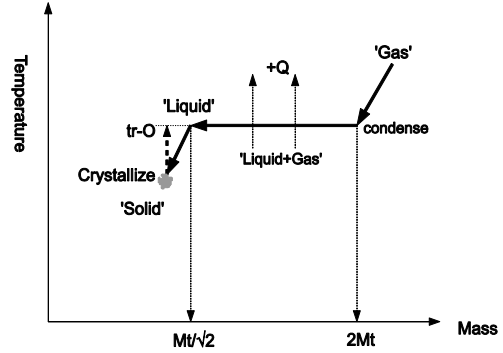


Fig.3 Diagram of Phase Transition into SM Higgs boson.

### 3.2 Bethe-Salpeter equation with Goldstein approximation

Tightly bound fermion-antifermion coupling which exchanges with vector particle by Bethe-Salpeter equation (BS) [6] has long been investigated.[7,8,9] Firstly, Salpeter and Bethe constructed the relativistic equation for two interacted nucleons. Goldstein studied its solution by ladder approximation and discovered the continuous spectral solution with relevant discrete ones.[6] There Goldstein argued the lack of physical interpretation for the continuous solution of highly singular behavior at the origin of coordinate space. Later, Kummer, Higashijima and Nishimura; Fukui and Seto; and others discussed the continuous spectral solution in the fermion-antifermion or in the spinor-spinor interactions. They excluded it from the reason of each difficulty of interpreting physical meaning, except that Higashijima and Nishimura considered it

as a renormalized vertex function of the solution for the homogeneous BS.[7] Thus we shall hereafter apply BS for tightly bound fermion (top quark)-antifermion (anti-top quark) coupling which exchanges with vector particle, and reconsider the physical meaning of continuous solution. The general form of BS is [9]

$$K_B \phi_{Br}(p, P_B) = I_B \phi_{Br}(p', P_B), \quad (17)$$

$$\text{where } K_B \equiv [\Delta'_{Fa}(\eta_a P_{B^+} \not{p}) \Delta'_{Fb}(\eta_b P_{B^-} \not{p})]^{-1} \quad (18)$$

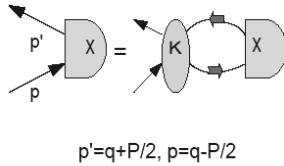
$$I_B \equiv \int d^4 p' I(p, p'; P_B). \quad (19)$$

Then BS for fermion-antifermion bound state with total four momentum is given explicitly [6] in the Bjorken-Drell metric, with  $x_{\alpha\beta}$  as  $4 \times 4$  matrix BS-amplitude of spinor having two legs;

$$\begin{aligned} & \left[ S^{-1} \left( q + \frac{1}{2} P \right) \mathcal{X}(q, P) S^{-1} \left( q - \frac{1}{2} P \right) \right]_{\alpha\beta} \\ &= \int \frac{d^4 q'}{(2\pi)^4} K_{\alpha\beta; \alpha'\beta'}(q, q'; P) \mathcal{X}_{\alpha'\beta'}(q', P), \end{aligned} \quad (20)$$

where

$$\mathcal{X}_{\alpha\beta}(q, P) = \int d^4 x \left\langle 0 \left| T \left[ \psi_\alpha \left( \frac{1}{2} x \right) \bar{\psi}_\beta \left( -\frac{1}{2} x \right) \right] P_\mu \right. \right\rangle. \quad (21)$$



Hence we will have the Goldstone equation for abelian vector gluon model with the ladder approximation, putting  $P_\mu = 0$

and  $x_{\alpha\beta}(q, 0) \equiv (\gamma_5)_{\alpha\beta} F(q)$ , [7]

$$(m^2 - q^2) F(q) = \frac{\lambda}{4\pi^2 i} \int \frac{d^4 q'}{-(q - q')^2 - i\epsilon} F(q'). \quad (22)$$

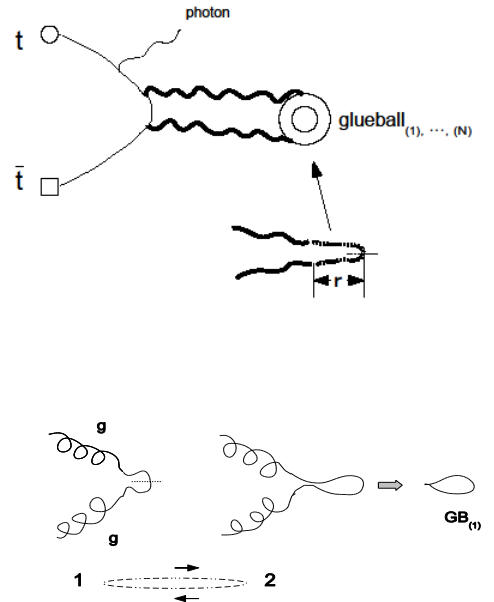
After the Wick rotation and then the Fourier transform regarding Eq.(22), we will see that it has the continuous spectrum solution for  $\lambda > 0$ , putting  $K$  as modified Bessel function of  $\nu$ -th order,

$$f(r) = (mr)^{-1} K_\nu(mr). \quad (23)$$

On the other hand, recently Iritani et al calculated gluon propagator's functional forms [10] in the Landau gauge in SU(3), which fit their result of lattice QCD (LQCD), one of whose candidates has similar form of Eq.(23). However they abandoned this form because of the deviation from LQCD for small  $r$ . We consider in this case, it should rather be adopted that

$$\begin{aligned} \nu &\equiv \sqrt{1 - \lambda}, \quad \text{where } 0 < \lambda \leq 1, \\ \lambda &\equiv (\xi + 3) g^2 / 4\pi^2 = (\xi + 3) \alpha / \pi, \end{aligned} \quad (24)$$

according to the continuous solution of Goldstein for BS and also the behaviors of modified Bessel function of  $K_\nu(x)$ . Since the value of  $K_\nu(x)$  is more gradually decreasing along with decreasing of  $\nu$ , the relevant value of  $\nu$  will be certainly obtained by comparing to LQCD. We prefer that  $\nu = 0$ ,  $\lambda = 1$ . So this is just the case that Higashijima and Nishimura have interpreted it as a renormalized vertex function [7]. Then we expect that the glueball would be produced successively by two gluons each of which has a length of and is made by the gluon from the vertex respectively, as shown in Fig.4.



**Fig.4** Glueball producing process

Therefore the final stage of massive gluon propagator into glueball ( $\leq r$  of Fig. 4) in the Landau gauge in SU(3) which is in accord with LQCD is determined from Eq.(23) as

$$D(r) = \frac{m^2}{4\pi^2} \left\{ \frac{1}{(\hat{m}r)} K_0(\hat{m}r) \right\} [\text{GeV}/c^2]^2, \quad (25)$$

where  $[m] = \text{GeV}/c^2$ ,  $[r] = \text{fm}$ ,  $\hat{m} = 1[\text{fm}]^{-1}$ .

Here the Compton wavelength of glueball in ground state is

$$\lambda_c \equiv \frac{\hbar}{mc} = 0.393[\text{fm}], \text{ provided } m = 502.55[\text{MeV}/c^2] \quad (26)$$

Thus  $r \leq \lambda_c$ , and the value of  $m$  will be actually given in next subsection.

### 3.3 Clustering force between glueballs

Since the number of the kind for colored gluon is 8 (eight), the color valence of glueball should be 4 (four) according to Fig. 5, which is same as carbon and is expected to be self-assembling force between GBs.

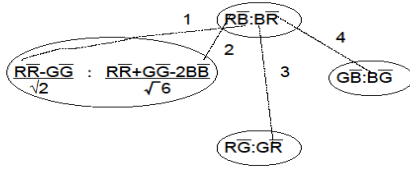


Fig.5 Color valence of glueball (= 4)

In LQCD it is now believed that there might be several scalar mesons of  $f_0(1370)$ ,  $f_0(1500)$ ,  $f_0(1710)$  all of which are supposed to have some contents of glueball of ground state. Then we can expect similar structure of the carbon fullerenes for these scalar mesons. After setting  $N_1, N_2, N_3$  as the fullerene number whose fullerene consists of  $f_0$  mesons above respectively, under the consideration of similar structure to the carbon fullerenes of  $C_{90}$ ,  $C_{80}$  and  $C_{70}$ , we put

$$M_{(f_0)_i} \equiv M_{\text{ur-H}^0} / N_i = M_{\text{GB}} / \eta_i, \quad (27)$$

$$\sum_{i=1}^3 \eta_i = 1, \quad \therefore N_{\text{GB}} = \sum_{i=1}^3 N_i.$$

From Eq. 27 with  $M_{\text{ur-H}^0} \equiv M_{\text{H}^0} = 120.611 \text{ GeV}/c^2$ , [1]

$$M_G \approx 502.55 \text{ MeV}/c^2, \text{ as an element of } GB_{240}\text{-fullerene};$$

$$\eta_1 \approx 0.292, \quad \eta_2 \approx 0.333, \quad \eta_3 \approx 0.375. \quad (28)$$

Table 1. Comparison of  $f_0$  mesons' mass values [ $\text{MeV}/c^2$ ]

$f_0$ meson	Our calculation	Experiment [4]
$f_0(500)$	502.55	400-1200
$f_0(1370)$	1340.1	1200-1500
$f_0(1500)$	1507.6	$1505 \pm 6$
$f_0(1710)$	1723.0	$1720 \pm 6$

It should be noted that  $f_0(1500)$  may be also a glueball for each element of  $GB_{80}$  since  $0.333 \times 3 = 1$ . And  $f_0(1370)$ ,  $f_0(1500)$  and

$f_0(1710)$  will have respective fullerene structure according to their  $N_i$ -number: 90, 80, 70. The calculated  $f_0$  mesons' masses by Eq.(27) and Eq.(28) are well consistent with (or at least within) experimental values as shown in Table 1.

### 3.4 Fullerene structure of ur-SM Higgs Boson

The fullerene structure of  $GB_{240} (\equiv \text{ur-H}^0)$  of  $I_h$  symmetry [11] which consists of 110-hexagons and 12-pentagons, obeying Euler's theorem [12], as shown in Fig. 6 with whose each hexagon of GB from two gluons on it.

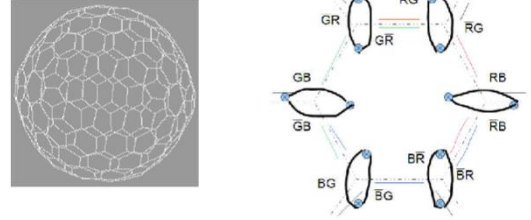


Fig.6 Fullerene structure of ur-Higgs Boson

As far as carbon fullerenes,  $C_{20}$ ,  $C_{60}$ ,  $C_{80}$ ,  $C_{180}$ ,  $C_{240}$  have a common point group:  $I_h$  which is of the icosahedral symmetry. Thus we expect that  $GB_{80}$  ( $f_0(1500)$ ) and  $GB_{240}$  ( $\text{ur-H}^0$ ) also have  $I_h$ . Therefore, inversely, we could expect that the  $f_0$  meson which has a fullerene structure of  $I_h$  symmetry, it may consist of pure GB.

### 3.4 A representation of certain $f_0$ masses

Over three decades ago, Rossi and Veneziano, also Igarashi et al [13] have described gauge invariant junction type baryonium

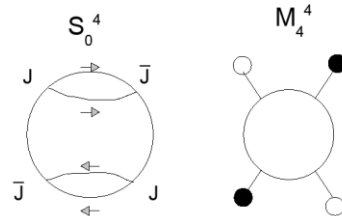


Fig.7 Junction type of baryonium

of  $S_0^{j=2i}$  and  $M_{2m}^{j=2m}$ , where  $j$ ,  $i$  and  $m$  denote respective number of junction, junction-pair (equivalent to GB) and quark-pair (meson), as shown in Fig.7. Recently, Csörgó et al [14] showed gluon junction networks of truncated-polyhedrons. Therefore we shall apply this junction type to be able to describe  $H^0$  with  $f_0$  mesons of the fullerene structure. For  $H^0$  with  $f_0(1500)$  of pure GB's we may construct it as an aggregation of 12 units of  $S_0^{10}$  and 30 units of  $S_0^{12}$ . For  $H^0$  with  $f_0(1370)$  or with  $f_0(1710)$  we must construct it as mixed one from a GB and several certain light pseudo-scalar mesons, because  $f_0(1370)$  and  $f_0(1710)$  have both been interpreted that they are not consisted only of GB's. We propose their mass structure formulae:

$$m_{f_0(1370)} = \left[ \text{GB} + \left(\frac{3}{90}\right)\eta_0 + \left(\frac{70}{90}\right)K^0 + \left(\frac{2}{3} \times \frac{70}{90}\right)K^\pm + \left(\frac{75}{90}\right)\pi^\pm + \left(\frac{40}{90}\right)\pi^0 \right]_{m_i},$$

$$m_{f_0(1500)} = [3\text{GB}]_{m_i},$$

$$m_{f_0(1710)} = \left[ \text{GB} + K^0 + \left(\frac{1}{3}\right)K^\pm + 4\pi^\pm \right]_{m_i}.$$
(29)

which give mass values of 1340.1 and 1723.0 MeV/c<sup>2</sup> respectively, reproducing the calculated values in Table 1 which have been obtained from Eq.(27) and Eq.(28), as already we have seen. Here we should remind that  $H^0$  would be constructed by  $70 \times f_0(1710)$ - or  $90 \times f_0(1370)$ -fullerene. So the factor (70/90)

for  $K^0$  or  $K^\pm$  in Eq. (29-1) is considered. While the factor (1/3) in Eq.(29-3) is expected from that  $3 \times [\text{fullerene number of } H^0 \text{ with } f_0(1500) \text{ of pure GB}] = 240$ . Because,  $H^0$  with  $f_0(1500)$  might have resonant mass of  $3 \times (\text{mass of GB for ground state})$  at each 80 vertexes.

### 3.5 Transformation (decay) of $ur-H^0$ into $H^0$

Reminding Eq.(10) and Eq.(12):

$(\bar{t}t)^* \Rightarrow \underline{\underline{\gamma}}H^0$ , where  $H^0$  condensates into GB fullerene ( $ur-H^0$ ), and

$$M_{H^0}(3\eta_c) \equiv \sum_{M_i} \left[ 3\eta_c, 10\pi^+\pi^- + 4 \left\{ \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right\} \right],$$

Then, 1)  $3\eta_c$ :

$$3\{\gamma\eta_c(1S)\} \leftarrow 3J/\Psi \leftarrow 3\{\gamma 6(f_0(500))\},$$

$$\therefore 3\eta_c(1S) \leftarrow 18f_0(500) \Leftrightarrow 6f_0(1500).$$
(30)

2)  $10\pi^+\pi^-$ :

$$5\pi^+\pi^- \leftarrow f_0(1500) \Leftrightarrow 3f_0(500),$$

$$\therefore 10\pi^+\pi^- \leftarrow 6f_0(500) \Leftrightarrow 2f_0(1500).$$

$$3) 4 \left\{ \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right\}:$$

$$\left( B_s^0 \bar{B}_s^0 \right) \leftarrow 22f_0(500), \left( B_c^+ B_c^- \right) \leftarrow 24f_0(500),$$

$$\left( D_s^+ D_s^- \right) \leftarrow 8f_0(500),$$

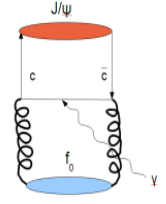
$$\therefore 4 \left\{ \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right\} \leftarrow 216f_0(500) \Leftrightarrow 72f_0(1500).$$

After all, we have the transformation under mass invariance that

$$H^0 \leftarrow 240f_0(500) \Leftrightarrow 80f_0(1500) \quad (31)$$



$$240\text{GB} \equiv ur-H^0.$$



## 4. Concluding remarks

So far, we have shown the condensation (molecular self-assembly) of  $H^0$  into certain fullerene structure which is to be constructed from a number of glueballs ( $f_0(500)$ ) of ground state as well as of heavier  $f_0$  mesons, with phase transition diagram and then Bethe-Salpeter equation. And we proposed a representation by which certain  $f_0$  mesons masses are reproduced respectively with masses of several light pseudo-scalar mesons. The relation between recent experiments of LHC and our result for  $H^0$  mass will be discussed in next paper. [15]

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