Reservoir computing (RC) systems are computational tools for information processing which can be fully implemented in optics. Here, we experimentally and numerically show that an optically pumped laser subject to optical delayed feedback can yield similar results as those obtained for electrically pumped lasers. Unlike the previous implementations, the input data is injected at a time interval which is much larger than the time-delay feedback. This data is directly coupled to the feedback light beam. Our results illustrate new possible avenues for RC implementations for prediction tasks.

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OCIS codes: (0000.0000) Lasers.
used [4–14, 16]. We explore here whether time-delays less or comparable to laser relaxation time can also be suitable for RC systems. This approach is interesting since short delay lines are desirable for on-chip implementations because it consumes less wafer space. The time-delays will be typically fixed during the manufacturing phase for real-world systems while the processing speeds can be controlled. We also investigate the effect of such a change for fixed time-delay. (iii) we also want to explore whether the data can be directly coupled to the feedback light beam instead of using an additional laser for optically injecting the electrical data into the reservoir as done in [7].

RC system can be conceptually divided into three blocks, which are schematically shown in Fig. 1: an input layer, a reservoir and an output layer. The input layer is the stage where the input data is provided and preprocessed before being injected into the reservoir for their processing. The preprocessing includes the rescaling of the input signal and its multiplicative by a mask matrix $M$ that defines the coupling weights from input to the reservoir. The role of the mask is to ensure the variability of the signal over the different virtual nodes where the information is read out. These virtual nodes are formed by sampling the delay line at a fixed time interval $\theta$. The output layer is the stage where the different node responses are weighted and linearly summed up. The optimal values of these weights are those for which the summation of all the different node responses always approaches the associated target as closely as possible. They can be typically determined with an off-line training procedure using digital computers [5–8, 11–14] or an on-line training procedure using an FPGA (Field-Programmable Gate Array) [17]. In our case, the training is done off-line. Three time scales are relevant: the data injection time $T_d$, the feedback time-delay $T$ and the mask length given by $N\theta$ where $N$ is the number of virtual (temporal) nodes. We consider $T_d = N\theta$ and apply this same duration during the postprocessing of each data point.

As benchmark task, we will use the Santa time series to evaluate the prediction performance of the system. The Santa-Fe data are intensity time series experimentally recorded from a far-infrared laser operating in a chaotic state [18]. The goal for this task is to predict the sample one-step ahead in a chaotic time trace before it has been injected into the reservoir computer. The system performance is evaluated by calculating the normalized mean square error (NMSE) between the predicted value $\hat{y}$ and the expected value $\hat{y}$:

$$\text{NMSE}(\hat{y}, \hat{y}) = \frac{\sum (y(n) - \hat{y}(n))^2}{\sum (\bar{y}(n) - \hat{y}(n))^2},$$

where $n$ is a discrete time index while $\| \cdot \|$ and $\langle \cdot \rangle$ stand for the norm and the average over time, respectively. Note that NMSE = 0 means perfect prediction while NMSE = 1 indicates no prediction at all. Our results will be compared to those in [7] as it is a similar RC system which was, however, built using a different type of laser and a large time-delay.

The Experimental setup is shown in Fig. 2. It is composed of a diode-pumped Erbium doped microchip laser with optical delayed feedback. The microchip laser is a sample provided by the laboratory LETI-CEA (Technologies Avancées) [19]. The threshold currents of the diode-pump and the micro laser are $I_p \approx 24.2$ mA and $I_s \approx 100$ mA, respectively. The microchip laser’s stability threshold is close but above its lasing threshold. The micro laser is bi-mode which, above $I_s$, emits a total radiation of a few milliwatts at the wavelength $\lambda_1 \sim 1532$ nm and $\lambda_2 \sim 1535$ nm with a side mode suppression of $\approx 26$ dB. The emitted light has been collected using collimating lens fiber with a collection efficiency of $\approx 50\%$. To implement the delayed feedback, the collected light beam is split into two parts using a $50\%$ optical coupler: one part is directly read out by a photodiode with 1 GHz-bandwidth while the second part is connected to port number 2 of an optical circular. The circulator couples light from port 2 to port 3 and from port 3 to port 1 with minimal power loss. The signal at port number 3 is delayed by a single mode optical fiber of length $\approx 126$ m (i.e., time-delay of $\approx 630$ ns). The light beam at the fiber output is split into two paths. One path passes through a 10 GHz LiNbO$_3$ phase modulator (PM) (half-wave voltage $V_{th} = 5$ V, maximum input power 28 dBm). The PM output signal is combined with the light beam of the second path using a $2 \times 2$ optical coupler implementing an intensity modulation, of the light beam. One output port of this coupler is connected to port number 1 of the circulator while the second output port is connected to an optical spectrum analyzer. So, taking into account the losses about $10\%$ of the collected power is re-injected into the laser.

The original data to be processed is first convoluted with a random mask which has 4 discrete values ($-1, -0.25, 0.25, 1$) generated randomly with equal probability at the time interval of $\theta = 24$ ns. The resulting signal is uploaded in a two channel arbitrary waveform generator (AWG) (Tektronix AFG3102C, dual channel, 1 GS/s, 100 MHz) from where it is injected into the reservoir via the PM radio-frequency electrode. The amplitude of the injected signal is rescaled such as to obtain an
optimal contrast of the modulation at the laser output. In our case, the peak-to-peak voltage from the AWG was 10 V. The data is recorded using a digital oscilloscope (LeCroy 200 MHz, 2.5 GS/s). The AWG is perfectly synchronized with the oscilloscope so that the original data and the laser response to this data can be simultaneously recorded. We experimentally use 1000 steps for training and 250 steps for testing. The samples are injected at the time interval $T_d = 2.4 \mu s$ which is 3.8 times larger than the time-delay, i.e. $T = 630$ ns. This corresponds to a reservoir with $N = T_d/\theta = 100$ virtual nodes. As such, only a fraction of the virtual nodes (i.e., $\sim N/3.8$) are connected through the feedback to nodes in a previous state (i.e. nodes that receive an input corresponding to a previous time).

Figure 3 shows the temporal profiles of the original data (black) and the laser response at the photodiode output (red) for two values of the diode-pump currents with one close the threshold ($I = 120$ mA) and another far away from the threshold ($I = 220$ mA). For $I = 120$ mA, the micro laser emits a low power laser response signal has been shifted for clarity. The laser response signal has been rescaled with its maximum value in each case. The laser response signal has been shifted for clarity.

This noise is mainly readout noise from photodiode. For large values of $\mu$ (i.e., large laser output signals), the SNR is large and the influence of readout noise can be minimized. For such cases, we obtain a lowest NMSE of 0.12 ± 0.04 at $\mu \approx 2.58$ (note that for clarity, the error bars are not plotted) while a lowest NMSE of 0.108 was obtained in [7] for the same task. The performance of the two systems is therefore similar although we consider a time delay comparable to the laser’s relaxation oscillation time, $T_{RO}$ [note that $T_{RO} \approx (\gamma \gamma_c (\mu - 1))^{-1/2} \approx 750$ ns at $\mu \approx 2.5$].

For real-world applications, the delay length would be fixed during the manufacturing phase of the device while the length of the mask could be independently varied by changing, for example, the temporal node’s separation. To investigate this fact, we show in Fig. 4(b) the NMSE values for different values of $\theta$ when $N$ and $\mu$ are kept fixed to 100 and 2.5 respectively. The optimal value lies around $\theta = 24$ ns. These are therefore the values of $\theta$ for which the optimal coupling between the virtual nodes is obtained.

For further insights, we perform numerical simulations using the model of diode-pumped Erbium microchip laser subject to optical delayed feedback introduced in [20, 21]. The relevant variables of the model are the slow-varying complex envelope of the electric field $E_m(t)$ associated to the longitudinal mode $m$ (with $m = 1, 2$) and its corresponding carrier number $N_m(t)$:

$$
\dot{E}_{1,2} = \frac{1}{2} \left[ B \delta_{1,2} N_{1,2} - \gamma E_{1,2} + i \frac{\alpha}{2} B N_{1,2} \right] + \frac{n}{2} \left[ 1 + e^{i \left( \omega_{e} - \omega_{2} + \omega_{T} \right) T} \right] E_{1,2}(t - T) e^{i \omega_{1} T} + \xi_{1,2}(t),
$$

$$
\dot{N}_{1,2} = \gamma_c (N_{p} - N_{1,2}) + g B N_{1,2} \left( |E_{1,2}|^2 + \varepsilon |E_{2,1}|^2 \right) + \xi_{1,2}(t),
$$

where the parameters are the Einstein coefficient: $B$, the decay rate of the population inversion: $\gamma_c$, the pumping rate: $\gamma_c N_p$, the laser cavity decay rate: $\gamma$, the linewidth enhancement factor: $\alpha$, the feedback rate: $\eta$, the time delay: $T$, the cross-saturation parameter: $\phi$, the solitary laser frequency for mode $m$: $\omega_m$. $S(t)$ is the signal which results from convolution between the original data to be processed and the mask. The noise is modeled as the Langevin forces $\xi_{1,2}(t)$ and $\xi_{1,2}(t)$ which describe the quantum fluctuations of the laser population and the radiation field. These forces are defined as having a zero mean value and white-noise-type correlation functions: $\langle \xi_{i}(t) \xi_{j}^{*}(t') \rangle = D \delta_{ij} (t - t')$ and $\langle \xi_{i}(t) \xi_{j}(t') \rangle = D \gamma \gamma_c \mu_0 \delta_{ij} (t - t')/B$ for $i, j = 1, 2$ where $D$ is the spontaneous emission factor.

From experiments, we have retrieved the parameters $\gamma = 53.66 \text{ ps}^{-1}$, $\gamma_c = 0.217 \text{ ps}^{-1}$, $\phi = 0.43$. We also consider other parameters from [20, 21]: $B = 122.1$, $\alpha = 1$, $\eta = 0.3 \text{ ps}^{-1}$, $g = 0.95$, $N_p = \mu \gamma / B$, $\omega_m T = 0$, $\omega_m \delta T = -0.9 \pi$ and $D = 10^{-4}$.

**Fig. 3.** Original data (black) and the corresponding experimental recorded signal of the laser response by the photodiode (color) for the diode pump current (a) $I = 120$ mA and (b) $I = 220$ mA. The amplitude of the input data and the laser response have been rescaled with its maximum value in each case. The laser response signal has been shifted for clarity.

**Fig. 4.** Prediction errors NMSE as a function of (a) the pump rate $\mu$ for virtual node’s separation $\theta = 24$ ns; (b) $\theta$ for $\mu = 2.5$. The time interval resolution of the time-delay is $\theta$. 

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**Table 1.** Parameters used in the model of diode-pumped Erbium microchip laser subject to optical delayed feedback.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>122.1</td>
</tr>
<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>0.3</td>
</tr>
<tr>
<td>$g$</td>
<td>0.95</td>
</tr>
<tr>
<td>$N_p$</td>
<td>$\mu \gamma / B$</td>
</tr>
<tr>
<td>$\omega_m T$</td>
<td>0</td>
</tr>
<tr>
<td>$\omega_m \delta T$</td>
<td>$-0.9 \pi$</td>
</tr>
<tr>
<td>$D$</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>
We numerically use 3000 steps in the data set for training and 1000 other steps for testing. For each value of \( \mu \), we run the simulations 10 different times and record for each run the laser output as \( |E(t)|^2 = |E_1(t) + E_2(t)|^2 \). Additional Gaussian white noise is added to \( |E(t)|^2 \) to model the readout noise from the photodiode. Thus, the weights are optimized from \( |E(t)|^2 + D_{\text{out}} s_{\text{out}}(t) \) instead of \( |E(t)|^2 \), where \( D_{\text{out}} \) is the noise amplitude while \( s_{\text{out}}(t) \) is a Gaussian white noise with zero mean and correlation \( \langle s_{\text{out}}(t) s_{\text{out}}(t') \rangle = \delta(t - t') \). \( D_{\text{out}} = 0 \) refers to noiseless photodetectors. We choose \( D_{\text{out}} \approx 4 \) as it is the value for which numerical and experimental SNRs calculated as \( \text{SNR} = 10 \log_{10}\left(\frac{|E(t)|^2}{D_{\text{out}}}\right) \) are similar. The SNR is \( \approx 10 \) dB at \( \mu = 1.2 \) and \( \approx 26 \) dB at \( \mu = 2.5 \). The value of \( D_{\text{out}} \) is kept fixed for the rest of the Letter.

The numerical obtained values of the NMSE as a function of the pump rate are also shown in Fig. 4 in comparison with the experimental ones. Again, the error bars are not plotted. In both cases, an excellent agreement is found between experimental and numerical results. In particular, it is confirmed that, because of noise, the system performance strongly degrades for small values of \( \mu \) [see Fig. 4(a)] and also for high processing speeds [see Fig. 4(b)]. We obtain, for example, an NMSE\( \approx 0.43 \) for \( \theta = 6 \) ns although all the \( N \) virtual nodes are connected through the feedback to nodes in the previous state for this \( \theta \). Note that for fixed \( N \), the processing speed increases with the decrease of \( \theta \). By comparing the NMSE values obtained from a system with (●) and without readout noise (○), it turns out that the degradation of the NMSE for low values of \( \mu \) is mainly due to the readout noise. Through numerical simulations, we have also found that the calculation of the weights from \( |E_1(t)|^2 \) or \( |E_2(t)|^2 \) (i.e., considering only one mode) yields similar results as from \( |E_1(t) + E_2(t)|^2 \). This is not surprising since the mode spacing is large such that the heterodyne signal is outside the detection bandwidth of the system. In such a case, the choice of either mono- or bi-mode laser is not of importance.

The memory capacity of the system may decrease when only a fraction of the virtual nodes is connected through the feedback to the previous input states. To investigate this fact, we compare in Fig. 5 (a) the simulated results of the memory function (calculated from uniformly distributed random signal drawn in the interval \([-0.5;0.5]\)) when \( N/3.8 \) virtual nodes (●) and when all the \( N \) virtual nodes (●) are connected to nodes in a previous state considering \( \theta = 24 \) ns and \( \mu = 2.5 \). Effectively, it is seen that the memory slightly decreases for \( N/3.8 \) connected to nodes in a previous state. The fading slope of the memory is also slow in this case. But, the calculation of the NMSE shows that these changes in the memory do not significantly degrade the system performance for one-step ahead prediction [Fig. 5(b)]. However, the low memory in Fig. 5 (a) suggests that this system may not be suitable for some tasks requiring large memory capacity.

In conclusion, we have experimentally and numerically shown that diode-pumped Erbium doped microchip lasers subject to optical feedback can be used to implement RC systems for prediction tasks. Using Santa-Fe time series as benchmark, we found a best prediction error similar to that obtained with a similar system (but with a long time-delay [7]) although the use of a short time-delay. Our results also have shown that even multi-mode lasers with large mode spacing can be used.

**FUNDING**

This work has been supported by the F.N.R.S. (Belgium). R.M.N acknowledges the support CNRS during his stay at LIPhy. This work also benefited from the support of the Belgium Science Policy Office under Grant No IAP-7/35 “photonic@be”.

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