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► **To cite this version:**

Reinoud J. Bootsma, Simon Ledouit, Remy Casanova, Frank T. J. M. Zaal. Fractional-order information in the visual control of lateral locomotor interception.. *Journal of Experimental Psychology: Human Perception and Performance*, 2016, 42 (4), pp.517 - 529. 10.1037/xhp0000162 . hal-01439633

**HAL Id: hal-01439633**

**<https://hal.science/hal-01439633>**

Submitted on 21 Apr 2022

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# **Fractional-order information in the visual control of lateral locomotor interception**

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Manuscript accepted for publication in

*Journal of Experimental Psychology: Human Perception and Performance*

## Abstract

Previous work on locomotor interception of a target moving in the transverse plane has suggested that interception is achieved by maintaining the target's bearing angle (often inadvertently confused and/or confounded with the target-heading angle) at a constant value. However, dynamics-based model simulations testing the veracity of the underlying control strategy of nulling optical velocity (i.e., the rate of change in bearing angle) have been restricted to limited conditions of target-motion and only few alternatives have been considered. Exploring a wide range of target motion characteristics with straight and curving ball trajectories in a virtual-reality setting, we examined how soccer goalkeepers moved along the goal line to intercept long-range shots on goal, a situation in which interception is naturally constrained to movement along a single dimension. Analyses of the movement patterns suggested reliance on combinations of optical position and velocity for straight trajectories and optical velocity and acceleration for curving trajectories. As an alternative to combining such standard integer-order derivatives, we demonstrate with a simple dynamical model that nulling a single informational variable of a self-tuned fractional (rather than integer) order efficiently captures the timing and patterning of the observed interception behaviors. This new perspective could fundamentally change the conception of what perceptual systems may actually provide, both in humans and other animals.

**Keywords:** Perception-action, interception, movements, information, fractional order, model, dynamics, human

## Introduction

Interceptive actions are paradigmatic examples of our behavioral interaction with dynamic elements of the environment. In the present contribution we address the functional organization of locomotor interception, operationally defined as making contact with a moving target by means of whole-body displacement. More specifically, we address the issue of the visual information used to organize such interceptive locomotor actions. While our study examines the example of soccer goalkeepers moving laterally to intercept balls shot at goal from long range, locomotor interception is of course encountered in many different situations and animal species, notably in the form of chasing prey, mates or invaders.

Studying freely moving agents chasing natural or artificial targets, a large body of locomotor interception work has focused on the correspondence of the behavioral patterns to one of two interception heuristics that we will refer to as classical pursuit and classical interception, respectively. In the case of unconstrained movement (see Fig. 1A), classical pursuit is characterized by moving, at every instant, in the current direction of the target. In the terms of Fajen and Warren (2007), classical pursuit thus entails steering (i.e., changing heading) to align the locomotor velocity vector with the target's bearing direction such that the target-heading angle  $\beta$  (= heading angle  $\phi$  – bearing angle  $\theta$ ) is equal to zero. For classical pursuit to allow the agent to make contact with the target, agent velocity needs to be larger than target velocity (i.e.,  $v_a > v_t$ ). Classical pursuit has been documented in houseflies (Land & Collett, 1974), blowflies (Boeddeker, Kern, & Egelhaaf, 2003), honeybees (Gries & Koeniger, 1996) and tiger beetles (Gilbert, 1997; Haselsteiner, Gilbert, & Wang, 2014). The alternative, classical interception, is characterized by moving in a direction ahead of the target “on a course that will keep the angular deviation of the target from the line of travel constant”

(Lanchester & Mark, 1975, p. 628), that is, by maintaining target-heading angle<sup>1</sup>  $\beta$  at a constant non-zero value. For classical interception to allow the agent to make contact with the target, the agent's transverse velocity needs to match the target's transverse velocity (i.e.,  $v_{t,a} = v_{t,t}$ ) while ensuring that the distance between target and agent decreases (i.e.,  $v_{r,a} > v_{r,t}$ )<sup>2</sup>. Such classical interception has been documented in teleost fish (Lanchester & Mark, 1975), dragonflies (Olberg, Worthington, & Venator, 2000), bats (Ghose, Horiuchi, Krishnaprasad, & Moss, 2006), falcons (Kane & Zamani, 2014), hawks (Kane & Zamani, 2014), dogs (Shaffer, Krauchunas, Eddy, & McBeath, 2004) and humans (Fajen & Warren, 2004).

\*\*\*\*\* Figure 1 about here \*\*\*\*\*

It has often been assumed that finding that over the course of interception the target-heading angle  $\beta$  remains close to zero or close to a particular non-zero value provides evidence for the operation of a control strategy based on, respectively, classical pursuit or classical interception heuristics. It is important to realize, however, that such evidence is but circumstantial. Direct evidence requires specification of how such a result is brought about, that is, specification of how the agent dynamically controls the locomotor velocity vector. If control is based on the state of target-heading angle  $\beta$ , as suggested by the classical pursuit and interception heuristics, then the steering dynamic driving target-heading angle  $\beta$  to zero (thereby instantiating a classical pursuit strategy) is provided by nulling  $\beta$ , and the steering dynamic driving target-heading angle  $\beta$  to a constant value (thereby instantiating a classical interception strategy) by nulling the rate of change of  $\beta$ . Simulations using such dynamic

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<sup>1</sup> Many studies have erroneously referred to the angle subtended at the point of observation by the target's angular position with respect to the agent's direction of locomotion as the target's bearing angle rather than the target-heading angle. However, by definition (Fajen & Warren, 2003; 2007; Klatzky, 1998), bearing is defined with respect to an exocentric (allocentric) frame of reference.

<sup>2</sup> Following Fajen and Warren (2007), we assume that  $v_r$  is positive in the direction extending from the agent to the target, such that  $v_{r,a} > 0$  and  $v_{r,t} < 0$  in Fig. 1A.

steering models led Fajen and Warren (2007) to conclude that a  $d\beta/dt$ -based classical interception strategy was in fact underconstrained: it resulted in the agent leading the target under some conditions but lagging it under other conditions. The lag solution was not consistent with their empirical observations. Fajen and Warren (2007) subsequently demonstrated that, consistent with their empirical observations, a steering dynamic based on nulling the rate of change in bearing angle  $\theta$  reliably led the agent to steer onto a straight interception path leading the target. Of course, on a straight path (i.e., with a fixed heading angle  $\phi$ ) a constant value of bearing angle  $\theta$  is accompanied by a constant value of target-heading angle  $\beta$ . Yet, Fajen and Warren's (2007) modeling work revealed that control (i.e., in their case how to get to the straight path and remain there) relies on nulling  $d\theta/dt$  and not on nulling  $d\beta/dt$  (also see Olberg et al., 2000).

Whereas animal work has typically studied freely moving agents, human locomotor interception of targets moving in the transverse plane has often been studied in direction-constrained paradigms, with participants riding a tricycle along a ground-fixed rail (Lenoir et al., 1999; Lenoir, Musch, Thiery, & Savelsbergh, 2002), walking along a designated line (Chohan, Savelsbergh, van Kampen, Wind, & Verheul, 2006; Chohan, Verheul, Van Kampen, Wind, & Savelsbergh, 2008) or walking on a treadmill (Bastin, Craig, & Montagne, 2006; Bastin, Jacobs, Morice, Craig, & Montagne, 2008; Chardenon, Montagne, Buekers, & Laurent, 2002; Chardenon, Montagne, Laurent, & Bootsma, 2004; 2005; Morice, François, Jacobs, & Montagne, 2010). Compared to the situation where agents can move freely (Fig. 1A), constraining locomotor displacement to a fixed direction (Fig. 1B) has a number of consequences. First, as steering is no longer possible, the agent can only control locomotor speed  $v_a$ . Second, the agent can no longer influence the location where contact with the target may be established; this location is now exclusively determined by the characteristics of the target's trajectory. Finally, for interception to be possible at all in a direction-constrained

setting the target's trajectory must cross the agent's displacement axis. In other words, it must have an orthogonal velocity component  $v_{o,t}$  reducing its distance to the agent's displacement axis. A direction-constrained setting thereby sets up a task-defined exocentric reference direction for locomotor interception that is orthogonal to the locomotor axis. (Fig. 1B).

In the light of the foregoing, it is perhaps surprising that studies of direction-constrained locomotor interception have systematically focused on the target-heading angle  $\beta$  rather than on the bearing angle  $\theta$ . Moreover, all concluded that locomotor interception was (at least to a certain extent) controlled by nulling  $d\beta/dt$  (Bastin et al., 2006; 2008; Chardenon et al., 2002; 2004; 2005; Chohan et al., 2006; 2008; Lenoir et al., 1999; 2002; Morice et al., 2010)<sup>3</sup>. Not only does our analysis of the literature suggest that for a first-order interception strategy agents would rely on nulling  $d\theta/dt$  rather than  $d\beta/dt$ , but focusing on  $\beta$  also eradicates the potential use of a pursuit-like strategy. Indeed, in a direction-constrained paradigm nulling  $\beta$  is not a viable interception strategy: Unrelated to the agent's position (and thus to the establishment of contact), target bearing angle  $\beta$  reaches zero when the target crosses the agent's axis of displacement. Nulling  $\theta$ , on the other hand, does allow instantiating a viable pursuit strategy, because it leads the agent to continuously track the target and thereby intercept it when the target crosses the agent's displacement axis. Finally, while for a stationary agent target-heading angle  $\beta$  is undefined, this is not the case for bearing angle  $\theta$ .

In order to make the link with the literature on (direction-constrained) lateral manual interception studies (Arzamarski, Harrison, Hajnal, & Michaels, 2007; Dessing & Craig, 2010; Dessing, Beek, Caljouw, & Peper, 2004; Dessing, Bullock, Peper, & Beek, 2002; Dessing, Oostwoud Wijdenes, Peper, & Beek, 2009; Dessing, Peper, Bullock, & Beek, 2005;

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<sup>3</sup> When moving over an earth-fixed ground surface target-heading angle  $\beta$  and bearing angle  $\theta$  are of course complementary and nulling the one is therefore equivalent to nulling the other. Chardenon et al. (2004) however demonstrated that lateral translation of the visual ground surface during locomotor interception gave rise to adaptations in locomotor velocity that were consistent with the visually-induced change in  $\theta$  but, under the assumption that heading direction was perceived from optic flow, not with the concomitant visually-induced change in  $\beta$ .

Jacobs & Michaels, 2006; Ledouit, Casanova, Zaal, & Bootsma, 2013; Michaels, Jacobs, & Bongers, 2006; Montagne, Fraise, Ripoll, & Laurent, 2000; Montagne, Laurent, Durey, & Bootsma, 1999; Peper, Bootsma, Mestre, & Bakker, 1994), the present study focused on lateral locomotor interception. As can be seen from Fig. 1C, the target's bearing angle  $\theta$  coincides with its egocentric orientation in this particular situation. Using a large-scale virtual reality set-up participants were immersed in the setting of a soccer goalkeeper that could move laterally along the goal line (displacement axis) so as to intercept balls shot at goal from long range. In order to elucidate the information-based control strategy used in such lateral locomotor interception, we studied participant behavior when intercepting both straight and curving ball trajectories.

With the relevant information for lateral locomotor interception thus defined by the state of the optical angle  $\theta$ , we can distinguish at least three potential control strategies based on the temporal-derivative order of  $\theta$  that is being nulled: a zeroth-order control strategy based on nulling optical position  $\theta$  (i.e., nulling  $d^0\theta/dt^0$ ), a first-order control strategy based on nulling optical velocity  $d\theta/dt$  (i.e., nulling  $d^1\theta/dt^1$ ), and a second-order control strategy based on nulling optical acceleration (i.e., nulling  $d^2\theta/dt^2$ ). Given that the latter strategy, proposed within the framework of catching fly balls<sup>4</sup>, has been questioned on the basis of the human visual system's low sensitivity to acceleration (Brouwer, Brenner, & Smeets, 2002; Calderone & Kaiser, 1989; Werkhoven, Snippe, & Toet, 1992; Zaal, Bongers, Pepping, & Bootsma, 2012), it does not seem reasonable to consider strategies based on even higher orders.

Based on the consensus in the literature (Chardenon et al., 2002; 2004; 2005; Chohan et al., 2006; 2008; Fajen & Warren, 2007; Lenoir et al., 1999; 2002), lateral locomotor

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<sup>4</sup> Rotating the reference frame of Fig. 1B by 90° around the agent's displacement axis allows examining the situation of target motion in the sagittal plane (Chapman, 1968; McLeod & Dienes, 1996; McLeod, Reed, & Dienes, 2003; Michaels & Oudejans, 1992; Todd, 1981; Zaal & Michaels, 2003). Because gravity will eventually lead inanimate targets initially moving upward to fall back onto the agent's locomotor surface, the task still defines an appropriate exocentric reference direction (the gravity-defined vertical).



interception of balls following straight trajectories would be expected to rely on a first-order  $d\theta/dt$ -nulling strategy. However, studies on lateral manual interception have demonstrated that balls following straight trajectories starting from the same distance from the interception axis and arriving at the same lateral position after the same flight duration give rise to ball-trajectory-dependent interception patterns (Arzamarski et al., 2007; Ledouit et al., 2013; Montagne et al., 1999). If also present in locomotor interception, this angle-of-approach effect (Ledouit et al., 2013) would militate against a  $d\theta/dt$ -nulling strategy.

The present study tested the adequacy of the first-order  $d\theta/dt$ -nulling interception strategy to capture the behavioral patterns observed not only for straight but also for curving ball trajectories. The phenomena encountered when ball trajectories curve (e.g., under the influence of sidespin) indeed provide some qualitative evidence in favor of a first-order strategy. For instance, even top-level soccer players make errors in judging whether balls following laterally curving trajectories will end up in the goalmouth (Craig, Berton, Rao, Fernandez, & Bootsma, 2006; Craig et al., 2009). Moreover, in direction-constrained paradigms, locomotor interception of curving ball trajectories has been demonstrated to give rise to ball-trajectory-specific changes in locomotor velocity (Bastin et al., 2006; 2008; Morice et al., 2010), sometimes even leading to reversal of movement direction (Casanova, Borg, & Bootsma, 2015; Lenoir, Vansteenkiste, Vermeulen, & De Clercq, 2005). Similar results have been reported for manual lateral interception (Craig, Bastin, & Montagne, 2011; Dessing & Craig, 2010). However, the generally-drawn conclusion that participants relied on a first-order strategy for the interception of balls following curved trajectories is, for the moment, predominantly supported by the capability of such an interception strategy to capture certain qualitative aspects of the behavioral patterns observed. Modeling of a first-order strategy, on the other hand, yielded rather modest levels of fit, even when parameters were allowed to vary over experimental conditions (Bastin et al., 2006; 2008; Morice et al., 2010).

Going beyond the qualitative character of earlier studies focusing on the general form of the interception patterns observed, in the present contribution we also examined the potential of informational variables of different orders (i.e.,  $\theta$ ,  $d\theta/dt$  and  $d^2\theta/dt^2$ ) to structure the movement characteristics at key moments of the interceptive action. As these analyses demonstrated that none of the three interception strategies considered (nulling  $d^0\theta/dt^0$ ;  $d^1\theta/dt^1$  or  $d^2\theta/dt^2$ ) could capture the behavioral patterns observed, we propose a new perspective for the control of interceptive actions based on nulling information of a fractional rather than integer temporal-derivative order (Beek, Peper, Daffertshofer, van Soest, & Meijer, 1998; Jacobs, Vaz, & Michaels, 2012; Podlubny, 1999; 2002).

## **Method**

### *Participants*

Twenty-one experienced male soccer players (age  $25.8 \pm 7.7$  years) voluntarily participated. All participants had at least four years of experience in competitive weekly matches.

### *Task and Procedure*

Using a model incorporating aerodynamic Magnus-Robins lift and drag forces fully described earlier (Craig et al., 2006), we calculated a set of trajectories departing 30 m from the goal line at lateral distances of -4 m and +4 m with respect to the center of the goal and arriving in the goal at lateral distances of -1.8, -0.6, +0.6, and +1.8 m, at a height of around 1.2 m. For each combination of departure and arrival positions one straight (No Spin, NS) and two curving ( $10 \text{ rot}\cdot\text{s}^{-1}$  around the ball's vertical axis Clockwise Spin, CS, and Counter-Clockwise spin, CCS) trajectories were derived, for a total of 24 trajectories (see Fig. 2A). To allow sufficient player displacement, we slowed down the ball trajectories to flight durations of  $2.66 \pm 0.07$  s, doubling the original ball flight durations using linear interpolation between

successive positions. A block of trials included all 24 trajectories presented in random order. Each participant performed five blocks of trials.

The experiment took place in a large virtual-reality facility ([crvm.ism.univ-amu.fr](http://crvm.ism.univ-amu.fr)) comprising four projection surfaces, each served by two projectors: a 3 x 3 m floor and three 4-m high x 3-m wide walls. The sidewalls were opened at 45° angles with respect to the front wall (theatre configuration) allowing participants to move laterally over a distance of 7.2 m. Stereopsis was ensured with passive Infitec® filter technology. Participants' stereo glasses were equipped with a configuration of reflective markers. An eight-camera ART® optical system enabled real-time motion capture of head position. The visual scene was refreshed at 60 Hz, taking into account the position of the participant's eyes relative to the virtual environment.

Using in-house developed software (ICE) we simulated a soccer stadium with a grass pitch. Positioned within a regulation-sized goal (7.32 m wide and 2.44 m high), participants could see the pitch with the goal line and other regulation white chalk lines. At the start of each trial an arrow on the front screen guided the participant to the initial position on the goal line, halfway between the goalposts. After the participant had remained at the initial position for 1 s, the 0.22-m diameter plain white ball (Casanova et al., 2015) started its flight trajectory. Participants were instructed to move along the goal line to try to intercept the ball with the torso (mid-line of the body), with the two arms jointly cupping the ball. While this did not prevent participants from using their hands every now and then when they realized that they were somewhat too late, a series of 20 randomly selected practice trials allowed them to correctly understand the whole-body movement task.

### *Data Analysis*

The position of the participant's head (obtained by tracking the markers placed on the participant's stereo glasses and sampled at 100 Hz) was filtered using a dual-pass second-order Butterworth filter with a cut-off frequency of 3 Hz. Movement initiation was determined by identifying the first 0.11-m (ball radius) change in position and then scanning backwards in time for the moment participant velocity exceeded a threshold of  $0.05 \text{ m}\cdot\text{s}^{-1}$ . All data series were aligned with the moment of arrival of the ball in the goal. For each ball trajectory, ensemble averages of participant movement were obtained by averaging for each point in time all trials of the 21 participants (Fig. 2B).

The target's bearing angle  $\theta$  was defined as the azimuthal eccentricity (Michaels et al., 2006) of the ball with respect to the participant's straight-ahead (perpendicular to the goal line).

## Results

Ball trajectory (Fig. 2A) systematically influenced players' interception movements (Fig. 2B), for both straight and curving ball trajectories.

\*\*\*\*\* Figure 2 about here \*\*\*\*\*

### *Straight ball trajectories*

As can be seen from Fig. 2D, straight ball trajectories converging onto the same interception location (Fig. 2C) revealed influences of ball (departure) position on interception patterns, thereby confirming the presence of the angle-of-approach effect (Ledouit et al., 2013) in locomotor interception. This observation was corroborated by an ANOVA on player position at 1.0 s before ball arrival that revealed not only the expected significant main effect of Ball

Arrival Position ( $F(3, 60) = 256.1, p < .001, \eta^2_p = .93$ ), but also a significant effect of Ball Departure Position ( $F(1, 20) = 132.9, p < .001, \eta^2_p = .87$ ) as well as a significant interaction between the two ( $F(3, 60) = 3.2, p < .05, \eta^2_p = .14$ ). Post-hoc Newman-Keuls analysis of this interaction demonstrated a significant influence of Ball Departure Position on player position at each of the four Ball Arrival Positions ( $p$ 's  $< .001$ ), even though the effect was somewhat smaller for balls arriving at the inner ( $\pm 0.6$  m) than at the outer ( $\pm 1.8$  m) arrival positions. The angle-of-approach effect observed on player position was accompanied by variations in the moment of movement initiation over straight ball trajectories arriving at the same lateral distance (Fig. 2C): an ANOVA on movement initiation time demonstrated a significant main effect of Ball Arrival Position ( $F(3, 60) = 24.8, p < .001, \eta^2_p = .55$ ) as well as a significant interaction between the factors of Ball Arrival Position and Ball Departure Position ( $F(3, 60) = 9.4, p < .001; \eta^2_p = .32$ ). Post-hoc Newman-Keuls analysis revealed that movement was initiated later ( $p$ 's  $< .05$ ) for balls arriving at the inner ( $\pm 0.6$  m) than at the outer ( $\pm 1.8$  m) arrival positions. While for each ball arrival position balls departing from the opposite side of the player's initial position tended to give rise to longer movement initiation times, this effect reached significance only for the ball arrival position  $+0.6$  m ( $p < .001$ ). Because at least up to movement initiation the time course of optical velocity  $d\theta/dt$  was nearly identical for balls converging along straight trajectories onto the same arrival position, the observation of systematic trajectory-specific differences in movement characteristics at each ball arrival position demonstrated that participants did not fully rely on optical velocity  $d\theta/dt$ ; that is, optical position  $\theta$  also appeared to be involved.

### *Curving ball trajectories*

Inspection of Fig. 2B revealed that 12 of the 16 curving trajectories led to reversals of movement direction during the players' interceptive actions. For four of these 12 trajectories

this change in direction occurred toward the end of the movement, following a slight overshoot. The eight remaining trajectories gave rise to players initially moving away from the future ball arrival position before reversing movement direction. Focusing on the characteristics of these latter reversal movements brought out the following points (Fig. 3 and 4).

\*\*\*\*\* Figures 3 and 4 about here \*\*\*\*\*

First, while present in the ensemble averages of the full data set presented in Fig. 2B, reversal movements (RM) were not always observed at the level of the individual trials. Separating RM trials and NoRM trials<sup>5</sup> for each ball trajectory revealed bimodal distributions of initiation times, indicating that RM trials were initiated considerably earlier than NoRM trials (see histogram insets in Figs. 3 and 4). This pattern of results was not due to between-participants differences: the presence of both RM and NoRM trials was observed within the individual participants. The number of trials with reversals varied over trajectories, for averages of 56.4% and 76.7% for balls arriving at the outer ( $\pm 1.8$  m) and inner ( $\pm 0.6$  m) arrival positions, respectively (see pie-chart insets in Figs. 3 and 4).

Second, in NoRM trials the movement appeared to be driven by optical acceleration  $d^2\theta/dt^2$  rather than by optical velocity  $d\theta/dt$ : at the time of movement initiation the latter specified movement in the opposite direction (see indicator bars in Figs. 3 and 4). This reliance on optical acceleration  $d^2\theta/dt^2$  was observed in 95.4% of the NoRM trials. RM trials, on the other hand, appeared to be initially driven by optical velocity  $d\theta/dt$  (consistently specifying movement away from the future arrival position at the time of movement initiation for the full set of RM trajectories). However, the subsequent reversing of movement direction

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<sup>5</sup> Identification of RM and NoRM trials was based on the criterion for movement initiation described in the methods section: trials were qualified as RM when an initial movement of at least 0.11 m was made in a direction away from the future ball arrival position.

again appeared to be driven by optical acceleration  $d^2\theta/dt^2$  as even at the time of peak excursion optical velocity  $d\theta/dt$  was still specifying movement in the same direction, away from the future ball arrival position (see indicator bars in Figs. 3 and 4). This characteristic was observed in 79.2% of the RM trials.

## Discussion

Overall, the above analyses demonstrated that the control of lateral interception seemed to require concurrent reliance on several informational variables.

For straight ball trajectories the behavioral effects observed (Fig. 2C and 2D) implicated both optical position  $\theta$  and optical velocity  $d\theta/dt$ . Interestingly, similar results were reported by Fajen and Warren (2004, 2007; also see Land & Collet, 1974). Fajen and Warren (2004, 2007) noted that in their experimental setting targets following straight trajectories that led them to cross the participants' initial movement direction gave rise to S-shaped bends in the interception paths. In order to capture such (target-position driven) effects within their steering model of locomotor interception, they incorporated a 0.5-s sigmoidal latency function on  $d\theta/dt$  suggested to correspond to "a visual delay to detect that the target is moving and a locomotor delay to overcome the inertia of the body" (Fajen & Warren, 2007, p. 311). When balls moved along straight trajectories in the present study, participants initiated their movements after 0.97 s of ball flight on average, for a range of 0.82 to 1.21 s. Thus, without considerably lengthening of its rise time (well beyond 1 s), the latency function proposed by Fajen and Warren (2007) cannot explain the angle-of-approach effects observed here. The question why a combination of optical position  $\theta$  and optical velocity  $d\theta/dt$  information would be used over an extended period of time, rather than the presumably available  $d\theta/dt$  information *per se*, therefore remains unanswered.

For curving ball trajectories the behavioral effects observed (Figs. 3 and 4) implicated both optical velocity  $d\theta/dt$  and optical acceleration  $d^2\theta/dt^2$ . While these observations could be captured by a model of control based on (perhaps time-evolving) combinations of  $d\theta/dt$  and  $d^2\theta/dt^2$  information, such a combinatorial approach appears to lack a principled foundation. It also begs the question why interception of curving trajectories would not simply rely on  $d^2\theta/dt^2$  information: if indeed accessible, this would allow avoiding the advent of reversal movements altogether. Given the consensus in the literature that optical acceleration cannot be detected accurately (Brouwer et al., 2002; Calderone & Kaiser, 1989; Werkhoven et al., 1992; Zaal et al., 2012), it is not clear how  $d^2\theta/dt^2$  could be used at all, either alone or in combination with other informational variables.

We suggest an alternative account. Rather than limiting potential informational variables to integer-valued time derivatives of  $\theta$ , we propose that information may cover the full continuum of states offered by fractional-order time derivatives (Beek et al., 1998; Jacobs et al., 2012; Podlubny, 1999; 2002). In this perspective, interception is controlled by nulling an informational variable  $d^\alpha\theta/dt^\alpha$  of a trajectory-specific fractional order  $\alpha$ , where  $\alpha \in \mathbb{R}^+$ .

### Model

We developed the following model as a proof of concept for movement control based on trajectory-specific fractional-order information. We started from the attractor dynamics:

$$d^2X/dt^2 = G \cdot t_c^\alpha \cdot d^\alpha\theta_{\Delta t}/dt^\alpha \quad (1)$$

in which  $d^2X/dt^2$  is player acceleration along the interception axis,  $G$  is a gain coefficient, and  $t_c$  is a characteristic time constant raised to the power  $\alpha$ ;  $d^\alpha\theta_{\Delta t}/dt^\alpha$  is the  $\alpha^{\text{th}}$ -order time derivative of  $\theta$  at a visuomotor delay  $\Delta t$  earlier. The gain coefficient  $G$  has the same



dimension as  $d^2X/dt^2$ . Because  $\alpha$  determines the dimension of  $d^\alpha\theta/dt^\alpha$ , including  $t_c^\alpha$  on the right-hand side of the equation balances dimensions.

For each trajectory the fractional order  $\alpha$  is determined at each point in time by having  $\alpha$  start at 0 and increase linearly (slope  $s_\alpha$ ) over time until the magnitude of  $d^\alpha\theta/dt^\alpha$  becomes smaller than a criterion value:

$$\left|d^\alpha\theta/dt^\alpha\right| < \frac{\varphi}{t_c^\alpha} \quad (2)$$

in which  $\varphi$  is the optical size of the ball, increasing during approach. Note that prior to movement initiation the instantaneous value of  $d^\alpha\theta/dt^\alpha$  is determined by the way the ball trajectory unfolds with respect to the stationary player. After movement initiation the characteristics of ball and player motion co-determine the instantaneous value of  $d^\alpha\theta/dt^\alpha$ .

In parallel to the order progression that leads to the fractional order that the system settles on for the specific ball trajectory, after a dead period  $t_m$ , the moment of movement initiation is determined when the magnitude of required player acceleration exceeds a threshold:

$$\left|d^2X/dt^2\right| > \frac{k_{ini}}{t - t_m} \quad (3)$$

in which  $k_{ini}$  is a gain coefficient,  $t$  is time and  $t_m$  is the duration of an initial dead period during which movement cannot be initiated.

In order to implement the model, fractional derivatives of  $\theta$  needed to be calculated. For reasons of simplicity we used an algorithm based on Euler's generalization (Dalir & Bashour, 2010):

$$\frac{d^n x^m}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n} \quad (4)$$

in which  $\Gamma$  is the gamma function. Fractional derivatives of time series of  $\theta$  can thus directly be computed using polynomials<sup>6</sup>. To this end, for each point in time, a fourth-order polynomial was fitted through the values of  $\theta$  at the preceding time steps. The coefficients  $c_m$  of the obtained polynomial were used to compute the fractional derivative according to:

$$\frac{d^\alpha}{dt^\alpha} \theta(t) = \sum_{m=0}^4 c_m \left( \frac{\Gamma(m+1)}{\Gamma(m-\alpha+1)} t^{m-\alpha} \right) \quad (5)$$

in which  $\alpha$  is the fractional order of the time derivative of  $\theta$ ,  $t$  is time, and  $m$  is the order of each term in the polynomial. Until sufficient values were available after onset of ball motion, the polynomial was fit through the first 34 time steps (during which the player never moved) for the computation of the fractional derivative at each time step.

\*\*\*\*\* Figure 5 about here \*\*\*\*\*

The model (i.e., Equations 1 to 3) was fitted to the ensemble averages of the interception patterns for all 24 ball trajectories, including only the RM trials for trajectories with RM and NoRM behaviors. Two parameters were set at fixed values ( $\Delta t = 0.1$  s and  $t_m = 0.23$  s) while the remaining four parameters were optimized ( $s_\alpha = 5.4$  s<sup>-1</sup>,  $G = 160$  m·s<sup>-2</sup>,  $t_c = 0.23$  s and  $k_{ini} = 0.255$  m·s<sup>-1</sup>) using sequential quadratic programming. Subsequent model simulations using this unique set of parameter values provided adequate overall results (Fig. 5). Simulations satisfactorily reproduced the interception patterns observed for all trajectories ( $R^2 = 0.976 \pm 0.038$  and RMSE =  $0.054 \pm 0.021$  m). Additionally, timing was accurately captured. For each ball trajectory, predicted initiation time closely matched observed initiation time,  $R^2 = 0.950$  and mean error =  $-0.014 \pm 0.046$  s. For the trajectories giving rise

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<sup>6</sup> In order to ascertain that our results did not depend on the specific algorithm used for calculating the fractional-order derivatives of  $\theta$ , we also ran the model with another, computationally more intensive matlab routine (fgl\_deriv.m; MathWorks reference 45982-fractional-derivative) based on a vectorized Grunwald-Letnikov definition (Podlubny, 1999). The algorithm used did not qualitatively affect the results obtained.

to movement reversals, the moment of peak excursion was accurately predicted,  $R^2 = 0.973$  and mean error =  $0.031 \pm 0.029$  s. When the model was initiated at the mean movement initiation times of the NoRM trials, the simulated trajectories showed no reversal movements.

\*\*\*\*\* Table 1 about here \*\*\*\*\*

In the model, order  $\alpha$  generally increased monotonically before settling onto a trajectory-specific value that was then maintained throughout the interceptive action (see Table 1 for final values of  $\alpha$ ). However, due to the spatiotemporal characteristics of certain ball trajectories, the temporal evolution of  $\alpha$  revealed a short-duration, transitory plateau for the four CS ball trajectories departing from  $-4$  m and for their mirror counterparts, the four CCS trajectories departing from  $+4$  m. This plateauing of  $\alpha$  at values between 0.7 and 0.8 observed for these particular (curving) ball trajectories resulted from small absolute  $d^\alpha\theta/dt^\alpha$  values occurring in the early stage of ball flight, when the ball was on one side of the player while its velocity was driving it to the other side. As over these trajectories the ball quickly approached and subsequently crossed the (still stationary) player's initial position, the magnitude of  $d^\alpha\theta/dt^\alpha$  rapidly began to grow again. Driven by Equation (2),  $\alpha$  therefore started to increase again and continued to do so until it settled onto a trajectory-specific value that was then maintained throughout the interceptive action.

The observed angle-of-approach effect for balls following straight trajectories to the same arrival position while coming from different departure positions (Fig. 5C and D) resulted from the presence of trajectories with  $\alpha < 1$ . The apparent initial dependence on optical velocity  $d\theta/dt$  and subsequent switch to optical acceleration  $d^2\theta/dt^2$  underlying the observed RM behavior for curving trajectories in fact resulted from continuous reliance on information with  $1 < \alpha < 2$ .

## General Discussion

The purpose of the model developed in the present contribution was to provide proof of concept for our hypothesis of control of interception based on fractional-order information nulling. Equation (1) captures the information-based attractor dynamics, defining how player acceleration ( $d^2X/dt^2$ ) depends on information ( $d^\alpha\theta/dt^\alpha$ ), in its simplest form: beyond a general gain  $G$  it only includes a time constant  $t_c$  raised to the power  $\alpha$  that is required to maintain dimensional homogeneity. Equation (1) is the part of the model that can be compared to existing behavioral dynamics models of interception, in both their direction-constrained (Chardenon et al., 2005, Eq. 1, p. 53; Bastin et al., 2006, Eq. 1, p. 720; Bastin et al., 2008, p. 302; Morice et al., 2010, Eq. 1, p. 398) and speed-constrained versions (Fajen & Warren, 2007, Eq. 5, p. 310 combined with the latency function described in Eq. 6, p. 311). While most models (Bastin et al., 2006; 2008; Fajen & Warren, 2007; Morice et al., 2010) incorporate a damping term in the attractor dynamics, ours (currently) does not, for a simple reason: Fitting systematically yielded a damping coefficient of negligible size. Future work will have to demonstrate whether the inconsequential role of damping observed in the present context is related to task constraints or to other factors. As mentioned earlier, in order to capture the observed target-position-dependent effects on interception patterns, Fajen and Warren (2007) included a latency function on  $d\theta/dt$  in their steering model. Our model based on fractional-order information does not require such an extension<sup>7</sup>.

What makes our model conceptually different from existing models is not only that we open up the potential information space from isolated (category-like) integer time derivatives

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<sup>7</sup> Finally, Fajen and Warren's (2007) model also incorporates a distance term (weighting the influence of a distant target more heavily) to avoid sluggish turns when the target is far away. Note that they demonstrate that inclusion of the distance term is not required for the range of target distances and speeds explored in their interception experiments (see Fajen & Warren, 2004).

to the continuum of fractional-order time derivatives (Jacobs et al., 2012). Taking the idea of a continuum seriously<sup>8</sup>, we also allow different situations to give rise to reliance on information of a different fractional order. Thus, in our model we suggest that on each occasion the system tunes in to (i.e., self-selects) the appropriate order of the information to be nulled. This self-tuning to an appropriate fractional order of the information is captured in our model by Equation (2), stating that the order  $\alpha$  of information increases when  $d^\alpha\theta/dt^\alpha$  exceeds a threshold, defined by the ratio of current optical target size  $\varphi$  and  $t_c^\alpha$ . Note that substitution of Equation (2) in Equation (1) reveals that the threshold corresponds to a (required) player acceleration of  $G$  times  $\varphi$ . Thus, when the acceleration required to intercept the target is too large, the order  $\alpha$  of the information is increased. Because very short stimulus exposure times only allow detection of position, in building our model we assumed that  $\alpha$  starts from zero. The choice to increase  $\alpha$  following a linear function, characterized by slope  $s_\alpha$ , is at present also but a simplicity assumption. For the set of conditions explored in the present study, after movement onset the value of  $\alpha$  nulled by model did not change over the course of movement. However, a change in the characteristics of ball motion (such as a sudden change in ball speed and/or direction) may be expected to lead to a change in  $\alpha$ . The present formulation of the  $\alpha$ -tuning mechanism (Equation (2)) allows for an increase but not for a decrease in  $\alpha$  during the interceptive action. Future work will have to determine whether  $\alpha$  may indeed change when ball trajectory characteristics change and, if so, whether  $\alpha$  then may decrease as well as increase. Experimental evidence in favor of a decrease in  $\alpha$  over the course of an interceptive action would, of course, require a reformulation of the  $\alpha$ -tuning mechanism.

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<sup>8</sup> When limited to isolated integer-order derivatives, it may seem parsimonious to choose, for a given set of conditions, between categorically different interception strategies such as classical pursuit (nulling zeroth-order information) and classical interception (nulling first-order information). Allowing for fractional-order derivatives eliminates these categorical differences and, thereby, the need of having to choose between them.

With the information-based attractor dynamics being defined by Equations (1) and (2) at each moment in time for each ball trajectory, we were able to determine an effective criterion for movement initiation captured by Equation (3), the last element of our model: the player begins to move when the magnitude of required acceleration exceeds a threshold value. Incorporating a dead period during which movement simply cannot be initiated, the threshold value decreases over time so that movement is eventually initiated even for a ball trajectory requiring only a small amount of movement for interception. Note that such an action-relevant criterion (Bootsma, Fayt, Zaal, & Laurent, 1997) allows the whole action sequence (from not moving, via starting to move at a particular moment, to moving in a particular way) to be understood within the same logic. It is indeed this logic that allows our model to reproduce the observed behavioral patterns (Fig. 5). Surprisingly, most existing studies (and *a fortiori* models) of locomotor interception remain silent on the timing of movement initiation (see Collett & Land, 1978, for an exception); at best they incorporate a sigmoidal time-dependent activation function (Bastin et al., 2006; 2008; Morice et al., 2010). We suggest that this lack of interest with respect to the timing characteristics of locomotor interception is related to the specifics of the experimental paradigms used in which analysis begins when agents are already moving (e.g., Bastin et al., 2006; 2008; Boeddeker et al., 2003; Chardenon et al., 2002; 2004; 2005; Fajen & Warren, 2004; Haselsteiner et al., 2014; Kane & Zamani, 2014; Kane, Fulton, & Rosenthal, 2015; Lenoir et al., 1999; 2002; Morice et al., 2010; Olberg et al., 2000; Tucker, Tucker, Akers, & Enderson, 2000). Moreover, for walking participants the data are generally binned over considerable time intervals in order to remove the cyclical variations of the step cycle; Fajen and Warren (2004, 2007) in fact time-normalized their data. Thus, by including a criterion for movement initiation such as the one provided by Equation (3), we hope that our model may incite future work on locomotor interception to consider not only the patterning but also the timing of behavior.

We conclude that control based on fractional-order information nulling provides a unifying framework, explaining the ball-trajectory effects observed here and in the literature for straight (Chardenon et al., 2004; 2005; Fajen & Warren, 2004) and curving trajectories (Bastin et al., 2006; 2008; Casanova et al., 2015; Lenoir et al., 2005; Morice et al., 2010). Breaking away from the traditional combinatorial approach (e.g., Beek, Dessing, Peper, & Bullock, 2003; also see the sequence of model developments in Dessing et al., 2002; 2004; 2005; 2009), fractional derivatives fill the space between classical integer-order derivatives, thereby providing a powerful tool to identifying intermediate informational states (Jacobs et al., 2012). The neurophysiological plausibility of the fractional-order dynamics was first demonstrated by Anastasio (1994; 1998; Anastasio & Correia, 1994) in his work on the vestibulo-ocular reflex. More recently, the firing rate of neocortical pyramidal neurons has been reported to encode slowly varying stimulus statistics through fractional-order differentiation (Lundstrom, Higgs, Spain, & Fairhall, 2008). We therefore conclude that reliance on fractional-order information as proposed in the present contribution is, at least, neurophysiologically plausible.

Fractional-order information may also be used in the locomotor interception of balls following curving trajectories along the sagittal plane, for which a heuristic of keeping constant the rate of change of the tangent of optical elevation (often referred to as optical velocity) has been proposed (Chapman, 1968; Fink, Foo, & Warren, 2009; McLeod, Reed, & Dienes, 2006; Michaels & Oudejans, 1992). However, the plausibility of a control strategy implementing this interception heuristic in the form of nulling vertical optical acceleration (McLeod et al., 2006; Michaels & Oudejans, 1992) has been questioned on the basis of the human visual system's low sensitivity to optical acceleration (Brouwer et al., 2002; Calderone & Kaiser, 1989; Werkhoven et al., 1992; Zaal et al., 2012). Human observers' ability to detect changes in optical velocity has led to the formulation of alternative perceptual candidates,

generally based on changes in velocity scaled to average velocity (Babler & Dannemiller, 1993; Brouwer et al., 2002; Calderone & Kaiser, 1989; Schmerler, 1976). To date, however, the variable that the visual system relies on to detect changes in optical velocity has not been identified (Zaal et al., 2012). The current framework intimates that reliance on information of a fractional order of about 1.8, as identified for the laterally curving trajectories of the present study, might characterize the informational variable underlying detection of change in velocity.

The proposed account suggests that combinations of ball trajectory characteristics and timing constraints may lead to fractional orders of the informational variable filling a continuum of possibilities. Generalizing, this implies that conceptual models of motion perception (Van Doorn & Koenderink, 1982) could benefit from considering sensitivity to fractional (rather than exclusively integer) order derivatives. The account may even provide a simple solution to the 40-year old enigma that constant velocity is not perceived as such (Runeson, 1974).



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## **Acknowledgments**

Exchanges between the authors were facilitated by a French-Dutch collaborative travel grant (PHC Van Gogh 29353PF) awarded to Reinoud Bootsma and Frank Zaal. We thank Joanne Smith and Laure Fernandez for their help in data collection and in initial analyses. Cedric Goulon provided essential support in implementing the virtual reality environment and experimental conditions. Viktor Jirsa's constructive comments are gratefully acknowledged. Reinoud Bootsma is a senior member of the Institut Universitaire de France.

## Figure Captions

**Fig. 1: Plan view of an agent moving through an environment containing a target moving in the transverse plane.** (A) Definition of variables for a freely moving agent. Agent heading  $\phi$  and target bearing  $\theta$  are defined with respect to an exocentric reference direction (dashed-blue line). Target-heading angle  $\beta$  is defined by the eccentricity of the target with respect to the agent's direction of locomotion. The radial components of agent and target velocities (i.e.,  $v_{r,a}$  and  $v_{r,t}$ ) determine the evolution of agent-target distance. (B) Situation for a direction-constrained agent moving forward along a fixed displacement axis. For interception to be possible, the target must have an orthogonal velocity component  $v_{o,t}$  reducing its distance to the interception axis. The task thus defines an exocentric reference direction perpendicular to the agent's displacement axis. (C) Situation for a direction-constrained agent moving laterally along a fixed displacement axis. In this situation the exocentric reference direction defined by the task, again perpendicular to the agent's displacement axis, coincides with the agent's egocentric reference direction. In each of the three situations  $\beta = \phi - \theta$ .

**Fig. 2: Ball trajectories and corresponding interception movements.** (A) Bird's-eye view of NS (red), CS (blue), and CCS (green) ball trajectories departing from -4 m (left side of panel) and +4 m (right side of panel) lateral positions, all converging onto the same four arrival positions on the interception axis. Open circles indicate average movement initiation times. (B) Ensemble averages of player position along the interception axis as a function of time until ball arrival for all trajectories. Color code as in (A). Note the mirror symmetry in the interception patterns of the two sides of the panel. (C) Pooling NS ball trajectories starting from -4 m (red) and +4 m (orange) reveals that (D) during the interception movement player

position was systematically shifted towards the ball departure position. Same color code as in (C).

**Fig. 3: Details of interception movements for the four curving trajectories with ball departure at -4 m giving rise to reversal movements.** (A) CCS ball trajectory arriving at +0.6 m. (B) CS ball trajectory arriving at -0.6 m. (C) CCS ball trajectory arriving at +1.8 m. (D) CS ball trajectory arriving at -1.8 m. Each panel shows the ensemble averages of player position (solid lines) and velocity (dashed lines) as a function of time until ball arrival for RM (red) and NoRM (blue) trials, accompanied on the same time line by indicator bars of the instantaneous direction of attraction of  $\theta$ ,  $d\theta/dt$  and  $d^2\theta/dt^2$ . Horizontal lines indicate the moments of movement initiation for RM trials (red) and NoRM trials (bleu) as well as the moment of RM peak excursion (red). Pie-chart insets indicate percentages of RM (red) and NoRM (blue) trials, requiring (full color) or not requiring (faded color) attraction to  $d^2\theta/dt^2$  in order to explain timing of RM. Histogram insets present frequency distributions of movement initiation times of RM (red) and NoRM (blue) trials.

**Fig. 4: Details of interception movements for the four curving trajectories with ball departure at +4 m giving rise to reversal movements.** (A) CS ball trajectory arriving at -0.6 m. (B) CCS ball trajectory arriving at +0.6 m. (C) CS ball trajectory arriving at -1.8 m. (D) CCS ball trajectory arriving at +1.8 m. Each panel shows the ensemble averages of player position (solid lines) and velocity (dashed lines) as a function of time until ball arrival for RM (red) and NoRM (blue) trials, accompanied on the same time line by indicator bars of the instantaneous direction of attraction of  $\theta$ ,  $d\theta/dt$  and  $d^2\theta/dt^2$ . Horizontal lines indicate the moments of movement initiation for RM trials (red) and NoRM trials (bleu) as well as the moment of RM peak excursion (red). Pie-chart insets indicate percentages of RM (red) and

NoRM (blue) trials, requiring (full color) or not requiring (faded color) attraction to  $d^2\theta/dt^2$  in order to explain timing of RM. Histogram insets present frequency distributions of movement initiation times of RM (red) and NoRM (blue) trials.

**Fig. 5: Ball trajectories and corresponding observed and simulated interception movements, including only the RM trials for trajectories giving rise to RM and NoRM behaviors.** (A) Bird's-eye view of NS (red and orange), CS (blue), and CCS (green) ball trajectories departing from -4 m (left side of panel) and +4 m (right side of panel) lateral positions, all converging onto the same four arrival positions on the interception axis. Colored circles indicate average movement initiation times; black circles indicate model-predicted movement initiation times. (B) Ensemble averages (fat lines) and model-generated simulations (thin lines) of player position along the interception axis as a function of time until ball arrival for all trajectories. Color code as in (A). (C) Pooling straight ball trajectories starting from -4 m (red) and +4 m (orange) reveals that (D) the model adequately captured the observed angle-of-approach effect. Same color code as in (C).

**Table 1:** Values of the fractional order  $\alpha$  being nulled for the 24 ball trajectories resulting from the model simulations.

	<b>BDP -4 m</b>				<b>BDP +4 m</b>			
	<b>BAP (m)</b>				<b>BAP (m)</b>			
	<b>-1.8</b>	<b>-0.6</b>	<b>+0.6</b>	<b>+1.8</b>	<b>-1.8</b>	<b>-0.6</b>	<b>+0.6</b>	<b>+1.8</b>
NS	1.03	0.97	0.97	0.92	0.92	0.97	0.97	1.03
CS	1.78	1.78	1.73	1.67	1.84	1.78	1.78	1.73
CCS	1.73	1.78	1.78	1.84	1.67	1.73	1.78	1.78

BDP: Ball Departure Position. BAP: Ball Arrival Position. NS: No spin. CS: Clockwise spin.

CCS: Counter-clockwise spin. Note the mirror symmetry in the  $\alpha$  values corresponding to the mirror symmetry in the trajectories departing from -4 m and +4 m.











