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► **To cite this version:**

Jha Mayank Shekhar, Geneviève Dauphin-Tanguy, Belkacem Ould Bouamama. Particle Filter Based Hybrid Prognostics for Health Monitoring of Uncertain Systems in Bond Graph Framework. Mechanical Systems and Signal Processing, 2016, 75, pp.301-329. 10.1016/j.ymssp.2016.01.010 . hal-01449915

**HAL Id: hal-01449915**

**<https://hal.science/hal-01449915>**

Submitted on 30 Jan 2017

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# Particle Filter Based Hybrid Prognostics for Health Monitoring of Uncertain Systems in Bond Graph Framework

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## Abstract

The paper's main objective is to address the problem of health monitoring of system parameters in Bond Graph (BG) modeling framework, by exploiting its structural and causal properties. The system in feedback control loop is considered globally uncertain and parametric uncertainty is modeled in interval form. The system parameter is undergoing degradation (prognostic candidate) and its degradation model is assumed to be known *a priori*. The detection of degradation commencement is done in a passive manner which involves interval valued robust adaptive thresholds over the nominal part of the uncertain BG-derived Interval Valued Analytical Redundancy Relations (I-ARRs). The latter forms an efficient diagnostic module. The prognostics problem is cast as joint state-parameter estimation problem, a hybrid prognostic approach, wherein the fault model is constructed by considering the statistical degradation model of the system parameter (prognostic candidate). The observation equation is constructed from nominal part of the I-ARR. Using Particle Filter (PF) algorithms; the estimation of state of health (state of prognostic candidate) and associated hidden time-varying degradation progression parameters is achieved in probabilistic terms. A simplified variance adaptation scheme is proposed. Associated uncertainties which arise out of noisy measurements, parametric degradation process, environmental conditions etc. are effectively managed by PF. This allows the production of effective predictions of the remaining useful life of the prognostic candidate with suitable confidence bounds. The effectiveness of the novel methodology is demonstrated through simulations and experiments on a mechatronic system.

**Keywords:** Prognostics, Bond Graph, Intervals, Particle Filter, Remaining Useful Life, Robust Fault Detection

## Abbreviations & Acronyms

RUL	Remaining Useful Life	I-ARR	Interval Valued Analytical Redundancy Relations
EOL	End of Life	DPP	Degradation Progression Parameter
DM	Degradation Model	RMAD	Relative Median Absolute Deviation
BG	Bond Graph	RA	Relative Accuracy
BG-LFT	Bond Graph in Linear Fractional Transformation	RMSE	Root Mean Square Error
PF	Particle Filters	SIR	Sampling Importance Resampling
		ARR	Analytical Redundancy Relations

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**Notations**

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$\theta$	System parameter	$\gamma^d$	Degradation progression parameter associated to $\theta^d$
$\theta^d$	System parameter under degradation (prognostic candidate)	$\gamma^{d*}$	True value of $\gamma^d$
$\theta_n^d$	Nominal value of $\theta^d$	$p$	probability
$\Delta\theta$	Additive uncertainty on $\theta$	$\sigma_X$	Standard deviation value of random specie $X$
$\delta_\theta$	Multiplicative uncertainty on $\theta$	$\sigma_X^2$	Variance of population values of $X$
$[\delta_\theta]$	Multiplicative uncertainty in interval form, equivalent to $[\underline{\delta}_\theta, \overline{\delta}_\theta]$	$N$	Number of particles in PF
$[w_\theta]$	Uncertain effort or flow brought by interval uncertainty on $\theta$ , to the system.	$w_k^i$	Weight of $i^{\text{th}}$ particle at discrete time $k$
$r_n(t)$	Numerical evaluation of the nominal part of I-ARR	$y^d$	Measurement of prognostic candidate $\theta^d$
$\hat{X}$	Estimated value of species $X$	$w^d(t)$	Noise associated with measurement of $\theta^d$
$[\underline{R}, \overline{R}]$	Interval valued ARR (I-ARR)	$\xi^d$	Normally distributed random walk noise for $\gamma^d$
$\Psi_2$	Interval function (uncertain part of I-ARR)	$P^d$	Proportional gain constant in variance adaptation of $\xi^d$
$\Psi_1$	Point-valued nominal part of I-ARR	$v^{\xi^d}$	RMAD (spread) of $\xi^d$
$\Psi_2$	Interval function $\Psi_2$ with point valued arguments	$v^{\gamma^{d*}}$	Reference RMAD (spread) involved in variance adaptation scheme
$b(t)$	Numerical evaluation of $\Psi_2$	$[\gamma_l^{d*}, \gamma_u^{d*}]$	Interval containing $\gamma^{d*}$
$[\underline{B}(t), \overline{B}(t)]$	Range of interval function $\Psi_2$	$\overline{\gamma}_k^d$	Moving average of mean estimations of $\gamma^d$

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33 **1. Introduction**

34 Health Monitoring aims at ensuring system safety, reliability and efficient functionality and deals with *fault detection*  
35 and prediction of the *Remaining Useful Life* (RUL) of the system in a holistic way. While the former two is mainly dealt  
36 by using a diagnostic module, the latter is performed by a prognostic module. The primary focus lies in scheduling the  
37 maintenance actions according to progression of the system to a time where it may be considered beyond the limits of  
38 certified functionalities [1]. Such a time-horizon of interest is termed as the *End of Life* (EOL) and the time remaining  
39 until that point is called RUL of the system [2, 3]. Prognostics are focused on the study of fault (or damage) evolution and  
40 prediction of the RUL of the system/component. Accurate prediction of EOL/RUL enables efficient and optimal planning  
41 of the future maintenance actions, and renders the capability of assessing reliability of the system [4]. This leads to sys-  
42 tem/component's life extension by modification of the system demand, operating conditions, workload etc. [5].

43 The failures of most systems can be attributed to the degradation of a given component, subsystem or material with  
44 time, environmental and operational conditions etc. Such system components/sub-systems can be identified as the poten-

45 tial prognostic candidates through Failure Modes, Mechanisms and Effect Analysis or through other ways [6]. The under-  
46 lying physical degradation is usually captured by *Degradation Model(s)* (DM) that can be obtained based upon physics of  
47 degradation or statistical (experimental) modeling approach as described in Gebraeel et al.[7] and Guo et al.[8]. In cases  
48 where physics of degradation is not available or reliable, the respective DM can be obtained statistically by finding a  
49 mathematical model that best fits a given set of degradation data. In this context, commonly employed DMs to fit the data  
50 are of linear, logarithmic, power or exponential form [8]. For example, approximation of degradation model by a linear  
51 part and logarithmic/exponential part [9], employment of exponential fit growth models [10], log-linear model for current  
52 drain degradation process [11] and stochastic degradation model [12].

53 Prognostic approaches are broadly divided into three categories [3, 13]: model-based prognostics [14], data-driven  
54 prognostics [15, 16] and hybrid prognostics [9, 17]. In model based approach, the degradation model is physics based and  
55 requires a detailed understanding of the underlying phenomenon [1]. Inadequate modeling information, variation in behav-  
56 ioral physics or environmental conditions, un-modeled/unclassifiable sources of noise etc., result in limiting its adequacy.  
57 Data-driven methods tend to learn the damage progression. However, they generalize damage progression over large sets  
58 of component population and remain unreliable in assessing the variability of damage progression trend from component  
59 to component in a population [1]. As such, they provide inferior results especially in absence of complete data and large  
60 unit to unit variations. Hybrid approaches on the other hand, benefit from the fusion of the advantages of the former two  
61 [9]. They employ physics or statistical based degradation models and use measured information to adapt the damage pro-  
62 gression, accounting for un-modeled variations, environmental changes, external noise etc.

63 Prognostic approaches set as a joint state-parameter estimation problem [18], have been widely useful and may fall  
64 under hybrid approach wherein, the prediction of RUL is based on current estimate of damage state and state of damage  
65 propelling hidden parameters. Prediction of the RUL is obtained as probability distribution and accounts for the various  
66 uncertainties involved [18-21].

67 Choice of the filter for estimation and prediction process depends on the assumptions that can be made about the sys-  
68 tem, and desired performance [22]. Well-known Kalman filter, an optimal estimator for linear systems, has been used for  
69 prognostics in[23, 24]. *Extended Kalman Filter* (EKF) [25]or Unscented Kalman filter [26], may also be used for param-  
70 eter estimation posing the problem as joint state-parameter estimation or as Expectation-Maximization problem [27] etc.  
71 However, they remain restricted to additive Gaussian noise. Also, EKF being sub-optimal diverges quickly if the initial  
72 estimate of state is significantly far from true value, or the model considered for estimation is not correct [10]. Compre-  
73 hensive comparative studies of filters for prognostic purposes are found in [10, 22, 28].

74 Set in Monte-Carlo framework, *Particle Filters* (PF) or Sequential Monte Carlo methods [29] form a suitable filter  
75 choice in this context, as it can be applied to non-linear systems corrupted with non-Gaussian noises for which, optimal  
76 solutions may be unavailable or intractable. Recently, particle filters have been exploited voraciously for prognostic meth-  
77 ods [30, 31]. Significant works include prediction of end of discharge and EOL in lithium-ion batteries [32], battery health  
78 monitoring [33], prediction of battery grid corrosion [34], estimation and prediction of crack growth [35-38], fuel cell  
79 prognostics [9], application to damage prognostics in pneumatic valve from the Space Shuttle cryogenic refueling system  
80 [18, 39], estimation-prediction of wear as concurrent damage problem in centrifugal pumps with a variance control algo-  
81 rithm [20], employment in distributed prognosis [21], exploring uncertainty management options for prognostics [40] etc.  
82 Particle filters attract considerable attention [41], owing to the ever growing efforts being made for enhancement of per-  
83 formances and computational efficiency, such as the use of correction loops[42], fixed-lag filters [43] and the recently  
84 proposed adaption of the degradation model with a kernel smoothing method [44]. Although a large amount of research

85 exists in model based diagnostics and prognostics, very few promise the achievement/implementation of all key require-  
86 ments in a common framework and the realization through a unified modeling paradigm [45, 46].

87 In this context, *Bond Graph* (BG) modeling technique becomes a very suitable tool to deal with dynamic systems, es-  
88 pecially those that belong to multi-energetic domains. A very brief and non-exhaustive introduction is given here. BG is a  
89 topological modeling language, where the exchange of energy between the different components of a dynamic system is  
90 captured in a graphical form. The energy exchange link is called a bond and there are two generic power variables named  
91 *effort*  $e$  and *flow*  $f$ , associated with every bond, such that  $e \times f = \text{Power}$ . The set of elements  $\{I, C, R\}$ , model the system  
92 parameters/component where  $I$ ,  $C$ , and  $R$  are the *inertial* element, *capacitance* element and *dissipation* element respective-  
93 ly. The latter along with the elements  $\{0, 1, TF, GY\}$  define the junction structure (global structure of the system) where  $TF$   
94 and  $GY$  are the *transformer* element and *gyrator* element respectively. Junction  $\mathbf{0}$  (or  $\mathbf{1}$ ) implies that all the connected  
95 bonds have same *effort* (or *flow*) and the sum of *flows* (or *efforts*) equals zero. For efficient simulation of the physical be-  
96 havior of the system, the computational order of the variables ( $e$  and  $f$ ) must be decided systematically. For that purpose,  
97 the cause and effect decisions are described by the notion of *causality*. *Causality* in BG models is depicted by a perpen-  
98 dicular stroke on a bond. It determines whether the *flow* for a bond is computed from the *effort* or vice versa. The end of  
99 the bond which receives the *effort* is represented by the perpendicular stroke at that end of the bond. If all of the energy  
100 storage elements in a model are in integral form, the system is in *integral causality*. The constitutive equations of  $I$  and  $C$   
101 respectively, in integral causality are<sup>2</sup> (linear case):  $f(t) = (1/I) \int e(t) dt$  and  $e(t) = (1/C) \int f(t) dt$ . For diagnosis task,  
102 where the initial conditions are unknown in real processes, the BG model is constructed in preferred *derivative causality*  
103 where the term “preferred” implies “wherever it is possible”. *Derivative causality* dictates the constitutive equation of  $I$   
104 and  $C$  respectively, to be as (linear case):  $e(t) = I d(f(t))/dt$  and  $f(t) = C d(e(t))/dt$ . For a detailed introduction from  
105 the *ab initio* and various related behavioral, structural and causal properties, the readers are referred to following works  
106 [47-49] and [50].

107 For uncertain dynamic systems too, BG has been used extensively for modeling purposes and development of supervi-  
108 sion techniques. This includes *Fault Detection and Isolation* (FDI) of complex systems [51], highly non-linear and com-  
109 plex thermo-chemical systems [52], non-linear mechatronic systems [53], intelligent and autonomous systems [54-56],  
110 industrial chemical reactors [57], hybrid systems [58] etc. In BG framework, the FDI is mainly based upon *Analytical Re-*  
111 *dundancy Relations* (ARRs)[59, 60] or by usage of the algebraic observers [55, 61]. Specifically, for uncertain systems,  
112 BG in *Linear Fractional Transformation* (BG-LFT)[62, 63] has been widely implemented for robust diagnosis by generat-  
113 ing adaptive thresholds with respect to parametric uncertainties[64-66]. Very recently in Jha et al. [67], the authors have  
114 proposed modeling of parametric uncertainties in interval form. Unlike BG-LFT, where the threshold limits are simply the  
115 summation of the absolute values of each of the induced uncertain *effort/flow* at the junction [64] leading to an over-  
116 estimation of threshold bounds, the interval valued thresholds consider the sensitivity of each uncertain candidate to the  
117 respective residual. Even though there has been wide implementation of BG for robust diagnosis of complex processes  
118 [65], there have been very little efforts if none, towards the development/integration of prognostic techniques in BG  
119 framework.

120 The few motivations propelling the development of this work are:

- 121 • Initial steps towards system level prognostics in BG framework: There are many benefits of using BG, including but  
122 not limited to: systematic graphical representation of the governing differential equations, efficient decomposition of

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<sup>2</sup> For any non-linear function  $\Theta_x$  with respect to BG element  $X$ , integral causality:  $f(t) = \Theta_x \left( \int e(t) dt \right)$  and  $e(t) = \Theta_x \left( \int f(t) dt \right)$ ; derivative cau-  
sality:  $e(t) = \Theta_x^{-1} d(f(t))/dt$  and  $f(t) = \Theta_x^{-1} d(e(t))/dt$

123 large scale multi-energetic systems into subsystems based upon functionality, an efficient understanding of the under-  
124 lying physics, explicit knowledge of cause-effect relationships, validated FDI techniques at global as well as local  
125 level etc. On the other hand, benefits of system level prognostics are many [68]. For almost all practical purposes,  
126 any plant (the dynamic system of interest) is a feedback closed loop system such that the system outputs follow a de-  
127 sired reference. As such, the system level prognostics present unique challenges in that incipient parametric degrada-  
128 tion may progress unnoticed in presence of controller compensated system outputs, resulting in non-estimation of the  
129 same till the saturation limit of controller is reached. BG derived ARR's being sensitive to system parameters and con-  
130 trol inputs can be exploited for the same at local component level while being in closed loop regime. Such a kind of  
131 BG enabled health monitoring, can be achieved in a unified framework at global system level.

- 132 • Very few and inefficient existing residual based prognostic approaches: Most of the previous residual based attempts  
133 in BG framework consider damage progression deterministic in nature, incapable of adapting to the current damage  
134 progression and hence, reliability of prediction is minimal [69-72]. Moreover, uncertainties associated with measure-  
135 ments, operating conditions, process noise etc. have not been taken into account. This results in prediction of RUL  
136 without any associated confidence bounds, rendering it virtually useless for industrial certification and critical appli-  
137 cations [73, 74].
- 138 • Inclusion of the recently developed diagnostic methodology: Recently, [67, 75] proposed a methodology of modeling  
139 uncertainties in interval form and the subsequent diagnosis through interval valued thresholds. This generates a genu-  
140 ine interest in its inclusion in an appropriate health monitoring framework.

141 This paper's main objective is to address the problem of prognostics in BG modeling paradigm while the system is  
142 considered globally uncertain and parametric uncertainty is modeled in interval form. This is achieved by casting the prob-  
143 lem as a joint state-parameter estimation problem, a hybrid prognostic approach, wherein the fault model is constructed by  
144 considering the statistical degradation model of the system parameter. The system parameter is known *a priori* to be un-  
145 dergoing degradation. Measurements are obtained from BG derived residuals (evaluation of ARR's). Using PF algorithms,  
146 estimation of state of the system parameter under degradation (prognostic candidate) along with the associated unknown  
147 hidden time varying *Degradation Progression Parameters*(s) (DPPs) is achieved and tracked to obtain the state of damage  
148 in probabilistic terms which is used for prediction of RUL of the system with respect to that parameter.

149 After this section, Section 2 details a fault detection algorithm where parametric uncertainty is modeled in interval  
150 form and *Interval valued ARR's* (I-ARRs) are derived systematically. The latter lead to the development of Interval valued  
151 robust thresholds over the nominal point-valued part of the I-ARRs. In Section 3, a novel methodology is proposed which  
152 includes construction of fault model and a novel way of obtaining the observation equation from the concerned nominal  
153 residual. In Section 4, the state of prognostic candidate and associated DPPs are estimated from the nominal part of the I-  
154 ARR's, sensitive to the latter and control inputs, in Monte-Carlo framework using PF algorithms. A novel variance control  
155 algorithm is proposed which ensures a suitable adaptation of random walk noise variance, once convergence is achieved in  
156 the estimation process. Prediction of the RUL is achieved in PF framework. Section 5 provides various evaluation metrics  
157 employed. In Section 6, the methodology is demonstrated through simulation and various issues are highlighted. In Sec-  
158 tion 7, the methodology is tested experimentally by variation of frictional torque on a mechatronic system and Section 8  
159 draws the conclusions. The various novel contributions of the paper are listed as follows:

- 160 • Integration of BG modeling framework and Monte Carlo framework for estimation of state of health and predic-  
161 tion of RUL.
- 162 • Exploitation of nominal part of I-ARRs derived in (BG framework) for detection of degradation beginning and  
163 prognosis of incipient parametric degradation in Monte Carlo framework using PF.

- 164 • Obtaining the observation equation from the nominal part of I-ARRs and construction of local fault model such  
165 that state of the prognostic candidate and RUL prediction is obtained while system outputs are feedback con-  
166 trolled or otherwise.
- 167 • Accounting the various noises associated with degradation process and nominal residual output, for estimation  
168 and RUL prediction.
- 169 • Proportional control type variance adaptation algorithm with novel feedback condition that ensures a sustained  
170 convergence with low estimation variance (spread).

171 Major assumptions are:

- 172 • Only the system parameters are considered uncertain. Sensors are considered non-faulty.
- 173 • The system parameter (prognostic candidate) that undergoes degradation is assumed to be known *a priori*. The  
174 issue of isolation or *isolability* of the faulty candidate is assumed resolved.
- 175 • DM of the prognostic candidate is assumed to be known *a priori*.
- 176 • Single fault (degradation) hypothesis is followed for an I-ARR considered.
- 177 • Noise associated with measurements (residuals) is assumed additive and normally distributed Gaussian in nature.

## 178 2. Bond Graph Based Uncertain System Modeling

179 Nominal model of any deterministic physical system may be modeled in BG form, in preferred integral causality, with  
180 nominal system parameters composed of basic elements  $\theta \in \{C, I, R, TY, GY\}$  with  $\theta \in \mathbb{R}^{N_\theta}$ . Sub-script  $n$  denotes the nom-  
181 inal value of the parameters. The sensor vector is formed by  $\mathbf{Y}(t) \in [De(t), Df(t)]^T$  with  $De(t) \in \mathbb{R}^{N_{De}}$  being *effort sensor*  
182 vector and  $Df(t) \in \mathbb{R}^{N_{Df}}$  being the *flow sensor* vector. The control/input vector is formed by  $\mathbf{U}(t) \in [Se(t), Sf(t)]^T$  with  
183  $Se(t) \in \mathbb{R}^{N_{Se}}$  and  $Sf(t) \in \mathbb{R}^{N_{Sf}}$  being respectively the *source of effort* and *source of flow* vectors. There is no uncertainty  
184 considered on system input (actuator/load/control input). The global system is considered uncertain with system param-  
185 eters in interval form  $[\underline{\theta}, \bar{\theta}] \in \mathbb{R}^{N_m}$  where  $N_m \leq N_\theta$ . The system parameter  $\theta$  is modeled in interval form as  $\theta \in [\underline{\theta}, \bar{\theta}]$ ,  
186 where  $\forall \theta \in [\underline{\theta}, \bar{\theta}] \Rightarrow \underline{\theta} \leq \theta \leq \bar{\theta}$  and  $[\underline{\theta}, \bar{\theta}] \in [\underline{\theta}, \bar{\theta}]$ . Then,

$$187 \quad [\underline{\theta}, \bar{\theta}] = [\theta_n - \Delta\theta_l, \theta_n + \Delta\theta_u] \quad (1)$$

$$188 \quad [\underline{\theta}, \bar{\theta}] = [\theta_n, \theta_n] + [-\Delta\theta_l, \Delta\theta_u] \quad (2)$$

189 Here,  $\Delta\theta_l \geq 0$  and  $\Delta\theta_u \geq 0$  are the additive uncertainty/deviation on the left and right sides, over the nominal value  $\theta_n$ .  
190  $[\theta_n, \theta_n]$  is a *degenerate interval* with equal upper and lower bounds. For any additive uncertainty  $\Delta\theta$  over  $\theta$ , the multipli-  
191 cative uncertainty is defined as:  $\delta_\theta = \Delta\theta / \theta_n$ . *Multiplicative interval uncertainty*  $[\underline{\delta}_\theta, \bar{\delta}_\theta]$  is expressed as in (3) such that

192  $\delta_\theta \in [\underline{\delta}_\theta, \bar{\delta}_\theta]$ . Then, the uncertain  $\theta$  may be expressed as shown in (4).

$$[\underline{\delta}_\theta, \bar{\delta}_\theta] = [-\Delta\theta_l / \theta_n, \Delta\theta_u / \theta_n] \quad (3)$$

$$[\underline{\theta}, \bar{\theta}] = \theta_n + [\underline{\delta}_\theta, \bar{\delta}_\theta] \theta_n \quad (4)$$

193 Interval valued parameters are represented on an uncertain BG closely following the BG-LFT representation details of  
194 which can be referred in [63].

195

196 **2.1.1. Uncertainty Modeling and Representation on BG**

197 Uncertain system parameters can be represented in interval form on an uncertain BG by decoupling the nominal pa-  
 198 rameter value  $\theta_n \in \{C_n, I_n, R_n, TY_n, GY_n\}$ , from the uncertain interval part  $[\delta_\theta] \theta_n$  where for the notational simplicity,  
 199  $[\underline{\delta}_\theta, \overline{\delta}_\theta] \equiv [\delta_\theta]$ . The additional uncertain *effort* (or *flow*) is brought-in at the junction 1 (or 0) by interval uncertainty  $[\delta_\theta]$ .  
 200 It is represented on uncertain BG by a combination of: virtual *effort* (or *flow*) detectors  $De^* : z_\theta$  (or  $Df^* : z_\theta$ ) and fictitious  
 201 source of *effort* input  $MSe : [w_\theta]$  (or fictitious source of *flow* input  $MSf : [w_\theta]$ ) (see (7) and Fig. 1 for illustration). In fact,  
 202 the fictitious sources  $MSe : [w_\theta]$  (or  $MSf : [w_\theta]$ ) are added to represent the introduction of an additional *uncertain effort*  
 203 (or *uncertain flow*) generated by the interval uncertainty on the system. The virtual detectors  $De^*$  (or  $Df^*$ ) are used to  
 204 represent the information exchange/transfer. The star ‘\*’ is added as super-script for distinguishing the fictitious detectors  
 205 (signals) from the real ones. In general, symbol ‘:’ is used alongside a generic BG element to indicate the value in its re-  
 206 spective characteristic equation. For instance, (see Fig. 1b)  $R : R_n$  indicates that the system component modeled as *resistor*  
 207  $R$  has the resistance value of  $R_n$  in its characteristic equation  $e_R = R \cdot f_R$ .

208 For pedagogical illustration, a resistor element  $R$  in *resistance* (imposed *flow*) causality is considered.

- 209 • Nominal case (see Fig. 1a): The characteristic equation with parameter in nominal state (without any uncertainty) is  
 210 expressed as:

211 
$$e_R = R \cdot f_R \tag{5}$$

- 212 • Uncertain case (see Fig. 1b): With multiplicative interval uncertainty  $[\underline{\delta}_R, \overline{\delta}_R]$ , the characteristic law is expressed as:

213 
$$[e_R, \overline{e}_R] = [\underline{R}, \overline{R}] \cdot f_R = R_n \left( 1 + [\underline{\delta}_R, \overline{\delta}_R] \right) \cdot f_R \tag{6}$$

214 
$$[e_R, \overline{e}_R] = \underbrace{[R_n, R_n]}_{e_{R,n}} \cdot f_R - \underbrace{[w_R]}_{-e_{R,unc}} \tag{7}$$

215 where  $[w_R] = -[\underline{\delta}_R, \overline{\delta}_R] \cdot z_R = -[\underline{\delta}_R, \overline{\delta}_R] \cdot R_n \cdot f_R$ . Interval valued uncertain *effort*  $e_{R,unc}$  is brought at the 1-junction by  $[w_R]$   
 216 . For the notational simplicity,  $MSe : (-[\underline{\delta}_R, \overline{\delta}_R]) \cdot R_n \cdot f_R \equiv MSe : [w_R]$ . As it is clear, the associated *effort* (or *flow*) infor-  
 217 mation  $z_R = R_n \cdot f_R$ , is brought to  $MSe : [w_R]$  by the virtual detector  $De^*$ .

218 For better illustration of the *effort/flow* transfer, Fig. 1c shows the equivalent block diagram representation of the uncertain  
 219 BG in Fig. 1b.

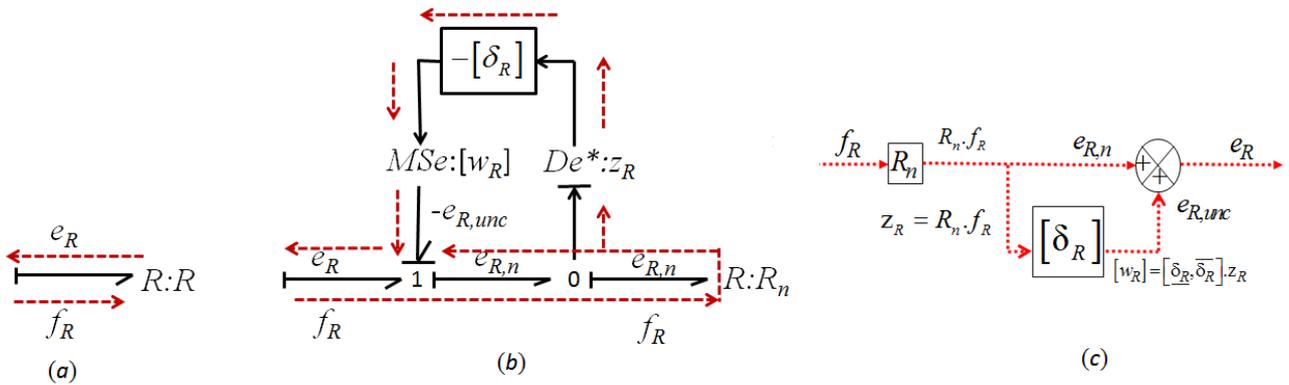


Fig.1. (a). Nominal  $R$  element (resistance causality), (b) Uncertain  $R$  element (resistance causality) in Interval form, (c) Equivalent Block Diagram Representation of Uncertain  $R$  element (Illustration of Signal Transfer)

221 Similarly, interval uncertainty can be modeled and represented for the other BG elements  $I, C, GY, TF, RS$  etc. For di-  
 222 agnosis based on ARR generation, the detectors are *dualized* such that *effort* detector  $De$  becomes a *source of effort signal*  
 223  $SSe$  and imposes the *effort* signal at the 0-junction connected to the detector. *Flow* detector  $Df$  becomes a *source of flow*  
 224 *signal*  $SSf$  and imposes *flow* at the 1-junction connected to the detector [76].

### 225 2.1.2. Interval Valued ARR Generation

226 Classically, an ARR is a constraint relation derived from an over-constrained system/subsystem. It is expressed in terms of  
 227 only known variables of the process [77]. For any function  $f$  and set of known variables  $k$ , it has the form:  $f(k) = 0$ . In the  
 228 context of BG modeling, an ARR:  $f(SSe(t), SSf(t), Se(t), Sf(t), \theta) = 0$ , where  $\theta$  is vector of system parameters. For  
 229 deterministic systems, the properties and ARR generation algorithm are detailed in [59]. BG model in preferred derivative  
 230 causality with *dualized* sensors are utilized to avoid unknown initial condition problem. For the uncertain systems, robust  
 231 FDI is achieved by generation of uncertain ARRs with perfectly separable nominal part and uncertain part[64]. The ap-  
 232 proach of latter is described in [64] and here, it is adapted to obtain *interval valued ARRs* (I-ARRs) in presence of interval

233 valued uncertainties. Consider the uncertain parameter vector  $[\underline{\theta}, \bar{\theta}] \in \mathbb{R}^{N_m}$  and  $N_m \leq N_\theta$ , following steps are taken to gen-  
 234 erate I-ARRs:

235 Step 1: Preferred derivative causality is assigned to the nominal model.

236 Step 2: Parametric uncertainties are modeled in interval form and represented on the nominal BG, as explained in section  
 237 2.1.1 to obtain uncertain BG.

238 Step 3: The candidate ARRs are generated from “1” or “0” junction, where power conservation equation dictates that sum  
 239 of *efforts* or *flows*, respectively, is equal to zero, as shown in (8) and (9) with  $s$  being the sign rendered to the *bond* due to  
 240 energy convention.

$$241 \quad \bullet \quad \text{for } 0\text{-junction: } s \sum [f, \bar{f}] + \sum Sf + \sum_{i=0}^{i \leq N_m} s_i \cdot MSf : [w_i] = 0 \quad (8)$$

$$242 \quad \bullet \quad \text{for } 1\text{-junction: } \sum s \cdot [e, \bar{e}] + \sum Se + \sum_{i=0}^{i \leq N_m} s_i \cdot MSe : [w_i] = 0 \quad (9)$$

243 Step 4: The unknown *effort* or *flow* variables are eliminated using covering causal paths from unknown variables to known  
 244 (measured) variables (*dualized* detectors), to obtain the I-ARRs,  $[\underline{R}, \bar{R}]$  consisting of known variables only as shown in  
 245 (10). The nominal part is characterized by point valued function  $\Psi_1$ , with point valued nominal parameters as coefficients  
 246 of point valued measured variables (cf.(12)). It is separated from the interval valued part which is identified as an interval  
 247 function  $\Psi_2$  (see Definition A.1 in Appendix A), sensitive to interval valued uncertainties (cf.(13)).

$$[\underline{R}, \bar{R}] : \Psi \left( \theta_n, [\underline{\theta}, \bar{\theta}], [w_i], \sum Se, \sum Sf, SSSe(t), SSSf(t) \right) \quad (10)$$

$$[\underline{R}(t), \bar{R}(t)] : r_n(t) + [\underline{B}(t), \bar{B}(t)] \quad (11)$$

$$r_n(t) = \Psi_1 \left( \theta_n, SSSe(t), SSSf(t), \sum Se, \sum Sf \right) \quad (12)$$

$$[\underline{B}(t), \bar{B}(t)] = \Psi_2 \left( [\underline{\theta}, \bar{\theta}], [\underline{\delta}_0, \bar{\delta}_0], SSSe(t), SSSf(t) \right) \quad (13)$$

248 Hereafter,  $r_n(t)$  being the numerical evaluation of the point-valued nominal part  $\Psi_1$ , will be referred to as *nominal resid-*  
 249 *ual* and  $\Psi_2$  will be termed as *Uncertain Residual Interval Function* (URIF).

## 250 2.2. Residual Based Determination of Degradation Model

251 In BG framework, the DM of a system parameter under degradation  $\theta^d \in \Theta$ ,  $\Theta \in \mathbb{R}^{N_0}$  can be obtained from the time  
 252 evolution profile of the respective ARR to which it is sensitive, assuming that the rest of the system parameters which are  
 253 sensitive to the same, do not undergo any kind of progressive fault or degradation [69],[78]. Here, consider the point val-  
 254 ued part of the  $d^{th}$  I-ARR  $r^d(t)$ , such that with  $\Theta' = \Theta \setminus \theta^d(t)$ ,  $t > 0, r_n^d(t) \neq 0$ :

$$255 \quad r^d(t) = \Psi_1^d \left( \theta^d(t), \Theta'_n, SSe(t), SSf(t), Se(t), Sf(t) \right) \quad (14)$$

256 Here, the sub-script  $n$  denotes nominal value. The computed values of  $r^d(t)$  at time sample points give an implicit relation  
 257 of the degradation profile of  $\theta^d(t)$  in time. Assuming that *implicit function theorem* is satisfied [79], (14) gives a real val-  
 258 ued function  $\psi_d$  such that:

$$259 \quad \theta^d(t) = \psi_d \left( r^d(t), \Theta'_n, SSe(t), SSf(t), Se(t), Sf(t) \right) \quad (15)$$

260 Residual based DM should be obtained prior to prognostics i.e. prior to the phase when system's health monitoring is of  
 261 interest.

### 262 2.3. Fault Detection Using Interval Valued Thresholds

263 In this work, the fault detection module is constructed for the robust detection of degradation commencement.

#### 264 2.3.1. Interval Valued Thresholds

265 Consider point valued parametric deviation(s)  $\delta_\theta$  such that  $\forall \left[ \underline{\theta}, \bar{\theta} \right] \in \left[ \underline{\Theta}, \bar{\Theta} \right] \left| \theta(t) \in \left[ \underline{\theta}, \bar{\theta} \right], \delta_\theta \in \left[ \underline{\delta}_\theta, \bar{\delta}_\theta \right] \right.$ ,  $b(t)$  is the  
 266 numerical evaluation of function  $\Psi_2$  with point valued arguments (cf.(16)).

$$b(t) = \Psi_2 \left( \delta_\theta, \theta(t), SSe(t), SSf(t) \right) \quad (16)$$

267  $\Psi_2$  can be considered as the *Natural Interval Extension Function* of point valued function  $\Psi_2$  (see Definition A.2 in Ap-  
 268 pendix A), with the point valued arguments and operators replaced by the corresponding interval arguments (time-  
 269 invariant here) and interval operators in the syntactic expression of the function  $\Psi_2$  [80, 81]. Then,  $\Psi_2$  can be expressed  
 270 as finite sequence of interval arithmetic operations (evaluated as class code during implementation [81]) so that it is con-  
 271 sidered as a *Rational Interval Function* of  $\Psi_2$  and hence, is inclusion isotonic (see Definition A.3 and Definition A.4 in  
 272 Appendix A). Then, through *Fundamental Theorem of Interval Analysis* (see Theorem A.1 in Appendix A) the inclusion  
 273 of (17) can be verified.

$$b(t) \subseteq \left[ \underline{B}(t), \bar{B}(t) \right] \quad (17)$$

274 Now, at all times, due to power conservation at the junction,

$$\begin{aligned} r_n(t) + b(t) &= 0 \\ \Rightarrow b(t) &= -r_n(t) \end{aligned} \quad (18)$$

275 Thus, the change in *effort/flow* brought by deviation at any instant in system is given by negative value of the nominal  
 276 residual at that time. From (16),(17) and(18), following is used for fault detection:

277 Under nominal conditions:

$$-r_n(t) \subseteq \left[ \underline{B}, \bar{B} \right](t) \quad (19)$$

278 Fault is detected if:

$$-r_n(t) \notin [\underline{B}, \overline{B}](t) \quad (20)$$

### 2.3.2. Robust Fault Detection

The thresholds are generated by the range evaluation of URIF as shown in (19) and (20). In discrete time step  $k$ , the algorithm for fault detection is given in Table I. It should be noted that therein, the bounds of URIF  $\Psi_2$  is computed by expressing them as a sequence (computational graph or code list) of real valued functions [81].

## 3. Fault model

In this work, the system parameter that undergoes degradation is assumed to be known *a priori*. Let  $\theta^d(t) \in \Theta$  be such a prognostic candidate. The objective in this paper is to estimate the state of  $\theta^d(t)$  based upon information (measurement) provided by the values of nominal residual sensitive to  $\theta^d(t)$ ,  $r_n^d(t)$ .

### 3.1. State Equation

The parameter under degradation  $\theta^d(t)$  is included as a tuple  $(\theta^d, \gamma^d, g^d)$  to model the progressive fault where  $g^d(\cdot)$  denotes the linear/non-linear *degradation progression function* (DPF) obtained from the corresponding DM. The latter models the way degradation progresses in  $\theta^d(t)$ :

$$\theta^d(t) = g^d(\gamma^d(t), \mathbf{v}^{\theta^d}(t)) ; \theta^d(t=0) = \theta_n^d \quad (21)$$

where,  $\gamma^d(t) \in \mathbb{R}^{N_{\gamma^d}}$  is DPP vector and  $\mathbf{v}^{\theta^d}(t) \in \mathbb{R}^{N_{v^{\theta^d}}}$  is the respective associated process noise vector. The *fault model* for  $(\theta^d, \gamma^d, g^d)$  is constructed in state-space form by considering the parameter  $\theta^d(t)$  as the state variable augmented with the DPP vector as,

$$\dot{\mathbf{x}}^d(t) = \mathbf{f}^d(\mathbf{x}^d(t), \mathbf{v}^{\mathbf{x}^d}(t)) \quad (22)$$

where,  $\mathbf{x}^d(t) = [\theta^d(t), \gamma^d(t)]^T$  is the augmented state vector and  $\mathbf{f}^d$  is state transition function following the Markovian assumption.

### 3.2. Residual Based Observation Equation

Here, the objective is to exploit the nominal residual for the estimation of state variables. This way, the nominal residual used for detection of degradation beginning can be further used further for estimation of state of health of the prognostic candidate and associated DPPs. This is possible if the ARR expression is altered to obtain the observation equation in an appropriate way, such that the nominal residual provides the measurements of state variables. For this purpose, a simple algebraic approach is proposed.

**Theorem:** *Under the single degradation hypothesis, assuming the nominal part  $r_n^d(t)$  of an I-ARR can be expressed as a linear combination of non-linear functions of  $\theta^d(t)$ , the measurement of the state  $\theta^d(t)$  can be obtained from the negative value of  $r_n^d(t)$ .*

**Proof:** Let  $\theta^d(t)$  be the prognostic candidate and  $\Theta' = \Theta \setminus \theta^d(t)$ . Assuming  $r_n^d(t)$  can be expressed as,

$$r_n^d(t) = \Xi \left( \Theta'_n, \text{SSe}(t), \text{SSf}(t), \text{Se}(t), \text{Sf}(t) \right) + \mathbf{A}^T \boldsymbol{\varphi}(\theta_n^d) \quad (23)$$

306 where  $\forall i | i = 1, 2 \dots m$ ,  $A^{m \times 1} = [a_1 \ a_2 \dots a_m]^T$  is a vector of known (measured system variables) with  
 307  $a_i = \phi_i(\theta'_n, \mathbf{SSe}(t), \mathbf{SSF}(t), \mathbf{Se}(t), \mathbf{Sf}(t))$  and  $\boldsymbol{\varphi}^{m \times 1}(\theta^d(t)) = [\varphi_1(\theta^d(t)), \varphi_2(\theta^d(t)), \dots, \varphi_m(\theta^d(t))]^T$  is the vector of non-linear  
 308 functions of  $\theta^d(t)$ . Then,  $\forall t \geq 0$  power conservation at the BG *junction* where the corresponding I-ARR is derived, gives,

$$r^d(t) = \Xi(\theta'_n, \mathbf{SSe}(t), \mathbf{SSF}(t), \mathbf{Se}(t), \mathbf{Sf}(t)) + A^T \boldsymbol{\varphi}(\theta^d(t)) = 0 \quad (24)$$

309 or,

$$\begin{aligned} r^d(t) &= \Xi(\theta'_n, \mathbf{SSe}(t), \mathbf{SSF}(t), \sum Se, \sum Sf) + A^T \boldsymbol{\varphi}(\theta_n^d) + (A^T \boldsymbol{\varphi}(\theta^d(t)) - A^T \boldsymbol{\varphi}(\theta_n^d)) = 0 \\ r^d(t) &= r_n^d(t) + A^T (\boldsymbol{\varphi}(\theta^d(t)) - \boldsymbol{\varphi}(\theta_n^d)) = 0 \\ A^T (\boldsymbol{\varphi}(\theta^d(t)) - \boldsymbol{\varphi}(\theta_n^d)) &= -r_n^d(t) \end{aligned} \quad (25)$$

310 Thus, degradation state  $\theta^d(t)$  can be linked implicitly to the measurements of  $-r_n^d(t)$ . Observation equation can be ob-  
 311 tained as,

$$y^d(t) = -r_n^d(t) = A^T (\boldsymbol{\varphi}(\theta^d(t)) - \boldsymbol{\varphi}(\theta_n^d)) \quad (26)$$

312 **Corollary:** When  $\boldsymbol{\varphi}(\theta_n^d) = \boldsymbol{\varphi}(\theta^d) = \theta_n^d$ , the vector  $A = a_1$ ,  $a_1 = \phi_1(\theta'_n, \mathbf{SSe}(t), \mathbf{SSF}(t), \sum Se, \sum Sf)$ , can be understood as the  
 313 coefficient function linking the fault value to the residual. It can be found as,

$$a_1 = \frac{\partial(r_n^d(t))}{\partial(\theta^d(t))} \quad (27)$$

314 The observation equation argument in (26) includes known variables (sensor measurements, system parameters, inputs  
 315 etc.) and their derivatives. It is heavily corrupted with noise, especially due to presence of derivative(s) of measured varia-  
 316 bles. In this work, the noise is considered additive, *independent and identically distributed* (i.i.d.) drawn from a zero mean  
 317 normal distribution. It is assumed uncorrelated to  $\mathbf{x}^d(t)$ . Thus, from (26), observation equation is formed as,

$$y^d(t) = h^d(\mathbf{x}^d(t)) + w^d(t) \quad (28)$$

318 where  $h^d(\cdot)$  is a nonlinear observation function obtained from (26) and  $w^d(t) \sim \mathcal{N}(0, \sigma_{w^d}^2)$ . The standard deviation  $\sigma_{w^d}$ , is  
 319 approximated from residual measurements.

#### 320 4. Degradation Estimation And RUL Prediction

321 In discrete time step  $k \in \mathbb{N}$ , the fault model  $(\theta^d, \boldsymbol{\gamma}^d, g^d)$  can be described in stochastic framework as,

$$\mathbf{x}_k^d = \mathbf{f}_k^d(\mathbf{x}_{k-1}^d, \mathbf{v}_{k-1}^d) \quad (29)$$

$$y_k^d = h^d(\mathbf{x}_k^d) + w_k^d \quad (30)$$

322 where  $\mathbf{x}_k^d = [\theta_k^d, \boldsymbol{\gamma}_k^d]^T$ ,  $\mathbf{f}_k^d$  is state transition function (possibly non-linear) and is described by first order Markov model.

323 Measurements  $y_k^d$  are assumed conditionally independent, given the state process  $\mathbf{x}_k^d$ . The likelihood function becomes as,

$$p(y_k^d | \theta_k^d, \boldsymbol{\gamma}_k^d) = \frac{1}{\sigma_{w_k^d} \sqrt{2\pi}} \exp\left(-\frac{(y_k^d - h^d(x_k^d))^2}{2\sigma_{w_k^d}^2}\right) \quad (31)$$

324 With the beginning of degradation being detected by the FDI module as a fault at time step  $k_d$ , the prediction of  
 325 EOL/RUL at prediction time  $k$ , requires the estimate of  $\theta_k^d, \boldsymbol{\gamma}_k^d$ . This problem is cast as joint state-parameter estimation

326 problem in particle filter (PF) framework, where the estimation at time  $k$  is obtained as *probability density function* (pdf)  
 327  $p(\theta_k^d, \Upsilon_k^d | y_{k_d:k}^d)$ , based upon history of measurements from the time of beginning of degradation  $k_d$  up to  $k$ ,  $y_{k_d:k}^d$ . In the  
 328 following section, the method employed for degradation estimation and consequent prognostics is explained assuming that  
 329 degradation begins at the start. In reality, information about  $k_d$  will be given by fault detection module as described in Sec-  
 330 tion 3.

#### 331 4.1. Degradation Estimation

332 In this section, concise details about PF are provided. The related concepts mentioned here can be found detailed in [29]  
 333 and [82]. The state distribution is approximated by set of discrete weighted samples or particles,  $\{(\theta_k^{d,i}, \Upsilon_k^{d,i}), w_k^i\}_{i=1}^N$ , where  
 334  $N$  is the total number of particles and for  $i^{\text{th}}$  particle at time  $k$ ,  $\theta_k^{d,i}$  is the estimate of the state (*system faulty* parameter  
 335 here) and  $\Upsilon_k^{d,i}$  is the estimate of *fault progression parameters*. The weight associated with each particle is denoted by  $w_k^i$ .  
 336 The posterior density at any time step  $k$  is approximated as,

$$p(\theta_k^d, \Upsilon_k^d | y_{0:k}^d) \approx \sum_{i=1}^N w_k^i \delta_{(\theta_k^d, \Upsilon_k^d)}(d\theta_k^d d\Upsilon_k^d) \quad (32)$$

337 where  $\delta_{(\theta_k^d, \Upsilon_k^d)}(d\theta_k^d d\Upsilon_k^d)$  denotes the Dirac delta function located at  $(\theta_k^d, \Upsilon_k^d)$  and sum of the weights  $\sum_{i=1}^N w_k^i = 1$ . In this paper,  
 338 *sampling importance resampling* (SIR) PF is employed for estimation of  $p(\theta_k^d, \Upsilon_k^d | y_{0:k}^d)$ , assuming that particles  
 339  $\{(\theta_{k-1}^{d,i}, \Upsilon_{k-1}^{d,i}), w_{k-1}^i\}_{i=1}^N$  are available as realizations of posterior probability  $p(\theta_{k-1}^d, \Upsilon_{k-1}^d | y_{0:k-1}^d)$  at time  $k-1$ , with the follow-  
 340 ing main steps:

- 341 • Realizations of *prediction*  $p(\theta_k^d, \Upsilon_k^d | y_{0:k-1}^d)$ , is obtained in form of new set of particles  $\{(\theta_k^{d,i}, \Upsilon_k^{d,i}), w_k^i\}_{i=1}^N$ , with  
 342 weights being chosen using the principle of *importance sampling*. The proposal *importance density* is chosen as the  
 343 transitional prior  $p(\mathbf{x}_k^{d,i} | \mathbf{x}_{k-1}^{d,i})$ , such that particles are generated by sampling from probability distribution of system  
 344 noise  $\mathbf{v}_{k-1}^{x_d}$  and simulation of the system dynamics of(29).
- 345 • Each sampled particle  $(\theta_k^{d,i}, \Upsilon_k^{d,i})$  is then *updated*. The weight  $w_k^i$  is associated to each of the particles based on the  
 346 likelihood of observation  $y_k^d$  made at time  $k$  as,

$$w_k^i = p(y_k^d | \theta_k^{d,i}, \Upsilon_k^{d,i}) / \sum_{j=1}^N p(y_k^d | \theta_k^{d,j}, \Upsilon_k^{d,j}) \quad (33)$$

347 Note that with the choice of *importance density* as the prior, the weights were obtained as,

$$w_k^i \propto w_{k-1}^i p(y_k^d | \theta_k^{d,i}, \Upsilon_k^{d,i}) \quad (34)$$

- 348 • To avoid the *degeneracy problem*, a new set of particles is *resampled* (with replacement) from the approximation of  
 349 posterior distribution  $p(\theta_k^d, \Upsilon_k^d | y_{0:k}^d)$  constructed on weighted samples previously drawn, such that weights are reset  
 350 equally to  $w_k^i = 1/N$ . The objective behind *resampling* is the elimination of particles with small weights and focus  
 351 on particles with large weights, for estimation. In this work, *systematic resampling* scheme is preferred as it is easy to  
 352 implement and takes  $O(N)$  time and the algorithm can be referred in [29].
- 353 • The *prediction*, *update* and *resample* procedures form a single iteration step and are applied at each time step  $k$ . The  
 354 algorithm for SIR filter is given in Table II. Details about other variants of *sequential importance sampling* PFs can  
 355 be referred in [29].

### 356 4.1.1. Random Walk Noise Variance Adaptation

357 Consider the DPP vector  $\boldsymbol{\gamma}^d \in \mathbb{R}^{N_{\gamma^d}}$  such that  $\forall j \in \{1, \dots, N_{\gamma^d}\}, \gamma^{d,j} \in \boldsymbol{\gamma}^d, \hat{\gamma}^{d,j}$  is the estimated value, and  
358  $\gamma^{d,j*} \in \boldsymbol{\gamma}^{d*}, \gamma_u^{d*} \in \mathbb{R}^{N_{\gamma^d}}$  being the respective true value. Also consider the interval vector  $[\boldsymbol{\gamma}_l^{d*}, \boldsymbol{\gamma}_u^{d*}] \in \mathbb{R}^{N_{\gamma^d}}$ , consisting of  
359 intervals  $[\gamma_l^{d,j*}, \gamma_u^{d,j*}] \in [\boldsymbol{\gamma}_l^{d*}, \boldsymbol{\gamma}_u^{d*}]$ , that contain the true value  $\gamma^{d,j*} \in [\gamma_l^{d,j*}, \gamma_u^{d,j*}]$ . Moreover, for every  $\gamma^{d,j} \in \boldsymbol{\gamma}^d$ , con-  
360 sider an associated constant (proportional gain)  $P^{d,j}$  such that  $P^{d,j} \in \mathbf{P}^d, \mathbf{P}^d \in \mathbb{R}^{N_{\gamma^d}}$ .  
361  $\gamma^{d,j}$  is modeled as a *random walk process*  $\gamma_k^{d,j} = \gamma_{k-1}^{d,j} + \xi_{k-1}^{d,j}$  where,  $\xi_{k-1}^{d,j}$  is sampled from an artificial random zero-mean  
362 Gaussian distribution i.e.  $\xi_{k-1}^{d,j} \sim \mathcal{N}(0, \sigma_{\xi_{k-1}^{d,j}}^2)$ . Here,  $\sigma_{\xi_{k-1}^{d,j}}^2$  denotes the associated variance  $v_{k-1}^{\xi^{d,j}}$  at time  $k-1$  i.e.  $\sigma_{\xi_{k-1}^{d,j}}^2 \equiv v_{k-1}^{\xi^{d,j}}$   
363 for notational simplicity where,  $\forall j \in \{1, \dots, N_{\gamma^d}\}, v^{\xi^{d,j}} \in \mathbf{v}^{\xi^d}$  and  $\mathbf{v}^{\xi^d} \in \mathbb{R}^{N_{\gamma^d}}$ . Moreover, associated with every  $v^{\xi^{d,j*}}$ , consider  
364 a reference variance (spread)  $v^{\xi^{d,j*}}, v^{\xi^{d,j*}} \in \mathbf{v}^{\xi^{d*}}$ . The artificial random walk noise permits the estimation of  $\theta^d(t)$  to con-  
365 verge to its true value during the estimation process. Selection of the variance of the random walk noise is essentially a  
366 tradeoff between values that are big enough to allow the convergence in reasonable amount of time, yet small enough to  
367 let the parameter values be tracked smoothly once convergence is reached [18]. One of the efficient ways of ensuring good  
368 estimation of  $\theta^d(t)$  is to reduce the random walk noise variance  $v_{k-1}^{\xi^{d,j}}$ , once a suitable convergence is reached. In this re-  
369 gard, performance enhancement has been achieved by the usage of proportional control law type variance adaptation  
370 method; it is proposed, demonstrated and implemented in [20]. Therein,

- 371 • Variance (spread) is quantified by the statistically robust metric *Relative Median Absolute Deviation* (RMAD)  
372 obtained as,

$$373 \quad RMAD(X) = \frac{\text{Median}_i(|X_i - \text{Median}_j(X_j)|)}{\text{Median}_j(X_j)} \quad (35)$$

373 where,  $X_i$  is an element for a data set  $X$ .

- 374 • The variance is adapted in a proportional control law way where the normalized error between the current  
375 RMAD  $v_k^{\xi^{d,j}}$  (e.g. 80%) and a reference  $v^{\xi^{d,j*}}$  (e.g. 10%) is multiplied by a proportional gain constant  $P^{d,j}$ . Cur-  
376 rent RMAD  $v_k^{\xi^{d,j}}$  is then increased or decreased by that amount. Thereafter, current random walk noise  $\xi_k^{d,j}$  is  
377 sampled from a zero mean Gaussian distribution with the modified variance  $v_k^{\xi^{d,j}}$ .

378 However, there-in, the adaptation that progresses in arbitrarily decided multiple stages, requires a proper tuning of refer-  
379 ence value  $v^{\xi^{d,j*}}$  and proportional gain constant  $P^{d,j}$ , for each stage. Such a procedure can be a tedious task especially in  
380 presence of multiple DPP. Although the objective that rests in achieving proper convergence and subsequent smooth  
381 tracking is clearly achievable, availability of no guidelines for a proper selection of number of stages and  $v^{\xi^{d,j*}}$ , makes  
382 the task complicated. In this paper, random walk variance is controlled in similar fashion as in [20], however, with the  
383 distinguishing feature that variance adaptation is triggered by  $\bar{\gamma}_k^{d,j}$ :

$$384 \quad \bar{\gamma}_k^{d,j} = \begin{cases} \frac{1}{L+1} \sum_{l=0}^{l=L} \text{mean}(\hat{\gamma}_{k-l}^{d,j}) & \text{if } k \geq L \\ \text{mean}(\hat{\gamma}_k^{d,j}) & \text{if } k < L \end{cases} \quad (36)$$

385 with  $\theta^d$  at time  $k$ , being the average of the estimation mean  $\hat{\gamma}^{d,j}$  in a running window of previous  $L$  estimates. Fig. 2  
 386 shows the schematic of the proposed algorithm. The fact that degradation model of  $\theta^d(t)$  is known, leads to an approxi-  
 387 mate knowledge of the true value of  $\gamma^{d,j*}$ . The adaptation of  $\xi^{d,j}$  is triggered when  $\bar{\gamma}_k^{d,j} \in [\gamma_l^{d,j*}, \gamma_u^{d,j*}]$ . The interval  
 388  $[\gamma_l^{d,j*}, \gamma_u^{d,j*}]$  can be decided based upon the approximate knowledge of  $\gamma^{d,j*}$ , obtained from the DM. The main objective  
 389 rests in letting the variance be regulated in an automatic way.  
 390 The corresponding pseudo-algorithm of the variance adaptation scheme followed in this paper is given in Table III.

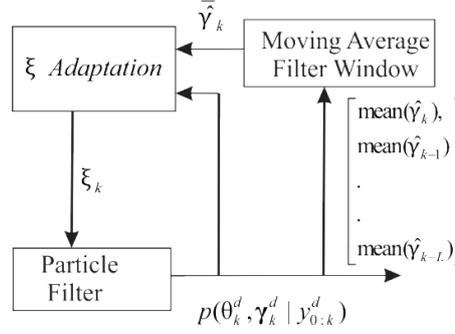


Fig. 2. Schematic of variance control scheme

391

## 392 4.2. Remaining Useful Life Prediction

393 The critical/failure value of  $\theta^d(t)$  is  $\theta_{fail}^d$ ; it is specified beforehand. The corresponding RUL prediction at time  $k$  is  $RUL_k^{\theta^d}$ ;  
 394 it is framed as generation of  $l^d$ -step ahead long term prediction  $p(\theta_{k+l^d}^d, \gamma_{k+l^d}^d | y_{0:k}^d)$  based upon the current joint state-  
 395 parameter estimate  $p(\theta_k^d, \gamma_k^d | y_{0:k}^d)$ . The latter is obtained with  $l^d = 1, \dots, T^d - k$ , where  $T^d$  is the time horizon of interest  
 396 i.e. time until  $\theta_{k+l^d}^d \geq \theta_{fail}^d$ . The  $l^d$ -step ahead state distribution is computed by propagating each of the particles

397  $\{(\theta_k^{d,i}, \gamma_k^{d,i}), w_k^i\}_{i=1}^N$ ,  $l^d$  steps ahead until  $\theta_{k+l^d}^{d,i} \geq \theta_{fail}^d$  [9, 18-20] as:

$$p(\theta_{k+l^d}^d, \gamma_{k+l^d}^d | y_{0:k}^d) \approx \sum_{i=1}^N w_k^i \delta_{(\theta_{k+l^d}^{d,i}, \gamma_{k+l^d}^{d,i})} (d\theta_{k+l^d}^d, d\gamma_{k+l^d}^d) \quad (37)$$

398 where for the  $i^{th}$  particle, the corresponding weight during the  $l^d$ -step propagation is kept equal to weight  $w_k^i$ . For  $i^{th}$   
 399 particle,  $RUL_k^{\theta^d} = k + l^d - k = l^d$ ; the corresponding  $RUL_k^{\theta^d}$  is obtained as:

$$p(RUL_k^{\theta^d} | y_{0:k}^d) \approx \sum_{i=1}^N w_k^i \delta_{(RUL_k^{\theta^d, i})} (dRUL_k^{\theta^d}) \quad (38)$$

400 The prediction  $RUL_k^{\theta^d}$  is done in the absence of future observations  $y_{k+1:k+l^d}^d$  which are not available. Pseudo algorithm for  
 401 RUL prediction is given in Table IV.

## 402 4.3. Health Monitoring Algorithm

403 The beginning of degradation is detected by the fault detection module described in Section 2. Subsequently, the joint  
 404 estimation and RUL prediction is triggered. As the thresholds are sensitive to other uncertain parameters,  $\theta_{t=t_d}^d = \theta_n^d$  cannot  
 405 be assured. Thus, the initial value of the state estimate is assumed uniformly distributed as,

$$\theta_{t=t_d}^d \sim U(\theta_n^d - \Delta\theta_l, \theta_n^d + \Delta\theta_u) \quad (39)$$

406 where  $t_d$  is the time of degradation commencement. The associated uncertainty interval limits  $[-\Delta\theta_l, \Delta\theta_u]$  decide the  
 407 bounds of the uniform distribution as shown in (2). Such an approximation guarantees to include the true initial state of

408  $\theta^d(t)$ . The complete algorithm is shown in Table V. Fig. 3 shows the schematic description of the proposed methodolo-  
 409 gy.

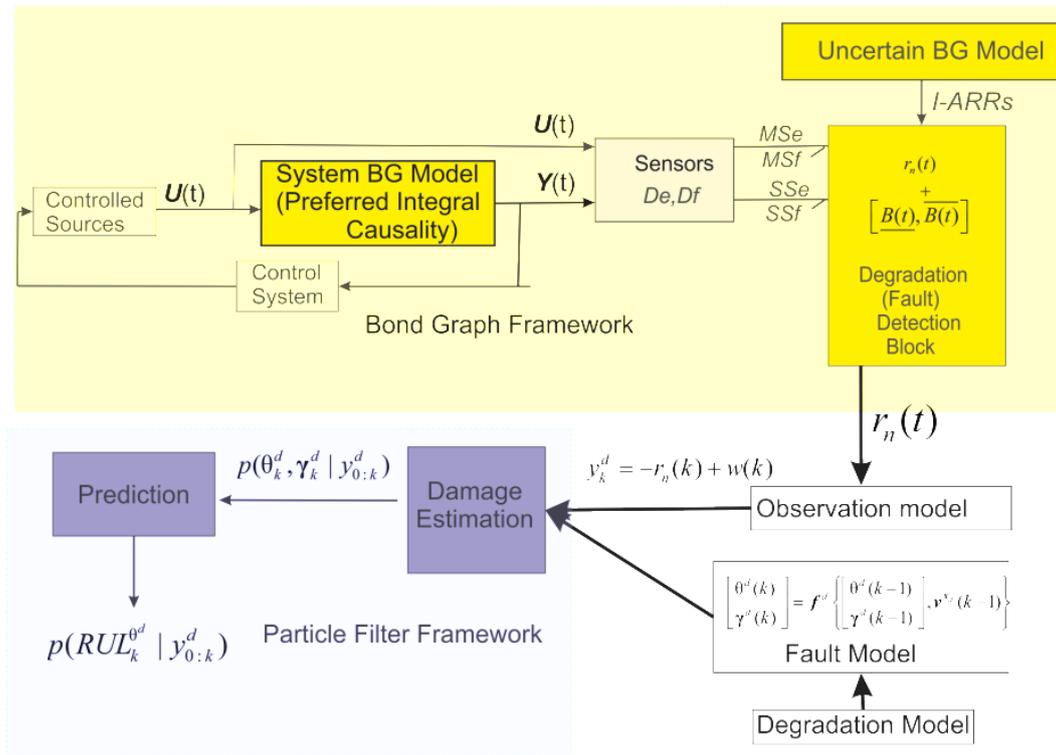


Fig. 3. Schematic description of the Health Monitoring Methodology

410

## 411 5. Evaluation Metrics

412 In this section, various metrics employed to evaluate the performance of estimation, prediction etc. are briefly dis-  
 413 cussed. [74] can be referred for details and [18, 20] for implementation of the same.

### 414 5.1. Estimation performance

415 The estimation performance is evaluated using two metrics that quantify the accuracy and spread.

416 *Root mean square error (RMSE)*: This metric expresses the relative estimation accuracy as:

$$417 \quad RMSE_X = \sqrt{Mean_k \left[ \left( \frac{X - X^*}{X^*} \right)^2 \right]} \quad (40)$$

418 Here,  $X \in \{\theta^d, \gamma^d\}$  and the corresponding true values are denoted by.  $Mean_k$  denotes mean over all the values of  $k$ .

419 *Relative median absolute deviation (RMAD)*: As detailed in Section 4.1.1, RMAD expresses the spread of estimation rela-  
 420 tive to median as a percentage. It is averaged over multiple values of  $k$  to obtain,

$$421 \quad \overline{RMAD}_{\gamma^d} = Mean_k (RMAD_{\gamma^d, k}) \quad (41)$$

422 where  $RMAD_{\gamma^d, k}$  is the RMAD of  $\gamma^d$  at time  $k$ .

### 423 5.2. Prediction performance

424 For a particular prediction time point  $k_p$ , the prediction accuracy is evaluated by Relative Accuracy (RA) metric as,

$$425 \quad \text{RA}_{\theta^d, k_p} = \left( 1 - \frac{|RUL_{\theta^d, k_p}^* - \text{Mean}(RUL_{\theta^d, k_p})|}{RUL_{\theta^d, k_p}^*} \right) \quad (42)$$

426 where  $RUL_{\theta^d, k_p}^*$  denotes the true  $RUL$  at time  $k_p$ , with respect to  $\theta^d$ . The overall accuracy is determined by averaging  
 427  $\text{RA}_{\theta^d, k_p}$  over all the prediction points. The latter is denoted as  $\overline{\text{RA}}_{\theta^d}$ ; it is determined as shown in (43). The associated  
 428 spread at  $k_p$  is denoted as  $\text{RMAD}_{RUL_{\theta^d}}$ . The overall spread is determined by finding the corresponding mean which is de-  
 429 noted as  $\overline{\text{RMAD}}_{RUL_{\theta^d}}$

$$430 \quad \overline{\text{RA}}_{\theta^d} = \text{Mean}_{k_p}(\text{RA}_{\theta^d, k_p}) \quad (43)$$

### 431 5.3. Prognostics Performance

432  $\alpha - \lambda$  metric[74] is employed to assess the prognostic performance. Here,  $\alpha \in [0,1]$  defines the bounds of true  $RUL$   
 433 as  $(1 \pm \alpha)RUL_{\theta^d, k_p}^*$  and  $\lambda \in [0,1]$  denotes the fraction of time between the initial prediction time point and the true EOL.  
 434 The third parameter  $\beta \in [0,1]$  signifies the desired (pre-fixed) fraction of the  $RUL$  prediction probability mass percentage  
 435 that must fall between the cones of accuracy determined by  $\alpha$ , for the respective  $RUL$  prediction to be acceptable. In this  
 436 paper, for all  $\lambda$  (all  $k$ ),  $\beta = 0.5$  which translates to the requirement of 50% of probability mass distribution of  $RUL_{\theta^d, k_p}$  fall-  
 437 ing within  $[(1 - \alpha)RUL_{\theta^d, k_p}^*, (1 + \alpha)RUL_{\theta^d, k_p}^*]$  for the prediction at  $k_p$ , to be acceptable.

## 438 6. Case Study on Mechatronic System

439 The method presented in this paper is applied on a mechatronic Torsion Bar 1.0 system shown in Fig.4. [83, 84]; it is  
 440 integrated with 20SIM, a BG dedicated software [85]. Real time implementation is achieved through 20 SIM 4C 2.1, a  
 441 prototyping environment that enables C-code implementation in real time on ARM-9 processor based torsion bar  
 442 system[86]. The interval computations, estimation, variance control and prediction algorithms are written in *Matlab Func-*  
 443 *tion Block* in *Simulink*. The embedded code is generated through *Simulink Coder* in *Matlab2013a*<sup>®</sup>. *INTLAB*[81, 87] is  
 444 used to implement interval calculations during simulation. For real time C-code generation, relevant/required functionali-  
 445 ties are borrowed from *INTLAB*.

### 446 6.1. Nominal System

447 The schematic model of the mechatronic system (detailed in [84]) is shown in Fig. 5. It consists of the Maxon<sup>®</sup> servo  
 448 motor that provides the controlled actuation (rotation) to the disks; it is equipped with voltage amplifier  $A_m$ , inductance  $L_a$ ,  
 449 resistance  $R_a$ , rotor inertia  $J_m$ . The associated motor friction coefficient is  $f_m$  and torque constant is  $k_m$ . The high stiffness  
 450 transmission belt provides the torque transmission with the transmission ratio of  $k_{belt}$ , to the motor disk with rotational  
 451 inertia  $J_{Md}$ . The motor disk is connected to load disk with rotational inertia  $J_{Ld}$ , through a flexible shaft that constitutes  
 452 the drive train. The shaft is modeled as spring-damper element having damping coefficient  $b_s$  and spring constant as  $k_s$ .  
 453 The friction in the bearings of the motor disk and load disk is modeled as viscous friction with damping parameters as  
 454  $b_{Md}$  and  $b_{Ld}$ , respectively. Friction arising due to belt action is lumped with the viscous friction coefficient at motor disk  
 455 in  $b_{Md}$ . The setup is equipped with motor encoder and load encoder that measure, respectively, the angular position of  
 456 motor shaft and load disk (2000 pulses per revolution). Angular position of the motor disk is obtained by dividing the mo-  
 457 tor encoder counts by belt ratio.

458 The BG model of the nominal system in integral causality[83] is given in Fig. 6. The control input from PI controller  
 459 (controlled variable: motor speed  $\omega_m$ ) modulates the input voltage  $MSe:U_{PI}$ . The measured angular velocities (obtained  
 460 from angular position measurements) of motor shaft and load disk are represented as  $Df:\omega_M$  and  $Df:\omega_{Ld}$  respectively.  
 461 Belt is considered of high stiffness and the rigidity is not considered in the model. Also, the frictional loss due to the ac-  
 462 tion of belt is lumped with frictional loss at motor bearing; it is modeled as a resistor element  $R:b_{Md}$ . *GY* element models  
 463 the conversion of electrical current to electrical torque in the DC motor with corresponding coefficient of gyration being  
 464  $m_{GY} = k_m$ . *TF* element models the transmission of velocity through the belt from motor shaft to the motor disk. The corre-  
 465 sponding coefficient of transformation  $m_{TY} = 1/k_{belt}$  where  $k_{belt}$  is the ratio between number of teeth on motor disk to  
 466 motor shaft [84]. The electrical part of the DC motor is not monitorable as there is no sensor installed in it.  
 467

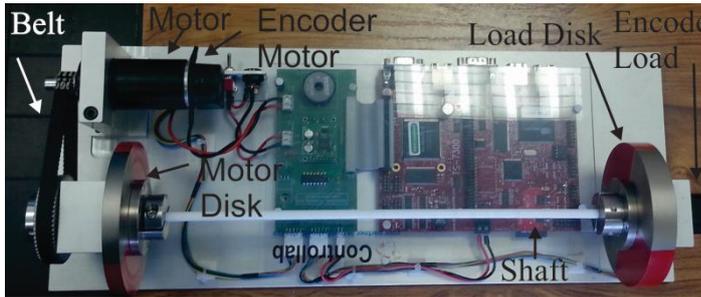


Fig. 4. (a). Mechatronic Torsion Bar 1.0 system

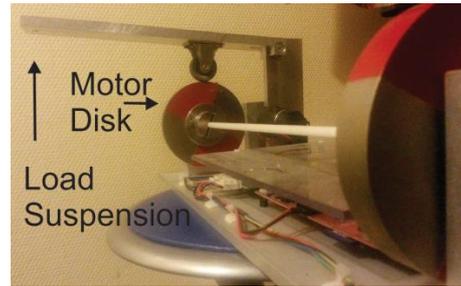


Fig. 4. (b). Fabricated Mechanical Lever type arrangement for Load (Mass) Suspension

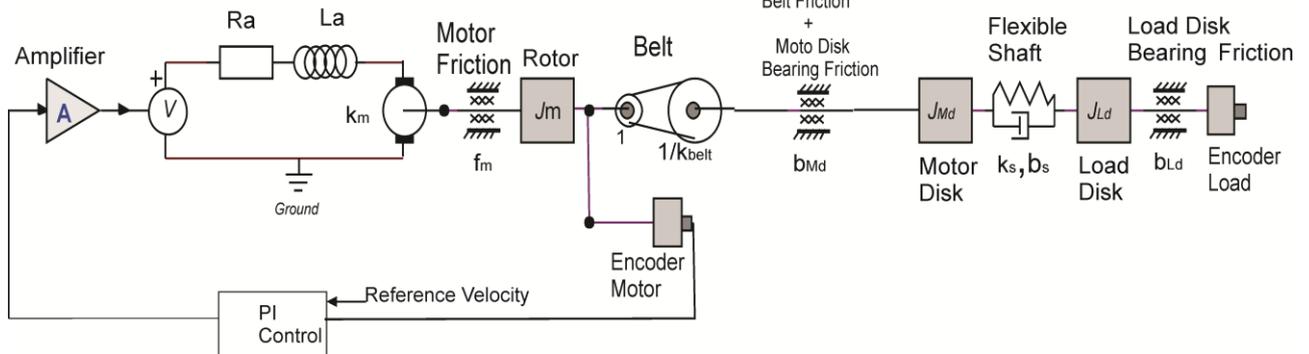


Fig. 5. Schematic Model Of The Mechatronic System

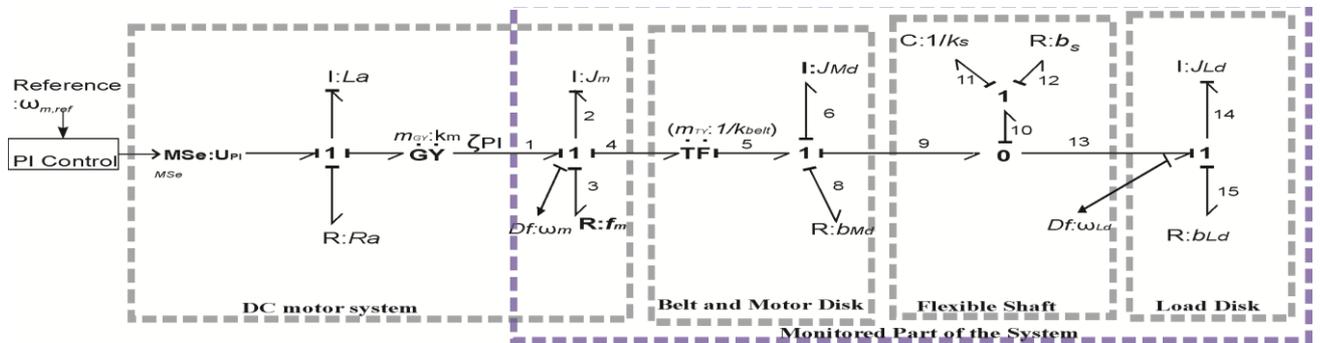


Fig. 6. Bond Graph Model ( Preferred Integral Causality) of The Nominal System

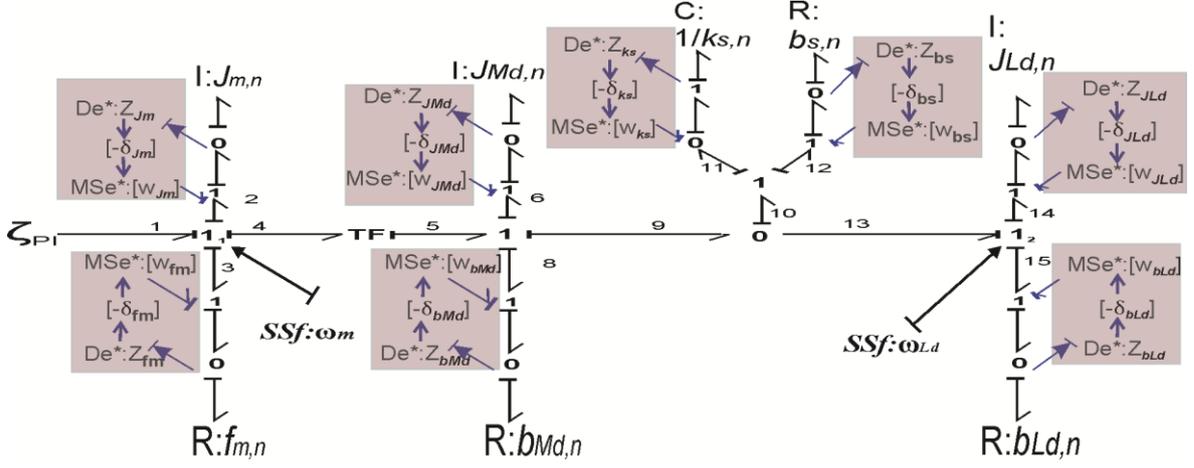


Fig. 7. Bond Graph Model Of Monitorable Part In Preferred Derivative Causality With Parametric Uncertainties As Intervals

468 Only the monitorable part (marked in Fig. 6) is used for analysis. It must be noted that the system is operating in feed-  
 469 back closed loop (Proportional-Integral (PI) control) regime. Analysis or development of the control strategy is not de-  
 470 scribed, as the main interest of the paper does not lie in the same.

## 471 6.2. Uncertain BG and System Validation

472 The uncertain BG of the monitorable part in preferred derivative causality is shown in Fig. 7. The parametric uncer-  
 473 tainties are modeled and represented in interval form. The global system is considered uncertain with uncertain parameter

$$474 \text{ vector } [\underline{\theta}, \bar{\theta}] : [\underline{\theta}, \bar{\theta}] = \left[ [J_m, \bar{J}_m], [f_m, \bar{f}_m], [J_{Md}, \bar{J}_{Md}], [b_{Md}, \bar{b}_{Md}], [J_{Ld}, \bar{J}_{Ld}], [b_{Ld}, \bar{b}_{Ld}], [k_s, \bar{k}_s], [b_s, \bar{b}_s] \right]^T \quad (44)$$

475 The monitorable part has input in form of the controlled electrical torque input generated by the DC motor. Both the sen-  
 476 sors ( $Df$ ) are *dualized* to corresponding *source of flows* as  $Y(t) = [SSf_1 : \omega_m, SSf_2 : \omega_{Ld}]^T$ .  $C$  element remains in integral  
 477 causality with the initial condition given by the *flow* at respective  $\mathbf{0}$ -junction, provided by encoder readings as  
 478  $f_{10} = f_9 - f_{13} = (\omega_m / k_{belt}) - \omega_{Ld}$ .

### 479 6.2.1. Interval valued ARR and Robust Thresholds

480 From the steps described in Section 2, I-ARR can be generated from the detectable junction  $\mathbf{1}_1$  of Fig. 7 as:

$$481 \left[ \underline{R}, \bar{R} \right]_1 = \tau_{PI} - \left( J_{m,n} \dot{\omega}_m + [\underline{\delta}_{J_m}, \bar{\delta}_{J_m}] J_{m,n} \dot{\omega}_m \right) - \left( f_{m,n} \omega_m + [\underline{\delta}_{f_m}, \bar{\delta}_{f_m}] f_{m,n} \omega_m \right) \quad (45)$$

$$- (1/k_{belt}) \left( \left( J_{Md,n} (\dot{\omega}_m / k_{belt}) + [\underline{\delta}_{J_{Md}}, \bar{\delta}_{J_{Md}}] J_{Md,n} (\dot{\omega}_m / k_{belt}) \right) + \left( b_{Md,n} (\omega_m / k_{belt}) + [\underline{\delta}_{b_{Md}}, \bar{\delta}_{b_{Md}}] b_{Md,n} (\omega_m / k_{belt}) \right) \right)$$

$$+ \left( k_{s,n} \int (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) dt + [\underline{\delta}_{k_s}, \bar{\delta}_{k_s}] k_{s,n} \int (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) dt + b_{s,n} (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) + [\underline{\delta}_{b_s}, \bar{\delta}_{b_s}] b_{s,n} (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) \right)$$

482 Electrical torque  $MSe : \tau_{PI}$  is the PI controlled input to the monitorable part of the system; it is given as,

$$483 MSe : \tau_{PI} = k_m i_m = k_m \cdot \frac{(U_{PI} - k_m \cdot \omega_m)}{Ra} (1 - e^{-(Ra/La) \times t}) \quad (46)$$

484 where  $U_{PI}$  is the PI controlled voltage input and  $i_m$  is the motor stator current. The nominal part  $r_{1,n}(t)$  is formed by col-  
 485 lecting point valued nominal parameters as coefficients of known (measured) variables. The I-ARR is expressed as,

$$486 \left[ \underline{R}, \bar{R} \right]_1 = r_{1,n}(t) + \left[ \underline{B}(t), \bar{B}(t) \right]_1 \quad (47)$$

487 where,

$$r_{1,n}(t) = \tau_m - J_{m,n} \dot{\omega}_m - f_{m,n} \omega_m - \frac{1}{k_{belt}} \left( J_{Md,n} \frac{\dot{\omega}_m}{k_{belt}} + b_{Md,n} \frac{\omega_m}{k_{belt}} + k_{s,n} \int \left( \frac{\omega_m}{k_{belt}} - \omega_{Ld} \right) dt + b_{s,n} \left( \frac{\omega_m}{k_{belt}} - \omega_{Ld} \right) \right) \quad (48)$$

$$\left[ \underline{B}(t), \overline{B}(t) \right]_1 = - \left( \left[ \underline{\delta}_{J_m}, \overline{\delta}_{J_m} \right] J_{m,n} \dot{\omega}_m - \left( \left[ \underline{\delta}_{f_m}, \overline{\delta}_{f_m} \right] f_{m,n} \omega_m \right) - \frac{1}{k_{belt}} \left( \left[ \underline{\delta}_{J_{Md}}, \overline{\delta}_{J_{Md}} \right] J_{Md,n} \frac{\dot{\omega}_m}{k_{belt}} + \left[ \underline{\delta}_{b_{Md}}, \overline{\delta}_{b_{Md}} \right] b_{Md,n} \frac{\omega_m}{k_{belt}} + \left[ \underline{\delta}_{k_s}, \overline{\delta}_{k_s} \right] k_{s,n} \int \left( \frac{\omega_m}{k_{belt}} - \omega_{Ld} \right) dt + \left[ \underline{\delta}_{b_s}, \overline{\delta}_{b_s} \right] b_{s,n} \left( \frac{\omega_m}{k_{belt}} - \omega_{Ld} \right) \right) \right) \quad (49)$$

Only one I-ARR has been derived here at  $\mathbf{1}_1$ ; it serves the purpose of approach-demonstration. Similarly, another independent I-ARR may be derived from  $\mathbf{1}_2$  junction.

### 6.3. Study by Simulation

#### 6.3.1. Nominal conditions

The nominal parameter values and respective *multiplicative interval uncertainty* is tabulated in Table VI. Fig. 8 shows the nominal outputs where motor velocity  $\omega_m$  is PI controlled with the reference  $\omega_{m,ref} = 112.5 \text{ rad/s}$ . Then, the motor disk velocity  $\omega_{Md}$  is regulated to  $\omega_{Md,ref} = \omega_{m,ref} / k_{belt} = 30 \text{ rad/s}$ . Noise is added to sensor outputs. It corrupts the residual and is approximated as  $w^d(t) \sim \mathcal{N}(0, \sigma_{w^d}^2)$ ;  $\sigma_{w^d} = 0.01 \text{ V}$ . Negative value of residual  $-r_{1,n}^d(t)$  is contained within the interval threshold bounds determined in (49).

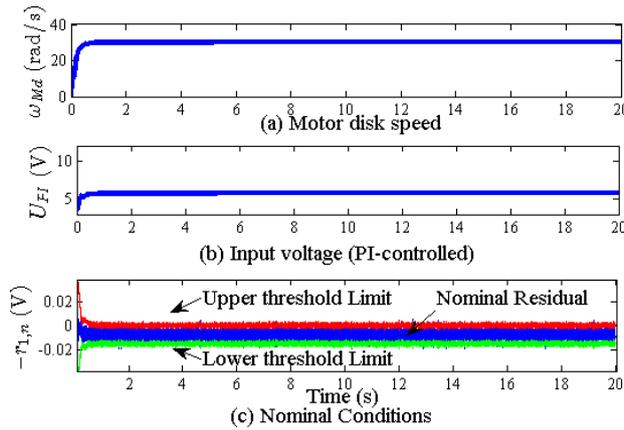


Fig. 8 Nominal Conditions (a) Motor disk speed (b) Input voltage (c) Nominal residual and Interval valued thresholds

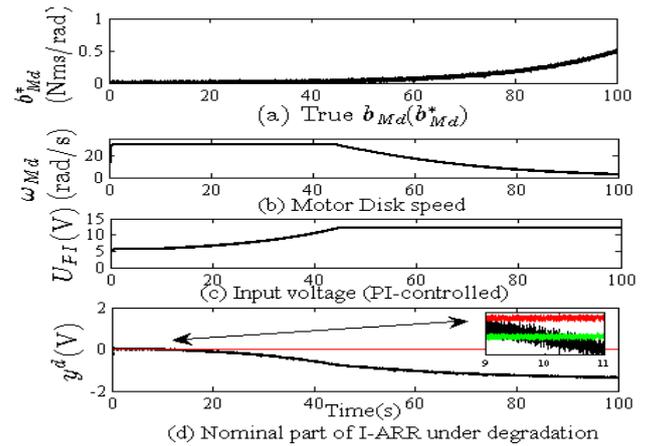


Fig. 9. Simulation of Degradation (a) Injected Degradation (b) Motor Disk Speed (c) Input Voltage to the System (d) Nominal Residual  $r_{1,n}(t)$

#### 6.3.2. Generation of Parametric Degradation

Degradation of motor disk bearing friction parameter  $b_{Md}$ , is simulated by considering the degradation model exponential in nature as,

$$b_{Md}(t) = \begin{cases} g_1(b_{Md}, \gamma_1) + v_{b_{Md}} \\ b_{Md,n} e^{\gamma_1(t)} + v_{b_{Md}} \end{cases} \quad (50)$$

where,  $g_1$  is the DM,  $\theta^d(t) = b_{Md}$  is the state variable and DPP vector  $\gamma^d = \{\gamma^d\} = \gamma_1$  and  $v_{b_{Md}} \sim \mathcal{N}(0, \sigma_{v_{b_{Md}}}^2)$  is the process noise. Fig. 9 shows the corresponding outputs. The fault is detected at  $t_d = 10\text{s}$  when residual crosses the interval thresholds. Note that  $\omega_{Md}$  is controlled at 30 rad/s until  $t = 44.2\text{s}$  while the PI controller is effective. Thereafter, as the saturation value of actuator (motor) input voltage (12V) is reached, the speed  $\omega_{Md}$  starts to decrease and reaches  $\omega_{Md,stop} = 3\text{rad/s}$  at

507  $t=100$ s. The latter is the time point at which system is considered to have obtained the failure state. The residual is sensi-  
 508 tive to the input torque and hence, the input voltage captures the degradation evolution throughout the system's lifetime.

### 509 6.3.3. Fault model

510 State measurement is obtained from the observation equation which is developed using the Nominal Part of I-ARR  
 511  $[\underline{R}, \overline{R}]_1$ ,  $r_{1,n}(t)$  (cf. (25) and (27)) as,

$$512 \quad 0 = r_{1,n}(t) + (b_{Md}(t) - b_{Md,n}). \frac{\partial(r_{1,n}(t))}{\partial(b_{Md})} \quad (51)$$

$$513 \quad y^d = -r_{1,n}(t) + w^d(t) = (b_{Md}(t) - b_{Md,n}) \frac{-\omega_m(t)}{k_{belt}^2} + w_1(t) \quad (52)$$

514 where  $w_1(t) \sim \mathcal{N}(0, \sigma_{w_1}^2)$  approximates the noise which corrupts  $-r_{1,n}(t)$ . For estimation, the fault model denoted as tuple  
 515  $(b_{Md}(t), \gamma_1, g_1)$ , is formulated as,

$$516 \quad \begin{aligned} \hat{b}_{Md,k} &= b_{Md,k-1} e^{\gamma_{1,k-1} \Delta t} + v_{b_{Md,k-1}} \\ \gamma_{1,k} &= \gamma_{1,k-1} + \xi_{1,k-1} \\ y_k^d &= (b_{Md,k} - b_{Md,n}) \frac{-\omega_{m,k}}{k_{belt}^2} + w_{1,k} \end{aligned} \quad (53)$$

517 where,  $\xi_1(t) \sim \mathcal{N}(0, \sigma_{\xi_1}^2)$  is the additive random walk noise. The estimation of state of parameter  $b_{Md}(t)$  is triggered at  
 518  $t_d=10$ s. Initial estimate  $b_{Md,t_d=10s} \sim U(0.045, 0.055)$  N.m.s/rad, contains  $b_{Md,n} = 0.005$  N.m.s/rad. The true value of DPP is  
 519 kept as  $\gamma_1^* = 0.05$  Nm/rad so that  $\omega_{Md,stop}$  is reached at 100s. Here,  $\Delta t = 0.1$ s and  $N=500$ . Simulation is run until  $t_f=100$ s  
 520 when  $\hat{b}_{Md}$  reaches the failure value  $b_{Md,fail} = 0.45$  N.m.s/rad.

### 521 6.3.4. Degradation Estimation

522 Estimated  $\hat{b}_{Md}$  is shown in Fig. 10. The true state  $b_{Md}^*$  is estimated accurately with  $RMSE_{b_{Md}} = 4.21\%$ . In fact, estima-  
 523 tion spread decreases as the estimation progresses, indicating the desirable performance. Estimation of  $b_{Md}$  largely de-  
 524 pends upon quality of estimation achieved with  $\gamma_1$ . Fig. 11 shows the estimation of  $\gamma_1$  achieved with  
 525  $[\gamma_{1,l}^*, \gamma_{1,\mu}^*] = [0.03, 0.07]$  N.m./rad, proportional gain  $P_I = 0.001$ ,  $v^{\xi_1^*} = 10\%$ , initial artificial noise variance  $\sigma_{\xi_1, k=0}^2 = 0.02^2$ .  
 526 The particle filter assumes measurement noise variance equal to 4 times that of residual noise variance  $\sigma_{w_1}^2 = 0.01^2$ . The  
 527 convergence is achieved very quickly but with large initial estimation spread. This is due to the high artificial noise vari-  
 528 ance set for the desirable quick convergence. As shown in Fig. 11b, the estimation spread is reduced (effective from  
 529  $t=20$ s) until  $v^{\xi_1^*} = 10\%$  is achieved at around  $t=50$ s and thereafter,  $\gamma_1^*$  is tracked smoothly with controlled spread and  
 530  $RMSE = 3.02\%$ . For comparison, Fig. 11c shows performance with no variance adaptation. Therein, the estimation con-  
 531 tinues with large spread even after the convergence is achieved.

### 532 6.3.5. RUL prediction

533 Using  $\alpha = 0.1, \beta = 0.5$  and for all  $\lambda_k$ , RUL prediction is shown in Fig. 12. The RUL predictions obtained until  $t=52$ s  
 534 are not good and suffer with large variance spread due to the large corresponding spread in  $\hat{\gamma}_1$  (see Fig. 11a), making them  
 535 virtually useless. However, after  $t=52$ s, the RUL distributions are well within accuracy cone (more than 50% of RUL

536 probability mass lies within accuracy cone). Ignoring the initial period of convergence, the overall prediction performance  
 537 is very good with  $\overline{RMAD}_{RUL} = 9.8\%$  and  $\overline{RA} = 97.15\%$ .

#### 538 6.4. A Qualitative Analysis

539 As seen in the previous section, accuracy and spread of RUL predictions are directly influenced by the estimation  
 540 quality of DPP. This in turn, depends on choice/initial setting and tuning of the several parameters involved. They are dis-  
 541 cussed here qualitatively. Note that estimation obtained in Fig. 11a forms the most desirable performance. In this section,  
 542 only the specified PF parameters are played with; rests are kept same as for Fig. 11a. Although some of the things dis-  
 543 cussed are intuitive from the perspective of estimation in a state space model, authors have felt the necessity to highlight  
 544 their concurrence when the residual is used as measurement.

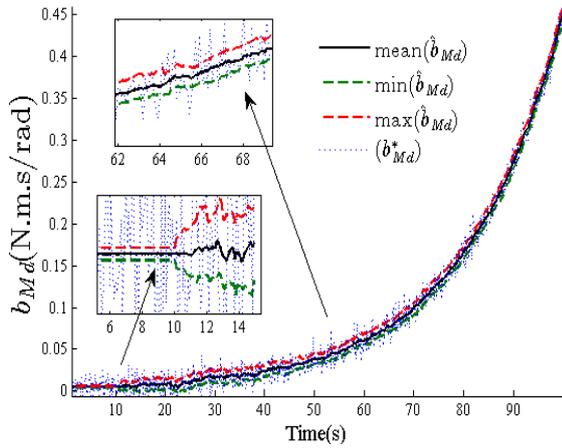


Fig. 10. State estimation of the prognostic candidate system parameter  $b_{Md}$

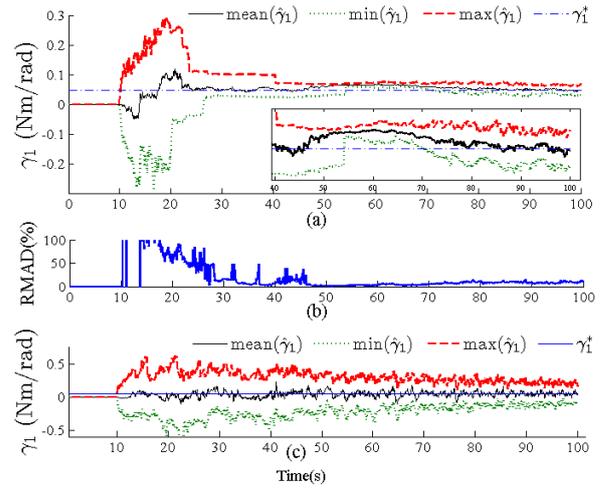


Fig. 11. Estimation performance (a) Estimation of DPP with variance adaptation (b) Estimation spread associated (c) Estimation performance without variance adaptation only for comparison purpose

545

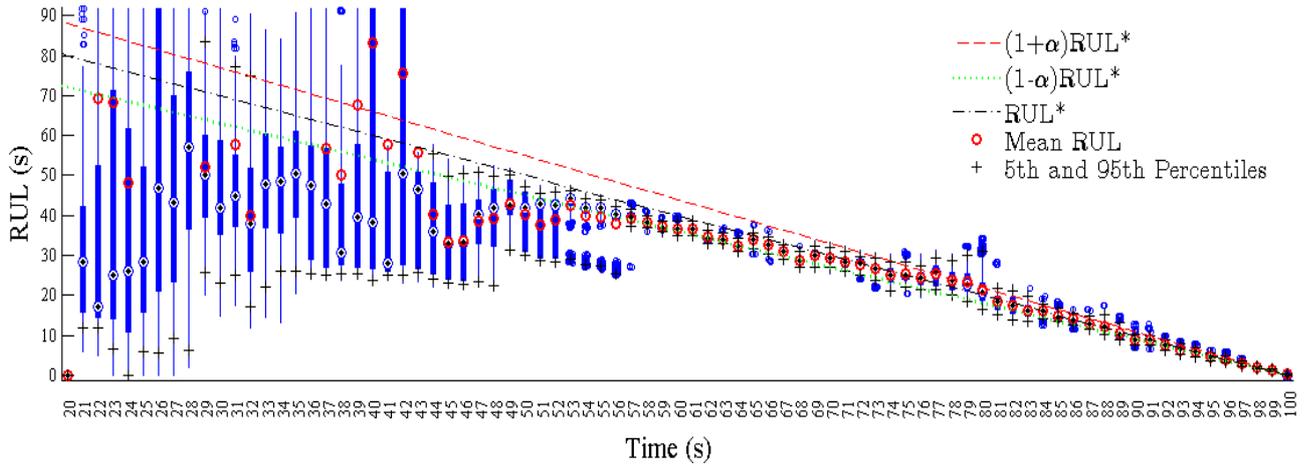


Fig. 12. RUL Prediction performance with respect to estimation in Fig. 10 and Fig. 11. (a).

546

- 547 • **Initial variance of the artificial random walk noise,  $\sigma_{\xi_1, k=0}^2$  (or  $v_{k=0}^{\xi_1}$ ):** The initial variance of random walk noise  
 548 is set according to the magnitude order of DPP  $\gamma^{d, j*}$ . It is kept high enough so that  $\hat{\gamma}_{1, k}^*$  is captured quickly as

549  $\bar{\gamma}_{1,k} \in [\gamma_{1,l}^*, \gamma_{1,u}^*]$ . It is the most important factor that determines good tuning of parameters in succession. A very  
550 high value of the latter, results in bad estimation performance. Fig. 13a shows estimation with  $\sigma_{\xi_1, k=0}^2 = 0.04^2$   
551 (read high) wherein, although, quick convergence of mean  $\hat{\gamma}_1$  is seen, the estimation continues further with a very  
552 wide spread for a long time before it is gradually reduced, owing to variance adaptation scheme. On the contrary,  
553 a very low variance will result in very late convergence, if at all. Fig. 13b has  $\sigma_{\xi_1, k=0}^2 = 0.001^2$  leading to a very  
554 late convergence. For tuning of other related parameters in this paper, an initial high value of variance  $\sigma_{\xi_1, k=0}^2$  is  
555 chosen.

556 • **Proportional Gain  $P_I$** : Proportional gain determines how fast the estimation spread is reduced to the reference  $v^{\xi^*}$   
557 . As observed in Fig. 11a, an appropriate choice of latter was found as  $P_I = 0.001$ . It resulted in smooth tracking  
558 after convergence was achieved. A high gain value results in quick reduction of estimation spread; however, it is  
559 accompanied with continuous shrink and expansion as shown in Fig. 13c with  $P_I=0.005$ . The latter has also been  
560 demonstrated in [20]. Although, a very high gain value may bring down variance spread quickly; however, it may  
561 be followed by poor convergence results as shown in Figs. 14c and d, with  $P_I=0.01$ . On the contrary, a very low  
562  $P_I$  renders a non-effective variance adaptation as shown in Fig. 13d with  $P_I = 0.0001$ , adding no significant bene-  
563 fits in RUL prediction.

564 • **Desired RMAD ( $v^{\xi^*}$ )**: The pre-fixed  $v^{\xi^*}$  for  $\xi_1$ , determines how much freedom is given to  $\gamma_1$  after the estima-  
565 tion spread is brought under control. An appropriate choice of  $v^{\xi^*}$  gives enough freedom for convergence even  
566 after actual variance is well under  $v^{\xi^*}$ , as shown in Fig. 11a between  $t=50s$  and  $t=80s$  with  $v^{\xi^*}=10\%$ .  
567 In extreme cases, where  $P_I$  is chosen of high value (rate of RMAD reduction depends on  $P_I$ ) and  $v^{\xi^*}$  is set very  
568 low, the estimation may remain stagnant near, but not equal to  $\gamma_1^*$ . This is shown in Fig. 14c with  $P_I=0.01$  (read  
569 high) and desired RMAD  $v^{\xi^*}=6\%$  (read very low).

570 • **True DPP interval  $[\gamma_{1,l}^*, \gamma_{1,u}^*]$** : The main objective of the latter remains in triggering the variance adaptation. As  
571 such, if width of  $[\gamma_{1,l}^*, \gamma_{1,u}^*]$  is kept too tight around  $\gamma_1^*$ ,  $\bar{\gamma}_{1,k}$  may never be captured inside the  $[\gamma_{1,l}^*, \gamma_{1,u}^*]$  band. This  
572 may lead to a very insignificant effect of variance adaptation on the estimation performance. Fig. 14a shows the  
573 estimation with  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.04, 0.06]$  Nm/rad, which can considered “too tight” around  $\gamma_1^* = 0.05$  Nm/rad.  
574 Here, the variance adaption is not effective enough. On the contrary, if the interval width is appropriately set (as-  
575 suming that initial estimate is outside of it),  $\bar{\gamma}_{1,k}$  is captured quickly and variance control is triggered early, as  
576 shown in Fig. 14b with  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.01, 0.09]$  Nm/rad. This leads to early reduction in variance. However, a bad  
577 choice of  $P_I$  (read high) and early variance adaptation, may lead to a rapid reduction in spread, followed by stag-  
578 nation of estimation around  $\gamma^{d,j^*}$ , before converging slowly to the same, as shown in Fig. 14d with  $P_I = 0.005$   
579 and  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.01, 0.09]$  Nm/rad.

580 • **Residual noise variance (measurement noise) assumed by PF**: Noise that corrupts the residual measurements  
581 can be non-Gaussian due to presence of derivative terms. Such noises can be dealt by PF effectively without any  
582 restrictions. In this work, the explicit distribution of the residual noise is not found. Instead, it is approximated as

583 normally distributed Gaussian in nature. The related standard deviation and variance is found out from residual  
 584 measurements. Moreover, generally, the variance of measurement noise (residual noise here) assumed by PF, is  
 585 greater than the approximated measurement noise. This is done to counter the sample impoverishment problem  
 586 which happens while very few particles have significant weights and most of the other particles with non-  
 587 significant weights are abandoned during the resampling process [88]. Higher residual noise variance assumed by  
 588 PF allows higher particles being sampled for estimation, thus, reducing the problem of sample degeneracy and  
 589 consequent impoverishment. As followed in other works [20], in this work too, the residual noise assumed by PF  
 590 is greater than actual residual noise.

## 591 6.5. Computational complexity

592 The time taken per step for estimation and RUL prediction depends on the number of particles used. With  $N=500$ , on  
 593 an average, 0.03s was consumed per step. Fig. 15 shows the RUL prediction computation time per step for the RUL pre-  
 594 diction performance of Fig. 12. In addition to the number of particles  $N$ , computational time for RUL prediction varies:

- 595 • Inversely with the time at which prediction is made: The farther is the time from EOL at which RUL predic-  
 596 tion is made, the longer it takes to simulate to EOL. This makes the computational time large.
- 597 • Inversely with estimated DPP  $\hat{\gamma}$  : At a certain time of prediction, higher is the rate of damage progression,  
 598 smaller is time taken to simulate to EOL. As seen in Fig. 11a, before  $t=50s$ , the estimation value of  $\hat{\gamma}_1$  is  
 599 lower than true value accompanied with large variance. Therefore, for a specific  $N$ , the computation time  
 600 per step before  $t=50s$  is higher and with large variations. After  $t=50s$ , with a nearly uniform  $\hat{\gamma}_1$  estimation  
 601 and lesser spread (see Fig. 11a), the computation time follows an almost uniform monotonic decreasing  
 602 trend (see Fig. 15).

603 Simulations were run on a 2.49-GHz dual core processor with 8GB RAM. With  $N=500$ , and sample time of 0.1s  
 604 (which translates to 10 computational steps per second); it took on an average 32 minutes to simulate system dynamics,  
 605 estimation and RUL prediction till 100s. With  $N=50$ , the same took 110 seconds. This indicates that through employment  
 606 of lesser number of particles, the RUL predictions could be achieved in real time, for experimental purposes. Moreover,  
 607 for real experiments run on compiled C, the run time reduces drastically by an order of magnitude.

608

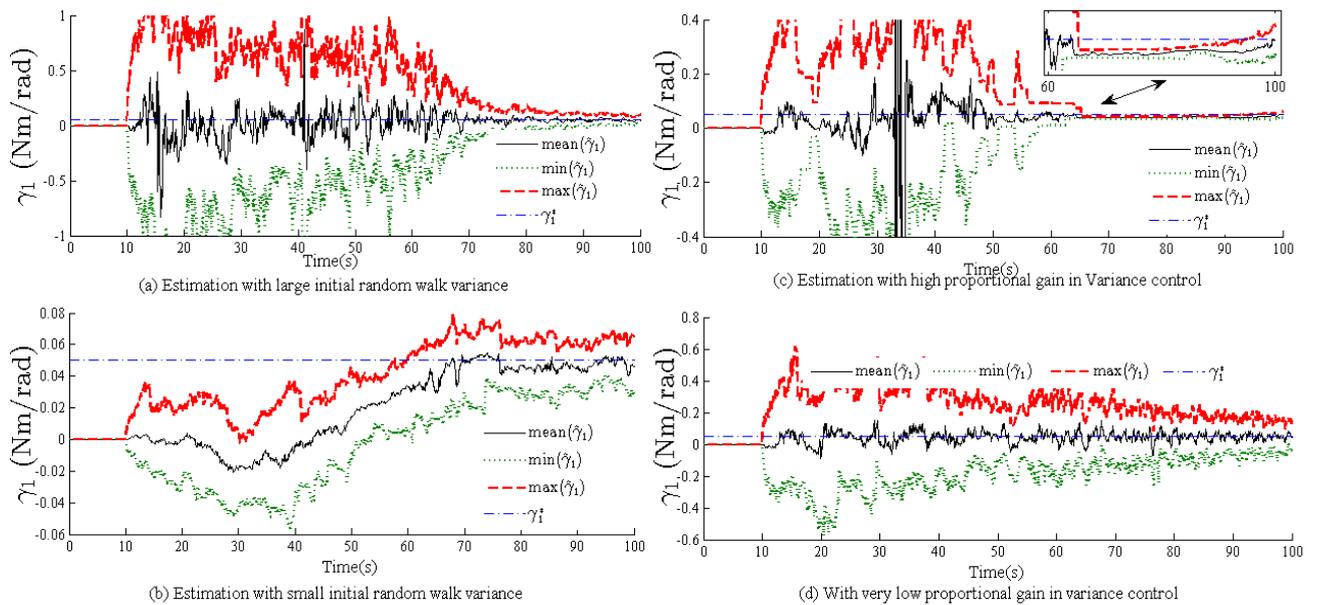


Fig. 13. Estimation of DPP  $\gamma_1$  for qualitative analysis (a) Large initial random walk variance  $\sigma_{\xi_1, k=0} = 0.04$  Nm/rad (b) Small initial random walk

variance  $\sigma_{\xi_i, k=0} = 0.001 \text{ Nm/rad}$  (c) High proportional gain in variance control  $P_I=0.005$  (d) Very low proportional gain in variance adaptation

$$P_I=0.0001$$

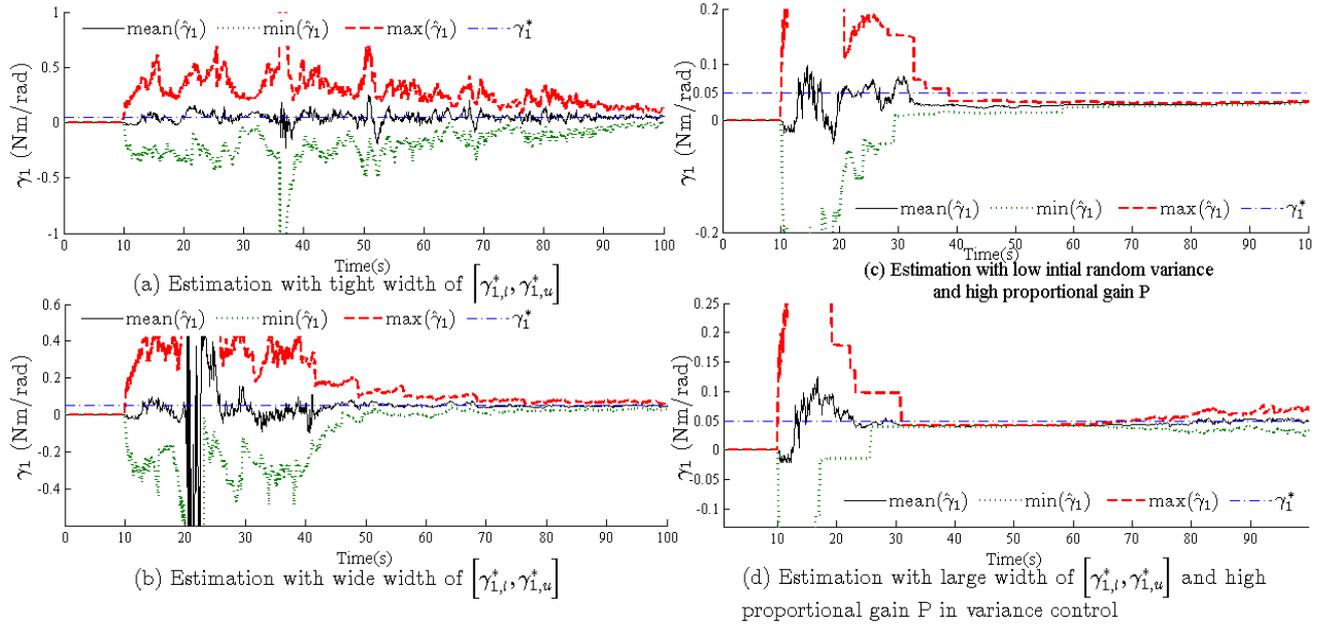


Fig. 14. Estimation of DPP  $\gamma_1$  for qualitative analysis (a)Tight width  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.04, 0.06] \text{ Nm/rad}$  (b) Wide width with  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.01, 0.09] \text{ Nm/rad}$  (c) High gain value,  $P_I=0.01$  and very low desired RMAD  $v^{s*} = 6\%$  (d) High gain  $P_I=0.005$  and large width  $[\gamma_{1,l}^*, \gamma_{1,u}^*] = [0.01, 0.09] \text{ Nm/rad}$

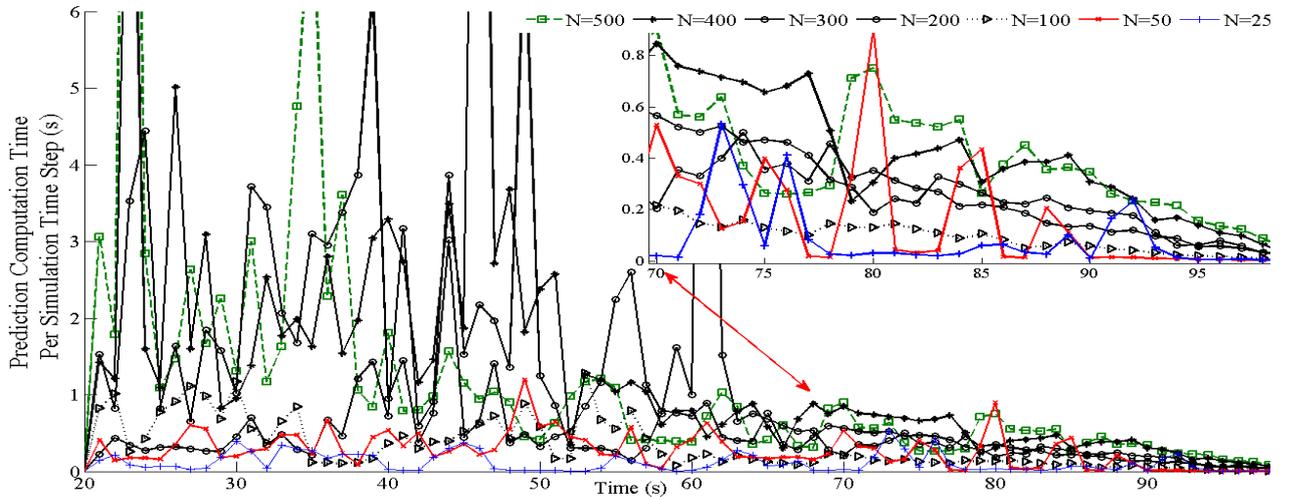


Fig. 15 Prediction computational time for per step for different number of particles

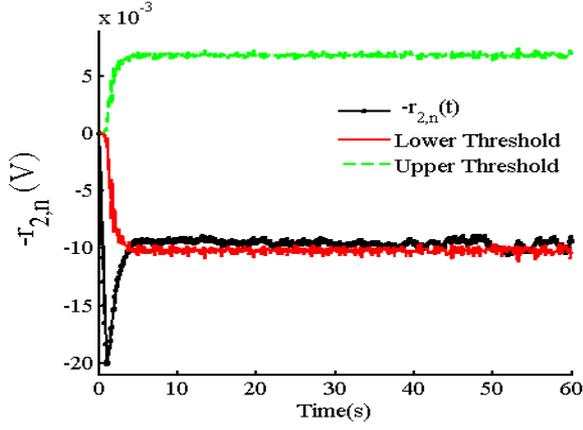


Fig. 16. Nominal residual  $-r_{2,n}(t)$  under nominal conditions

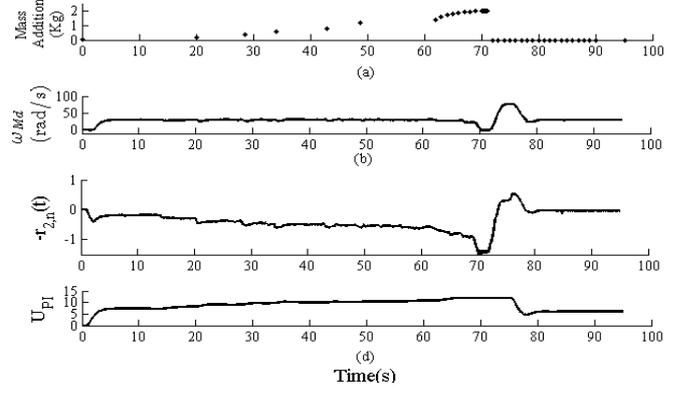


Fig 17. (a) Addition of mass discretely to introduce degradation (b). Motor disk speed (c) Nominal residual  $-r_{2,n}(t)$  (d) Input voltage (PI controlled)

## 609 7. Experiments and Results

610 For the experiments, a mechanical lever type arrangement is fabricated as shown in Fig. 4b; it introduces frictional  
 611 torque  $\tau_{Mech}$  over the motor disk by suspension of load in form of sand. The frictional torque manifests due to Coulomb  
 612 friction existing between the surfaces ( $\mu$  being friction coefficient); it is modulated by the suspended load of mass  $M$  kg  
 613 as,

$$614 \begin{aligned} \tau_{Mech} &= f_{mech} \cdot r_{Md} \\ f_{mech} &= \mu M g (\omega_{Md} / |\omega_{Md}|) \end{aligned} \quad (54)$$

615 with  $r_{Md}$  as the radius of the motor disk. In the BG model, it is incorporated as non-linear resistance element at motor disk  
 616 as shown in (55); the corresponding characteristic equation is obtained as shown in (56).

$$617 R = b_{Md} + \mu M (t) \cdot r_{Md} g / |\omega| \quad (55)$$

$$618 e_8 = R(f_8) = b_{Md} \omega_{Md} + \mu M (t) \cdot r_{Md} g \times (\omega_{Md} / |\omega_{Md}|) \quad (56)$$

619 The corresponding I-ARR  $[\underline{R}, \bar{R}]_1$  in (45) changes to  $[\underline{R}, \bar{R}]_2$  as,

$$620 r_{2,n}(t) = \tau_{in} - J_{m,n} \dot{\omega}_m - f_{m,n} \omega_m - \frac{1}{k_{belt}} \left( \begin{aligned} & J_{Md,n} \frac{\dot{\omega}_m}{k_{belt}} + b_{Md,n} \frac{\omega_m}{k_{belt}} + \mu_n M_n g r_{Md} \text{sgn}(\omega_m / k_{belt}) \\ & + k_{s,n} \int (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) dt + b_{s,n} (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) \end{aligned} \right) \quad (57)$$

$$621 [\underline{B}(t), \bar{B}(t)]_2 = - \left( \begin{aligned} & \left[ \underline{\delta}_{J_m}, \bar{\delta}_{J_m} \right] J_{m,n} \dot{\omega}_m - \left[ \underline{\delta}_{f_m}, \bar{\delta}_{f_m} \right] f_{m,n} \omega_m - \frac{1}{k_{belt}} \left( \begin{aligned} & \left[ \underline{\delta}_{J_{Md}}, \bar{\delta}_{J_{Md}} \right] J_{Md,n} \frac{\dot{\omega}_m}{k_{belt}} + \left[ \underline{\delta}_{b_{Md}}, \bar{\delta}_{b_{Md}} \right] b_{Md,n} \frac{\omega_m}{k_{belt}} + \left[ \underline{\delta}_{\mu}, \bar{\delta}_{\mu} \right] \mu_n M_n g r_{Md} \text{sgn}(\omega_m / k_{belt}) \\ & + \left[ \underline{\delta}_{k_s}, \bar{\delta}_{k_s} \right] k_{s,n} \int (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) dt + \left[ \underline{\delta}_{R_s}, \bar{\delta}_{R_s} \right] R_{s,n} (\frac{\omega_m}{k_{belt}} - \omega_{Ld}) \end{aligned} \right) \end{aligned} \right) \quad (58)$$

622 The nominal value of  $\mu$ ,  $\mu_n$  is found out from  $r_{2,n}(t)$  and (57) by suspending a known load mass. Fig. 16 shows the residual  
 623 profile under nominal conditions. Fig. 17 shows the effect of adding load (or frictional torque) in a discrete way on the  
 624 system.  $\omega_{Md}$  is controlled at 30 rad/s. Each time load is added, there is PI controller enabled compensation due to which  
 625  $\omega_{Md}$  settles to the reference velocity. However,  $-r_{2,n}(t)$  being sensitive to increase in current (and thus, voltage) decreases  
 626 and settles to a different value. Saturation value for input voltage is reached around  $t=65s$  as the total load suspended is 1.6  
 627 Kg. Thereafter ( $t>65s$ ), controller is unable to compensate the change in  $\omega_{Md}$ . Addition of more sand leads to reduction in  
 628 motor disk speed; it stops at around  $t=70s$ . For safety reasons, the disk is kept at stop condition for few seconds after

629 which the load is removed; this brings back the controller action into play. It is clearly visible that residual captures the  
 630 variation of friction (variation of mass) while controller remains effective or otherwise.

631 The experiments involve only non-destructive procedures so that there is no degradation (wear) of the surfaces. In oth-  
 632 er words,  $\mu$  is assumed constant. Experiments involve variation of suspended load mass  $M$  in a uniform way till the limit  
 633  $M_{fail}$ , is reached.  $M(t)$  is treated as system parameter under degradation. The experiments were conducted in two distinct  
 634 phases:

- 635 • Offline: In this phase, multiple tests were done with the load being added uniformly. As explained in Section  
 636 2.2, variations of  $M(t)$  were obtained from the evolution of  $r_{2,n}(t)$  found in (57). This provided the time de-  
 637 pendent DM of the system parameter  $M(t)$ .
- 638 • Online health monitoring: The maximum limit of additive load mass  $M_{fail}$  was pre-decided keeping in mind  
 639 the safety of the system. Load was varied until  $M_{fail}$ ; this was performed in the similar environment as of the  
 640 offline phase. In real time, estimation of  $M(t)$  and associated DPPs, and subsequent RUL predictions were  
 641 obtained.  
 642

### 643 7.1. Case I :Linear variation of mass

644 Linear degradation models are frequently employed where incipient degradation does not accelerate subsequent degra-  
 645 dation. Here, such a scenario was created through experiments and tested in real time.

#### 646 7.1.1. Degradation Test and Degradation model

647 Load is varied linearly. Ten experiments are carried out wherein; sand is poured with same environmental conditions to  
 648 maintain the uniformity. Fig. 18 shows the experimental data and the data mean found at each instant. A linear fit over  
 649 data mean is obtained using linear regression. The DM can be expressed as,

$$650 \begin{aligned} M(t) &= g_2(\gamma_2, t) + v_{M_2}(t) \\ &= \gamma_2 \times t + v_{M_2}(t) \end{aligned} \quad (59)$$

651 where  $g_2(\cdot)$  is the DM, DPP vector  $\gamma^d = \{\gamma^d\} = \gamma_2$  and  $v_{M_2}(t) \sim \mathcal{N}(0, \sigma_{v_{M_2}}^2)$ . An approximate  $\gamma_2^* = 0.005$  Kg/s is obtained.

652 Sum of squared errors provide an approximate standard deviation for process noise  $v_M$ ,  $\sigma_{v_{M_2}} = 1 \times 10^{-3}$  Kg.

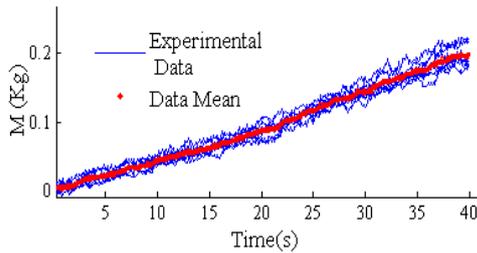


Fig. 18 Degradation Test Data (linear variation)

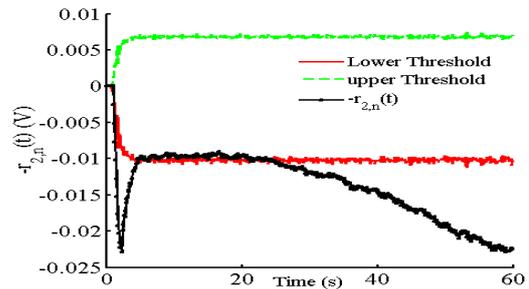


Fig.19 Nominal residual  $-r_{2,n}(t)$  while system is under degradation. (linear variation of mass)

653

#### 654 7.1.2. Fault model

655 The tuple  $(M(t), \gamma_2, g_2)$  is formulated in state space as in (60), with  $\xi_{2,k} \sim \mathcal{N}(0, \sigma_{\xi_2}^2)$  as the additive random walk  
 656 noise.

$$M_k = M_{k-1} + \gamma_{2,k-1} \times \Delta t + v_{M_{2,k-1}} \quad (60)$$

$$\gamma_{2,k} = \gamma_{2,k} + \xi_{2,k}$$

658 Observation equation is obtained from the nominal part of I-ARR  $[\underline{R}, \bar{R}]_2$ ,  $r_{2,n}(t)$ , as shown in (61) and (62) with  
 659  $w_{2,k} \sim \mathcal{N}(0, \sigma_{w_2}^2)$ .  $\sigma_{w_2}$  is determined from  $r_{2,n}(t)$  values during degradation tests of Fig. 18.

$$0 = r_{2,n}(t) + (M(t) - M_n) \cdot \frac{\partial(r_{2,n}(t))}{\partial(M)} \quad (61)$$

$$y_{2,k} = -r_{2,n,k} + w_{2,k}(t) = (M_k - M_n) \left( -\frac{\mu_n g r_{Md} \text{sgn}(\omega_{Md,k})}{k_{belt}} \right) + w_{2,k} \quad (62)$$

662 For the experiment, load is varied until  $M(t) = M_{fail} = 1.5 \text{Kg}$ . Fig. 19 shows the nominal residual profile under degra-  
 663 dation.

### 664 7.1.3. Estimation

665 The prognostics module is triggered at  $t=22$  s; estimation and predictions are performed with  $N=50$  particles,  $\Delta t=0.1$ s,  
 666  $\sigma_{\xi_{2,k=0}}^2=1 \times 10^{-6}$ ,  $\sigma_{v_{M_2}}=1 \times 10^{-3}$  Kg,  $\sigma_{w_2}=5 \times 10^{-3}$  V. For estimation, particle filter assumes measurement noise variance 9 times  
 667 that of measurement variance  $\sigma_{w_2}^2$  to counter sample *impoverishment* problem. Estimation of  $M$  is shown in Fig. 20a. The  
 668 true  $M^*$  is the residual based measurement of  $M(t)$  (as described in section 2.2, cf. (15)). State is estimated very accurately  
 669 with  $RMSE_M = 3.98\%$ .

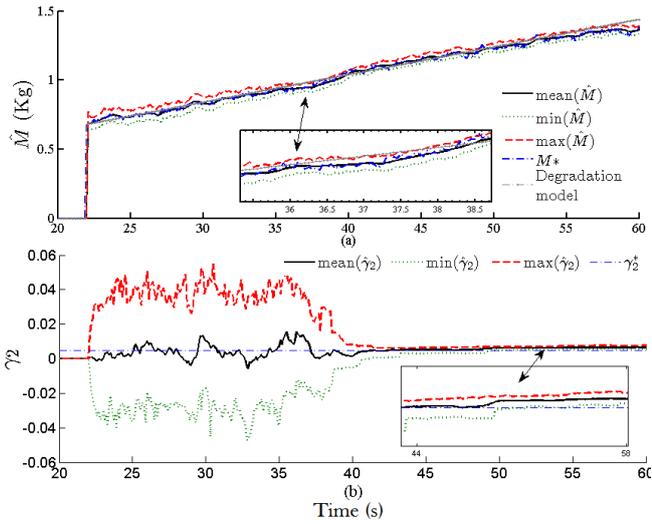


Fig. 20. (a). Estimation of  $M$  (b) Estimation of  $\gamma_2$

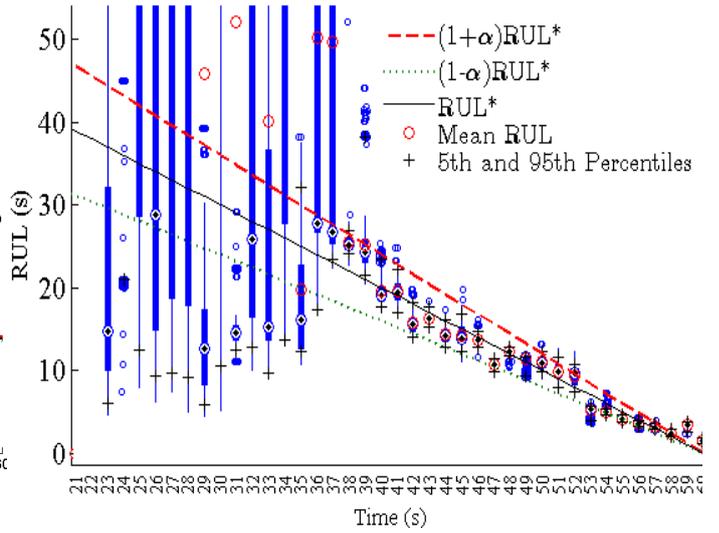


Fig 21: RUL prediction in experimental case: Linear variation of mass

670 Estimation of DPP  $\gamma_2$  is shown in Fig. 20b. Here, reference RMAD is set as  $v^{\xi_2^*} = 5\%$ , proportional gain  $P_2 = 0.007$ , true  
 671 DPP interval  $[\gamma_{2,l}^*, \gamma_{2,u}^*] = [3 \times 10^{-3}, 7 \times 10^{-3}]$  Kg/s around the approximately true  $\gamma_2^* = 0.005$  Kg/s. It should be noted that in  
 672 the real experiment,  $\gamma_2^*$  is not guaranteed to remain constant; the DM provides only an approximate idea of its magnitude  
 673 order. Fig. 20b shows the estimation with large initial variance. The estimation spread is reduced effectively from  $t=40$ s.  
 674 Thereafter, the estimation mean remains around  $\gamma_2^*$  with RMAD of 6%.

675

### 676 7.1.4. RUL prediction

677 Prediction of RUL is shown in Fig. 21 with  $\alpha = 0.2$  and  $\beta = 0.5$ . The initial predictions have a very large spread due to  
678 the large corresponding spread in  $\hat{\gamma}_2$ . However, after  $t=35$ s, the RUL is within the  $(1 \pm \alpha)RUL^*$  bounds with  
679  $\overline{RA} = 98.64\%$ ,  $\overline{RMAD}_{RUL} = 9.4\%$ . During the last 3 seconds of experimentation, the sand inflow is stopped gradually (and  
680 not abruptly) bringing in certain non-uniformity. As such, RUL predictions at  $t=58$ s,  $59$ s and  $60$ s, do not fall under the  
681  $(1 \pm \alpha)RUL^*$  bounds that are based upon the ideal linear degradation model.

682

## 683 7.2. Case II :Exponential variation of mass

684 Load is varied in an exponentially. Eight experiments are carried out in total. The considered DM is given in (63)  
685 where  $g_3(\cdot)$  is the DM,  $\theta^d = M(t)$ , DPP vector  $\gamma^d = \{\gamma^d\} = \gamma_3$  and  $v_{M3}(t) \sim \mathcal{N}(0, \sigma_{v_{M3}}^2)$ .

$$686 \begin{aligned} b_{Md}(t) &= g_3(M, \gamma_3) + v_{M3} \\ &= M_n e^{\gamma_3(t)} + v_{M3} \end{aligned} \quad (63)$$

687 Fig. 22a shows the experimental data. Fig. 22b shows the exponential fit over the experimental data mean from which the  
688 approximate value of DPP  $\gamma_3^* = 0.05$  Kg/s, is obtained. Regression residuals provide  $\sigma_{v_{M3}} = 8 \times 10^{-4}$  Kg.

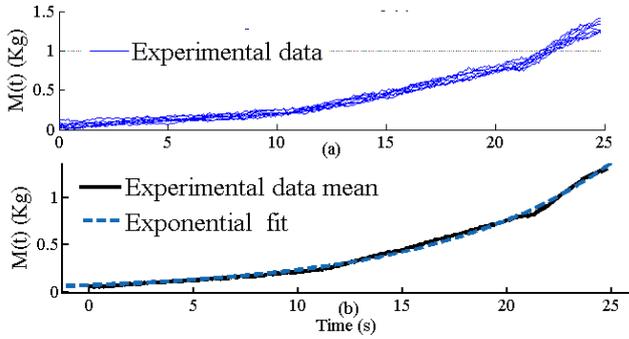


Fig. 22 Exponential variation of mass. (a) experimental data  
(b). Exponential fit over experimental data mean

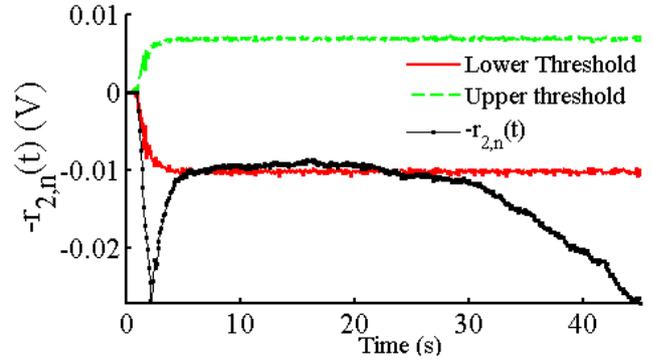


Fig. 23. Nominal residual  $-r_{2,n}(t)$  while system is under degradation (exponential case)

689

### 690 7.2.1. Fault Model

691 The tuple  $(M(t), \gamma_3, g_3)$  is formulated in state space as,

$$692 \begin{aligned} M_k &= M_{k-1} \cdot e^{\gamma_{3,k-1} \Delta t} + v_{M3,k-1} \\ \gamma_{3,k} &= \gamma_{3,k-1} + \xi_{3,k-1} \\ y_{3,k} &= -r_{2,n,k} + w_{3,k}(t) = (M_k - M_n) \left( -\frac{\mu_n g r_{Md} \text{sgn}(\omega_{Md,k})}{k_{belt}} \right) + w_{3,k} \end{aligned} \quad (64)$$

693 where  $\xi_{3,k} \sim \mathcal{N}(0, \sigma_{\xi_3}^2)$ ,  $w_{3,k} \sim \mathcal{N}(0, \sigma_{w_3}^2)$  and the approximation of  $\sigma_{w_3}$  is determined from  $r_{2,n}(t)$  values during degrada-  
694 tion tests. The structure of the observation equation remains same as in (62). For the experiment, load mass is varied until  
695  $M(t) = M_{fail} = 1.8$  Kg. Fig 23 shows the profile of nominal residual under exponential degradation.

### 696 7.2.2. Estimation and RUL Prediction

697 The prognostic module is triggered at  $t=22$  s. It is performed with  $N=50$ ,  $\Delta t=0.1$ s,  $\sigma_{\xi_3, k=0}^2 = 4 \times 10^{-6}$  and  $\sigma_{w_3} = 5 \times 10^{-3}$  V.  
698 For estimation, particle filter assumes measurement noise variance 9 times that of measurement variance  $\sigma_{w_3}^2$  to counter  
699 sample impoverishment problem during the experimentation. As shown in Fig. 24a, state of parameter is estimated accu-

700 rately with  $RMSE_M = 3.78\%$  . Fig. 24b shows the DPP  $\gamma_3$  estimation with reference RMAD set as  $v^{\xi_3^*} = 10\%$  , proportion-  
 701 al gain  $P_3 = 0.003$ , true DPP interval  $[\gamma_{3,l}^*, \gamma_{3,u}^*] = [1 \times 10^{-2}, 9 \times 10^{-2}]$  Kg/s. Estimation is achieved with  $RMSE_{\gamma_3} = 7.6\%$ . It  
 702 must be noted that in reality,  $\gamma_3^*$  cannot be claimed to be the accurate true value of  $\gamma_3$  . Fig. 24c shows the RUL prediction  
 703 with  $\alpha = 0.2, \beta = 0.5$ . Ignoring the initial predictions until  $t=32$  (due to large spread),  $\overline{RMAD}_{RUL} = 9.4\%$  and  
 704  $\overline{RA} = 97.02\%$  . In fact, the EOL is achieved slightly before than that predicted by DM.

705 It should be noted that  $RMSE_{\gamma}$  in real time experiments is higher than that obtained in simulations as  $\gamma^*$  does not re-  
 706 maining perfectly constant in real cases. Also, usage of lesser number of particles leads to worse estimation performance.  
 707 However, overall prediction and estimation performances are very good and satisfactory.

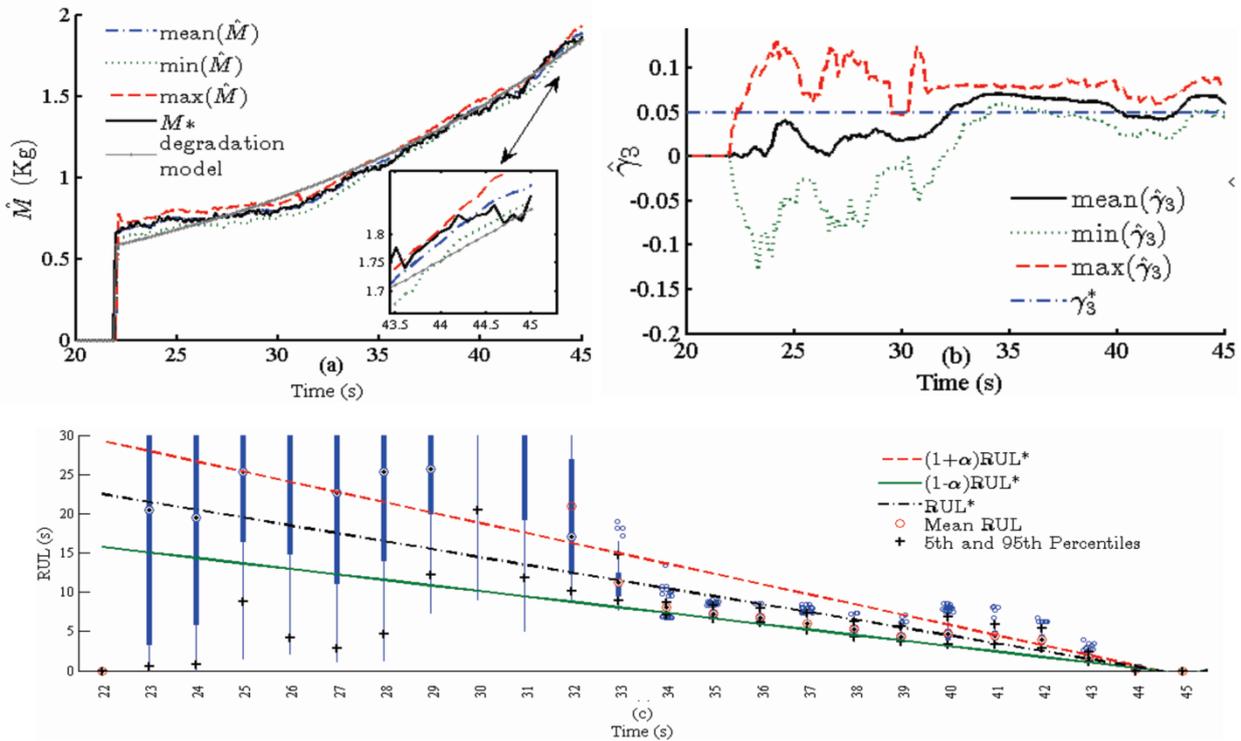


Fig. 24. (a) State estimation for Trail 1 (b) Estimation of DPP  $\gamma_3$  (c) RUL prediction for case II

708

## 709 8. Conclusions

710 It has been successfully demonstrated through simulation and experimental studies that under single degradation hypothe-  
 711 sis, the nominal part of Interval Valued Analytical Redundancy Relations (I-ARRs) derived from the Bond Graph (BG)  
 712 model of the uncertain system can be used for detection of system parameter's degradation. Subsequent estimation of the  
 713 state of health and associated degradation progression parameter(s), and prediction of the remaining useful life of the  
 714 prognostic candidate can be obtained using particle filtering algorithms. This leads to an efficient integration of the bene-  
 715 fits of BG modeling framework and Monte Carlo framework. The uncertain part of the I-ARRs is used for robust thresh-  
 716 old generation over the nominal part. This enables efficient detection of the degradation commencement, robust to para-  
 717 metric uncertainty. Further, the same nominal residual can be used for obtaining the measurements of state variables in the  
 718 fault model while the observation equation is developed from the nominal part of the I-ARR. For the latter, a novel alge-  
 719 braic approach is proposed so that the robust detection of degradation and further estimation of state variables of the fault  
 720 model can be achieved using the same nominal residual in a unified framework. Moreover, this methodology can be ex-  
 721 tended in presence of multiple degradations which forms a potential future work. In future, the work will be effectively  
 722 explored for large systems with multiple prognostic candidates. Being sensitive to the control inputs, nominal residual is

723 able to capture the parametric degradation profile even while the system outputs remain in feedback closed loop regime.  
 724 This makes the approach effective for system level health management. Approximation of the distribution of noise pre-  
 725 sent in residuals can be difficult or impossible, due to presence of derivative or integral terms in the arguments. As such,  
 726 employed Particle filter algorithms form the best choice in this regard, supporting non-Gaussian noises. The novel vari-  
 727 ance adaptation scheme leads to very good estimation results and involves less complexity in terms of tuning of the in-  
 728 volved factors. In future, the latter will be developed further and exploited for similar purposes. Through simulations, this  
 729 approach has the capability of generating long term and very long term predictions.

730 Through experiments, capability of obtaining RUL predictions in real time has been shown, although, in very short time  
 731 window. The associated computational complexity prevents the long and very long-term RUL predictions in real time.

732 In future, additional ways to obtain the same in sliding time windows will be explored. The method will be extended to  
 733 achieve very long term predictions in multiple stages, comprising of small time windows, in real time. Although, robust-  
 734 ness of the methodology has not been analyzed quantitatively, a qualitative analysis has been presented which helps in an  
 735 efficient tuning of the PF parameters. As this work forms an effective initial step towards prognostics in BG framework,  
 736 the same methodology will be applied over complex non-linear thermochemical-hydraulic systems such as fuel cells and  
 737 vapor generator systems.

### 738 Acknowledgements

739 The first author is thankful to Prof. Genevieve Dauphin-Tanguy and *Ecole Centrale de Lille*, for funding the research  
 740 work.

### 741 Appendix A

742 Given a real function  $f$  of real variables  $x = [x_1, x_2, \dots, x_n]^T$  belonging to intervals  $X = [X_1, X_2, \dots, X_n]^T$  [81]:

743 **Definition A.1:** The *interval extension function* (IEF),  $F(X)$ , is any interval valued function that satisfies

744  $F(x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n)$  . For degenerate interval arguments, the result must be the degenerate interval

745  $[f(x_1, x_2, \dots, x_n), f(x_1, x_2, \dots, x_n)]$ .

746 **Definition A.2:** *Natural interval extension* (NIE)  $F$ , of  $f$  is obtained, by replacing the real arguments with interval argu-  
 747 ments and real operators (arithmetic etc. ) by their equivalent interval operators, in the syntactic expression of the real  
 748 function  $f$ .

749 **Definition A.3:** We say that is  $F = F(X_1, X_2, \dots, X_n)$  *inclusion isotonic* if  $Y_i \subseteq X_i \forall i = 1, 2, \dots, n \Rightarrow$

750  $F(Y_1, Y_2, \dots, Y_n) \subseteq F(X_1, X_2, \dots, X_n)$  .

751 **Definition A.4:** A *rational interval function* is an interval-valued function whose values are defined by a specific finite se-  
 752 quence of interval arithmetic operations.

753 **Lemma A.3.1:** All rational interval functions are inclusion isotonic.

754 **Theorem A.1 (Fundamental Theorem of Interval Analysis):** If  $F$  is an inclusion isotonic interval extension of  $f$ , then

755  $f(X_1, X_2, \dots, X_n) \subseteq F(X_1, X_2, \dots, X_n)$  .

756 **Corollary A.1.1:** If  $F$  is a rational interval function and an interval extension of  $f$ , then  $f(X_1, X_2, \dots, X_n) \subseteq F(X_1, X_2, \dots, X_n)$  .

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942

Table I

**Algorithm 1: Fault detection with  $d^{\text{th}}$  I-ARR**

**Input:**  $\left\{ \begin{array}{l} \Psi_1^i (Se, Sf, \theta_n, SSe(k), SSf(k)) \\ \text{FC-}\Psi_2^i \left( [\underline{\theta}, \bar{\theta}], [\underline{\delta}_0, \bar{\delta}_0], SSe(k), SSf(k) \right) \end{array} \right\}$

**Output:** *fault detection*

$$r_n(k) = \Psi_1 (Se, Sf, \theta_n, SSe(k), SSf(k))$$

$$\left[ \underline{B}(k), \overline{B}(k) \right] = \Psi_2 \left( \begin{array}{l} [\underline{\theta}, \bar{\theta}], [\underline{\delta}_0, \bar{\delta}_0], SSe(k), \\ SSf(k) \end{array} \right)$$

**if**  $-r(k) \geq \underline{B}(k)$  **and**  $-r(k) \leq \overline{B}(k)$

*fault detection*  $\leftarrow$  **false**

**else**

*fault detection*  $\leftarrow$  **true**

**end if**

Table II

**Algorithm 2: Estimation using SIR filter**

**Inputs:**  $\left\{ (\theta_{k-1}^{d,i}, \gamma_{k-1}^{d,i}), w_{k-1}^i \right\}_{i=1}^N, y_k^d$

**Output:**  $\left\{ (\theta_k^{d,i}, \gamma_k^{d,i}), w_k^i \right\}_{i=1}^N$

**for**  $i=1$  **to**  $N$  **do**

$$\gamma_k^{d,i} \sim p(\gamma_k^{d,i} | \gamma_{k-1}^{d,i})$$

$$\theta_k^{d,i} \sim p(\theta_k^{d,i} | \theta_{k-1}^{d,i}, \gamma_{k-1}^{d,i})$$

$$w_k^i \sim p(y_k^d | \theta_k^{d,i}, \gamma_k^{d,i})$$

**end for**

$$W \leftarrow \sum_{i=1}^N w_k^i$$

**for**  $i=1$  **to**  $N$  **do**

$$w_k^i \leftarrow w_k^i / W$$

**end for**

$$\left\{ (\theta_k^{d,i}, \gamma_k^{d,i}), w_k^i \right\}_{i=1}^N \leftarrow \text{RESAMPLE} \left\{ (\theta_{k-1}^{d,i}, \gamma_{k-1}^{d,i}), w_{k-1}^i \right\}_{i=1}^N$$

Table III

**Algorithm 3:  $\xi$  Adaptation**

**Inputs:**  $\left\{ (\theta_k^{d,i}, \gamma_k^{d,i}), w_k^i \right\}_{i=1}^N, \mathbf{v}_k^\xi, [\gamma_l^{d*}, \gamma_u^{d*}], \mathbf{v}_{k=0}^\xi, \mathbf{v}^{\xi*}, \mathbf{P}^d$

**Outputs:**  $\xi_k$

**for all**  $j \in \{1, \dots, N_{\gamma^d}\}$  **do**

**if**  $k \geq L$

$$\overline{\gamma}_k^{d,j} \leftarrow \frac{1}{L+1} \sum_{l=0}^{l=L} \text{mean}(\hat{\gamma}_{k-l}^{d,j})$$

**else**

$$\overline{\gamma}_k^{d,j} \leftarrow \text{mean}(\hat{\gamma}_k^{d,j})$$

**end if**

**if**  $\overline{\gamma}_k^{d,j} \in [\gamma_l^{d,j*}, \gamma_u^{d,j*}]$  **then**

$$\mathbf{v}_k^{\xi^{d,j}} = \text{RMAD} \left\{ \gamma_k^{d,j,i} \right\}_{i=1}^N$$

$$\mathbf{v}_k^{\xi^{d,j}} = \mathbf{v}_k^{\xi^{d,j}} \left( 1 + \mathbf{P}^{d,j} \frac{\mathbf{v}_k^{\xi^{d,j}} - \mathbf{v}^{\xi^{d,j*}}}{\mathbf{v}^{\xi^{d,j*}}} \right)$$

**else**

$$\mathbf{v}_k^{\xi^{d,j}} = \mathbf{v}_{k=0}^{\xi^{d,j}}$$

**end if**

$$\xi_k^{d,j} \leftarrow \text{Sample} \mathcal{N}(0, \mathbf{v}_k^{\xi^{d,j}})$$

**end for**

Table IV

**Algorithm 4: RUL Prediction**

**Inputs:**  $\left\{ (\theta_k^{d,i}, \gamma_k^{d,i}), w_k^i \right\}_{i=1}^N$

**Variable:**  $l$

**Outputs:**  $\left\{ RUL_k^{\theta^{d,i}}, w_k^i \right\}_{i=1}^N$

**for**  $i=1$  **to**  $N$  **do**

$l=0$

**while**  $\theta_{k+l}^{d,i} \leq \theta_{fail}^d$  **do**

$$\gamma_{k+l}^{d,i} \sim p(\gamma_{k+l}^{d,i} | \gamma_k^{d,i})$$

$$\theta_{k+l}^{d,i} \sim p(\theta_{k+l}^{d,i} | \theta_k^{d,i}, \gamma_k^{d,i})$$

$l \leftarrow l+1$

**end while**

$$RUL_k^{\theta^{d,i}} \leftarrow l$$

**end for**

**Table V**


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Algorithm 5: Health monitoring of  $\theta_0^d$  with respect to  $r_n^d$

---

**while** *system is running* **do**  
 Detect the beginning of degradation using **Algorithm 1**  
**if** *fault detection* =true **then**  
 //set initial conditions  
 $\theta_0^d \sim U(\theta_n^d - \Delta\theta_l, \theta_n^d + \Delta\theta_u)$   
 $\gamma_0^d = 0$   
 $y_0^d = -r_n^d(k)$   
**do** Estimation using **Algorithm 2**  
**do**  $\xi$  Adaptation using **Algorithm3**  
**do** RUL prediction using **Algorithm4**  
**end if**  
**end while**

---

**Table VI List of parametric values**

Parameter	Nominal Value	Multiplicative Uncertainty $[\underline{\delta}_\theta, \overline{\delta}_\theta]$
$\theta$	$\theta_n$	
$J_m$	$6.76 \times 10^{-6} \text{ kg.m}^2 / \text{rad}$	[-0.02,0.02]
$f_m$	$2 \times 10^{-6} \text{ N.m.s/rad}$	[0,0.3]
$J_{Md}$	$9.07 \times 10^{-4} \text{ kg.m}^2 / \text{rad}$	[-0.1,0.1]
$b_{Md}$	$5.025 \times 10^{-3} \text{ N.m.s/rad}$	[0,0.2]
$J_{Ld}$	$1.37 \times 10^{-3} \text{ kg.m}^2 / \text{rad}$	[-0.1,0.1]
$b_{Ld}$	$2.5 \times 10^{-5} \text{ N.m.s/rad}$	[0,0.2]
$k_s$	1.786 N.m/rad	[-0.1,0.1]
$R_s$	$5.11 \times 10^{-4} \text{ N.m/rad}$	[-0.1,0.1]
$k_m$	$3.89 \times 10^{-4} \text{ Nm/A}$	-
$k_{belt}$	3.75	-
$La$	$1.34 \times 10^{-3} \text{ H}$	-
$Ra$	1.23 $\Omega$	-
$\mu$	0.27	[-0.1,0.1]

---

