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Chuan Xu, Janna Burman, Joffroy Beauquier

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Power-Aware Population Protocols

Chuan Xu
Université Paris Sud
LRI, France
Email: chuan.xu@lri.fr

Janna Burman
Université Paris Sud
LRI, France
Email: janna.burman@lri.fr

Joffroy Beauquier
Université Paris Sud
LRI, France
Email: joffroy.beauquier@lri.fr

Abstract—The contribution of the paper is twofold. First, it introduces a model for analyzing energy consumption in networks of mobile sensors. Second, it uses this model for studying energy complexity of distributed protocols for the task of *data collection*. The proposed model is quite general and can be used for other tasks. It can be considered as the first extension of this kind, taking into account energy consumption, of the classical model of *population protocols*. In population protocols, anonymous and bounded memory agents (sensors) move unpredictably and communicate in pairs when two of them are in proximity. The interest of the extended model is to allow a purely analytical analysis of the energy complexity of a protocol, in the same spirit as for time and space complexity, without appealing to simulations. This approach allows to exhibit energy functions and to draw their curves, from which the optimal values of various parameters can be deduced.

In order to illustrate the power and the usefulness of this model, we consider the issue of determining and adjusting the amount of agents' initial energy necessary and sufficient for being able to perform a given task. This issue is crucial for choosing, in practice, a category of sensors (in respect with their power capacities) adapted both to the task and to the number of times it should be repeated (before the sensors are replaced or recharged). In this context, the natural chosen metric is the maximum energy spent by an agent (for accomplishing the overall collaborative task). This metric is directly related to the minimum necessary initial energy (and also to the lifetime of the network). Contrary to most of the energy studies in networks of mobile agents, our approach is completely deterministic. The reason is that due to the nature of the considered problem, the analysis has to be done in the worst case (in particular, such analysis is impossible when the interactions between agents are supposed to be probabilistic).

The specific task we consider in this work is data collection, which is known to be fundamental in sensor networks. In this problem, initially, each sensor has an input (a sensed) value, and eventually, every input should be delivered exactly once, to a base station. Transfers of inputs between sensors are possible in order to optimize time and energy metrics. In this context, our first contribution is the energy complexity analysis of an already known time optimal protocol. The second contribution is a new power-aware protocol, which improves the previous one in terms of the maximum energy spent by an agent. Finally, we present a lower bound concerning energy consumption of any possible data collection protocol and we show the cases where this lower bound is reached by the presented protocols.

I. INTRODUCTION

In wireless mobile networks, routing is a complex problem, mainly due to node mobility resulting in a frequently changing network topology. The rate of the topology changes depends on many factors including node mobility patterns and speeds, and characteristics of the terrain and of the communication

medium. Typical metrics used to evaluate routing protocols like shortest-hop, shortest-delay, and local stability may have a negative effect on a wireless network, because they result in an overuse of energy resources of a small set of nodes, decreasing network life. Consequently, it is essential to consider routing algorithms from a power-aware perspective, in addition to such traditional metrics.

In this work, we are interested in a special type of mobile sensor networks, where the sensors are extremely simple and cheap. They have very limited computing and communication capabilities. They have no identifiers and execute all the same code. The size of their memory is uniformly bounded, independently of the total number of sensors in the network. As their communication power is limited, they do not communicate by broadcasting, but only, in pairs, when two agents come close to each other. Moreover, the number of nodes in the network is unknown and may be very large. Though, they generally move in a limited area.¹

In this framework of very simple mobile sensors, we are interested in a restricted, but natural (for sensor networks), form of routing, toward a special node, called *data collection*. One may assume that each sensor has got a value from the environment (temperature, pressure, altitude, etc.), and that all values must be routed towards a base station, where they will be analyzed and treated. As the considered mobile sensors interact unpredictably and only in pairs, the data collection is by essence opportunistic, meaning that it has to be performed even though there are no permanent routes from mobile sensors to the base station. Then, when a node meets another node, it has to decide, according to its protocol, whether it is better or not to transfer its sensed value to the other or to keep it. Better usually refers to the global time for performing data collection. Several factors may be relevant for reducing the data collection duration: speeds of the mobile sensors, fixed trajectories, available memory, etc.

The issue that we address here is how to route the sensors' values towards the base station and spend as few energy as possible, while retaining good performances in terms of data collection time. Our approach is at the algorithm level. That means that, beside the classical time complexity, we define formally an energy complexity, the goal being to compute

¹As examples to such networks, one can consider the ZebraNet project deploying a network of sensors attached to zebras [1] or the Pigeon Air Patrol Network project, with pigeons carrying very simple sensors.

time and energy complexities in a purely analytical way, independent of simulations.

The distributed computing model that we choose for the considered networks is the (classical) model of *population protocols* (PP) [2]. This model has been designed to study asynchronous networks of finite state anonymous mobile sensors (called agents) interacting in pairs. We enhance this model with an energy consumption framework. Basically, an agent consumes energy when: (1) interacting and exchanging data, and when (2) signaling its presence and sensing the wireless medium continuously (to allow detection of other agents in proximity). An interacting agent is said to be in *awake* mode and spends an amount E_{wkp} of energy. When it does not interact, but senses the medium, it is in *sleep* mode, and spends E_{slp} every global interaction (that can be seen as a time unit). We also consider an additional mode called *terminated*, in which an agent consumes no energy. This mode can allow saving energy of agents having terminated their part in a task.

To adapt this general model to the specific type of networks we consider here, we make some additional considerations and assumptions detailed in Sect. II. For instance, as the data exchanged during an interaction is bounded by the small agent’s memory, we can assume that any transmitted data fits in one packet and thus the energy spent in a communication is constant. In addition, as the agents communicate only in pairs, when they come close to each other, it is likely to assume that the sensing of the medium is done by proximity sensors (cf. [3], [4]) and thus, agents signal their presence only passively, spending no energy.

In this enhanced model of PP, the basic goal is to perform the complexity analysis of the energy spent by the protocols. In this study, we analyze the maximum amount of energy spent by an agent for performing a one-time collaborative task (data collection in our case). This metric is clearly related to the lifetime of the network and to the amount of initial energy for each agent. The formal analysis of this metric allows, in advance, to adjust the network both in terms of size and of battery power allocated to the agents. It is particularly important in the case of networks where it is difficult or undesirable to access the sensors frequently (cf., Bird Species Recognition [5] and Underwater Networks [6] projects). However, it is important to note that, if no guarantees are imposed on the agents’ interactions, such analysis is in general impossible (for non-trivial protocols). Consider, for example, one of the classical assumptions in PP where any pair of agents is required to interact only infinitely often. In this case, protocols converge only eventually, consuming an arbitrary energy till convergence. At the contrary, with another classical assumption where pairs of agents interact according to some probability distribution, an average complexity analysis of energy consumption can be relevant.

Here, for having a bounded convergence and still being able to perform a worst case analysis of the maximum energy spent, we do not adopt the probabilistic approach, but we assume a sort of partial synchrony. We assume that an agent interacts with all the others with a certain “frequency” (in terms of

interactions), like in the model proposed in [7]. Each agent has its own “frequency” expressed by a *cover time* parameter. This is an upper bound on the “time”, counted in number of global interactions (pairwise meetings), for an agent to interact with all the others. The “faster” an agent is to meet all the others, the smaller is its cover time.² Note that the “time” bounds imposed by the cover times allow to compute the time and energy complexities *deterministically*. This would be impossible with above-mentioned simpler classical assumptions on interactions.

As shown by recent experimental and analytical studies [11], [12], [13], [14], the assumption of bounded cover times is (maybe surprisingly) well adapted for modeling mobility in practical sensor networks. The relevant networks are those where agents have different mobility and communication capabilities, and move within a bounded area or have tendency to return periodically to some specific places, e.g., their homes. The experiments studied networks of students in a campus, participants in a network conference, visitors at Disney World and more [11], [15], [16]. The analytical studies of such networks show that the *inter-contact time* of two specific agents is practically bounded. Since a cover time of an agent can be expressed as a sum of inter-contact times of the agent with the others, bounded cover times can model well the mobility in such networks.

Contributions. The main general contribution of this paper is the framework allowing an analytical computation of energy consumption, presented for the first time in the context of population protocols (Section II). Using this energy model, we start by analyzing energy consumption of the data collection protocol proposed and proven to be time optimal in [7] (Section III). In this analysis, our approach is motivated by the observation that the energy spent is proportional to the length of an execution. However, further investigation shows that this is not the only factor that can affect the energy spent, and other factors, as the residual energy of an agent, if considered by a solution, can improve energy performance. This gives an idea for developing a better solution in terms of energy consumption, and we propose an energy-aware protocol. We prove its correctness and analyze its energy complexity (Section IV-A and IV-B). Not surprisingly, it appears that it needs less energy than the first protocol when correctly adjusting the protocol’s parameters. Having analytic formulas for energy consumption under the form of functions allows to draw graphs and plots, on which remarkable points, zones and limits appear immediately (Section IV-C). This is particularly advantageous when a great number of parameters are involved and when the exhibited

²The notion of cover time has some similarities with *partial synchrony* [8] or *timeliness* [9] (in a lock step message-passing or shared-memory communication model), in the sense that the cover times impose constraints on the relative (activation) speeds and on the communication of processes (here agents). This sort of synchrony is partial, because the agents have no access to the cover times.

In the context of population protocols, somewhat similar to cover times’ guarantees on agents’ interactions are assumed in the implementation of the cover time service proposed in [10], for augmenting population protocols with termination detection.

functions are hybrid (i.e., composed of several functions). Getting the same or approximative results from simulations would certainly be longer and costly, while here exact results can be obtained almost immediately. The paper presents such plots as an illustration of this methodology. Section V presents a lower bound concerning energy consumption for all possible data collection protocols. Cases where this lower bound is reached by the presented protocols are also given. Finally, Sect. VI is a summary of the paper results.

Note that, in most of the study, we consider the more difficult, but also more realistic case, where an agent can hold only a limited number of values (independent of the number of agents). Such an assumption is coherent with the spirit of population protocols.

Due to the lack of space, several proofs have been moved to the appendix.

Related Work.

There is a very large literature on power-aware network protocols. The reader could find general references in some surveys, like [17], [18], [19] for power-aware routing protocols in MANET, like [20] for data collection in mobile sensor networks, and like [21] for optimistic routing in highly dynamic networks. Most of the works on energy consumption do not present an analytical study of energy complexity, and the validation of the approaches is made by simulation. In this section, we will only mention the studies directly linked to our approach and restrict our attention to references for which a formal model of energy consumption is presented, and its mathematical analysis is done.

In this category, we should first mention [22] where authors study the problem of energy balancing between *non finite state* agents (interacting uniformly at random) in a model inspired by population protocols. The considered model is not an energy consumption one. An interacting agent can transfer some part of its energy to the other agent during the interaction (by using a corresponding energy equipment) and there may be a constant energy loss for each unit of the transferred energy. However, contrary to our study, energy consumption for interacting and sensing is not considered. Three protocols, for balancing (averaging) the distribution of energies given to agents, are presented and their analytical or simulation analysis is presented.

Then, one can mention [23], which studies a synchronized scheduling of wake-up times to improve the data collection in a non-mobile sensor network. It proposes protocols with energy consumption of at most twice the optimum. Synchronization of the wake-up times is also studied in [24], where several self-stabilizing protocols to solve the problem of temporal partition are presented. The techniques include using randomly chosen relatively prime sleep periods and occasional, and possibly random, probing of extra time slots.

In [25], a 2-approximation (energy consumption) centralized heuristic and a distributed power-aware heuristic are developed for wireless sensor networks, to ensure that all packets are delivered with the minimum energy cost within a required deadline. Authors develop an evaluation function of energy

cost for one transmission, considering residual energy, quality of link and the type of link nodes.

In [26], authors propose a randomized clustering algorithm for non-mobile wireless sensor networks which minimizes the energy spent for transmission. It is proven formally that the algorithm has a time complexity of much less than $O(n)$.

In [27], authors give an approximated cone-based topology scheme to increase ad-hoc network lifetime with adaptive transmission power control, while maintaining connectivity for routing. In an analytical way, it is shown that the power consumption of each route can be made arbitrarily close to the optimal.

[28] studies the performance of a class of simple and local algorithms for energy-efficient construction of minimum spanning trees in a wireless ad hoc setting. Bounds on the performance of these algorithms in instances obtained by uniformly distributed points in the unit square, are given.

For being complete, we should also mention that there exist studies presenting a mathematical model for optimizing the energy-consumption of a single entity in the network, independently of the protocol run, but they are very far from our preoccupations.

Even if some approaches, considering energy consumption, are in spirit similar to ours, no one of them is devoted to the specific type of networks that we consider, anonymous resource-limited mobile sensors moving unpredictably and constituting a highly dynamic opportunistic network.

II. MODEL AND NOTATIONS

A. Population Protocols

A system consists of a collection \mathcal{A} of pairwise interacting agents, also called a population. Each agent represents a finite state sensing and communicating mobile device. Among the agents, there is a distinguishable agent called the *base station* (BST), which can be as powerful as needed, in contrast with the resource-limited non-BST agents. The non-BST agents are also called *mobile*, interchangeably. The size of the population $|\mathcal{A}| = n$ is unknown to the agents.

A (*population*) *protocol* can be modeled as a finite transition system whose states are called *configurations*. A *configuration* is a function that associates each agent with its state. Each agent has a state taken from a finite set of states, the same for all non-BST agents, but generally different for BST. Every transition $C \rightarrow C'$ between two configurations C and C' is modeled by a single *transition* between two agents happening during an interaction. That is, when two agents x , in state p , and y , in state q , in C , interact (meet), they execute a transition rule $(p, q) \rightarrow (p', q')$. As a result, in C' , x changes its state from p to p' and y from q to q' .

An *execution* of a protocol is a sequence of configurations C_0, C_1, C_2, \dots such that C_0 is the starting configuration and for each $i \geq 0$, $C_i \rightarrow C_{i+1}$. In a real distributed execution, interactions could take place simultaneously, but when writing down an execution we can order those simultaneous interactions arbitrarily. An execution is said to be *finite*, iff from some point on, no applicable transition changes the configuration.

In this case, this non-changing configuration is said to be *terminal*. When a terminal configuration is reached, we say that the *termination* has occurred. Each execution corresponds to a unique sequence of interactions, also called *schedule*. If an execution e is finite, its length $|e|$ is the number of interactions until termination. For convenience, we consider the number of interactions in an execution as the time reference, i.e., each next interaction adds one time unit to the global time.

Cover Time Fairness. Every agent $i \in \mathcal{A}$ has (an unknown to agents) parameter $cv_i \in \mathcal{N}$ called *cover time* s.t. during any cv_i time units (consecutive interactions in an execution), agent i interacts with every other agent at least once. cv_i is the minimum such number of interactions.³ For two agents x and y , if $cv_x < cv_y$, then we say that x is *faster* than y , and y is *slower* than x . The minimum cover time value is denoted by cv_{min} and the maximum one by cv_{max} . A *fastest* (resp. *slowest*) agent z has $cv_z = cv_{min}$ (resp. $cv_z = cv_{max}$). We denote by F the set of fastest mobile agents (whose $cv = cv_{min}$), and by NF the set of non-fastest ones.

In the protocol analysis, we consider only the most time and energy consuming case (for data collection) where $cv_{BST} > cv_{min}$. In some cases, for simplicity, we also assume that $cv_{min} \gg O(n^2)$. This assumption holds for a large and natural family of cover time vectors. It is especially natural for the case of passively mobile sensor networks, which is exactly the type of networks modeled by population protocols. Intuitively, as cv_{min} (in this family) is much larger than the number of all the possible pair-wise interactions, agents are free to interact in many different ways, as it would be naturally expected with passive mobility. Still very particular schedules are possible and there are guarantees that agents interact according to the cover times.

We emphasize that agents are not assumed to know cover times (to conform with the finite state population protocol model). Instead, we do assume that two interacting agents can only compare their respective *cv*s. As is described in [7], this can be implemented with finite state agents, if e.g. the number of different *cv*s is finite too (see [7] for more details and justifications).

Data Collection. A *problem* is defined by a predicate \mathcal{D} on executions. A population protocol \mathcal{P} is said to *solve a problem* \mathcal{D} , if and only if every execution of \mathcal{P} satisfies the conditions defining \mathcal{D} .

The problem of *data collection* (or *data gathering*) is defined as follows. We assume that each agent, except BST, owns an input value (which is constant during one execution of the protocol). Eventually, in any execution, every input value has to be delivered to BST, and exactly once.

When it happens, we say that the protocol has *converged* or *terminated*. The *convergence time* of a protocol is the maximum length of a possible execution (until convergence).

³Note that there are mappings from agents to their cover times for which the cover time fairness cannot be satisfied (e.g. if $cv_1 = 4$, $cv_2 = 6$, $cv_3 = 9$). From now on, we assume mappings allowing at least one possible fair execution. For an additional discussion on the validity of the cover time values, refer to [7].

In a protocol, an input value may move between different agents before it is delivered to BST. We denote by M the number of values a mobile agent can receive from other agents (on top of its own initial value). When describing or analyzing a considered protocol, a term "to transfer a value" from agent x to y means to copy it to y 's memory, and remove it from the memory of x .

B. Energy Consumption Scheme and Metrics

During an execution each non-BST agent consumes energy according to three different modes: *sleep* (*idle*), *awake* (*active*) and *terminated*. The transitions between modes are depicted in Fig. 1 and the overall consumption scheme can be summarized by the following points:

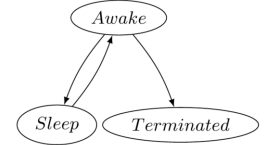


Fig. 1. Transitions between modes.

- 1) Each agent starts with the same amount of initial energy E_0 (e.g., all agents are fully charged). It is in *sleep* mode, and consumes E_{slp} per time unit (every interaction).

During this mode, an agent consumes a reduced amount of energy. It senses the vicinity for detecting other agents in proximity, and signals its presence. The signalling can be done either actively, by emission of small beacon messages (as, e.g., in IEEE 802.11), or passively, only by agent's presence. The latter can be carried out by sensing the vicinity using proximity sensors (cf. [3], [4]).

- 2) When two agents in *sleep* mode have successfully detected each other, the interaction happens and both turn into *awake* mode to proceed to a communication. For that, they have to switch on their main radio transceiver, which is generally a high energy consuming operation (cf. [29], [30]). During the interaction both agents stay *awake* and each consumes $E_{wkp} = E_{sw} + E_{tx} + E_{rx} + E_{slp}$, where E_{sw} is the energy consumed for switching to the main radio, E_{tx} and E_{rx} are consumed during the sending and reception of data, and E_{slp} is due to point 1.

Since in the considered types of networks (and in PP) non-BST agents have a small memory, independent of the network size, very few packets are transferred between agents in an interaction. Moreover, as two agents in an interaction should be in proximity in many considered networks, the transmission of data is done to a very short physical distance. Thus, we can assume that the energy consumed by the communication, $E_{tx} + E_{rx}$, is almost negligible compared to the switching energy E_{sw} , i.e., $E_{wkp} \approx E_{slp} + E_{sw}$.

- 3) During an interaction, two *awake* agents decide to turn either to *sleep* or to *terminated* mode, according to the protocol. No energy is consumed by a *terminated* agent.

This mode is useful in conserving energy of agents that have terminated their protocol activities. Together with

that, the scheme can be adapted such that a *terminated* agent could be restored into *sleep* mode, whenever it is required to restart a task. This can be done, for example, by a global signal from BST, which, at the same time, can also recharge the mobile agents, to prepare them for the next task [31].⁴

- 4) We assume that when a sleeping agent x meets a *terminated* agent y , x wakes up and thus, spends E_{wkp} . This assumption makes sense in networks deploying proximity sensors for sensing close agents. In this case, the *terminated* agent y is detected by the proximity sensor, and as x is not aware of y 's mode in this step, x wakes up and only then detects that y is *terminated*.⁵

Evaluation Metrics. There are many different power-aware metrics in the related literature on sensor networks. The term lifetime of a network is generally used and can be evaluated, e.g., as the maximum time until only one or all sensors have no energy, or until there are more than a ratio β of surviving nodes, or as the time until ‘‘connectivity’’ or ‘‘coverage’’ are lost. Alternatively, to analyze the lifetime, one can evaluate the maximum energy spent per node for a given task (e.g., per data collection from all nodes to BST, or per packet routing to the next node, or to the final destination). Refer to the surveys on such metrics [32], [33].

In this work, we choose to evaluate and minimize the maximum energy spent per node across the network in each data collection, thereby enhancing the lifetime performance. This metric is one of the most popular metrics studied in power-aware routing protocols for MANET [33].

Let $el_i(t)$ be the residual energy of a mobile agent i at time t (we use el_i if the time is clear from the context). As with cover times to conform with finite state assumption, we assume that agents have no access to the values corresponding to their residual energies, but two interacting agents can compare their respective energies (assuming that it is useful to consider only some finite number of values of the residual energy for comparisons).

Given a protocol \mathcal{P} , let $el_i(e)$ be the residual energy of a mobile agent i at the end of the execution e . Then $Es_{max}(e) = \max_{i \in \mathcal{A} \setminus \text{BST}} (E_0 - el_i(e))$ is the maximum energy spent by one agent at the end of e . To evaluate the lifetime of a system under protocol \mathcal{P} or to determine the required initial energy per agent, we analyze $Es_{max}(\mathcal{P})$ - the maximum energy spent by one agent on all executions of \mathcal{P} . Notice that for determining this value correctly for a given population, technically, the analysis should assume that the initial energy E_0 is large enough to accomplish the task in the original population (i.e., before any agent goes out of a power).

III. ENERGY CONSUMPTION OF *TTFM*

In this section, we consider an existing protocol, which was designed without energy consumption in mind, and we

⁴In this work, we do not consider this possibility, as we analyze energy consumption per one data collection only.

⁵However, with a beacon based approach for agents' detection (described in the first point), neither x , nor y spend E_{wkp} in such an interaction.

perform its energy consumption analysis, using the defined energy model. *TTFM* (Transfer To the Faster Marked) is a time-optimal protocol for data collection in PP [7]. The basic strategy for data transfer, in the protocol, is that a mobile agent only transfers its values either to BST, or to other agents which have never met any other mobile agent faster than themselves. When two fastest agents meet, there is no transfer of data. The idea of *TTFM* is to make use of the fastest agents, which are more likely to bring the values faster to BST.

For illustration of the usage of *terminated* mode, we adapt *TTFM* such that a mobile agent turns to *terminated* mode once it has transferred its values to a faster agent (excluding BST). Indeed, according to *TTFM*, once this happens, the slower agent does not receive any value till the end of the execution. Therefore, turning into *terminated* mode saves energy. For the fastest agents, as they never transfer values to other agents except to BST, they stay *non-terminated* until the termination. This adapted protocol, called *E-TTFM*, is presented below.

Protocol 1 *E-TTFM*

(for a mobile agent i interacting with an agent j)

- 1: $fastest_mark_x \in \{0, 1\}$ (* initialized to 1 *)
 - 2: **if** (j is not *BST*) **then**
 - 3: **if** ($cv_j < cv_i$ **and** $fastest_mark_j := 1$) **then**
 - 4: $fastest_mark_i := 0$
 - 5: **end if**
 - 6: **if** ($fastest_mark_j > fastest_mark_i$ **and** there is a free place in j 's memory) **then**
 - 7: agent i transfers to j as many values as would fit in the free part of j 's memory
 - 8: **if** (agent i has no value in its memory) **then**
 - 9: agent i turns to *terminated* mode
 - 10: **end if**
 - 11: **end if**
 - 12: **end if**
 - 13: **if** (j is *BST*) **then**
 - 14: agent i transfers all values to BST
 - 15: **end if**
-

E-TTFM has the same time complexity as *TTFM*, because the adaption we did does not influence the worst case convergence of data collection. Therefore, the longest execution in *E-TTFM* is of length $2cv_{min} \lceil \frac{|NF|}{|M|F} \rceil - 1$ (Th. 11 in [7]). The worst case on maximum energy spent by one agent is attained in the longest execution in which one fastest agent participates in a maximum number of interactions (being in *awake* mode as long as possible). This implies the following result (proven in the appendix):

Corollary 1: An upper bound on the maximum energy spent by one agent in the worst case of *E-TTFM* is $(2cv_{min} \lceil \frac{|NF|}{|M|F} \rceil - 1)E_{wkp}$. This bound is reached when $cv_{min} \gg O(n^2)$.

IV. ENERGY-BALANCED PROTOCOL *EB-TTFM*

A first step towards an energy efficient protocol would be to consider the following strategy. When an agent notes that it becomes very “low” in energy, it tries to transfer its values to a more powerful agent, as soon as possible. The issue raised by this strategy is that the convergence time of the data collection could possibly be considerably augmented, especially if some values are given to a slow agent. And an increased convergence time means more energy spent in *sleep* mode.

Therefore, the level of the amount of residual energy that is considered to be “low” should be carefully set. In the sequel, we will discuss the possible choices of such a level of residual energy. We make these choices depend on a single parameter, λ , by defining the “low” level by $\frac{E_0}{\lambda}$ where $\lambda \geq 1$ ⁶. Adding the test of the energy level to *E-TTFM* yields a protocol that we name *EB-TTFM* (for Energy Balanced *TTFM*). In this protocol, the energy test is performed and *E-TTFM* is executed only if the residual energy of the agent i is above the threshold or if the amount of energy el_j of the other agent j in the interaction is smaller.

Protocol 2 *EB-TTFM*(λ)

(for a mobile agent i interacting with an agent j)

- 1: **if** (j is not *BST*) **then**
 - 2: **if** ($el_i < \frac{E_0}{\lambda}$ **and** $el_i < el_j$ **and** there is a free place in j 's memory) **then**
 - 3: agent i transfers to j as many values as would fit in the free part of j 's memory
 - 4: **if** (agent i has no value in its memory) **then**
 - 5: agent i turns to *terminated* mode
 - 6: **end if**
 - 7: **else**
 - 8: execute *E-TTFM*
 - 9: **end if**
 - 10: **end if**
 - 11: **if** (j is *BST*) **then**
 - 12: agent i transfers all values to *BST*
 - 13: **end if**
-

A. Energy Consumption Analysis

Intuitively, by observing the code of *EB-TTFM*, it can be seen that, if the threshold $\frac{E_0}{\lambda}$ is too high, an important number of interactions will result in the execution of lines 3-5, since the condition in line 2 is satisfied easier. That concerns especially the fastest agents, since they interact more frequently. As a consequence, all the fastest agents will switch quickly to *terminated* mode and the values will be carried to *BST* by slower, even possibly slowest, agents. This will result in a worst case convergence time and energy consumption depending on cv_{max} and not on cv_{min} , which is worse than for *E-TTFM* (Cor. 1).

⁶To conform with the finite state population protocol model, we assume that $\frac{E_0}{\lambda}$ can only take a finite number of values.

On the other hand, if the threshold $\frac{E_0}{\lambda}$ is too small, *EB-TTFM* will execute line 8 more often, i.e., behave like *E-TTFM*, since the condition in line 2 will be difficult to satisfy. Thus, in the perspective of improving the performance of *E-TTFM*, we first study the conditions excluding the two previous (uninteresting) cases. More precisely, we study upper and lower bounds for E_0 with respect to λ (Lem. 1 and 2). Then, for fixed E_0 and λ , inside these bounds, we study the worst-case performance of *EB-TTFM* in terms of maximum energy spent by one agent (Th. 1). Next, we try to figure out, for a known E_0 , the best choice for λ (Eq. 2), according to the results on the worst-case performance.

As it is explained above, the number of the fastest agents turning into *terminated* mode during an execution plays an important role in the energy performance of *EB-TTFM*. Therefore, in the sequel, we classify the executions by the number of fastest agents in *terminated* mode at the end of the execution. Then, we study the length of the longest execution in each classification set (Prop. 1, 2 and 3) and use these results to obtain bounds for E_0 (Lem. 1 and 2) and determine the worst-case performance of *EB-TTFM* (Th. 1).

Definitions. We denote by Ξ_d the set of executions of *EB-TTFM*, s.t. $\forall e \in \Xi_d$, there are d fastest agents in *terminated* mode at the end of execution e , $d \in \{0, 1, \dots, |F|\}$. For a non-empty set Ξ_d , let $mni(d)$ be the minimum number of interactions until d fastest agents are in *terminated* mode in the executions of Ξ_d . And let $mnu(d)$ be the minimum number of values that have been delivered to *BST* at the $mni(d)^{th}$ interaction in the executions of Ξ_d .

We can see that, if $\Xi_1 = \emptyset$, no fastest agent ever turns into *terminated* mode and only *E-TTFM* (in line 6) is executed. In this case, *EB-TTFM* has the same performance as *E-TTFM*. On the contrary, if $\Xi_{|F|} \neq \emptyset$, there exists at least one execution, at the end of which all fastest agents are in *terminated* mode. Then, some values could be delivered to *BST* by a slow agent, increasing the convergence time of *EB-TTFM* (to the dependent on cv_{max}). The case of $\Xi_1 = \emptyset$ is uninteresting and the case of $\Xi_{|F|} \neq \emptyset$ is undesirable. Therefore, in the perspective of Ξ_d , we need to find sufficient conditions on E_0 and λ which ensure that $\Xi_1 \neq \emptyset$ and $\Xi_{|F|} = \emptyset$.

To ensure $\Xi_1 \neq \emptyset$, $mni(1)$ must be less than or equal to the length of the longest execution in *E-TTFM*. To ensure $\Xi_{|F|} = \emptyset$, $mni(|F|)$ must be greater than the length of the longest execution in the set $\cup_{i=0}^{|F|-1} \Xi_i$, i.e., the set of executions terminating before all the fastest agents are in *terminated* mode. Thus, we study the values of $mni(d)$ (Prop. 1) and the length of the longest execution in a non-empty Ξ_d (Prop. 3). And in order to give an upper bound for the length of the longest execution, we compute $mnu(d)$ in Prop. 2.

Proposition 1: The minimum number of interactions before d fastest agents are all in *terminated* mode, taken over all executions in a non-empty set Ξ_d of *EB-TTFM*, is

$$mni(d) = \left\lceil \frac{E_0 - \frac{E_0}{\lambda}}{E_{slp}} \left[1 - \left(1 - \frac{E_{slp}}{E_{wkp}} \right)^{\lceil \frac{d}{2} \rceil} \right] \right\rceil .$$

Proposition 2: The minimum number of values that have been delivered to BST at the $mni(d)^{th}$ interaction, taken over all executions in a non-empty set Ξ_d of $EB-TTFM$, is

$$mni(d) = M(|F| - 2\beta) \times \left\lfloor \frac{mni(d)}{2cv_{min}} \right\rfloor + 2M \times \left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor \times \left[1 + \frac{\beta}{\theta} - \frac{1-\theta}{\theta^2} (1 - (1-\theta)^\beta) \right],$$

where $\beta = \lfloor \frac{d-1}{2} \rfloor$ and $\theta = \frac{E_{slp}}{E_{wkp}}$.

Proposition 3: If $d < |F|$, the length of the longest execution \hat{e}_d in a non-empty set Ξ_d , is

$$\left\lceil \frac{|NF| - 2M \left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor \left[1 + \frac{1}{\theta} \left\lceil \frac{d}{2} \right\rceil - \frac{1-\theta}{\theta^2} (1 - (1-\theta)^{\lceil d/2 \rceil}) \right]}{M(|F| - d)} \right\rceil \times 2cv_{min} , \text{ where } \theta = \frac{E_{slp}}{E_{wkp}} .$$

Using the results above, to exclude the two previously described uninteresting cases for energy analysis, we compute the corresponding lower and upper bounds on the initial energy E_0 in $EB-TTFM$.

Lemma 1: [Upper Bound on Initial Energy E_0 ($\Xi_1 \neq \emptyset$)] If $E_0 \leq \left(2cv_{min} \left\lceil \frac{|NF|}{M|F|} \right\rceil - 1 \right) \frac{\lambda}{\lambda-1} E_{wkp}$, there exists at least one execution of $EB-TTFM$, in which a fastest agent turns into *terminated* mode.

Lemma 2: [Lower Bound on E_0 ($\Xi_{|F|} = \emptyset$)] If $E_0 \geq \left\lceil \frac{|NF|}{M \times |F|} \right\rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp}$ and $\theta \leq \frac{1}{2}$, in any execution of $EB-TTFM$, there is at least one fastest agent, which is not in *terminated* mode at the end of the execution. ⁷

Next, we consider the worst-case performance of $EB-TTFM$ with a fixed λ , supposing that every agent has an initial energy E_0 inside the bounds established above. Then from the analysis of the worst-case performance on energy consumption, we will determine the best choice for λ in $EB-TTFM$.

Notice that if there is only one fastest agent ($|F| = 1$), Lem. 1 and 2 cannot hold simultaneously. This case is not considered by Th. 1. However, with conditions of Lem. 2 only, the only fastest agent staying *non-terminated* during all the execution ensures a behavior of $E-TTFM$. In this case, the upper bound of Cor. 1 (depending on cv_{min}) holds. Otherwise, the bound is less interesting, as it depends on a cover time greater than cv_{min} .

Theorem 1: [Worst Case Energy Analysis] When $\left\lceil \frac{|NF|}{M \times |F|} \right\rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp} \leq E_0 \leq (2cv_{min} \left\lceil \frac{|NF|}{M|F|} \right\rceil - 1) \frac{\lambda}{\lambda-1} E_{wkp}$, the maximum energy spent by one agent in the

⁷Recall that $E_{wkp} > E_{slp}$. For simplicity and following the studies in [29] and [30], we choose $\theta \leq \frac{1}{2}$.

worst case of $EB-TTFM$, denoted by $Es_{max}(EB-TTFM)$, satisfies

$$Es_{max}(EB-TTFM) \leq \frac{\lambda-1}{\lambda} E_0 + (|\hat{e}_{|F|-1}| - mni(1)) E_{wkp} . \quad (1)$$

Proof: For an execution e of $EB-TTFM$:

- as $E_0 \geq \left\lceil \frac{|NF|}{M \times |F|} \right\rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp}$, $\Xi_{|F|} = \emptyset$ (Lem. 2), we have $|e| \leq |\hat{e}_{|F|-1}|$ if $\Xi_{|F|-1} \neq \emptyset$ and $|e| < |\hat{e}_{|F|-1}|$ otherwise.
- if $i \in F$ is the agent which consumes the most in e , there are two possibilities at the end of e :
 - 1) The energy spent by agent i is smaller or equal to $E_0 - \frac{E_0}{\lambda}$. Thus, the maximum energy spent by one agent in e is smaller or equal to $\frac{\lambda-1}{\lambda} E_0$.
 - 2) The energy spent by agent i is greater than $E_0 - \frac{E_0}{\lambda}$. Suppose that t_i is the number of already happened interactions when the residual energy of agent i decreases to $\frac{E_0}{\lambda}$. As $mni(1)$ is the minimum number of interactions when a fastest agent turns into *terminated* mode (Prop. 1), we have $t_i \geq mni(1)$. Thus, the interval $[t_i, |e|]$ of execution e is of length $|e| - t_i \leq |e| - mni(1)$. Consequently, the maximum energy spent by one agent in e is smaller or equal to $E_0 - \frac{E_0}{\lambda} + (|e| - t_i) E_{wkp} \leq \frac{\lambda-1}{\lambda} E_0 + (|e| - mni(1)) E_{wkp}$.

In conclusion, the maximum energy spent by one agent in the worst case of $EB-TTFM$, $Es_{max}(EB-TTFM)$, is $\frac{\lambda-1}{\lambda} E_0 + (|e| - mni(1)) E_{wkp}$ when $\Xi_{|F|-1} \neq \emptyset$. Otherwise, $Es_{max}(EB-TTFM) \leq \frac{\lambda-1}{\lambda} E_0 + (|e| - mni(1)) E_{wkp}$. ■

A natural issue is to determine the best value for λ , that is the value that maximizes the lower bound in (1), when $\Xi_{|F|-1} \neq \emptyset$. This determination is not easy because of the ceiling and floor functions appearing in (1). Thus, in order to get an exploitable expression, we drop, in the sequel, the assumption of bounded memory and we consider that an agent can hold an arbitrary number of values. Then we obtain a precise result on the maximum energy spent by one agent in the worst case.

B. Special Case of Non-Bounded Agent's Memory ($M \geq n - 1$)

The following improved bound for E_0 (with respect to Lem. 2) is obtained when $mni(|F|) \geq cv_{min}$. This condition ensures that at least one fastest agent is not *terminated* during the first cv_{min} . During this period, it meets all the non-fastest agents, collects their values, and they turn to *terminated* mode, as it is stated by Lem. 3.

Lemma 3: If $M \geq n - |F| - 1$ and $E_0 \geq \frac{\lambda}{\lambda-1} \left(\frac{\theta}{1 - (1-\theta)^{\lceil \frac{|F|}{2} \rceil}} \right) cv_{min} E_{wkp}$, in any execution of $EB-TTFM$, all non-fastest mobile agents are in *terminated* mode after the first cv_{min} interactions, where $\theta = E_{slp}/E_{wkp}$.

Similarly to the bounded case in the previous section, we compute $Es_{max}(EB-TTFM)$ for E_0 satisfying the bounds

of Lemmas 1 and 3.

Theorem 2: If $\frac{\lambda}{\lambda-1}(\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp} \leq E_0 \leq (2cv_{min} - 1)\frac{\lambda}{\lambda-1}E_{wkp}$, then $E_{s_{max}}(EB-TTFM) = \frac{\lambda-1}{\lambda}E_0 + (1 + \frac{1}{2-\theta})cv_{min}E_{wkp}$, where $\theta = E_{slp}/E_{wkp}$.

1) **Best Choice for λ :** Now, we consider the case where all agents have a sufficient initial energy E_0 for accomplishing one data collection. Then the issue is, for a known E_0 , to determine the best choice for λ for minimizing $E_{s_{max}}(EB-TTFM)$.

We could see from Th. 2 that $E_{s_{max}}(EB-TTFM)$ decreases when λ decreases. But λ must satisfy the condition $E_0 \geq \frac{\lambda}{\lambda-1}(\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp}$. Therefore, the smallest value of λ is obtained when $\frac{\lambda}{\lambda-1}(\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp} = E_0$. Solving this equation, we get the best choice for λ in $EB-TTFM$:

$$\tilde{\lambda} = \frac{E_0}{E_0 - (\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp}}. \quad (2)$$

Now, we study the consequences of setting λ to $\tilde{\lambda}$ (Eq. 2) in $EB-TTFM$. We denote by $El = \frac{E_0}{\lambda}$ the energy level in $EB-TTFM$. Since $\tilde{\lambda}$ is the best choice for λ , the best energy level is

$$\tilde{El} = \frac{E_0}{\tilde{\lambda}} = E_0 - (\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp}. \quad (3)$$

Eq. 3 expresses that the best choice for the energy level El is the initial energy minus a fixed amount, which is precisely $E_{fix} = (\frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}})cv_{min}E_{wkp}$. What is the explanation of that? Actually, in the analysis of $EB-TTFM$, an important point is the relation between E_0 and λ ensuring that there is at least one *non-terminated* fastest agent at the end of the first cv_{min} , i.e., $\Xi_{|F|} \neq \emptyset$. E_{fix} is the minimum energy consumption of an agent that ensures that $\Xi_{|F|} \neq \emptyset$. The interpretation of E_{fix} in the process of $EB-TTFM$ is the following. Once an agent has consumed more than E_{fix} energy, it transfers its values to more powerful agents and turns to *terminated* mode.

So, we can say that E_{fix} is the amount of energy to activate the condition in line 3 of $EB-TTFM$. In other words, the best strategy to increase the lifetime in the worst case of $EB-TTFM$ is to activate the condition as soon as an agent has consumed E_{fix} energy, i.e., when it is certain that there will be at least one *non-terminated* fastest agent at the end of the first cv_{min} . Thus, the best worst-case performance of $EB-TTFM$ is when λ is set to $\tilde{\lambda}$:

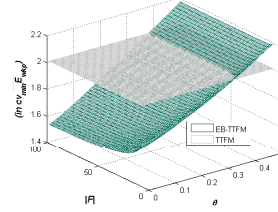
$$\begin{aligned} E_{s_{max}}^{\tilde{\lambda}}(EB-TTFM) &= E_{fix} + (1 + \frac{1}{2-\theta})cv_{min}E_{wkp} \\ &= (1 + \frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}} + \frac{1}{2-\theta})cv_{min}E_{wkp}. \end{aligned} \quad (4)$$

C. Interpretation of Results: Comparison of Energy Performance between $EB-TTFM$ and $E-TTFM$

In this section, we give a meaningful example of the interest to have analytical functions for describing the energy consumption performance. This example considers the unbounded memory case and illustrates conditions under which $EB-TTFM$ outperforms $E-TTFM$. Another example illustrating the relationship between the different parameters and the energy consumption in $EB-TTFM$ is given in the appendix.

According to Corollary. 1, when the memory is not bounded and $cv_{min} \gg O(n^2)$, $E_{s_{max}}(E-TTFM) = (2cv_{min} - 1)E_{wkp}$. From (4), we know that when λ is set to $\tilde{\lambda}$ (Eq. 2), $E_{s_{max}}^{\tilde{\lambda}}(EB-TTFM) = (1 + \frac{\theta}{1-(1-\theta)^{\lceil \frac{|F|}{2} \rceil}} + \frac{1}{2-\theta})cv_{min}E_{wkp}$.

Then, we obtain that when $|F| \geq 10$ and $\theta \leq \frac{3-\sqrt{5}}{2} \approx 0.38$, $E_{s_{max}}^{\tilde{\lambda}}(EB-TTFM) \leq E_{s_{max}}(E-TTFM)$, i.e., $EB-TTFM$ outperforms $E-TTFM$. The figure above represents the comparison of $E_{s_{max}}$ between $EB-TTFM$ and $E-TTFM$ with different $|F|$ and θ . The grey plane shows $E_{s_{max}}(E-TTFM)$ and the green surface represents $E_{s_{max}}^{\tilde{\lambda}}(EB-TTFM)$. We can see that $EB-TTFM$ is more energy efficient when θ is small, i.e., the energy spent in *sleep* mode is much less than the energy spent in *active* mode.



V. LOWER BOUND ON $E_{s_{max}}$

The idea of power-aware population protocols is to balance as much as possible the energy consumption among agents. According to our energy consumption scheme, each agent consumes E_{wkp} during each interaction and E_{slp} per time unit until either turning into *terminated* mode or until the termination of the execution. Thus, in order to reduce the energy consumption, being in *terminated* mode early, if possible, can prevent an agent from consuming too much. However, from the global point of view of the system, if less agents are involved for delivering the values, especially less fastest agents, the length of the execution is necessarily increased and slower remaining agents consume more. Thus an energy efficient protocol has to realize a good trade-off between the two strategies. A natural issue is to determine how far we can go, by establishing lower bounds.

In this section, we present a lower bound (Th. 3 and 4) on the maximum energy spent by an agent ($E_{s_{max}}$) for achieving data collection. The protocols considered here, like in the whole study, are those that can compare (but not use in any other way) cover times and the residual energies of any two interacting agents. Let us denote this class of protocols by $\mathbf{P}^{cv \cup e}$. Following the intuition above for strategies of energy efficient data collection protocols, we study the lower bound for different sub-classes of protocols in $\mathbf{P}^{cv \cup e}$, distinct in their strategies of turning agents into *terminated* mode.

Hence, we decompose $\mathbf{P}^{cv \cup e}$ into subsets \mathbf{P}_i , defined below. For that, we order the agents according to their cover times and denote by ns the number of different cover times and by F_i the set of mobile agents with the i^{th} smallest cover time. And we denote by cv^i the value of i^{th} smallest cover time. Thus the set of fastest agents is F_1 . Then, we define \mathbf{P}_i as the class of protocols for which, there is at least one *non-terminated* agent in $\cup_{j=1}^i F_j$ at the end of any execution (or turning into *terminated* mode during the last interaction of the execution). Moreover, in \mathbf{P}_i , when $i > 1$, there exists at least one execution in which all agents in $\cup_{j=1}^{i-1} F_j$ are *terminated* before the end of execution, i.e.,

$$\begin{aligned} \mathbf{P}_i &= \{ \mathcal{P} \in \mathbf{P}^{cv \cup e} \\ &\quad \forall e \in \mathbf{e}(\mathcal{P}), \exists f \in \cup_{j=1}^i F_j, \text{State}(f, e) \neq \text{Terminated} \wedge \\ &\quad \text{if } (i > 1) : \\ &\quad \exists e \in \mathbf{e}(\mathcal{P}), \forall f \in \cup_{j=1}^{i-1} F_j, \text{State}(f, e) = \text{Terminated} \}. \end{aligned}$$

where $\mathbf{e}(\mathcal{P})$ is the set of all executions in protocol \mathcal{P} and $\text{State}(f, e) \in \{ \text{Awake}, \text{Sleep}, \text{Terminated} \}$ indicates the state of agent f just before the last interaction in execution e . By definition, the \mathbf{P}_i 's are disjoint and $\cup_{i=1}^{ns} \mathbf{P}_i = \mathbf{P}^{cv \cup e}$. Moreover, $E\text{-TTFM}$ belongs to \mathbf{P}_1 . Note that $EB\text{-TTFM}$ can belong to different subsets depending on the value of $\frac{E_0}{\lambda}$. Under the conditions of Lem. 2, $EB\text{-TTFM}$ is in \mathbf{P}_1 .

We obtain our lower bound using in particular the lower bound result from [7]. This is a bound on time (the length of an execution) for any data collection protocol performing comparisons of the cover times of two interacting agents. Let us denote this class of protocols by \mathbf{P}^{cv} . The proof of the lower bound for \mathbf{P}^{cv} is based on constructing a possible worst case scenario, from the point of view of interactions that satisfy only the necessary conditions allowing data collection. For example, it is necessary that every mobile agent either meets BST by itself, or meets at least one other agent that meets BST , before termination. Such scenario cannot be improved by considering the comparisons of the residual energies of the interacting agents. Thus, it is easy to verify that the proof and the corresponding lower bound of [7] holds also for any protocol in $\mathbf{P}^{cv \cup e}$. This is stated by the following observation.

Observation: The lower bound on the length of the execution for any data collection in \mathbf{P}^{cv} is also correct for any protocol in $\mathbf{P}^{cv \cup e}$.

Recall that the bound in [7] is tight and equal to the upper bound on time (length of any execution) of protocol $TTFM$ also presented in [7]. We denote it here by $|e_{max}(TTFM)|$.

The lower bound presented below holds for the large and natural family of cover time vectors for which $cv_{min} \gg O(n^2)$ (see justifications in Sect. II-A).

Theorem 3: If $cv_{min} \gg O(n^2)$, for any protocol \mathcal{P} in \mathbf{P}_i , the maximum energy spent by one agent in the worst case, is greater than or equal to $\max\{ \frac{Es_{max}(E-TTFM)}{\lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil}, cv^i E_{wkp} \}$.

Proof: First, consider the following schedule which repeats the segment $X_1 X_2$, where $X_1 X_2$ is of length cv_{min} .

$X_1 \equiv [\text{repeat as much as possible all the possible interactions between agents in } \cup_{j=1}^i F_j],$

$X_2 \equiv [\text{all the other necessary interactions to satisfy } cv_s].$

Consider now the execution of $\mathcal{P} \in \mathbf{P}_i$ for this schedule, denoted by e . Since no agent consumes more than $Es_{max}(\mathcal{P})$ amount of energy in any execution of \mathcal{P} , when an agent consumes $Es_{max}(\mathcal{P})$ in e , either it turns to *terminated* mode or e reaches the final configuration. And as $\mathcal{P} \in \mathbf{P}_i$, there is at least one agent in $\cup_{j=1}^i F_j$ which is *non-terminated* at the end of e . Moreover, notice that $|X_2| \leq \frac{n(n-1)}{2} = O(n^2)$ and as $cv_{min} \gg O(n^2)$, the length of X_2 is negligible. Thus, following the above schedule, at time $t = \frac{Es_{max}(\mathcal{P})}{E_{wkp}} \times \lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil$, all agents in $\cup_{j=1}^i F_j$ consume $Es_{max}(\mathcal{P})$ amount of energy. However, the length of e must be smaller than or equal to t . Because otherwise, either there would be a *non-terminated* agent consuming more than Es_{max} , or all agents would be *terminated*, which contradicts the fact that $\mathcal{P} \in \mathbf{P}_i$. Thus, we have $t \geq |e|$, i.e.,

$$\frac{Es_{max}(\mathcal{P})}{E_{wkp}} \times \lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil \geq |e|.$$

And following the above schedule, there exists a ‘‘bad’’ execution e presented in [7] satisfying the lower bound on the length of the execution. Thus, we obtain $|e| \geq |e_{max}(TTFM)| = |e_{max}(E-TTFM)|$, as $E-TTFM$ has the same time performance as $TTFM$ (Sect. III). Moreover, by Cor. 1, when $cv_{min} \gg O(n^2)$, $|e_{max}(E-TTFM)| E_{wkp} = Es_{max}(E-TTFM)$. Then we have

$$\begin{aligned} Es_{max}(\mathcal{P}) &\geq \frac{|e| \times E_{wkp}}{\lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil} \\ &\geq \frac{|e_{max}(E-TTFM)| E_{wkp}}{\lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil} \\ &= \frac{Es_{max}(E-TTFM)}{\lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil}. \end{aligned} \quad (5)$$

Secondly, when $i > 1$, as $\mathcal{P} \in \mathbf{P}_i$, there exists at least one execution e in which all agents in $\cup_{j=1}^{i-1} F_j$ are terminated before the termination of the execution. Let t' indicate the time when all agents in $\cup_{j=1}^{i-1} F_j$ are *terminated* in e and let f be a *non-terminated* agent (with cover time cv_f), holding one of the values not yet collected by BST at time $t + 1$. We prove the existence of another execution e' such that $Es_{max}(e') \geq cv^i E_{wkp}$. The execution e' begins with the same prefix of t events of e and continues by meetings between agent f and the *terminated* agents until f meets BST at time $t + cv_f$. In our energy consumption scheme, when a sleeping agent meets a *terminated* agent, it consumes E_{wkp} and does not change its state (See Sect. II-B). Therefore and because $cv_f > cv_{min} \gg O(n^2)$, in the corresponding cv_f interactions in e' , we can choose interactions s.t. agent f consumes at least $cv_f E_{wkp}$ (the necessary interactions involving other agents than f during cv_f are negligible, as before). Since all the agents in $\cup_{j=1}^{i-1} F_j$ are in *terminated* mode at time t , $f \notin \cup_{j=1}^{i-1} F_j$ which implies

$cv_f \geq cv^i$. So, we have $E_{s_{max}}(e') \geq cv_f E_{wkp} \geq cv^i E_{wkp}$ which implies

$$E_{s_{max}}(\mathcal{P}) \geq cv^i E_{wkp} \quad (\text{when } i > 1) \quad (6)$$

Next, we consider the case where $i = 1$. We prove that $E_{s_{max}}(\mathcal{P}) \geq cv_{min} E_{wkp}$ by contradiction. Suppose that there exists a protocol $\mathcal{P} \in \mathbf{P}_1$ such that $E_{s_{max}}(\mathcal{P}) < cv_{min} E_{wkp}$. To contradict this, we construct an execution e of \mathcal{P} with $E_{s_{max}}(e) \geq cv_{min} E_{wkp}$. Consider again the schedule presented at the beginning of the proof. In the first cv_{min} , it is possible that an agent $j \in F_i$ consumes $E_{s_{max}}(\mathcal{P})$, since $t = \frac{E_{s_{max}}(\mathcal{P})}{E_{wkp}} < cv_{min}$. Let f be a *non-terminated* agent (with cv_f) holding one of the values not yet collected by *BST* at time $t + 1$. For the same reason as above, we can construct another execution e in which f meets the *terminated* agent j consecutively until it meets others at time $t + cv_f$. Thus, we have

$$E_{s_{max}}(\mathcal{P}) \geq cv_{min} E_{wkp} \quad (\text{when } i = 1) \quad (7)$$

Theorem 3 synthesises (5), (6) and (7) ■

Theorem 4: If $cv_{min} \gg O(n^2)$, for any protocol in $\mathbf{P}^{cv \cup e}$, the maximum energy spent by one agent in the worst case, is greater than or equal to $\max\{\frac{E_{s_{max}}(E-TTFM)}{\lceil \frac{|F|}{2} \rceil}, cv_{min} E_{wkp}\}$.

Proof: The result comes directly from Th. 3, since $\cup_i \mathbf{P}_i = \mathbf{P}^{cv \cup e}$ and

$$\begin{aligned} & \min_i \max\left\{ \frac{E_{s_{max}}(E-TTFM)}{\lceil \frac{|\cup_{j=1}^i F_j|}{2} \rceil}, cv^i E_{wkp} \right\} \\ &= \max\left\{ \frac{E_{s_{max}}(E-TTFM)}{\lceil \frac{|F|}{2} \rceil}, cv_{min} E_{wkp} \right\}. \end{aligned}$$

■

The following corollary identifies several cases where the bound of Th. 4 is reached. In these cases, $E-TTFM$ is energy-optimal.

Corollary 2: When $|F| = 1$ or 2 and $cv_{min} \gg O(n^2)$, $E-TTFM$ is *energy-optimal* in $\mathbf{P}^{cv \cup e}$, with respect to the maximum energy spent by one agent in the worst case.

Proof: From Th. 4, if $|F| = 1$ or 2 , $\forall \mathcal{P} \in \mathbf{P}^{cv \cup e}$, $E_{s_{max}}(\mathcal{P}) \geq E_{s_{max}}(E-TTFM)$. As $E-TTFM \in \mathbf{P}^{cv \cup e}$, $E-TTFM$ is energy optimal in $\mathbf{P}^{cv \cup e}$. ■

Corollary 3: When $|F| > 2$ and $cv_{min} \gg O(n^2)$, for any protocol in $\mathbf{P}^{cv \cup e}$, the maximum energy spent by one agent in the worst case is greater than or equal to $cv_{min} E_{wkp}$.

Proof: This result comes directly from Th. 4. ■

Corollary 3 actually indicates that, in the case where $M \geq n - |F| - 1$ and $cv_{min} \gg O(n^2)$, the presented lower bound differs from the upper bound on $E_{s_{max}}(EB-TTFM)$ (See Th. 2) only by a constant multiplicative factor. In this case, $EB-TTFM$ can be considered as approximately energy-optimal.

VI. CONCLUSION

This paper presents a formal model for power-aware distributed algorithms. Designing a good model is difficult. If

the model is too abstract (or general), it won't easily apply to reality, if it is too low level and considers too many parameters, it won't allow formal treatments. We believe that the model we introduced is a good compromise between the two approaches. On the one hand, it is based on practical information on how real resource-limited mobile sensor networks are functioning, and on the other hand, as shown in the paper, it allows to obtain analytically precise formulas, which are obviously not directly intuitive.

Moreover, the proposed model is adaptable to other types of mobile sensor networks and their applications (e.g., to the case where the transmitted data does not always fit into one packet). As already noticed, the assumption on the cover times does not particularize the model, since almost all real mobile sensor networks either operate in a bounded area (town, factory, security zone, etc.) or satisfy the home coming tendency (networks related to human or animal mobility). Nevertheless, other kinds of "periodic" conditions on the interactions (e.g., probabilistic interactions) can be considered to study energy consumption using the proposed framework. Finally, other important communication problems (as broadcast, all-to-all communication, routing, etc.) have many points in common with data collection (excepted that unique identifiers may be needed, like in community protocols [34]). Hence, the techniques developed here can be useful in future studies of these problems.

The second contribution of the paper consists in establishing analytical formulas related to the energy consumption of two data collection algorithms. We want to emphasize that having analytical formulas allows obtaining information that could not be obtained by the sole intuition, and that would be difficult to obtain by simulations. On one hand, the study of the curves and plots allows to get easily the conditions on the parameters (e.g., think of the value of $\frac{3-\sqrt{5}}{2}$ for θ in Sect. IV-C). On the other hand, the analytical approach allows obtaining general upper and (always difficult to establish) lower bounds.

To conclude, we summarize the main technical results of the paper. In Sect. IV-A, we obtain formulas for the worst-case performance of energy in $EB-TTFM$. Then, for the case of non-bounded memory, we compute the best value for λ with respect to E_0 , $|F|$, E_{wkp} , E_{slp} and cv_{min} (Sect. IV-B). Further analysis with the best λ shows that no matter how much initial energy an agent has, once it consumes more than some calculated amount E_{fix} , the best strategy is to switch to *terminated* mode, when interacting with a more powerful agent. We interpret the formulas graphically, and show that $EB-TTFM$ is more energy efficient in the case where the number of fastest agents are more than 10 and the ratio between E_{slp} and E_{wkp} is less than $\frac{3-\sqrt{5}}{2}$ (Sect. IV-C). At the end (Sect. V), we give a uniform lower bound for energy consumption for all possible data collection protocols. We identify the cases where this bound is reached by the presented protocols (Cor. 2), and where it differs only by a constant multiplicative factor from the energy upper bound of $EB-TTFM$ (Cor. 3).

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APPENDIX

Corollary 1. *An upper bound on the maximum energy spent by one agent in the worst case of E-TTFM is $(2cv_{min} \lceil \frac{|NF|}{M \times |F|} \rceil - 1)E_{wkp}$. This bound is reached when $cv_{min} \gg O(n^2)$.*

Proof: The upper bound is easy to obtain, as the longest execution in E-TTFM is of length $(2cv_{min} \lceil \frac{|NF|}{M \times |F|} \rceil - 1)$. Thus, no agent can consume more than $(2cv_{min} \lceil \frac{|NF|}{M \times |F|} \rceil - 1) \times E_{wkp}$ energy.

Next, we prove that this bound can be reached when $cv_{min} \gg O(n^2)$. Consider the following schedule which repeats the segment X_1X_2 , where X_1X_2 is of length cv_{min} . We denote by f to be one of the fastest agents.

$X_1 \equiv$ [repeat as much as possible all interactions between f and every agent in $A \setminus BST$],

$X_2 \equiv$ [all the other necessary interactions to satisfy cvs].

Consider the execution of E-TTFM for this schedule, denoted by e . This execution could be the worst case execution in time presented in [7]. Let us choose such e and thus, $|e| = (2cv_{min} \lceil \frac{|NF|}{M \times |F|} \rceil - 1)$.

In E-TTFM, a fastest agent never turns to *terminated*. Therefore, in e , agent f consumes the most (E_{wkp} energy in each interaction of X_1). Thus, $Es_{max}(e) = (|e| - \epsilon|X_2|)E_{wkp} + \epsilon|X_2|E_{slp}$ where $\epsilon|X_2|$ is a constant indicating the number of necessary interactions in X_2 not involving the agent f in e .

Notice that $|X_2| \leq \frac{n(n-1)}{2} = O(n^2)$ and as $cv_{min} \gg O(n^2)$, the length of X_2 is negligible. Therefore, when $cv_{min} \gg O(n^2)$, $Es_{max}(E-TTFM) = Es_{max}(e) = |e|E_{wkp} = (2cv_{min} \lceil \frac{|NF|}{M \times |F|} \rceil - 1)E_{wkp}$. ■

Proposition 1. *The minimum number of interactions before d fastest agents are all in terminated mode, taken over all executions in a non-empty set Ξ_d of EB-TTFM, is $mni(d) =$*

$$\left\lceil \frac{E_0 - \frac{E_0}{\lambda}}{E_{slp}} \left[1 - \left(1 - \frac{E_{slp}}{E_{wkp}} \right)^{\lceil \frac{d}{2} \rceil} \right] \right\rceil.$$

Proof: Consider an execution and a fastest agent i . In EB-TTFM, fastest agent i can only turn into *terminated* mode when the condition (in line 3) is satisfied, which means its residual energy is less than $\frac{E_0}{\lambda}$. During each interaction, agent i cannot spend more energy than E_{wkp} , corresponding to the cost of the *awake* mode. Then the minimum number of interactions until agent i turns into *terminated* mode, is $\lceil \frac{E_0 - \frac{E_0}{\lambda}}{E_{wkp}} \rceil = mni(1)$.

As both participants to one interaction consume energy, $mni(2) = mni(1)$, $mni(4) = mni(3)$, ..., and more generally $mni(d) = mni(d-1)$ when d is even. In addition, we have the following recursive equations when $d > 2$: $(mni(d) - mni(d-2)) \times (E_{wkp} - E_{slp}) + mni(d) \times E_{slp} = E_0 - \frac{E_0}{\lambda}$. Resolving this recursive equation yields:

$$mni(d) = \left\lceil \frac{E_0 - \frac{E_0}{\lambda}}{E_{slp}} \left[1 - \left(1 - \frac{E_{slp}}{E_{wkp}} \right)^{\lceil \frac{d}{2} \rceil} \right] \right\rceil. \quad (8)$$

We can see that $mni(d)$ is an increasing function of the ratio between E_{slp} and E_{wkp} (Eq. 8). When the ratio is small,

the configuration in which d fastest agents are in *terminated* mode will be reached earlier. ■

Proposition 2. *The minimum number of values that have been delivered to BST at the $mni(d)^{th}$ interaction, taken over all executions in a non-empty set Ξ_d of EB-TTFM, is*

$$mnv(d) = M(|F| - 2\beta) \times \left\lfloor \frac{mni(d)}{2cv_{min}} \right\rfloor + 2M \times \left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor \times \left[1 + \frac{\beta}{\theta} - \frac{1-\theta}{\theta^2} (1 - (1-\theta)^\beta) \right],$$

where $\beta = \lfloor \frac{d-1}{2} \rfloor$ and $\theta = \frac{E_{slp}}{E_{wkp}}$.

Proof: It has been shown that in any segment of $2cv_{min}$ consecutive interactions of an execution in TTFM, at least $M|F|$ values (tight bound) are transferred to the fastest agents (Th. 11 of [7]). Similarly, it is easy to see that every $2cv_{min}$ interactions, at least $M|F|$ values (tight bound) are delivered to BST. However, in EB-TTFM, since the fastest agents can turn into *terminated* mode (in line 3), this property is no longer true. But it still holds for the segments of $2cv_{min}$ consecutive interactions where all fastest agents are *non-terminated*. According to the definition of $mni(d)$, before the $mni(1)^{th}$ interaction, all fastest agents are surely *non-terminated* in any execution of EB-TTFM. So, we obtain at first the minimum number of values that have been collected at the $mni(1)^{th}$ interaction in EB-TTFM, which is: $mni(1) = M|F| \times \lfloor \frac{mni(1)}{2cv_{min}} \rfloor$. As $mni(1) = mni(2)$ (Prop. 1), we have $mni(1) = mnv(2)$.

Then, we consider the interval $[mni(1), mni(3))$ of an execution. It is certain that there are at least $|F| - 2$ fastest agents which are *non-terminated* in this interval. So we know that at least $M(|F| - 2) \times \left(\lfloor \frac{mni(3)}{2cv_{min}} \rfloor - \lfloor \frac{mni(1)}{2cv_{min}} \rfloor \right)$ values have been delivered to BST during this interval. Recursively, we obtain the following relation between $mni(d)$ and $mni(d-2)$:

$$\begin{aligned} mnv(3) &= mnv(4) \\ &= mnv(1) + M(|F| - 2) \left(\left\lfloor \frac{mni(3)}{2cv_{min}} \right\rfloor - \left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor \right) \\ &= M(|F| - 2) \times \left\lfloor \frac{mni(3)}{2cv_{min}} \right\rfloor + 2M \times \left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor ; \\ mnv(5) &= mnv(6) \\ &= mnv(3) + M(|F| - 4) \left(\left\lfloor \frac{mni(5)}{2cv_{min}} \right\rfloor - \left\lfloor \frac{mni(3)}{2cv_{min}} \right\rfloor \right) \\ &= M(|F| - 4) \times \left\lfloor \frac{mni(5)}{2cv_{min}} \right\rfloor \\ &\quad + 2M \times \left(\left\lfloor \frac{mni(1)}{2cv_{min}} \right\rfloor + \left\lfloor \frac{mni(3)}{2cv_{min}} \right\rfloor \right) ; \\ &\vdots \\ mnv(d) &= mnv(d-2) + M(|F| - 2 \lfloor \frac{d-1}{2} \rfloor) \\ &\quad \times \left(\left\lfloor \frac{mni(d)}{2cv_{min}} \right\rfloor - \left\lfloor \frac{mni(d-2)}{2cv_{min}} \right\rfloor \right) \\ &= M(|F| - 2 \lfloor \frac{d-1}{2} \rfloor) \times \left\lfloor \frac{mni(d)}{2cv_{min}} \right\rfloor \\ &\quad + 2M \sum_{j=1}^{\lfloor \frac{d-1}{2} \rfloor} \left\lfloor \frac{mni(2j-1)}{2cv_{min}} \right\rfloor . \end{aligned}$$

Let $\beta = \lfloor \frac{d-1}{2} \rfloor$, $\theta = \frac{E_{slp}}{E_{wkp}}$, by Prop. 1, we have:

$$\begin{aligned} mnv(d) &= M(|F| - 2\beta) \times \lfloor \frac{mni(d)}{2cv_{min}} \rfloor \\ &+ 2M \times \lfloor \frac{mni(1)}{2cv_{min}} \rfloor \times [1 + \frac{\beta}{\theta} - \frac{1-\theta}{\theta^2}(1 - (1-\theta)^\beta)] \end{aligned} \quad (9)$$

Proposition 3. *If $d < |F|$, the length of the longest execution \hat{e}_d in a non-empty set Ξ_d , is $2cv_{min} \times$*

$$\lceil \frac{|NF| - 2M \lfloor \frac{mni(1)}{2cv_{min}} \rfloor [1 + \frac{1}{\theta} \lfloor \frac{d}{2} \rfloor - \frac{1-\theta}{\theta^2}(1 - (1-\theta)^{\lfloor d/2 \rfloor})]}{M(|F| - d)} \rceil,$$

where $\theta = \frac{E_{slp}}{E_{wkp}}$.

Proof: The longest execution \hat{e}_d in Ξ_d is the one in which d fastest agents are in *terminated* mode as fast as possible, i.e., at the $mni(d)^{th}$ interaction. Moreover, the data is delivered to BST in the slowest way. It means that there are $|NF| - mnv(d)$ values that have not been delivered to BST at the $mni(d)^{th}$ interaction of \hat{e}_d . And from the $mni(d)^{th}$ interaction until the termination of \hat{e}_d , only $M(|F| - d)$ values are delivered to BST during every $2cv_{min}$ consecutive interactions. Thus, the longest execution is of length:

$$|\hat{e}_d| = mni(d) + \lceil \frac{|NF| - mnv(d)}{M \times (|F| - d)} \rceil \times 2cv_{min}$$

$$\stackrel{\text{Eq. 8, 9}}{=} \lceil \frac{|NF| - 2M \lfloor \frac{mni(1)}{2cv_{min}} \rfloor [1 + \frac{1}{\theta} \lfloor \frac{d}{2} \rfloor - \frac{1-\theta}{\theta^2}(1 - (1-\theta)^{\lfloor d/2 \rfloor})]}{M(|F| - d)} \rceil \times 2cv_{min}.$$

We can see that $|\hat{e}_d|$ is an increasing function of θ and d . As $\theta = \frac{E_{slp}}{E_{wkp}}$, when the energy spent in *sleep* mode increases, less interactions are needed for an agent to terminate (Eq. 8). Since agents turn to *terminated* mode more quickly, the convergence time of *EB-TTFM* is longer. On the other hand, if $\Xi_{|F|-1} \neq \emptyset$, the longest execution in $\cup_{i=0}^{|F|-1} \Xi_i$ is the longest execution in $\Xi_{|F|-1}$, i.e., $\hat{e}_{|F|-1}$. ■

Lemma 1. *If $E_0 \leq (2cv_{min} \lceil \frac{|NF|}{M|F|} \rceil - 1) \frac{\lambda}{\lambda-1} E_{wkp}$, there exists at least one execution of *EB-TTFM*, in which a fastest agent turns into *terminated* mode.*

Proof: We know that the time complexity of *E-TTFM* is $2cv_{min} \lceil \frac{|NF|}{M|F|} \rceil - 1$ (Corollary 1).

To ensure that $\Xi_1 \neq \emptyset$, i.e., there exists at least one execution of *EB-TTFM* in which a fastest agent i turns into *terminated* mode, $mni(1)$ must be less than or equal to $2cv_{min} \lceil \frac{|NF|}{M|F|} \rceil - 1$. Elsewhere, *EB-TTFM* would behave as *E-TTFM*. Thus, we have

$$\begin{aligned} mni(1) &\leq \left(2cv_{min} \lceil \frac{|NF|}{M|F|} \rceil - 1 \right) \stackrel{\text{Eq.8}}{=} \\ E_0 &\leq \left(2cv_{min} \lceil \frac{|NF|}{M|F|} \rceil - 1 \right) \frac{\lambda}{\lambda-1} E_{wkp}. \end{aligned}$$

Lemma 2. *If $E_0 \geq \lceil \frac{|NF|}{M \times |F|} \rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp}$ and $\theta \leq \frac{1}{2}$, in any execution of *EB-TTFM*, there is at least one fastest agent, which is not in *terminated* mode at the end of the execution.*

Proof: We know from Prop. 3 that when $d < |F|$, the length of the longest execution in a non-empty Ξ_d , is an increasing function of d . So, if $\Xi_{|F|-1}$ is not empty, the longest execution in $\cup_{i=0}^{|F|-1} \Xi_i$ is of length $|\hat{e}_{|F|-1}|$.

According to the definition of $mni(d)$, the configuration, where all fastest agents are in *terminated* mode can happen only at or after the $mni(|F|)^{th}$ interaction in an execution. Then, if all executions in $\cup_{i=0}^{|F|-1} \Xi_i$ are of length less than $mni(|F|)$, no execution would ever reach such a configuration, i.e., there is no execution e such that all fastest agents are in *terminated* mode at the end of e . Thus, $|\hat{e}_{|F|-1}| < mni(|F|)$ implies $\Xi_{|F|} = \emptyset$.

Substituting Eq. 8 and 3 in $|\hat{e}_{|F|-1}| < mni(|F|)$, we obtain the lower bound of E_0 , which is:

$$E_0 > \lceil \frac{|NF|}{M} \times \frac{\theta^2}{\theta^2 + (\lceil \frac{|F|+1}{2} \rceil + 1/2)\theta - 1} \rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp} = lb(\theta) \quad (10)$$

where $\theta = \frac{E_{slp}}{E_{wkp}}$. This lower bound can be seen as a function of θ , denoted by $lb(\theta)$.

Since $lb(\theta)$ is an increasing function of θ and $\theta \leq \frac{1}{2}$, we have $lb(\theta) \leq lb(\frac{1}{2}) < \lceil \frac{|NF|}{M} \rceil \times \lceil \frac{1}{|F|} \rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp}$. So, we have a sufficient condition for $\Xi_{|F|} = \emptyset$, which is: $E_0 \geq \lceil \frac{|NF|}{M \times |F|} \rceil \frac{\lambda}{\lambda-1} cv_{min} E_{wkp}$. ■

Lemma 3. *If $M \geq n - |F| - 1$ and $E_0 \geq \frac{\lambda}{\lambda-1} \left(\frac{\theta}{1 - (1-\theta)^{\lceil \frac{|F|}{2} \rceil}} \right) cv_{min} E_{wkp}$, in any execution of *EB-TTFM*, all non-fastest mobile agents are in *terminated* mode after the first cv_{min} interactions, where $\theta = E_{slp}/E_{wkp}$.*

Proof: We know that the state where all fastest agents are in *terminated* mode can happen only at or after the $mni(|F|)^{th}$ interaction. Thus, if $mni(|F|) \geq cv_{min}$, in any execution of *EB-TTFM*, there is at least one *non-terminated* fastest agent at the end of the first cv_{min} . Moreover, all the non-fastest mobile agents are in *terminated* mode, since every non-fastest mobile agent interacts with a *non-terminated* fastest agent during the first cv_{min} and has transferred all of its value (as the memory is unbounded) in this interval.

Substituting $mni(|F|)$ by Eq. 8 and expressing the formula in function of E_0 yields the formula appearing in Lemma 3. ■

Theorem 2. *If $\frac{\lambda}{\lambda-1} \left(\frac{\theta}{1 - (1-\theta)^{\lceil \frac{|F|}{2} \rceil}} \right) cv_{min} E_{wkp} \leq E_0 \leq (2cv_{min} - 1) \frac{\lambda}{\lambda-1} E_{wkp}$, then $Es_{max}(EB-TTFM) = \frac{\lambda-1}{\lambda} E_0 + (1 + \frac{1}{2-\theta}) cv_{min} E_{wkp}$ where $\theta = E_{slp}/E_{wkp}$.*

Proof: Assume that for an execution e , the energy of agent i decreases to $\frac{E_0}{\lambda}$ at time t_i . Then, there are two possible states for agent i after $t_i + cv_{min}$:

- Agent i is in *terminated* mode: It means that agent i has already transferred its value. Therefore, in this case, the residual energy of agent i at the end of execution, $el_i(|e|) = el_i(t_i + cv_{min}) \geq \frac{E_0}{\lambda} - cv_{min} E_{wkp}$.
- Agent i is not in *terminated* mode: If agent i is still alive after $t_i + cv_{min}$, for any interaction (i, j) at time $t \in [t_i, t_i + cv_{min}]$ with an agent j still *non-terminated*, $el_j(t) < el_i(t) \leq \frac{E_0}{\lambda}$. Thus, after $t_i + cv_{min}$,

agent i is the only *non-terminated* agent. And agent i is the agent that transfers the last value to the base station. Let agent j be the last agent turning into *terminated* mode during $[t_i, t_i + cv_{min}]$ and let t_j be the time where its energy decreases to $\frac{E_0}{\lambda}$. Let $t_{ij} \in [t_i, t_i + cv_{min}]$ be the time when *non-terminated* j interacts with i . We know that $t_{ij} > t_j$ since $el_j(t_{ij}) < el_j(t_j) = \frac{E_0}{\lambda}$. Denoting by x_i the number of interactions of agent i during $[t_i, t_{ij}]$ and by x_j the number of interactions of agent j during $[t_j, t_{ij}]$:

- 1) First, according to the definition of x_i and x_j , we have $x_i \leq t_{ij} - t_i, x_j \leq t_{ij} - t_j$.
- 2) As $el_j(t_{ij}) < el_i(t_{ij}), \frac{E_0}{\lambda} - x_j(E_{wkp} - E_{slp}) - (t_{ij} - t_j)E_{slp} < \frac{E_0}{\lambda} - x_i(E_{wkp} - E_{slp}) - (t_{ij} - t_i)E_{slp}$. Thus, we have $x_i < x_j + (t_i - t_j)\frac{\theta}{1-\theta}$ where $\theta = \frac{E_{slp}}{E_{wkp}}$.
- 3) There is no interaction between i and j during the interval $[\min(t_i, t_j), t_{ij}]$. Indeed, suppose that there is one at time $t' \in [\min(t_i, t_j), t_{ij}]$, $k_1 = \arg \min_{x \in \{i, j\}} t_x$ and $k_2 = \{i, j\} \setminus k_1$. We know that $el_{k_1}(t') \leq el_{k_1}(t_{k_1}) = \frac{E_0}{\lambda}$. If $el_{k_1}(t') < el_{k_2}(t')$, agent k_1 transfers its values to agent k_2 . If $el_{k_1}(t') > el_{k_2}(t')$, as $el_{k_2}(t') < el_{k_1}(t') \leq \frac{E_0}{\lambda}$, agent k_2 transfers its values to agent k_1 . Then, after the interaction at time t' , either agent k_1 or agent k_2 is in *terminated* mode which contradicts the interaction of (i, j) at time t_{ij} where both agents are *non-terminated*.

Since there is no interaction between agent i and j during the interval $[\min\{t_i, t_j\}, t_{ij}]$ and since there is at least one interaction between i and j for each cv_{min} time interval, according to the definition of a cover time, we have $t_{ij} \leq \min\{t_i, t_j\} + cv_{min}$. Thus, $x_i + x_j \leq cv_{min}$.

Taken all the inequalities obtained above into consideration, x_i reaches its maximum value $\frac{1-\theta}{2-\theta}cv_{min}$ when $t_{ij} - t_j = cv_{min}, t_{ij} - t_i = \frac{1-\theta}{2-\theta}cv_{min}, x_j = \frac{1-\theta}{2-\theta}cv_{min}$. Figure 2 gives an illustration of this case. As the ex-

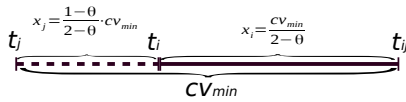


Fig. 2. Illustration of the case where x_i has the maximum value.

ecution ends before $t_{ij} + cv_{min}$, agent i interacts at most cv_{min} times after t_{ij} . Thus, from t_i to the end of execution, agent i interacts at most $x_i + cv_{min} = \frac{3-\theta}{2-\theta}$ times. Therefore, in the worst case, agent i will consume $\frac{3-\theta}{2-\theta}cv_{min}E_{wkp}$ energy at the end of execution e .

In conclusion, for any agent $i \in F$ whose energy decreases to $\frac{E_0}{\lambda}$ before the end of an execution e , we have proved that agent i should consume at most $\frac{3-\theta}{2-\theta}cv_{min}E_{wkp}$ energy. Therefore, we have $E_{smax}(EB-TTFM) = \frac{\lambda-1}{\lambda}E_0 + (1 +$

$$\frac{1}{2-\theta})cv_{min}E_{wkp} .$$

■

Interpretation of Results: Relationship between λ, E_0 and E_{smax} in $EB-TTFM$.

In the Section IV-B1, we discussed the best choice for λ in $EB-TTFM$. But what if the value of λ is not set to $\tilde{\lambda}$ (Eq. 2) and what is the performance of $EB-TTFM$ in this case? In this section, we use 3D plots to show the relation among λ, E_0 and $E_{smax}(EB-TTFM)$.

Firstly, from Lemma 3, we have a lower bound for E_0 depending on λ . Then, 3D plots of λ, E_0 and E_{smax} can be constructed following Th. 2. Figure 3 shows the performance of $EB-TTFM$ where $|F| = 10, \theta = 0.2$. x axis presents λ , y axis shows E_0 and z axis shows $E_{smax}(EB-TTFM)$. The blue polygon in Figure 3a presents the possible values of λ and E_0 for which $EB-TTFM$ does not behave like $E-TTFM$. As we can see, λ should be carefully chosen to a small value for $EB-TTFM$ to perform better than $E-TTFM$ in the worst case. The green border on the blue polygon represents the best choice for λ (Eq. 2). The red polygon in Figure 3a shows the region of values for λ and E_0 where $EB-TTFM$ behaves like $E-TTFM$.

Figure 3b gives a view of x, y axis which shows the feasible values of λ and E_0 (blue and red area).

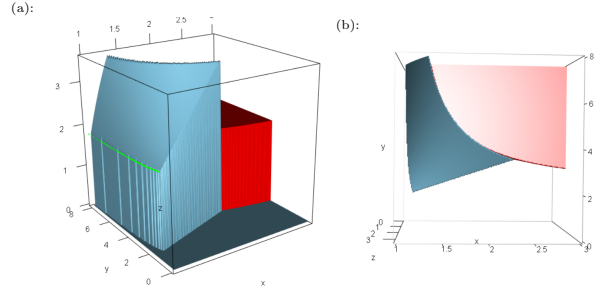


Fig. 3. 3D plot on the relationship of λ, E_0 and E_{smax}