Assessing Students' Competence in Algebra

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ASSESSING STUDENTS' COMPETENCE IN ALGEBRA[1]

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Abstract: The main focus of the LINGOT project is how to build systems that facilitate algebra learning. The starting point is to capture in a multi-dimensional way students knowledge from their behaviour when working with a computer on algebraic tasks. Once captured, this knowledge is used by humans teachers to propose to the students learning activities likely to help the evolution of their knowledge.
The originality of this on going project is to be based on a mathematical education research that had set up a multi-dimensional model of competence in algebra. This model is first applied to analyse and design tasks given by teachers to students, paper-and-pencil tasks as well as computerised tasks. Secondly this model is used to analyse the students' answers when performing those tasks.
We have built a first prototype, PÉPITE, that helps teachers to assess students in elementary algebra, setting up students' cognitive profiles. This system is yet tested in actual classrooms and yet used in teachers training. The research work on computer mediated remedial activities is just beginning. In this paper, we focus on the assessment system and we share our experience of designing classrooms tools in a participative way. The methodology adopted is a combination of work in mathematics education and user-centred design derived from Human-Computer Interaction research. Finally we discuss the validation process of such an assessment system and our research results.

Keywords: Elementary Algebra, Interface Design, Evaluation of Instructional System, Assessment of Student's Competence,.

1. Introduction

The aim of the PÉPITE project is to develop a tool to help teachers in assessing students' competence in elementary algebra. The 15 year-old students enter French general high schools coming from French college or vocational schools. Most of them encounter strong difficulties and the educational system fails to help them in overcoming those difficulties. As we started this research, our aim was to understand the reasons of such dysfunction, to identify the necessary conditions to a positive evolution, to create appropriate learning situations likely to help evolution of students' knowledge. The idea is to seek out, in the

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student's way of functioning, the *nuggets* of knowledge (in French, “*pépites*”) to use as a basis to build some new knowledge. One of the results of this study is tools enabling teachers to interpret students’ production in order to find starting points to modify students’ knowledge. The PÉPITE project is the first part of a larger project LINGOT (not presented here) which aims to assist teachers in choosing activities for students or groups of students corresponding to the starting points highlighted by PÉPITE.

As pointed by Conlon and Pain [3], applied AIED needs “a research methodology that gives a central place to collaboration among teachers, researchers and technologists”. HCI research proposes such methods (user-centred design, participatory design, usability engineering) [16] [14]. These methods suggest that users (actual students and teachers) must participate in the design process from the very beginning. We present here our experience in such a multidisciplinary approach and discuss about validation process of our assessment system.

In such an approach, the focus is on how to collect *relevant* and *reliable* data with a computer to *make sense* of students’ behaviour according to teachers’ needs. In this paper we assume that teachers’ needs are expressed in a model of competence derived from an educational research presented in section 2. This model gives the kind of results the diagnosis system has to produce (the students’ profiles). *Relevance* of the collected data refers to the model. *Reliability* refers to the biases introduced by using a computer and thus refers to interface design problems. *To make sense* refers usually in AI community to diagnosis techniques. It refers also to cognitive and epistemological assumptions about nature of competence in the domain.

In our work we dealt more with difficulty to clarify the model of competence and with interface design problems than with diagnosis techniques. In order to have a prototype we can test early in classrooms we made the choice of a width study rather than a depth study, that means that we first put emphasis on the multidimensional aspect rather than on sophisticated diagnosis techniques. So we focus in this text on how to ensure quality of incoming data that is, in our opinion, of most importance in relation with our objectives.

We here firstly present the educational basis of our work. We then introduce the research objectives of PÉPITE project and the general architecture of the system. We describe each prototype we have implemented and its validation. We point out that difficulties in designing and implementing such a software are not only a diagnosis problem as well known in AIED community, but first an interface designing problem. Finally, we discuss the methodology of validation of PÉPITE and our research results.

2. Educational basis

We begin by presenting what teachers needs to know about students, we then present our theoretical framework about mathematical learning and our model of competence in algebra on which our work is based. This section ends with the presentation of the paper-and-pencil diagnosis tool we built.

2.1. What do teachers want to know?

Assessment systems are very often short-item tests consisting of questions that can be answered in less than one minute each. Such systems give a description of student’s state of knowledge in term of rates of success / failure. A more popular approach in AIED community bases assessment on student modelling [20]. In those systems the representation of student’s knowledge consists of a set of rules, each expressing some small aspect of the domain. This set includes rules for most common misconceptions. A student model is a fine-grained report on student's skills. For example in OLAE [18], the student model reports the probability of mastery of around 290 rules.

Teachers and mathematics educational researchers of our project found inadequate the level of rule mastery to make decisions about elementary algebra teaching. It is not the only dimension of algebra competence.

Let's take an example. Figure 1 shows a student's solution for a classical problem. In term of rules we could say that Karine uses famous incorrect rules:

x + a → x a
a x ± b → (a ± b) x
a x - x → a – 1

A prestidigitator is self-confident while carrying out the following trick. He says to a player: "Think of a number, add 8, multiply by 3, subtract 4, add the number you thought of, divide by 4, add 2 and subtract the number you first thought of: you have found 7. " Is this affirmation true? Justify your answer.	x + 8 = 8x 8x 3 × (8x) = 24 + 3x = 27x 27x − 4 = 23x 23x + x = 24x 24x ÷ 4 = 6x 6x + 2 = 8x 8x − x = 7 la solution est bien égal à $\boxed{7}$ [2]

Figure 1a: The prestidigitator problem

Figure 1b: Karine's paper-and-pencil answer to the prestidigitator problem

Teachers in the PÉPITE project observe three points and then give an interpretation [7]:
– Karine reduces algebraic expressions in order to obtain a result without operator symbol at each right member of an equality. This difficulty is reported by Davis [4] as *process product dilemma*. Nonetheless, Karine's algebraic formulae keep meaning in relation with the problem and let her use incorrect rules but also correct ones: 3(x+8) → 3×8x →24 + 3x.
– Karine translates each sentence of statement into one symbolic expression: Teachers interpret this translation as an algebraic strategy yet very close to an arithmetic one.
– It is possible that knowing the result stirs Karine into using incorrect rules to obtain 7.
Karine has constructed malrules coherent with her conception about algebra as a formal tool to compute a result. In order to help her, it is not efficient enough to show her the right rules. Teachers have proposed to her problem situations involving algebra as a proving tool and emphasising the equivalence meaning of equal sign.

To adapt mathematical activity to student's state of knowledge, teachers need more than a quantitative description of student's behaviour expressed as success rates. Thus we intended to define a qualitative description in order to help teachers to choose adequate students' activities.

2.2. Assumptions about mathematical learning

Making sense of learner's behaviour is closely linked to a theoretical framework about mathematical learning. In this section we present assumptions that found our research.

In order to analyse the dysfunction mentioned above, we feel necessary to define a kind of reference for algebraic competence at this level. We made a synthesis of mathematical, epistemological, didactical and cognitive research works in algebra learning.

According to Douady [5] mathematical concepts have two non-independent dimensions: a tool status and an object status. As far as the tool dimension is concerned competence is expressed in terms firstly of ability to build algebraic expressions and relationships in order

[2] Translation: "the solution is actually 7".

to translate (for instance a verbal or graphical description of a problem) and to interpret them. Secondly it addresses the ability to choose adequate algebraic tools to solve problems. Different kinds of problems are involved with this tool dimension such as translating problem situations into equations. As far as the object dimension is concerned, we take into account the duality of the algebraic expressions when manipulating them formally: both semantics and syntactic objects. Competence is then expressed in terms of status of algebraic objects, manipulative ability and articulation between their semantic and syntactic attributes linked with other semiotic registers (algebraic, numerical, graphical and geometrical register and natural language) [6]. At this level, we need to consider that algebraic thinking requires a rupture with arithmetic thinking, requires abilities to interpret algebraic expressions both at a procedural and a structural level and requires to develop a necessary flexibility between the two kinds of interpretations [11] [15] [19].

2.3. The multidimensional model of competence in algebra

Based on this theoretical framework, we have been observing students' behaviour in mathematical classrooms activities during a long period (all the school year round) and have linked those observations with analysis of their exercise books of the previous year [7]. This study highlights that students' productions present coherence and regularities that correspond to their personal knowledge. From this study we kept four dimensions to have a qualitative description of students' algebraic behaviour (cf. figure 2). This model is used firstly to analyse tasks on which students are supposed to learn algebra and secondly to analyse students' productions on those tasks.

<table>
<tr>
<td rowspan="3">From arithmetic to algebra</td>
<td rowspan="3">– using equal sign
♦ announces a result
♦ expresses a symmetric and transitive relation
– calculating with arithmetic numbers
♦ correctly
♦ incorrectly
– using letters
♦ correctly (as unknown to write an equation, as variable to express a relationship or to prove a numerical property)
♦ incorrectly (as generalised number to substitute numerical values, as unspecified to manipulate formulae with incorrect rules, as label or shorthand for a concrete object)
♦ never using</td>
<td>Manipulating algebraic formulae</td>
<td>– good technical mastery
– weak technical mastery (e.g.: not recognising of remarkable identities)
– incorrect technique
♦ bad using of brackets (leading to good / bad result)
♦ using identified malrules
♦ sign errors while transforming
– confusing + et ×</td>
</tr>
<tr>
<td>Translating from a register to another</td>
<td>– correctly
– correctly but unexpected
– incorrectly (e.g.: square of sum→x^2+y^2)
– abbreviating</td>
</tr>
<tr>
<td>Justifying</td>
<td>– using algebra
– using legal rules
– using formal rules
– arguing in natural language
– using numerical example
– no explanation</td>
</tr>
</table>

Figure 2: Qualitative model of student's algebraic behaviour.

2.4. The paper-and-pencil diagnosis tool

Combining this multidimensional model with an ad hoc set of paper-and-pencil tasks we designed a tool enabling teachers to interpret the students' productions in order to establish their profile. This set of tasks has been carefully chosen by researchers and teachers to cover each model dimension. Three types of tasks are proposed to students during a test. *Technical exercises* aim to determine the level of mastery of formal manipulations. *Recognition exercises* aim to determine how students identify and interpret algebraic expressions. *Modelling exercises* aim to identify if students use the expected algebraic type of treatment, how they translate problems into algebraic register and how they use adapted tools to solve problems.

Matching students' answers to the model provides a diagnosis matrix of values (40x60) linking questions and dimensions of analysis. This very fine description of the behaviour is too detailed to be used by teachers. It is necessary to establish a higher level description: students' *cognitive profiles*. These profiles have three levels of description: a *quantitative description* of algebraic skills in terms of success rates for each type of tasks, a description of *flexibility between algebraic register and other registers* (represented by a diagram, and a *qualitative description* of functioning coherence. The Figure 3, 4, 5 give three screens from the Pépite prototype that give an idea of the aspects taken in account in the profile.

This paper-and-pencil diagnosis tool has been tested several times. It has in particular been tested in June 1996 on 600 students (21 classes and 7 teachers) of a third form class. This experiment has pointed out that it was difficult and boring for teachers to fill the diagnosis matrix for all their students because the encoding of the students' productions is a very difficult diagnosis task that needs an important didactical expertise. Moreover when several teachers encode same students' tests, diagnosis matrixes may be slightly different but cognitive profiles are in the end identical. It seems to indicate the soundness of the diagnosis tool with respects of teachers' expertise and their acceptance of the algebraic competence description. Furthermore, teachers involved in the experiment are excited at our project to computerise the boring part of the diagnosis.

At last, the students' paper answers coming from this experiment have been used as a corpus for the conception of the PÉPITE project described in next section.

3. The PÉPITE project

The PÉPITE project intends to demonstrate that it is possible to collect with a computer data on students' competence from which experts can build students' profiles, that it is possible to automate this diagnosis (at least for a major part) and that it is possible for teachers to use these profiles that they have built with the computer in order to make decisions in their classrooms.

Thus, PÉPITE software contains three modules: PÉPITEST collects students' answers to problems adapted from the paper-and-pencil tasks; PÉPIDIAG automatically fills the diagnosis matrix from data collected by PÉPITEST; PÉPIPROFIL from this diagnosis matrix computes the students' profiles and presents them to the users (teachers or researchers).

3.1. PÉPITEST

PÉPITEST is the student interface: it provides problems and gathers students' answers.

In PÉPITEST design, we firstly pay very much attention to *usability problems*. Indeed it is crucial in an assessment environment where collected data had to be interpreted as competence indicators and not to be biased by interface manipulation problems. Ease of learning and short learning times are paramount because students take the test only once. Iterative design is strongly recommended for ensuring interface usability throughout the HCI literature [13] [16]. In [97] we have discussed means to evaluate usability: ergonomic criteria, guidelines, expert walkthrough and pilot test with users [2].

Secondly, we had to create PÉPITEST tasks *as close as possible* to paper-and-pencil tasks in order to get answers *equivalent* to paper-and-pencil ones. Equivalent means that an expert or PÉPIDIAG could interpret them to fill the diagnosis matrix. Let us note that the multidimensional model of competence is used both to diagnose students' productions and to evaluate the PÉPITE tasks. Transferring pencil-and-paper exercises and tools to computational environment is not so obvious. It changes the tasks and has consequences on students' productions. Balacheff [1] calls this *computational transposition*. The main problem in PÉPITEST is that writing an algebraic expression with a pen is very different from typing it on keyboard. From students' point of view, without a specific editor, they have to translate a spatial representation of the expression (e.g. a fraction) into a linear one. From assessment point of view, this translation introduces a difficulty that can disturb diagnose acting as a *distorting mirror* (introducing bias) or that can make visible normally invisible indicators, acting as a *cognitive microscope* [17]. We can propose an algebraic expression editor but it is not yet so easy for student to use it. No PÉPITEST version presented here integrates this editor.

Thirdly, we bear in mind the difficulties in interpreting students' open answers. We could have use form-based user interface allowing students to express their approach without using natural language nor typing algebraic formulae. But it is necessary to allow students to express themselves without monitoring answers, in order to capture for instance their kind of justification or their writing of algebraic formulae. So, we have limited open questions but not too much in order to ensure the test completeness [8].

Presently PÉPITEST runs with 22 problems, with 32 closed questions, 26 answers requiring algebraic expressions and 31 answers using both algebraic expressions and natural language.

As a formative evaluation we first set up a pilot test on October 1996 with 25 students in a high school classroom. As far as the usability was concerned, some minor changes in the test rise to evidence: For instance the basic manipulations (such as carriage return, drag and drop etc.) have to be taught to some students. Nonetheless students have found easy using PÉPITEST 1. As designer, we enjoy that, in spite of difficulties in writing algebraic expressions, students have produced such expressions, and moreover, educational researchers succeed in interpreting them.

PÉPITEST 2 was tested on June 1997 with 43 students in two classes in order to validate PÉPITEST as data collector for diagnosis. Educational researchers in our team were enthusiastic: they were suspicious PÉPITEST would reduce the range of students' productions. For each question, we have found every kind of expected answer proposed in our model of competence in algebra. Thus it shows PÉPITEST *completeness in relation with the model of competence*. In regard to algebraic formulae, as we expected, students had difficulties in producing them. But, those difficulties do not prevent them from answering with algebraic formulae. According to teachers, only one student out of 43 seems to modify her answers. Thus it shows that the expression editor is welcome and useful but may be temporarily bypassed. Finally, educational researchers can fill the diagnosis matrix from students' answers to PÉPITEST problems and the teacher of the class could confirm the profiles thus obtained. So it shows the validity of PÉPITEST in relation with the paper-and-pencil diagnosis tool.

3.2. PÉPIDIAG

PÉPIDIAG is the diagnosis module that analyses answers to PÉPITEST and fills the diagnosis matrix [10]. Closed questions are easy to analyse because we manage to design the interface so that each choice matches expected skills in the competence model. Exercises requiring entering answers with algebraic formulae are more difficult to deal with. Besides linking them with the model skills it is necessary to apply transformations to students' productions in order to normalise them (commutativity, associativeness, etc.). In remaining exercises, in addition to the well-known difficulties processing natural language answers, we face with a segmentation problem, when algebraic formulae occur mixed with natural language.

For each exercise, a specific algorithm is written in order to match the students answer with the educational researcher's analysis. For algebraic expression a tree is built and the diagnostic is done working on this tree. For open questions, we do not attempt to understand the answer but only to look at some features : for instance searching key-word (it is necessary, you must etc. that denote a school child type of justification) or searching the equal status (announcing a result or relation of equivalence). For the moment the analyse is mostly ad hoc and the diagnosis techniques used rather simple.

In spite of that, for this module, we presently obtain two main results. Firstly, PÉPIDIAG is able to automatically analyse every closed answer and every simple algebraic expression answer. So we analyse 71 percent of students' answers to PÉPITEST problems and this analyse covers each dimension of the model. Secondly, we ran PÉPIDIAG on every student's production of our corpus: the system fills the diagnosis matrixes. In order to correlate this partial diagnosis with human assessment, we choose 5 students with different levels of competence and we asked an expert to fill manually the diagnosis matrix. PÉPIDIAG and the human assessor were in agreement. That means that we can already partially automate the diagnosis, but analysing the remaining 25 percent answers still has sense to obtain the completeness of the profiles.

3.3. PÉPIPROFIL

PÉPIPROFIL is the teacher interface: it computes students' profiles and presents them to the teacher.

3.3.1 Computing students' profiles

As we explained in 2.4, student's profile has three levels of description: success rates (Figure 3), flexibility between registers (Figure 4), functioning coherences (Figure 5). This results from algorithmic processes merging similar answers and applying thresholds to the content of the diagnostic matrix.

We yet obtain two results. In some case studies, with a manually filled matrix, PÉPIPROFIL computes same profile than teacher can do. And from the partial matrix yet filled by the system, PÉPIPROFIL builds partial profiles that are confirmed by the teachers that are members of the project. This can be explained by two kinds of reasons : the diagram representing flexibility between registers is very informative and the diagnosis gives information for each dimension.

In that time, we are making experiments with more student productions to validate the system.

3.3.2 Presenting profiles to teachers

A student profile presented to the teacher is a very sophisticated model. Stephanie Jean [10] built an hypertext to present the features characterising the students. For each feature, merely clicking on the feature, the teacher can access to the exercises and the student's answers that lead PÉPIDIAG to this item of the diagnostic. The teacher can also correct or complete the diagnostic and then have an abstract of the student profile (Figure 6).

When testing the system with teachers[3] we noticed difficulties for whose who did not know the model of competence. Even teachers trainers are not used to model student knowledge. For teachers who were already using the model of competence, they were enthusiastic and some decides to use PÉPITE in teachers training.

[3] This experiment took place in January 2000 with 16 teachers trainers and is yet unpublished

So before using PEPIPROFIL a training about the model of competence is necessary for the teachers.

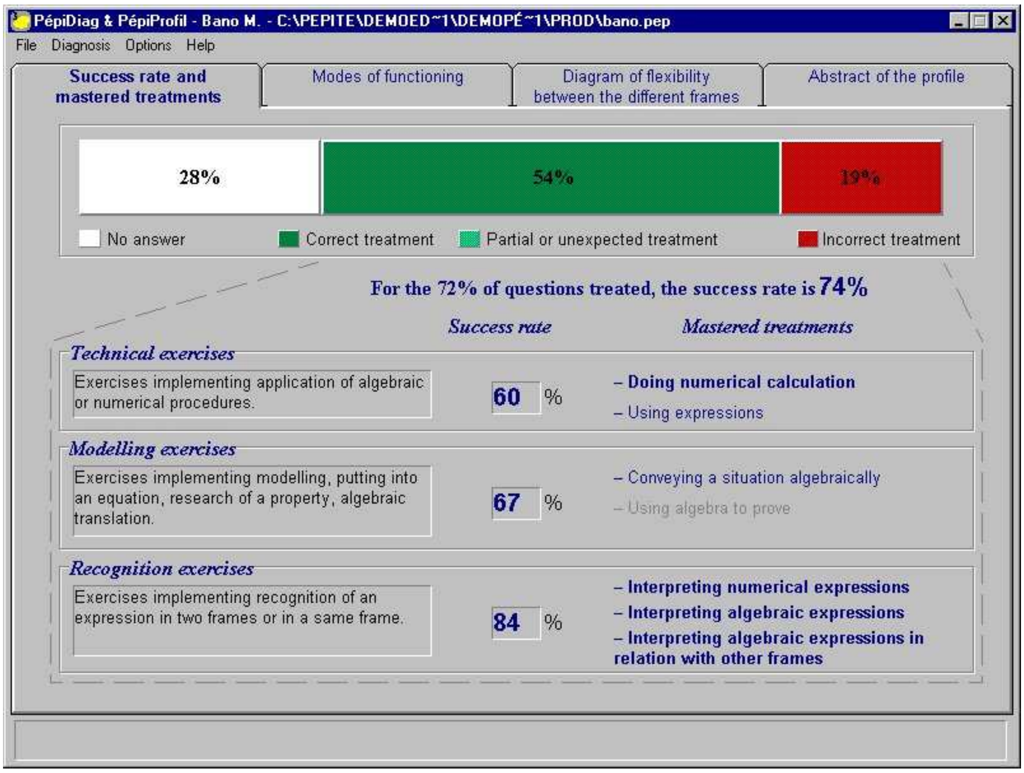

Figure 3: Bano's profil, success rates on algebraic tasks

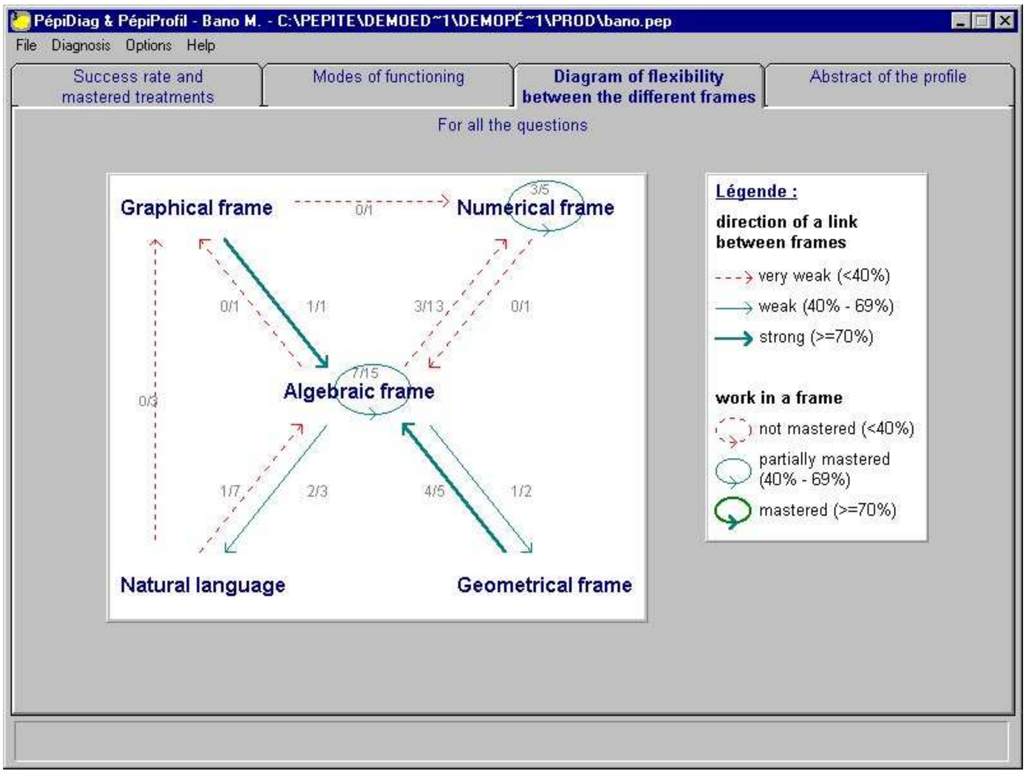

Figure 4: Bano's profile, diagram of flexibility between the different registers

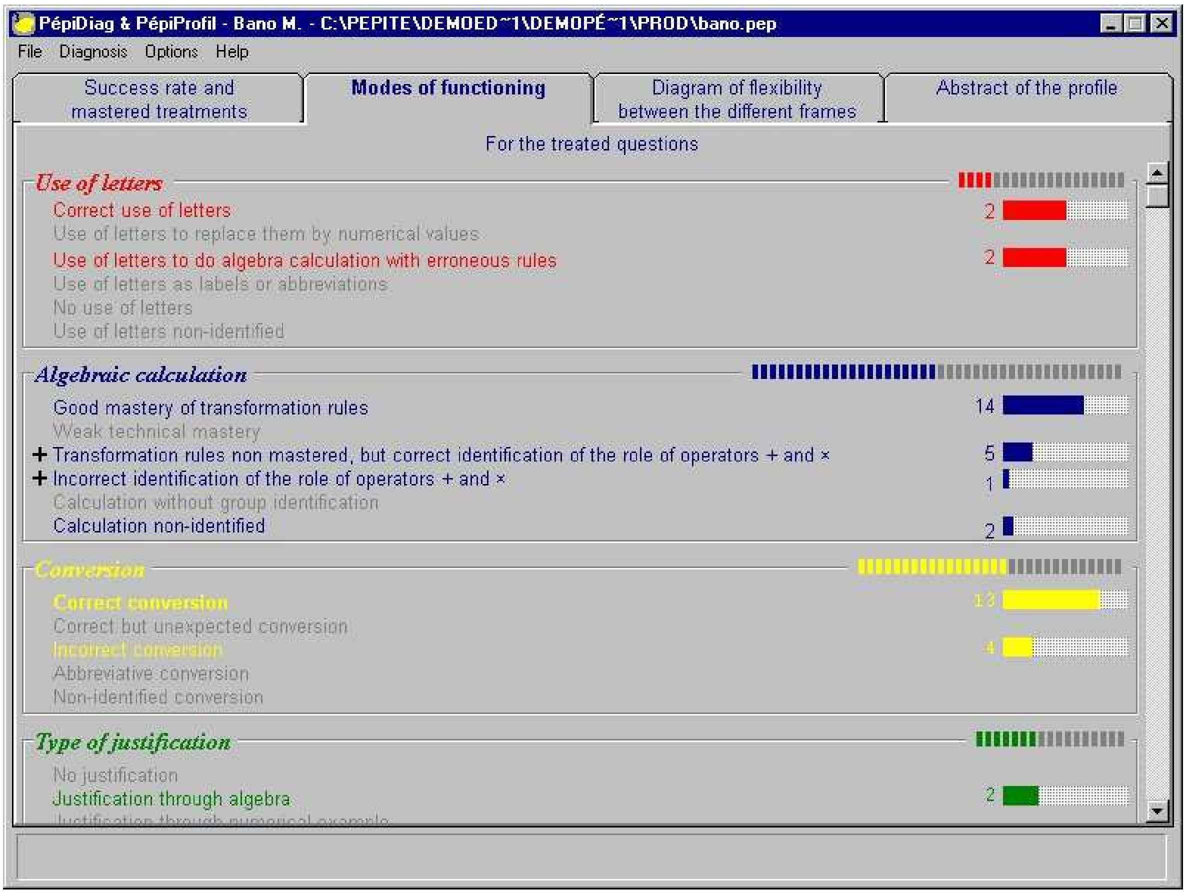

Figure 5: Bano's profile, different modes of functioning (part)

Bano M.'s profile based on the test done with PépiTest on the 23/06/99, according to the parameters selected in PépiProfil.

28% of the questions have not been treated.

54% of the questions have been correctly treated, -1% have been partially or unexpectedly treated.

In 19% of the questions, Bano M.'s answers are incorrect.

For the treated exercises, the results are as follows:

Bano M.'s success rate is 74%.

For technical exercises, 60% of the answers are correct. Bano do numerical calculation and partially succeed to using expressions.

For modelling exercises, 67% of the answers are correct. Bano partially succeed to conveying a situation algebraically .

For recognition exercises, 84% of the answers are correct. Bano is able to interpret numerical expressions, is able to interpret algebraic expressions and is able to interpret algebraic expressions in relation with another register.

The main modes of functioning spotted by Bano M. are:

Correct use of letters in 50% of the answers.

The mastery of algebraic calculation is still inadequate (45% of the questions). Difficulties are linked to:

- Transformation rules non mastered, but correct identification of the role of operators + and × in 23% of the answers (unsuitable use of brackets leading to correct result, use of identified transformation erroneous rules).
- Incorrect identification of the role of operators + and × in 5% of the answers (the transformation rules used gather the terms together).

Correct conversion in 76% of the answers.

School-child type of justification in 72% of the answers.

No information has been obtained concerning the numerical knowledge.

Links between the different register for all the exercises (treated or not):

Work in a specified register:

The numerical register is partially mastered.

The algebraic register is partially mastered.

Links between registers:

6 links are non-existent, but the following links exist:

Link from the register of graphical writings to the register of algebraic writings

Link from the register of algebraic writings to the register of numerical writings

Link from the register of numerical writings to the register of algebraic writings

Link from the register of algebraic writings to the register of natural language

Figure 6: Abstract of Bano's profile set up by PÉPIPROFIL

4. Discussion and conclusion

We intend to develop a tool to help teachers in assessing students' competence in elementary algebra. This tool is not yet completed but we have proved its feasibility.

In the early stage of the project the mathematical educational researcher war suspicious about the possibility to collect, with the computer data rich enough to build a profile. Until the algebraic expressions editor is not integrated, it's not so easy for students to enter such expressions. But in spite of that, students do produce algebraic expression and we collect with PEPITEST the whole range of answers studied by educational researchers.

Secondly, until we progress in interpreting natural language answers, the diagnosis would be partial. But as this partial diagnosis covers every dimension of the model, experts can propose remedial activities even with this partial diagnostic. Researchers can do that but not teachers, not even teachers trainers. So our second goal is achieved : we can for a major part automate the diagnosis. But the third one is only partially achieved : only experimented teachers can use PÉPITE to make decisions. In order to extend PÉPITE target audience we have to work on training the teachers to model student s' knowledge and then to work on a assistant system to choose remedial activity associated to the student profile.

With regards to the mathematical educational expertise the project was based on, our validation of PÉPITEST consists in verifying that we obtain equivalent answers in paper-and-pencil test and with PÉPITEST and that data obtained from the software allows experts to build profiles equivalent to paper-and-pencil ones. We evaluate PÉPIDIAG and PÉPIPROFIL by comparing how automatic profiles fit with human assessors ones.

It would be interested but we have not done that yet, to look at the differences: What does the software allow the paper-and-pencil test does not allow ?

The present PÉPITE prototype already gives results that some teachers are already using. In spite of its still simple diagnosis module, PÉPITE already performs by the quality of incoming data gathered by this interface. In our opinion this first success is due to teachers' and educational researchers' involvement from the very beginning of the project and to the carefully design of interfaces.

As research perspectives, this prototype allows us to incrementally develop the diagnosis module, working with corpus obtained from real students' interactions with PÉPITEST. It gives us an experimental platform to set up training in algebra for teachers and to develop with them remedial learning situations. In the future, we would like to adapt PÉPIPROFIL to present the profile to the student working with him/her on his/her own competence in algebra.

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