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► **To cite this version:**

Adrien Leygue, Michel Coret, Julien Réthoré, Laurent Stainier, Erwan Verron. Data Driven Constitutive identification. 2017. hal-01452494v2

**HAL Id: hal-01452494**

**<https://hal.science/hal-01452494v2>**

Preprint submitted on 21 Apr 2017

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# Data Driven Constitutive identification

Adrien Leygue, Michel Coret, Julien Rethoré, Laurent Stainier and Erwan Verron

April 21, 2017

# 1 Introduction

Constitutive equations constitute a key concept in mechanical engineering as it relates, for a given material, strain and stress. The parameters of a constitutive equation are usually adjusted considering a sufficient set of experimental data through an appropriate fitting procedure. Beyond the mere description of the mechanical response, a constitutive equation has several purposes:

- it provides a smoothed strain-stress relation in which experimental noise has been smeared out,
- for given loading conditions, *e.g.* uniaxial extension, it both interpolates and extrapolates between individual measurements,
- its tensorial nature naturally extends the stress-strain relation to multiaxial loading conditions that might be difficult to attain experimentally.

Recently, Kirchdoerfer and Ortiz [**Kirchdoerfer:2016ft**] introduced the concept of Data-Driven Computational Mechanics (DDCM) for elastic materials, in which constitutive equation vanishes and is replaced by a database of strain-stress couples (called states) which sample the mechanical response of the material. In this approach the regularization/smoothing/interpolation of experimental data are carried out during the computation of the numerical solution of the boundary value problem. The results presented by the authors are encouraging and open the door to many perspectives from the modeling point of view, since the necessity of an explicit or implicit strain-stress relation is relaxed.

Let us briefly recall the method of Kirchdoerfer and Ortiz for data-driven simulation in the particular case of truss structures. It seeks to assign to each truss element of the domain a mechanical state and a material state: a state being a strain-stress couple. Considering both mechanical equilibrium (involving stress) and compatibility conditions (involving strain) as non-questionable, the mechanical state of a given element  $e$  consists of a strain-stress pair  $(\epsilon_e, \sigma_e)$  which exactly satisfies the above equations which can be considered as constraints. The second state associated to  $e$ , denoted  $(\epsilon_{ie}^*, \sigma_{ie}^*)$ , is called the material state and is extracted from a collection of admissible material states for the material:  $(\epsilon_i^*, \sigma_i^*)$ , where  $i \in 1 : N^*$ . The index  $ie \in 1 : N^*$  specifies the material state of element  $e$ .

The proposed solver seeks, for every element simultaneously, a mechanical and a material state as close to each other as possible and such that the former satisfies mechanical equilibrium and compatibility conditions. This is formally expressed as:

$$\text{solution} = \arg \min_{\epsilon_e, \sigma_e, ie} \frac{1}{2} \sum_e w_e \|(\epsilon_e - \epsilon_{ie}^*, \sigma_e - \sigma_{ie}^*)\|_C^2, \quad (1)$$

subject to:

$$\sum_e w_e \mathbf{B}_{ej} \sigma_e = \mathbf{f}_j, \quad (2)$$

and:

$$\epsilon_e = \sum_j \mathbf{B}_{ej} \mathbf{u}_j. \quad (3)$$

In the above equations,  $\|(\epsilon, \sigma)\|_C$  is a given energetic norm, the matrix  $\mathbf{B}_{ej}$  encodes the connectivity and geometry of the truss, and  $w_e$  denotes the volume of the truss element  $e$ . Furthermore,  $\mathbf{u}_j$  and  $\mathbf{f}_j$  represent respectively the displacement and the force applied to truss nodes. Figure 1 illustrates the main ideas behind the method. For the particular choice

$$\|(\epsilon_e, \sigma_e)\|_C^2 = \frac{1}{2} (C_e \epsilon_e^2 + \frac{1}{C_e} \sigma_e^2), \quad (4)$$

the authors propose an efficient algorithm to solve this problem of combinatorial complexity. The constant parameter (possibly defined element-wise)  $C_e$  is the only parameter of the method and can be interpreted as a modulus associated to the mismatch of mechanical and material states.

Although there is no need for a constitutive equation, the database of material states is a mandatory pre-requisite of the method before starting the simulation.

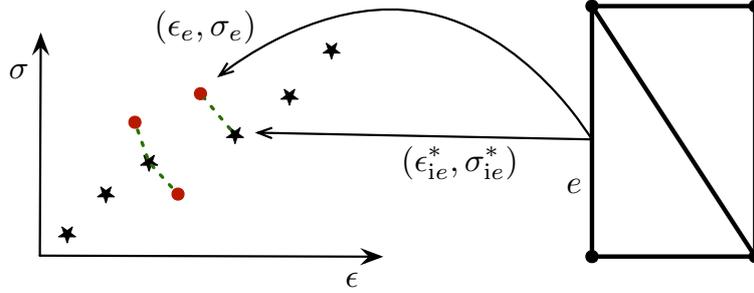


Figure 1: The two states  $(\epsilon_e, \sigma_e)$  and  $(\epsilon_{ie}^*, \sigma_{ie}^*)$  associated to a truss element  $e$ . The dashed line represents the energetic mismatch between the two states. On the left we see that the mapping  $ie$  between elements and material states can assign the same particular material state to two different elements.

Building this database computationally, for example through micro-macro approaches such as  $FE^2$  is computationally expensive and might require efficient model order reduction and high dimensional interpolation techniques. From an experimental point of view however it is far from trivial to be able to, somehow, measure the strain and stress over a wide range of deformations.

Here we propose a procedure, based on some inversion of the data-driven solver, to extract strain-stress couples characteristic of an elastic response. The method uses a collection of displacement and (non homogeneous) strain fields acquired, for example, using Digital Image Correlation techniques. It identifies simultaneously the stress part of the mechanical state for each loading condition and the full material states database, which is common to all loading conditions. The proposed method is first developed for truss structures. The convergence, the influence of measurement noise and the parameters influence are discussed. It is then generalized to a more general small strain elastic problem. In both cases the method is validated using manufactured data.

## 2 Data-driven identification for truss structures

In this section we propose a method, derived from the data-driven solver of Kirchdoerfer and Ortiz [Kirchdoerfer:2016ft], to build the material database by identifying strain-stress couples from experimental measurements.

### 2.1 Data-driven Identification

Consider a large database of measurements made on real truss structures subject to different loading conditions; all truss elements being of the same material. For each loading condition, or data item, indexed by  $X$ , we have access to the following quantities:

- the nodal displacements  $\mathbf{u}_j^X$ ,
- the truss geometry and connectivity, encoded through the matrix  $\mathbf{B}_{ej}^X$ . The mechanical strains are computed as  $\epsilon_e = \sum_j \mathbf{B}_{ej} \mathbf{u}_j$ ,
- the applied forces  $\mathbf{f}_j^X$ ,
- the prescribed nodal displacements.

Identifying the material response of the truss elements reduces to the determination of a finite number  $N^*$  of material states:  $(\epsilon_i^*, \sigma_i^*)$ , with  $i \in 1 : N^*$ , common to all the data items (thus independent of  $X$ ), and such that:

1. for each data item  $X$  the mechanical stress  $\sigma_e^X$  satisfies mechanical equilibrium for  $X$ ,
2. for each data item  $X$ , a material state  $(\epsilon_{ie^X}^*, \sigma_{ie^X}^*)$  is assigned to each element  $e$ , such that it is close to the mechanical state  $(\epsilon_e^X, \sigma_e^X)$  according to a given energetic norm.

It can be formulated as follows

$$\text{solution} = \arg \min_{\sigma_e^X, \epsilon_i^*, \sigma_i^*, ie^X} \mathcal{E}(\sigma_e^X, \epsilon_i^*, \sigma_i^*, ie^X), \quad (5)$$

with,

$$\mathcal{E}(\sigma_e^X, \epsilon_i^*, \sigma_i^*, ie^X) = \frac{1}{2} \sum_X \sum_e w_e^X \|(\epsilon_e^X - \epsilon_{ie^X}^*, \sigma_e^X - \sigma_{ie^X}^*)\|_C^2, \quad (6)$$

and subject to

$$\sum_e w_e^X \mathbf{B}_{ej}^X \sigma_e^X = \mathbf{f}_j^X \quad \forall X, j. \quad (7)$$

Unlike the data-driven solver of Kirchdoerfer and Ortiz, the mechanical strain  $\epsilon_e^X$  is not an unknown of the problem since it can be computed as  $\epsilon_e^X = \sum_j \mathbf{B}_{ej}^X \mathbf{u}_j^X$ .

At first glance, this problem seems to be quite difficult to solve, but we will show that it can be simplified a great deal. Let us first substitute the expression of the energetic norm in Eq. (5) and introduce a set of Lagrange multipliers  $\boldsymbol{\eta}_j^X$  to enforce the equilibrium constraint Eq. (7). Assuming that the material state mapping  $ie^X$  is known, we obtain the following stationary problem

$$\delta \left( \sum_X \sum_e \left( w_e^X C_e^X (\epsilon_e^X - \epsilon_{ie^X}^*)^2 + w_e^X \frac{1}{C_e^X} (\sigma_e^X - \sigma_{ie^X}^*)^2 - \sum_j (w_e^X \mathbf{B}_{ej}^X \sigma_e^X - \mathbf{f}_j^X) \cdot \boldsymbol{\eta}_j^X \right) \right) = 0 \quad (8)$$

Taking all possible variations yields the following set of equations

$$\delta \epsilon_i^* \Rightarrow \sum_X \sum_{ie=i} w_e^X C_e^X (\epsilon_e^X - \epsilon_{ie^X}^*) = 0 \quad \forall i, \quad (9)$$

$$\delta \sigma_i^* \Rightarrow \sum_X \sum_{ie=i} w_e^X \frac{1}{C_e^X} (\sigma_e^X - \sigma_{ie^X}^*) = 0 \quad \forall i, \quad (10)$$

$$\delta \sigma_e^X \Rightarrow w_e^X \frac{1}{C_e^X} (\sigma_e^X - \sigma_{ie^X}^*) - \sum_j w_e^X \mathbf{B}_{ej}^X \boldsymbol{\eta}_j^X = 0 \quad \forall e, X, \quad (11)$$

$$\delta \boldsymbol{\eta}_j^X \Rightarrow \sum_e (w_e^X \mathbf{B}_{ej}^X \sigma_e^X - \mathbf{f}_j^X) = 0 \quad \forall j, X. \quad (12)$$

In the above equations,  $\sum_{ie=i}$  stands for the sum over all elements  $e$  assigned to the material state  $i$  i.e.  $ie = i$ . Eq. (9) simply states that each material strain  $\epsilon_i^*$  is a weighted average of the mechanical strains in elements assigned to this specific material strain. Similarly Eq. (10) states that each material stress  $\sigma_i^*$  is a weighted average of the mechanical stresses in elements assigned to this specific material stress. The above equations can be simplified and re-interpreted through some simple manipulations. The combination of Eqs. (11) and (12) yields

$$\sum_k \sum_e w_e^X C_e^X \mathbf{B}_{ej}^X \mathbf{B}_{ek}^X \boldsymbol{\eta}_k^X + \sum_e w_e^X \mathbf{B}_{ej}^X \sigma_{ie^X}^* = \mathbf{f}_j^X \quad \forall j, X. \quad (13)$$

This equation simply states that for any data item  $X$ , the mechanical imbalance between applied forces  $\mathbf{f}_j^X$  and material stresses  $\sigma_{ie^X}^*$  is balanced by virtual nodal displacements  $\boldsymbol{\eta}_j^X$  considering the pseudo-stiffness  $C_e^X$  for the truss elements. Finally combination of Eqs. (10) and (11) yields:

$$\sum_{ie=i} \sum_X \sum_j w_e^X \mathbf{B}_{ej}^X \boldsymbol{\eta}_j^X = 0 \quad \forall i, \quad (14)$$

which merely states that the strains originating from all the virtual displacements  $\boldsymbol{\eta}_j^X$  and associated to a particular material state  $i$  have a zero  $w_e^X$ -weighted mean. The combination of Eqs. (13) and (14) is a symmetric linear system.

## 2.2 Solution procedure

To solve the previous set of equations Eqs. (9,11,13,14) and the material state mapping, similarly than in Kirchdoerfer and Ortiz, we consider the following decoupled algorithm:

1. randomly initialize  $ie$ ,
2. compute  $\epsilon_i^*$  from Eq. (9),
3. simultaneously compute  $\sigma_i^*$  and  $\boldsymbol{\eta}_j^X$  from Eqs. (13) and (14),
4. update the value of  $\sigma_e^X$  using Eq. (11),
5. compute a new state mapping  $ie^X$  with:

$$ie^X = \arg \min_{ie^X} \sum_X \sum_e w_e^X \|(\epsilon_e^X - \epsilon_{ie^X}^*, \sigma_e^X - \sigma_{ie^X}^*)\|_C^2, \quad (15)$$

6. iterate steps 2, 3, 4 and 5 until convergence of  $ie^X$ .

### Remarks

- Since all the mechanical strains  $\epsilon_e^X$  are known, a very good initialization of  $\epsilon_i^*$  and  $ie$  can be computed a priori from Eq. (9) through some kmeans-like algorithm [Lloyd:1982jj, MacQueen:1967uv]
- In this algorithm, Step 3 entails the solution of a large linear system for which all the diagonal blocks of the left hand side are constant pseudo-stiffness matrices as they do not depend on  $ie$ . Off-Diagonal blocks need to be recomputed at each iteration. This specificity opens the door to efficient resolution schemes that would re-use the initial Cholesky factorization of the diagonal blocks.
- Step 4 is computationally inexpensive as it reduces to a matrix vector product.
- Step 5 is expensive as it requires, for each mechanical state, to determine its closest (with respect to  $\|\cdot\|_C$ ) neighbor in the set of material states.
- Similarly to the data driven solver, the Data Driven Identification proposed here entails only few parameters: the number  $N^*$  of material states to be identified and the pseudo-stiffness  $C_e^X$ . It should be noted that  $C_e^X$  is not necessarily the same for all truss elements and data items and can therefore depend both on  $e$  and  $X$ . This provides some additional flexibility in the method and is a tool to weight the data according to some a priori confidence level.
- Even for non-linear material behaviors, the identification procedure only requires the solution of linear systems and simple database searches.
- The success of the proposed method relies on several ingredients. First, the richness of the experimental data over all data items  $X$ , *i.e.* the extent of  $\mathbf{u}_j^X$  and  $\mathbf{f}_j^X$  ensures the identification of the material behavior over a wide range of strains  $\epsilon_i^*$ . Second the richness of the individual data items, *i.e.* the range of  $\epsilon_i^X$  for a given  $X$ , which couples different material and mechanical states through mechanical equilibrium.

## 2.3 Numerical results

In this section, the DDI method is applied to the identification of the mechanical behavior of truss elements exhibiting a non linear strain-stress relation. We use a synthetic data set generated by applying different loading conditions to the 2D truss structure depicted in Figure 2. This structure is made of 249 nodes and 657 bar elements with a nonlinear strain-stress behavior of the form:  $\sigma = K(\epsilon + \epsilon^3)$ .  $N^X = 50$  different loading scenarios involving traction, compression and shear have been simulated. Some representative deformed configurations are depicted in Figure 3. The single parameter

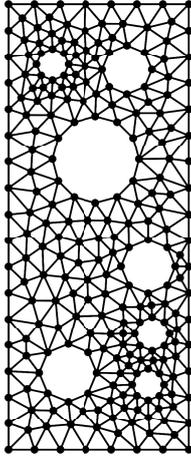


Figure 2: Undeformed truss structure used for the generation of manufactured data. The structure comprises 249 nodes and 657 nonlinear bar elements.

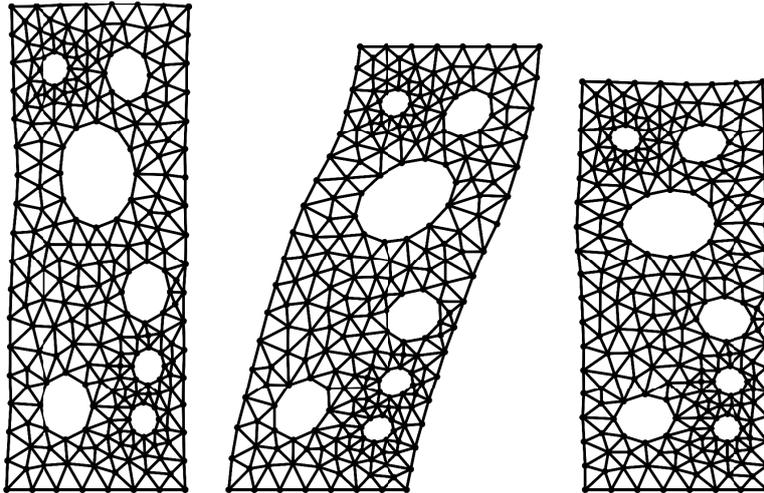


Figure 3: Three deformed configurations representative of the learning data-set: traction, shear and compression.

$C_e$  is set to 10 as we do not assume any noise on the mechanical strains  $\epsilon_e^X$  and therefore wish to put more weight on the strain part of the energetic norm (cf. Eq. (4)).

In Figure 4 we show, for  $N^* = 41$  the computed material and mechanical states together with the "true" constitutive relation used to generate the input data. Both states closely match the constitutive relation and that the density of material states is higher for small strains as there are more data points. The quantization of the material states can be observed through the clustering of the mechanical states around their corresponding material state, for example at high strains.

Figure 5 shows, for two different initializations of the state mapping  $ie$ , the convergence of the iterative process with respect to the minimized quantity  $\mathcal{E}(\sigma_e^X, \epsilon_i^*, \sigma_i^*, ie^X)$ . The initialization of  $ie^X$  and  $\sigma_i^*$  with a simple kmeans algorithm is so good that the first iteration already outperforms the converged result for a random initialization. All subsequent results are therefore computed with this more efficient initialization.

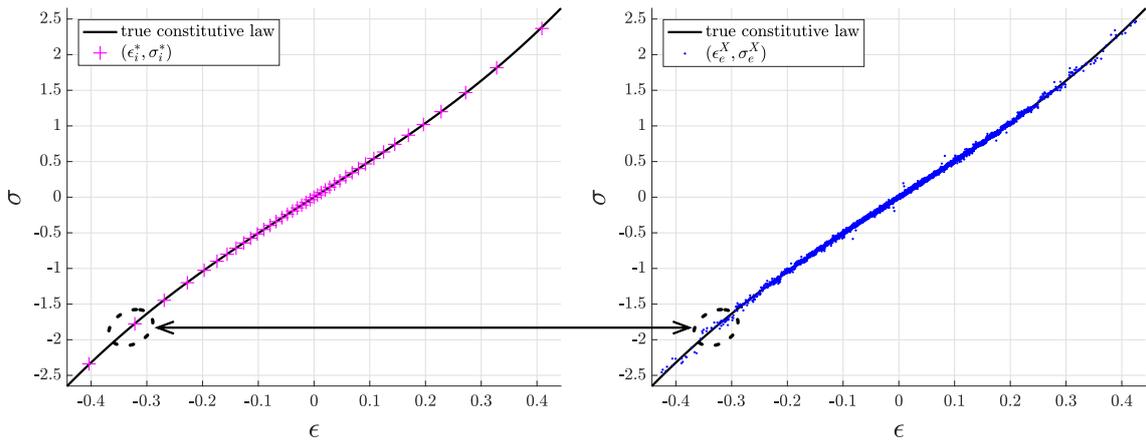


Figure 4: Material states (left) and mechanical states (right) computed for  $N^* = 41$ . The dotted ellipsoid and the arrows illustrates the clustering of the mechanical states around a particular material state.

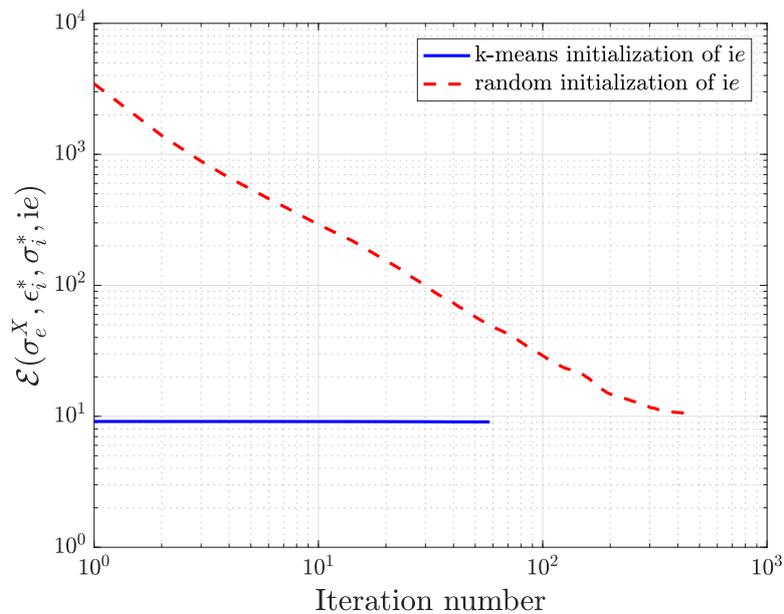


Figure 5: Comparison of the convergence for two different initialization of  $ie$  for  $N^* = 41$ .

Next, we investigate the influence of noisy input data. Considering that the manufactured data was gathered through digital image correlation with a 1000 by 1000 pixels grid, we add to the displacement fields  $\mathbf{u}_j^X$  a zero-mean uniform noise  $\delta\mathbf{u}_j^X$  with a one pixel amplitude (hence corresponding to a

standard deviation of  $\frac{1}{2\sqrt{3}} \approx 0.29$  pixel). This noise model leads to a greater relative influence on small displacements and mechanical strains. It is expected that the addition of noise will have the following influence:

- If the noise does not perturb the state mapping  $ie^X$ , it will only influence the determination of the material strains  $\epsilon_i^*$  because Eq. (9) becomes:

$$\sum_X \sum_{ie=i} w_e^X C_e^X (\epsilon_e^X + \delta\epsilon_e^X - \epsilon_{ie^X}^*) = 0 \quad \forall i.$$

As  $\delta\epsilon_e^X = \mathbf{B}_{ej}^X \delta\mathbf{u}_j^X$  has zero mean, it does not introduce any systematic bias in the value of  $\epsilon_i^*$ . Furthermore, the effect of noisy data is mitigated for large databases of experimental measurements.

- If the amount of noise is sufficient to perturb the state mapping, it will also affect the computed values of material stress  $\sigma_i^*$  and mechanical stress  $\sigma_e^X$ , thereby reducing the quality of the results.

In order to mitigate the influence of noisy mechanical strains on the state mapping, we see from Eqs. (4) and (15) that a simple possibility is to reduce the value of  $C_e$ . In Figure 6 some material states computed for  $C_e = 10$  and  $C_e = 0.1$  are shown. As expected large strain values are less perturbed by noise, and reducing the value of  $C_e$  improves results quality.

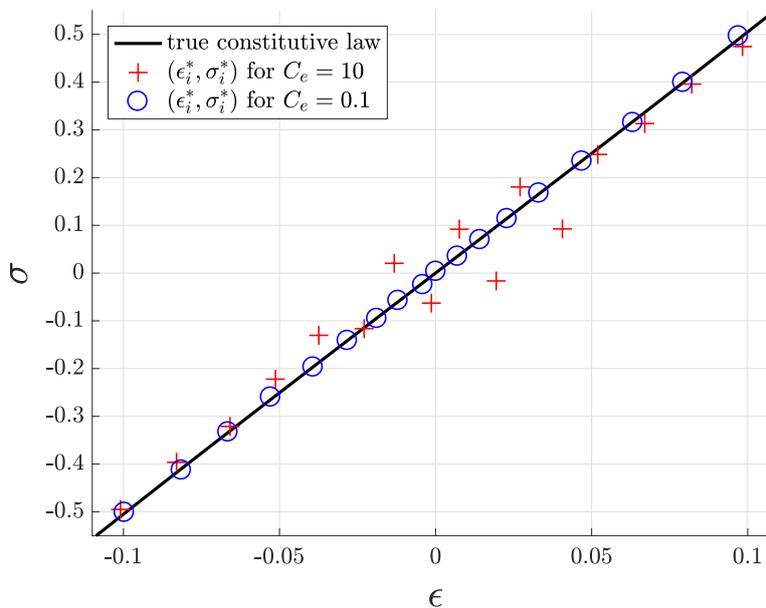


Figure 6: Influence of  $C_e$  with noisy data.

### 3 Data-Driven Identification for elastic materials

In this section we extend the method developed in Section 2 to the more general case of linear elasticity, in the limit of small strain. The major change is that the phase space of material and mechanical states is of much higher dimensionality: a state (mechanical or material) now consists in a linear strain tensor  $\epsilon$  and a Cauchy stress tensor  $\sigma$ , each belonging in a 6-dimensional space (after accounting for their symmetry).

#### 3.1 Data-Driven Identification

Again, we assume the existence of a large database of measurements, obtained by Digital Image Correlation or any related technique. Furthermore we consider a linearized kinematics discretized by a finite element mesh in which each quadrature point  $e$  admits an integration weight  $w_e$ . For each data item  $X$  (or snapshot), the following quantities are available:

- the nodal displacements  $\mathbf{u}_j^X$ ,
- the finite element geometry and connectivity, encoded through a matrix  $\mathbf{B}_{ej}^X$ , which can compute the mechanical strain  $\boldsymbol{\epsilon}_e^X = \sum_j \mathbf{B}_{ej}^X \cdot \mathbf{u}_j^X$ ,
- the applied nodal forces  $\mathbf{f}_j^X$ ,
- the prescribed nodal displacements.

The aim of the DDI technique is to compute a number  $N^*$  of material states  $(\boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*)$  such that:

- for each snapshot  $X$  and quadrature point  $e$ , we can compute the mechanical state  $\boldsymbol{\sigma}_e^X$  which satisfies mechanical equilibrium,
- for each snapshot, we can assign a material state  $(\boldsymbol{\epsilon}_{ie}^*, \boldsymbol{\sigma}_{ie}^*)$  to each quadrature point  $e$  which is the closest to the mechanical state according to a given energetic norm  $\|\cdot\|_{\mathbb{C}_e}^2$ .

Following Kirchdoerfer and Ortiz , we consider

$$\|(\boldsymbol{\epsilon}_e, \boldsymbol{\sigma}_e)\|_{\mathbb{C}_e}^2 = \frac{1}{2}(\boldsymbol{\epsilon}_e : \mathbb{C}_e : \boldsymbol{\epsilon}_e + \boldsymbol{\sigma}_e : \mathbb{C}_e^{-1} : \boldsymbol{\sigma}_e), \quad (16)$$

where  $\mathbb{C}_e$  is a (symmetric positive definite) fourth order stiffness tensor. Like in the truss case, we formulate the global minimization problem as:

$$\text{solution} = \arg \min_{\boldsymbol{\sigma}_e^X, \boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*, ie^X} \mathcal{E}(\boldsymbol{\sigma}_e^X, \boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*, ie^X), \quad (17)$$

with,

$$\mathcal{E}(\boldsymbol{\sigma}_e^X, \boldsymbol{\epsilon}_i^*, \boldsymbol{\sigma}_i^*, ie^X) = \sum_X \sum_e w_e^X \|(\boldsymbol{\epsilon}_e^X - \boldsymbol{\epsilon}_{ie^X}^*, \boldsymbol{\sigma}_e^X - \boldsymbol{\sigma}_{ie^X}^*)\|_{\mathbb{C}_e}^2, \quad (18)$$

and subject to the global equilibrium equations:

$$\sum_e w_e^X \mathbf{B}_{ej}^{XT} \cdot \boldsymbol{\sigma}_e^X = \mathbf{f}_j^X \quad \forall X, j. \quad (19)$$

All unknowns are continuous excepted the state mapping  $ie^X$  which is discrete. For an arbitrary state mapping, the equilibrium constraints Eq. (19) are enforced by means of Lagrange multipliers  $\boldsymbol{\eta}_j^X$ , leading to the following problem:

$$\delta \left( \sum_X \sum_e \left( w_e^X (\boldsymbol{\epsilon}_e^X - \boldsymbol{\epsilon}_{ie^X}^*) : \mathbb{C}_e^X : (\boldsymbol{\epsilon}_e^X - \boldsymbol{\epsilon}_{ie^X}^*) + w_e^X (\boldsymbol{\sigma}_e^X - \boldsymbol{\sigma}_{ie^X}^*) : \mathbb{C}_e^{X-1} : (\boldsymbol{\sigma}_e^X - \boldsymbol{\sigma}_{ie^X}^*) - \sum_j (w_e^X \mathbf{B}_{ej}^{XT} \cdot \boldsymbol{\sigma}_e^X - \mathbf{f}_j^X) \cdot \boldsymbol{\eta}_j^X \right) \right) = 0. \quad (20)$$

Taking all possible variations yields the following set of equations:

$$\delta \boldsymbol{\epsilon}_i^* \Rightarrow \sum_X \sum_{ie=i} w_e^X \mathbb{C}_e^X : (\boldsymbol{\epsilon}_e^X - \boldsymbol{\epsilon}_{ie^X}^*) = 0 \quad \forall i \quad (21)$$

$$\delta \boldsymbol{\sigma}_i^* \Rightarrow \sum_X \sum_{ie=i} w_e^X \mathbb{C}_e^{X-1} : (\boldsymbol{\sigma}_e^X - \boldsymbol{\sigma}_{ie^X}^*) = 0 \quad \forall i \quad (22)$$

$$\delta \boldsymbol{\sigma}_e^X \Rightarrow w_e^X \mathbb{C}_e^{X-1} : (\boldsymbol{\sigma}_e^X - \boldsymbol{\sigma}_{ie^X}^*) - \sum_j w_e^X \mathbf{B}_{ej}^X \cdot \boldsymbol{\eta}_j^X = 0 \quad \forall e, X \quad (23)$$

$$\delta \boldsymbol{\eta}_j^X \Rightarrow \sum_e (w_e^X \mathbf{B}_{ej}^{XT} \cdot \boldsymbol{\sigma}_e^X - \mathbf{f}_j^X) = 0 \quad \forall j, X \quad (24)$$

The interpretation of these equations is the same as in Section 2. Combining Eqs (22), (23) and (24) yields the following system that is solved to simultaneously determine  $\sigma_i^*$  and  $\sigma_e^X$  (through  $\eta_j^X$ ):

$$\sum_k \sum_e w_e^X \mathbf{B}_{ej}^{X^T} : \mathbb{C}_e^X : \mathbf{B}_{ek}^X \eta_k^X + \sum_e w_e^X \mathbf{B}_{ej}^{X^T} \sigma_{ie^X}^* = \mathbf{f}_j^X \quad \forall j, X, \quad (25)$$

and

$$\sum_{ie=i} \sum_X \sum_j w_e^X \mathbf{B}_{ej}^X \eta_j^X = 0 \quad \forall i. \quad (26)$$

We suggest the following algorithm for computing  $\epsilon_i^*, \sigma_i^*, \sigma_e^X$  and  $ie$ :

1. simultaneously initialize  $\epsilon_i^*$  and  $ie$  by a kmeans algorithm on  $\epsilon_e^X$ .
2. simultaneously compute  $\sigma_i^*$  and  $\eta_j^X$  from Eqs. (25) and (26),
3. update the value of  $\sigma_e^X$  using Eq. (23),
4. compute a new state mapping  $ie^X$  with:

$$ie^X = \arg \min_{ie^X} \sum_X \sum_e w_e^X \|(\epsilon_e^X - \epsilon_{ie^X}^*, \sigma_e^X - \sigma_{ie^X}^*)\|_{\mathbb{C}_e}^2, \quad (27)$$

5. update  $\epsilon_i^*$  from Eq. (21),
6. iterate steps 2, 3, 4 and 5 until convergence of  $ie^X$ .

## Remarks

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## 3.2 Results and discussion

Manufactured data, the method proposed in the previous section. The problem consists in the identification of the mechanical behavior of a non-linear incompressible material with the plane stress assumption. We consider a 2D finite element mesh with 1340 nodes and 2416 triangular elements, depicted in Figure 9, and subjected to  $N^X = 40$  different loading conditions. Representative deformed configurations are similar to the ones in Figure 3 in which the bars are now the edges of triangular elements. The constitutive equation used in the FE simulations is of the form:

$$\boldsymbol{\sigma} = G(\boldsymbol{\epsilon} + \alpha \boldsymbol{\epsilon}^3) - p \mathbf{I}, \quad (28)$$

$$p = -(\epsilon_{xx} + \epsilon_{yy}) - \alpha(\epsilon_{xx} + \epsilon_{yy})^3. \quad (29)$$

where  $G$  and  $\alpha$  are the material parameters chosen as  $G = 5$  and  $\alpha = 5$ . We consider a tensor  $\mathbb{C}_e$  corresponding to the above equation with  $\alpha = 0$  and  $G = 1$ . In this case, the kmeans initialization of  $ie^X$  is so efficient that only a few iterations are necessary to converge for  $N^* = 500$ .

Since the "true" constitutive law used to generate the data is isotropic, we first investigate the isotropy of the identified material states. To this end we compute, for each material state  $(\epsilon_i^*, \sigma_i^*)$ , the angle  $\theta_i$  between the dominant eigenvectors of the strain and stress tensors respectively. The distribution of these misalignment angles is shown in Figure 7. We observe that for most of the states, the misalignment angle is less than  $1^\circ$ . As the identified behavior can be considered isotropic, in Figure 8 we present the first eigenvalue of the material stress tensor ( $\sigma_I^*$ ) as a function of the eigenvalues of the corresponding material strain ( $\epsilon_I^*, \epsilon_{II}^*$ ). All points fall very close to the surface that can be built from the constitutive equation used to generate the input data:

$$\frac{\sigma_I}{G} = \epsilon_I + \alpha * \epsilon_I^3 + (\epsilon_I + \epsilon_{II}) + \alpha * (\epsilon_I + \epsilon_{II})^3. \quad (30)$$

The computed material states provide a discrete description of the "true" material response in the range of strains found in the input data. We now turn to the analysis of the mechanical stress distributions

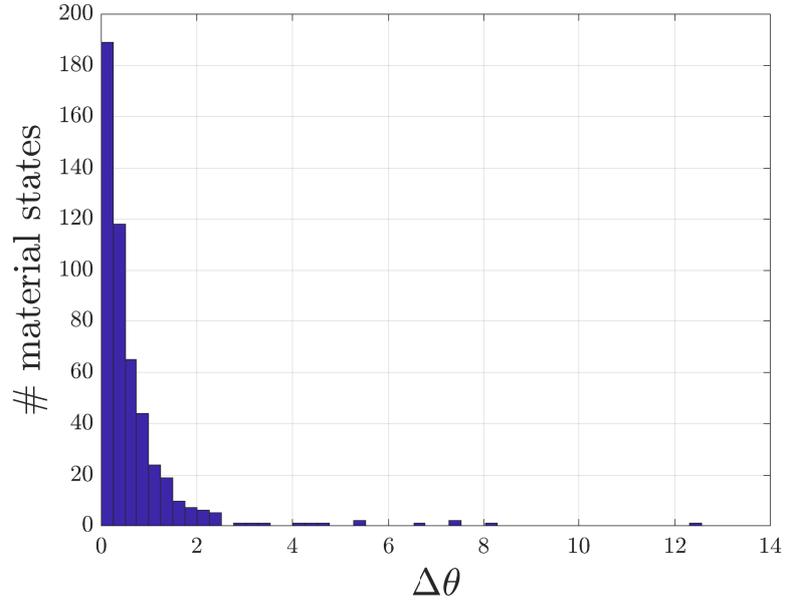


Figure 7: Distribution of misalignment angle  $\theta$  between  $\epsilon^*$  and  $\sigma^*$ .

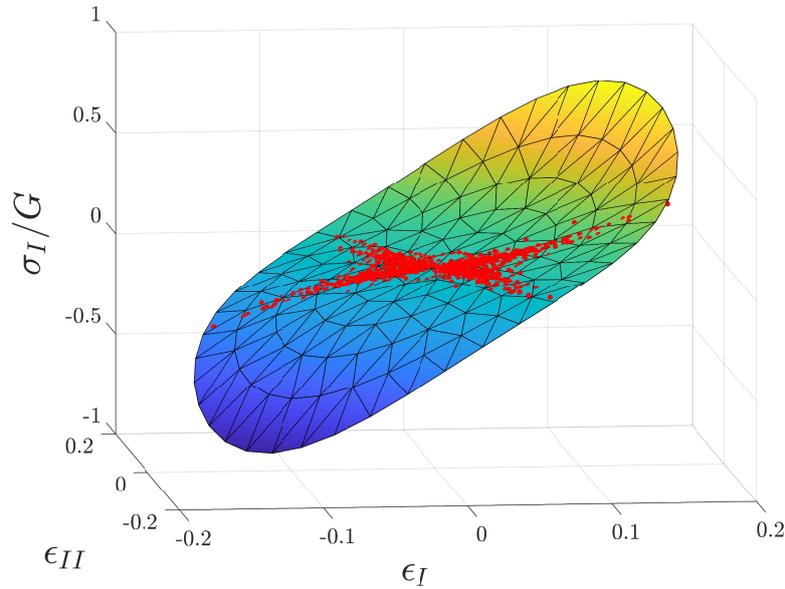


Figure 8: Largest eigenvalue of the material stress tensor as a function of the eigenvalues of the material strain tensor. The symbols are computed from the identified material states  $(\epsilon^*, \sigma^*)$  and the surface from Eq. (28).

predicted for all input snapshots. Figure 9 (left & middle) presents the predicted Von Mises stress for one of the snapshots, and with the relative error on it. The prediction is accurate to less than 10% in most of the computational domain and reaches 50% in only a few of the 2416 elements, confirming the accuracy of the DDI method. Figure 9 (right) depicts the distribution of the relative error on the Von Mises stress for all elements of the 40 snapshots. The observed narrow distribution, with a mean of 8.5% and a median of 5.3% further highlights that the computed mechanical stresses are close to the actual ones.

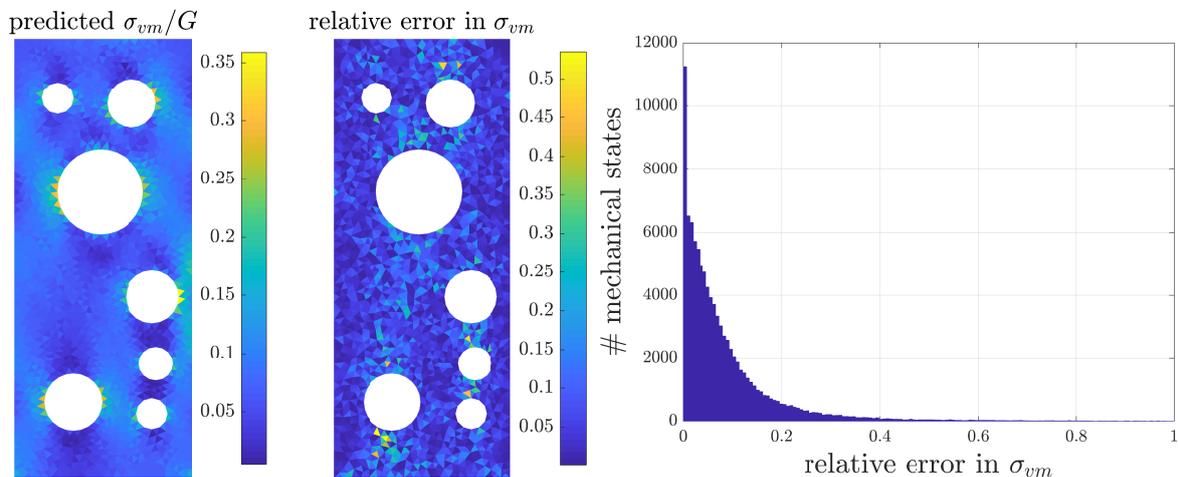


Figure 9: Left: predicted Von Mises stress field of a particular snapshot computed from the identified mechanical stress. Middle: relative error on the Von Mises stress with respect to the true underlying model. Right: distribution of the relative error of the Von Mises stress for all elements across all snapshots.

## 4 Summary and concluding remarks

In this paper, we demonstrate that it is possible to build a Data Driven Identification (DDI) method that computes admissible strain-stress couples from a set of experimental data, based on the Data-Driven Computational Mechanics (DDCM) framework recently proposed by Kirchdoerfer & Ortiz. The method only requires kinematics and applied forces, which are both accessible using Digital Image Correlation and reasonable assumptions. The computed strain-stress couples can then be used either as constitutive law surrogate in DDCM, or to fit a classical constitutive model. Stress fields of the experimental data are also obtained as a byproduct of the algorithm.

The proposed method is for now applicable to elastic behaviors only, where stress is uniquely determined by strain. Future developments will focus on more complex material responses such as viscoelasticity, plasticity and damage which involve strain history.

## 5 Bibliography