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Joint Inpainting of Depth and Reflectance with Visibility Estimation[☆]

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Abstract

This paper presents a novel strategy to generate, from 3-D lidar measures, dense depth and reflectance images coherent with given color images. It also estimates for each pixel of the input images a visibility attribute. 3-D lidar measures carry multiple information, e.g. relative distances to the sensor (from which we can compute depths) and reflectances. When projecting a lidar point cloud onto a reference image plane, we generally obtain sparse images, due to undersampling. Moreover, lidar and image sensor positions typically differ during acquisition; therefore points belonging to objects that are hidden from the image view point might appear in the lidar images. The proposed algorithm estimates the complete depth and reflectance images, while concurrently excluding those hidden points. It consists in solving a joint (depth and reflectance) variational image inpainting problem, with an extra variable to concurrently estimate handling the selection of visible points. As regularizers, two coupled total variation terms are included to match, two by two, the depth, reflectance, and color image gradients. We compare our algorithm with other image-guided depth upsampling methods, and show that, when dealing with real data, it produces better inpainted images, by solving the visibility issue.

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Keywords: Inpainting, Total Variation, Depth Maps, Lidar, Reflectance, Point Cloud, Visibility

1. Introduction

Image-based 3D reconstruction of static and dynamic scenes (Herbort and Wöhler, 2011; Seitz et al., 2006; Stoykova et al., 2007) is one of the main challenges in computer vision nowadays. In the recent years many efforts have been made to elaborate configurations and approaches, possibly requiring the employment of multiple sensors, with the final goal of generating plausible and detailed 3D models of scenes. To this end, typical optical cameras are often combined with non-visual sensors. The intermediate outputs of these hybrid systems, prior to the final scene rendering, are in general depth or depth+color images (RGB-D). Among the non-visual sensors, we can find Time-of-Flight (ToF) cameras (Kolb et al., 2010), which acquire low-resolution co-registered depth and color images at a cheap cost, and the famous Kinect (Zhang, 2012), capable to extract depth information by exploiting structural light. Another possibility is represented by lidar devices, which are used in a variety of applications and provide as output point clouds with measures of distance and reflectivity of the sensed surfaces.

This work lies in the context described and is particularly driven by the exploitation of data acquired by Mobile Mapping Systems (MMS), such as (Paparoditis et al., 2012). MMS systems are vehicles equipped with high-resolution cameras and at least one lidar sensor: their contained dimensions allow them to be driven through regular streets and acquire data of urban scenes. The data acquired is a set of calibrated and geolocated images, together with coherent lidar point clouds. The interest towards them comes from the possibility of having available, at a relatively small processing cost, the combination of depth and color information, without having to perform explicit (error-prone) reconstructions. Having a good depth estimate at each pixel, for example, would enable the possibility to perform depth-image-based rendering algorithms, e.g. (Chen et al., 2005; Schmeing and Jiang, 2011; Zinger et al., 2010). Similarly, the availability of depth information allows the insertion of virtual elements into the image, such as pedestrians or vehicles generated by a traffic simulation (Brédif, 2013). While MMS data sets do not include directly depth images aligned with the available color images, it is easy, by exploiting the known geometry, to project the lidar

34 point clouds onto each image. This operation produces initial depth images,
 35 which present three main issues (see Figure 1, where three parts of an input
 depth image are shown, together with the corresponding image parts).

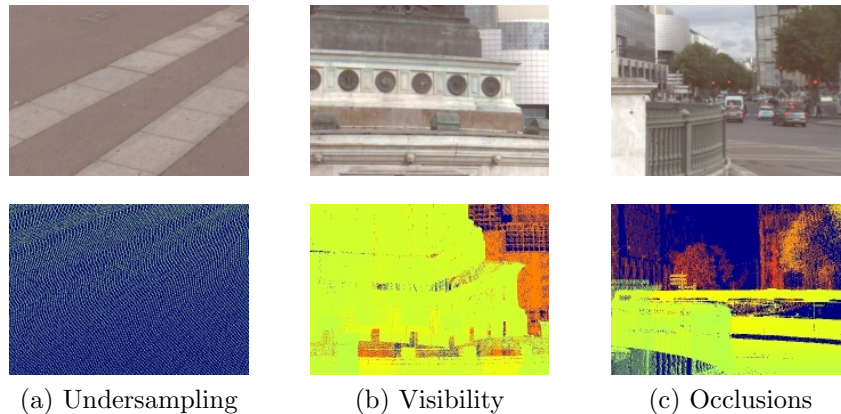


Figure 1: Examples of parts from a resulting input depth image (bottom row), with the corresponding parts from the reference color image (top row), showing the three issues mentioned: undersampling, appearance of hidden points, and presence of occlusions.

36

- 37 1. *Undersampling*: since lidar and image acquisitions are deeply different
 38 in terms of geometry and characteristics, the resulting depth images
 39 turn to be irregular. No points are present in the sky and on reflective
 40 surfaces. Moreover, the point density, which depends on the variable
 41 distances between the camera image plane and the positions of the lidar
 42 sensor, is generally significantly smaller than the pixel resolution. We
 43 can therefore talk about sparse input depth images (see for example
 44 Figure 1a, showing the low density of lidar points from the ground).
- 45 2. *Visibility* (hidden parts appear): since points that are not visible from
 46 the image view point (hidden points) can be occasionally “seen” by
 47 the moving lidar sensor, erroneous values referring to such points can
 48 appear in the input depth image. This occurs even when a Z-buffer
 49 approach (Greene et al., 1993) is used, i.e. only the closest depth values
 50 for each pixel are kept (in case multiple values end up in the same pixel
 51 location). E.g., Figure 1b shows that depth values from the building
 52 behind appear as foreground points.
- 53 3. *Occlusions* (visible parts disappear): for the same reason as above, i.e.
 54 the different acquisition timing and geometry between image and lidar

55 sensors, surfaces normally visible from the image view point do not get
56 a corresponding depth. This can happen when the lidar sensor suffers
57 occlusions at a given instant or because of the scene dynamics. E.g.,
58 in Figure 1c, a moving bus that is not present at the moment of the
59 image shot happens to appear in the depth image.

60 While there is a variety of methods in the literature that deal with the first
61 issue, i.e. that aim at upscaling an irregular input depth image possibly with
62 the guidance of a corresponding color image, little work has been performed
63 to address the last two issues. In this paper, while inpainting the input depth
64 image, we also intend to tackle the visibility problem. Moreover, we treat at
65 the same time an additional input: a sparse reflectance image derived in the
66 same way as the input depth image (i.e., by naively projecting the lidar point
67 cloud, considering the reflectance information carried out by each point). We
68 will show that the simultaneous use of a reflectance image, which is inpainted
69 jointly with the depth, improves the quality of the produced depth image
70 itself. To jointly inpaint depth and reflectance and concurrently evaluate the
71 visibility of each point (i.e. establish if a single point is reliable or, since
72 non-visible, must be discarded), we formulate an optimization problem with
73 three variables to estimate: depth, reflectance and a visibility attribute per
74 pixel. The inpainting process is also guided by the available color image, by
75 means of a two-fold coupled total variation (TV) regularizer.

76 The remainder of the paper is organized as follows. In Section 2, we
77 present our approach and mention the related works, in particular on the
78 image-guided depth inpainting problem. In sections 3 and 4 we describe the
79 model used and the primal-dual optimization algorithm that arises, respec-
80 tively. Finally, in Section 5 we bring experimental evidence that proves the
81 effectiveness of the proposed approach.

82 2. Problem addressed and related work

83 Figure 2 depicts the scheme of the proposed approach. Given an MMS
84 data set consisting of a lidar point cloud and a set of camera images, we
85 choose among the latter a reference color image (w), and we obtain input
86 depth (u_S) and reflectance (r_s) images by re-projecting the lidar points ac-
87 cording to the image geometry. The two lidar-originated images are sparse
88 images with irregular sampling and need to be inpainted. We propose to do
89 that jointly and simultaneously estimate the visibility of the input points,

90 within a variational optimization framework. The output of the algorithm
 91 are then three: the inpainted depth and reflectance (u and r , respectively),
 92 and a binary image expressing the visibility at each point (v).

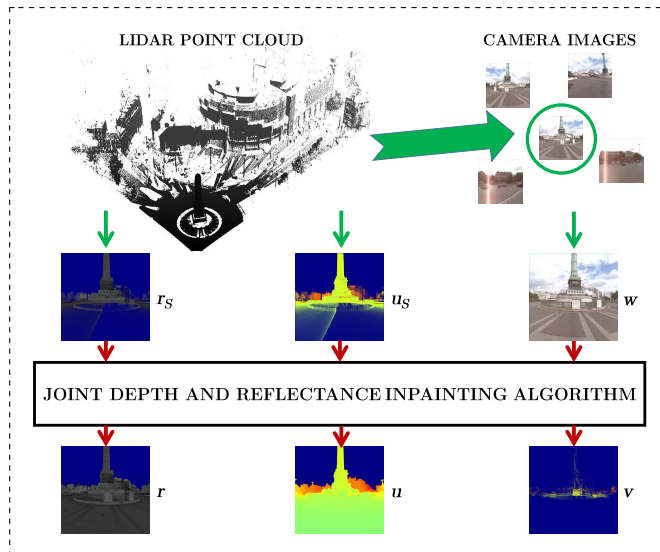


Figure 2: General scheme of the proposed approach. The final outputs of the algorithm are the inpainted reflectance and depth images, r and d respectively, and a binary visibility image v . To represent v , we show the original depth values that finally get $v \simeq 0$.

93 In the literature there is a variety of methods that aim at upscaling or
 94 inpainting an original sparse depth image. Most of them are presented in the
 95 context of ToF cameras; thus, a high quality color image is acquired at the
 96 same time and can be exploited. We refer to this problem as image-guided
 97 depth inpainting. The typical assumption, when exploiting the available image,
 98 is that image edges are related to depth edges. Following this principle,
 99 many approaches have been proposed, such as methods using different versions
 100 of multilateral filtering (Chan et al., 2008; Garcia et al., 2010; Yang
 101 et al., 2013), methods based on Markov Random Fields (Diebel and Thrun,
 102 2005), and methods using Non-Local Means (Huhle et al., 2010; Park et al.,
 103 2011). Another family relates to recent methods that make use of optimization
 104 (Ferstl et al., 2013; Harrison and Newman, 2010; Liu and Gong, 2013;
 105 Schneider et al., 2016; Schwarz et al., 2012). Among these, in (Harrison
 106 and Newman, 2010), a method to assign image pixel with a range value,
 107 using both image appearance and sparse laser data, is proposed. The problem
 108 is posed as an optimization of a cost function encapsulating a spatially

109 varying smoothness cost and measurement compatibility. In the same spirit,
 110 the authors of (Ferstl et al., 2013) present an optimization-based depth up-
 111 sampling method, which uses an Anisotropic Total Generalized Variation
 112 (ATGV) term to regularize the solution while exploiting the color image in-
 113 formation. Another recent algorithm for the upsampling of sparse depth data
 114 is presented in (Schneider et al., 2016). The key idea here is to exploit ad-
 115 ditional object boundary cues (via structured edge detection and semantic
 116 scene labelling) together with usual intensity cues in a unique optimization
 117 framework.

118 While presenting good results on images that are not particularly “prob-
 119 lematic”, in none of the mentioned methods the visibility issue is directly
 120 tackled, i.e. there is no explicit estimation of input depth measures to pos-
 121 sibly exclude from the inpainting process. We instead intend to estimate visi-
 122 bility, to be able to cope with realistic depth images. To this end, we build on
 123 our previous work on lidar-based depth inpainting (Bevilacqua et al., 2016).
 124 W.r.t. the latter, the model is significantly modified to include a reflectance
 125 image as well into a new optimization framework. We will show that depth
 126 and reflectance mutually benefit of each other in the inpainting process, thus
 127 leading to better output results for both. In the next section we present the
 128 novel model.

129 3. Model

130 Let $\Omega \subseteq \mathbb{R}^2$ be the “full” image support, and $\Omega_S \subseteq \Omega$ the sparse im-
 131 age support where the input images are defined (i.e., there is at least one
 132 lidar point ending up there after projection). Given an input depth image
 133 $u_S : \Omega_S \rightarrow \mathbb{R}$, an input reflectance image $r_S : \Omega_S \rightarrow \mathbb{R}$, and the luminance
 134 component of their corresponding color image $w : \Omega \rightarrow \mathbb{R}$ (defined in the
 135 complete domain), the goal is to fully inpaint the depth and reflectance in-
 136 put images to obtain $u : \Omega \rightarrow \mathbb{R}$ and $r : \Omega \rightarrow \mathbb{R}$, and concurrently estimate a
 137 visibility attribute $v : \Omega_S \rightarrow \mathbb{R}$. For each input pixel, v indicates whether it
 138 is visible from the image view point and should thus be taken into account in
 139 the inpainting process. Figure 3 reports an example of three possible input
 140 images - depth (u_S), reflectance (r_S) and camera images - and their respective
 141 gradient images.

142 We model our joint inpainting problem as an optimization problem with
 143 three variables, u , r , and v , to be estimated. Lower and upper bounds for the
 144 values of u and r are considered in the expression. The visibility attribute

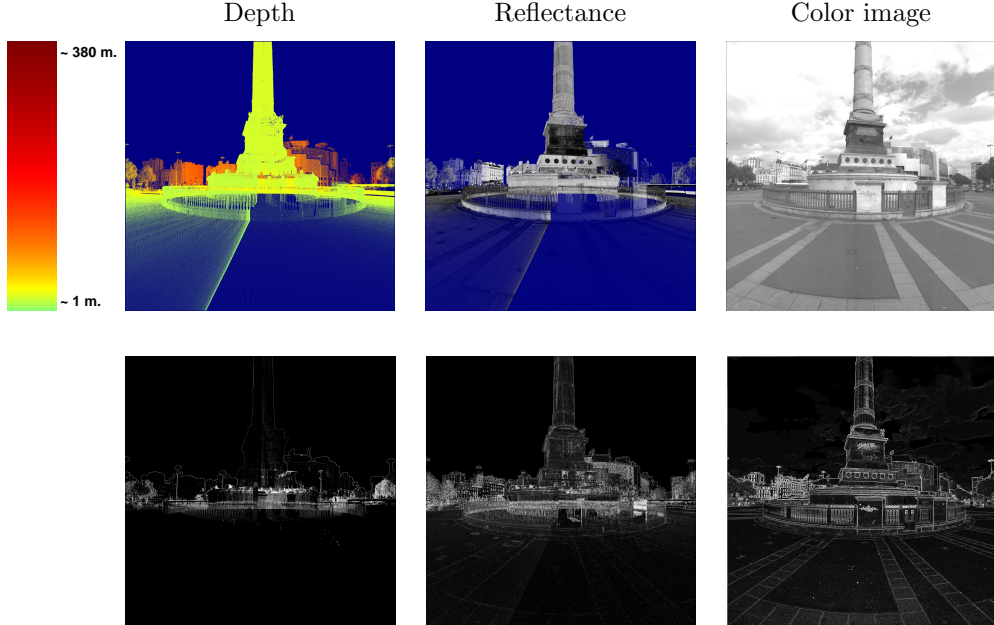


Figure 3: Example of input depth, reflectance and color images (top row), and their respective gradient images (bottom row). Besides the input depth image, the color map used to encode depth values is reported. Gradients of depth and reflectance are computed on the interpolated versions of the input sparse images, initially obtained by nearest neighbor interpolation.

145 v takes values in $[0, 1]$, where $v = 0$ stands for “hidden” and $v = 1$ means
 146 that the point is visible from the considered image view point. The model
 147 considered consists of four terms:

$$\min_{\substack{u \in [u_m, u_M] \\ r \in [r_m, r_M] \\ v \in [0, 1]}} F(u, v|u_S) + G(r, v|r_S) + H(v|u_S, r_S) + R(u, r|w). \quad (1)$$

148 $F(u, v|u_S)$ and $G(r, v|r_S)$ are two data-fidelity terms, for depth and re-
 149 flectance respectively. In both of them the visibility attribute v intervenes.
 150 $H(v|u_S, r_S)$ is a term depending exclusively on v , which represents the total
 151 cost of classifying input pixels as non-visible. Finally, $R(u, r|w)$ is a regular-
 152 ization term that penalizes the total variation of u and r , by also taking into
 153 account the color image w . In the next sections we will detail all the terms
 154 composing (1).

155 *3.1. Visibility-weighted data-fidelity terms*

156 The data-fitting terms in (1) are meant to enforce fidelity with the original
 157 values of depth and reflectance, u_S and r_S respectively. Deviations from the
 158 original values are more penalized if the points are considered “trustful”;
 159 conversely, for erroneous original measures (e.g., referring to hidden points)
 160 larger deviations are allowed. Therefore we use the visibility attribute v to
 161 weight the data terms. For the reflectance data-fidelity term $G(r, v|r_S)$ we
 162 have the following expression:

$$G(r, v|r_S) = \eta_2 \int_{\Omega_S} v|r - r_S| dx_1 dx_2, \quad (2)$$

163 where η_2 is a coefficient weighting the term within the model, and dx_1 and
 164 dx_2 express the differential lengths in the two image directions. Note that in
 165 (2) an ℓ_1 -norm error is used. The ℓ_1 norm is considered in substitution of the
 166 classical ℓ_2 measure of the error for its effectiveness in implicitly removing
 167 impulse noise with strong outliers (Nikolova, 2004) and its better contrast
 168 preservation (Chan and Esedoglu, 2005). As said, weighting by v relaxes the
 169 dependence on the input data for those points classified as hidden.

170 The depth data-fidelity term, weighted by the coefficient η_1 , is further
 171 divided into two terms, as follows:

$$\begin{aligned} F(u, v|u_S) &= \eta_1 \left(\int_{\Omega_S} \max(0, u - u_S) dx_1 dx_2 + \int_{\Omega_S} v(\max(0, u_S - u)) dx_1 dx_2 \right) \\ &= F_1(u|u_S) + F_2(u, v|u_S). \end{aligned} \quad (3)$$

172 The basic idea behind this separation is to treat differently over- and under-
 173 estimated depths. Points for which the estimated depth is greater than the
 174 original value ($u > u_S$) most likely correspond to correct input measures,
 175 where the over-estimation would be due to the surrounding presence of larger
 176 erroneous depths. The expression $\max(0, u - u_S)$ is meant to select this
 177 kind of points (over-estimated depths). As they are considered reliable, an
 178 unweighted data-fitting term, $F_1(u|u_S)$, is imposed. It is easy to see that
 179 for these points the visibility attribute v tends to converge to 1, i.e. they
 180 are the best candidates for being classified as visible points. Conversely, the
 181 hidden points to remove are sought among depth values which undergo under-
 182 estimation ($u < u_S$). These points are taken into account in the second term
 183 $F_2(u, v|u_S)$, where the ℓ_1 error is weighted by the visibility attribute. Ideally,
 184 a fraction of them, the most “problematic” ones, will be classified as hidden

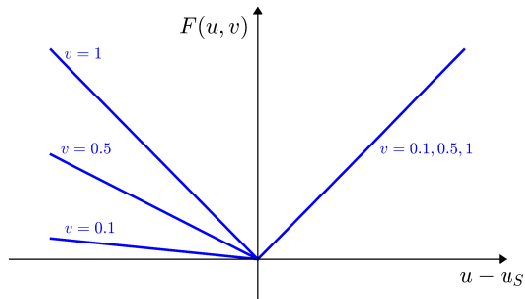


Figure 4: Depth data-fidelity cost $F(u, v|u_S)$ as a function of $u - u_S$ for different values of v ($\eta_1 = 1$ for simplicity). For over-estimated depths ($u - u_S > 0$) the cost is independent of v , whereas for $u - u_S < 0$ we have different lines as v varies.

185 ($v = 0$) and thus not considered in the data fitting cost. Figure 4 shows
 186 graphically the depth data-fidelity cost as a function of $u - u_S$. Depending
 187 on the value of the visibility attribute v , the ℓ_1 -type error $|u - u_S|$ is relaxed
 188 for negative depth deviations ($u < u_S$).

189 3.2. Removal cost

190 The second term of the model (1) is meant to penalize the total number
 191 of hidden points.

$$H(v|u_S, r_S) = \int_{\Omega_S} \alpha(u_S, r_S)(1 - v) dx_1 dx_2 . \quad (4)$$

192 The cost of a single pixel exclusion is proportional to $1 - v$, i.e. we have the
 193 highest cost for an input pixel when it is totally excluded in the data-fitting
 194 cost ($v = 0$). We individually weight each removal cost, in order to give
 195 different importance to each decision visible/hidden. Individual weighting is
 196 given by a coefficient dependent on the original depth and reflectance values,
 197 $\alpha(u_S, r_S)$. We generally choose $\alpha = k_1 u_S + k_2 r_S$. The linear dependence of α
 198 on the depth and the reflectance “balances” the three terms of (1) depending
 199 on v , such that k_1 and k_2 appear to be constants. We will discuss their choice
 200 later in this paper (Section 5.1).

201 3.3. Coupled Total Variation

202 Depth upsampling/inpainting methods that exploit corresponding camera
 203 images often relate image edges to depth edges. This has been shown to
 204 improve the quality of the reconstructed depth images.

205 To couple two images in a total variation framework, we adopt the *coupled*
 206 total variation (coupled TV) of (Pierre et al., 2015):

$$\text{TV}_\lambda(a, b) = \int_{\Omega} \sqrt{(\partial_{x_1}a)^2 + (\partial_{x_2}a)^2 + \lambda^2(\partial_{x_1}b)^2 + \lambda^2(\partial_{x_2}b)^2} dx_1 dx_2 . \quad (5)$$

207 where λ is a coupling parameter. When $\lambda \neq 0$ the minimization of TV_λ
 208 encourages the gradient “jumps” to occur at the same locations in a and b .
 209 The coupled TV is then a way to align the edges of an image with those of
 210 a given one.

211 In our problem we have three types of images: a color image w , a depth
 212 image u , and a reflectance image r . Figure 3 reports in the bottom row
 213 an example of gradient magnitudes related to three images. The gradients
 214 of the input depth and reflectance images have been computed after initial
 215 interpolation of the latter. As we can clearly see from the image, the color
 216 image gradient particularly matches the reflectance one, while being rather
 217 dissimilar to the depth gradient. In turn, the reflectance gradient shares
 218 some patterns, yet less prominently, with the depth one. See, e.g., the area
 219 at the base of the column, where multiple layers mix and produce a similar
 220 effect in the two gradient images. We therefore propose to match the three
 221 gradients two by two: depth with reflectance, and the same reflectance with
 222 the fixed color image. By using the previous definition of coupled TV (5),
 223 we express the regularization term as follows:

$$R(u, r|w) = \text{TV}_{\lambda_1}(u, r) + \text{TV}_{\lambda_2}(r, w) . \quad (6)$$

224 After detailing all the terms, our model (1) can therefore be rewritten as
 225 follows, the four terms being still distinct:

$$\begin{aligned} \min_{\substack{u \in [u_m, u_M] \\ r \in [r_m, r_M] \\ v \in [0, 1]}} & \underbrace{\eta_1 \left(\int_{\Omega_S} \max(0, u - u_S) + \int_{\Omega_S} v(\max(0, u_S - u)) \right)}_{F: \text{Data-fidelity for Depth}} + \underbrace{\eta_2 \int_{\Omega_S} v|r - r_S|}_{G: \text{Data-fidelity for Reflectance}} \\ & + \underbrace{\int_{\Omega_S} \alpha(u_S, r_S)(1 - v)}_{H: \text{Removal cost}} + \underbrace{\text{TV}_{\lambda_1}(u, r) + \text{TV}_{\lambda_2}(r, w)}_{R: \text{TV regularization}} . \end{aligned} \quad (7)$$

226 In the next section we detail a primal-dual approach to solve (7).

227 4. Algorithm

228 The optimization problem (7) turns out to be convex, but not smooth,
 229 due to ℓ_1 -type data-fidelity terms, $F(u, v|u_S)$ and $G(r, v|r_S)$, and the total

230 variation regularization term $R(u, r|w)$. Recently, in (Chambolle and Pock,
 231 2011) a primal-dual first-order algorithm has been proposed to solve such
 232 problems. In Section 4.1 we provide the necessary definitions for the algo-
 233 rithm, which is subsequently described in Section 4.2.

234 4.1. Discrete setting and definitions

235 Images, considered in Section 3 as continuous functions in \mathbb{R}^2 , are here
 236 converted into real finite-dimensional vectors. Let M and N be the image
 237 dimensions in this discrete setting, and (i, j) the indices denoting all possible
 238 discrete locations in the Cartesian grid of size $M \times N$ ($1 \leq i \leq M$, $1 \leq j \leq$
 239 N). We then have u, u_S, r, r_S, v, w , and $\alpha \in X = \mathbb{R}^{MN}$, where X is a finite
 240 dimensional vector space equipped with a standard scalar product:

$$\langle u, v \rangle_X = \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} u_{i,j} v_{i,j}, \quad u, v \in X. \quad (8)$$

241 The gradient of an image $u \in X$, ∇u , is a vector in the vector space X^2
 242 with two components per pixel:

$$(\nabla u)_{i,j} = ((\nabla_V u)_{i,j}, (\nabla_H u)_{i,j}). \quad (9)$$

243 We compute the gradient components via standard finite differences with
 244 Neumann boundary conditions, i.e.:

$$\begin{aligned} (\nabla_V u)_{i,j} &= \begin{cases} u_{i+1,j} - u_{i,j} & i < M \\ 0 & i = M \end{cases} \\ (\nabla_H u)_{i,j} &= \begin{cases} u_{i,j+1} - u_{i,j} & j < N \\ 0 & j = N \end{cases} \end{aligned} \quad (10)$$

245 From the definition of gradient, it follows the expression of discrete cou-
 246 pled total variation, which matches the continuous one (5):

$$\text{TV}_\lambda(a, b) = \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} \sqrt{(\nabla_H a_{i,j})^2 + (\nabla_V a_{i,j})^2 + \lambda^2 (\nabla_H b_{i,j})^2 + \lambda^2 (\nabla_V b_{i,j})^2}. \quad (11)$$

247 As first suggested by (Chan et al., 1999), a total variation optimization
 248 problem can be recast into a primal-dual form that makes its solution eas-
 249 ier, by rewriting the gradient norm by means of a vector-valued dual vari-
 250 able. To this end, in our case we first define a ‘‘coupled gradient’’ operator

251 $\mathcal{K}_{\lambda b} : X \rightarrow Y$ ($Y = X^4$), which, applied to an image $a \in X$, expands its
 252 gradient to include the one of a reference image b according to a coupling
 253 parameter λ . I.e., we have the following element-wise definition:

$$(\mathcal{K}_{\lambda b}a)_{i,j} = ((\nabla_H a)_{i,j}, (\nabla_V a)_{i,j}, \lambda(\nabla_H b)_{i,j}, \lambda(\nabla_V b)_{i,j}) . \quad (12)$$

254 The coupled gradient operator $\mathcal{K}_{\lambda b}$ can be further decomposed as $\mathcal{K}_{\lambda b} =$
 255 $\tilde{\mathcal{K}} + \beta_\lambda(b)$, according to the following element-wise definition:

$$\begin{aligned} (\mathcal{K}_{\lambda b}a)_{i,j} &= (\tilde{\mathcal{K}}a)_{i,j} + (\beta_\lambda(b))_{i,j} \\ &= ((\nabla_H a)_{i,j}, (\nabla_V a)_{i,j}, 0, 0) + (0, 0, \lambda(\nabla_H b)_{i,j}, \lambda(\nabla_V b)_{i,j}) . \end{aligned} \quad (13)$$

256 $\tilde{\mathcal{K}}$ is the usual gradient operator “padded” with two zero components and it
 257 is linear in a ; $\beta_\lambda(b)$ is a bias term, depending on the gradient of the fixed
 258 variable b , which determines the last two components of the global coupled
 259 gradient operator.

260 Thanks to the definitions above, we can express alternatively the coupled
 261 total variation (11), by introducing the dual variable $p \in Y$:

$$\begin{aligned} \text{TV}_\lambda(a, b) &= \max_{p \in Y} \langle \mathcal{K}_{\lambda b}a, p \rangle_Y - \delta_P(p) \\ &= \max_{p \in Y} \langle \tilde{\mathcal{K}}a, p \rangle_Y + \langle \beta_\lambda(b), p \rangle_Y - \delta_P(p) , \end{aligned} \quad (14)$$

where the scalar product in Y is defined as

$$\begin{aligned} \langle p, q \rangle_Y &= \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} p_{i,j}^1 q_{i,j}^1 + p_{i,j}^2 q_{i,j}^2 + p_{i,j}^3 q_{i,j}^3 + p_{i,j}^4 q_{i,j}^4 , \\ p &= (p^1, p^2, p^3, p^4), \quad q = (q^1, q^2, q^3, q^4) \in Y \end{aligned}$$

262 δ_P denotes the indicator function of the set P

$$\delta_P(p) = \begin{cases} 0 & \text{if } p \in P \\ +\infty & \text{if } p \notin P \end{cases} , \quad (15)$$

263 and the feasibility set P for the dual variable p , is defined as

$$P = \{p \in Y \mid \|p_{i,j}\|_2 \leq 1, \forall i, j\} , \quad (16)$$

264 i.e. $\|p\|_\infty \leq 1$.

265 We can now finally express the regularization term of our model $R(u, r|w)$
 266 (6) as the maximization over two dual variables. We then have:

$$\begin{aligned} R(u, r|w) &= \max_{p \in Y} \max_{q \in Y} \langle \mathcal{K}_{\lambda_1} u, p \rangle_Y + \langle \mathcal{K}_{\lambda_2} w r, p \rangle_Y - \delta_P(p) - \delta_Q(q) \\ &= \max_{p \in Y} \max_{q \in Y} \langle \tilde{\mathcal{K}} u, p \rangle_Y + \langle \beta_{\lambda_1}(r), p \rangle_Y + \langle \tilde{\mathcal{K}} r, q \rangle_Y + \langle \beta_{\lambda_2}(w), q \rangle_Y - \delta_P(p) - \delta_Q(q). \end{aligned} \quad (17)$$

267 This will let us formulate a discrete version of our joint inpainting problem
 268 (7), which falls into the primal-dual optimization framework. As for the
 269 other terms in (7), rewritten in discrete notation, we have:

$$\begin{aligned} F_1(u|u_S) &= \eta_1 \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} \Phi_{i,j} \max(0, u_{i,j} - u_{S \ i,j}) \\ F_2(u, v|u_S) &= \eta_1 \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} \Phi_{i,j} v_{i,j} \max(0, u_{S \ i,j} - u_{i,j}) \\ G(r, v|r_S) &= \eta_2 \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} \Phi_{i,j} v_{i,j} |r_{i,j} - r_{S \ i,j}| \\ H(v|u_S, r_S) &= \sum_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}} \Phi_{i,j} \alpha_{i,j} (1 - v_{i,j}) \end{aligned} \quad (18)$$

270 where Φ is a binary mask indicating the initial known pixels, i.e. belonging
 271 to the sparse image support Ω_S .

272 4.2. A primal-dual algorithm

Thanks to the previous definitions, we can express our model (7) in the form of the following saddle-point problem, which is an extension (including two extra variables) of the one presented in (Pierre et al., 2015):

$$\begin{aligned} \min_{u \in X} \min_{r \in X} \min_{v \in X} \max_{p \in Y} \max_{q \in Y} \{ \langle K_1 u, p \rangle + \langle K_2 r, q \rangle - D_1^*(p) - D_2^*(q) \\ + A(u) + B(r) + a(u, v) + b(r, v) + C(v) \} . \end{aligned} \quad (19)$$

273 It is a primal-dual problem with three primal variables (u , r , and v) and
 274 two dual variables (p and q) that evolve independently. Each dual variable
 275 is particularly linked to the gradient of a primal variable, i.e. p to u , and
 276 q to r . D_1^* , D_2^* , A , B , and C are convex functions; a and b are convex
 277 w.r.t. each of its respective variables. Globally, the functional is not convex

278 w.r.t. the triplet (u, r, v) . By relating (7) and (19), and using the primal-dual
 279 expression of the regularization term reported in (17), we have the following
 280 equivalences:

$$\begin{aligned}
 281 \quad & \bullet K_1 u = \tilde{\mathcal{K}}u; & \bullet K_2 r = \tilde{\mathcal{K}}r; \\
 282 \quad & \bullet D_1^*(p) = -\langle \beta_{\lambda_1}(r), p \rangle_Y + \delta_P(p); & \bullet D_2^*(q) = -\langle \beta_{\lambda_2}(w), q \rangle_Y + \delta_Q(q); \\
 283 \quad & \bullet A(u) = F_1(u|u_S) + \delta_{[u_m, u_M]}(u); & \bullet B(r) = \delta_{[r_m, r_M]}(r); \\
 284 \quad & \bullet a(u, v) = F_2(u, v|u_S); & \bullet b(r, v) = G(r, v|r_S); \\
 285 \quad & \bullet C(v) = H(v|u_S, r_S) + \delta_{[0,1]}(v).
 \end{aligned}$$

286 An algorithm to solve (19) can be derived within the primal-dual opti-
 287 mization framework of (Chambolle and Pock, 2011). It consists in a unique
 288 loop, where all variables are alternatively updated via proximal operators
 289 (see Algorithm 1). The algorithm takes as inputs the initial estimates of
 290 the complete depth and reflectance images (u_0 and r_0 , respectively), and the
 291 reference intensity image w . It also requires three parameters inherent to the
 292 algorithm: σ and τ , which are related to each other by the relation $16\tau\sigma \leq 1$
 293 (Chambolle and Pock, 2011), and ρ , which is a parameter regulating the
 294 update speed of v .

295 Algorithm 1 involves the computation of the adjoints to the linear oper-
 296 ators K_1 and K_2 (the “zero-padded” gradient operators). It is known
 297 that the adjoint of the gradient operator is the negative divergence operator
 298 ($\nabla^* = -\text{div}$). In our case, the adjoint to the operator $K_1 : X \rightarrow Y$ is a
 299 linear operator $K_1^* : Y \rightarrow X$ consisting in the negative divergence computed
 300 only on the two first components of a four-component dual variable $p \in Y$,
 301 and by taking finite differences in the opposite direction than the gradient
 302 operator (10). These components are in fact the ones related to the primal
 303 variable to which the coupled gradient operator has been applied. We then
 304 have the following element-wise definition for K_1^*p (the same definition stands
 305 for K_2^*q):

$$(K_1^*p)_{i,j} = - \begin{cases} p_{i,j}^1 - p_{i-1,j}^1 & \text{if } 1 < i < M \\ p_{i,j}^1 & \text{if } i = 1 \\ -p_{i-1,j}^1 & \text{if } i = M \end{cases} - \begin{cases} p_{i,j}^2 - p_{i,j-1}^2 & \text{if } 1 < j < N \\ p_{i,j}^2 & \text{if } j = 1 \\ -p_{i,j-1}^2 & \text{if } j = N \end{cases} . \quad (20)$$

Algorithm 1 Primal-dual based algorithm for depth and reflectance joint inpainting.

1: **Inputs:**

$$u_0, r_0, w, \sigma, \rho, \tau$$

2: **Initialize:**

$$u^0, \bar{u}^0 \leftarrow u_0, r^0, \bar{r}^0 \leftarrow r_0, v_{i,j}^0 \leftarrow 0.5, \\ p^0 \leftarrow (\nabla u_0, \lambda_1 \nabla r_0), q^0 \leftarrow (\nabla r_0, \lambda_2 \nabla w)$$

3: **for** $n = 0, 1, \dots$ **do**

$$4: \quad p^{n+1} \leftarrow \text{prox}_{\sigma D_1^*}(p^n + \sigma K_1 \bar{u}^n) \\ 5: \quad q^{n+1} \leftarrow \text{prox}_{\sigma D_2^*}(q^n + \sigma K_2 \bar{r}^n) \\ 6: \quad v^{n+1} \leftarrow \text{prox}_{\rho a(\bar{u}^n, \cdot) + \rho b(\bar{r}^n, \cdot) + \rho C}(v^n) \\ 7: \quad u^{n+1} \leftarrow \text{prox}_{\tau A + \tau a(\cdot, v^{n+1})}(u^n - \tau K_1^* p^{n+1}) \\ 8: \quad r^{n+1} \leftarrow \text{prox}_{\tau B + \tau b(\cdot, v^{n+1})}(r^n - \tau K_2^* q^{n+1}) \\ 9: \quad \bar{u}^{n+1} \leftarrow 2u^{n+1} - u^n \\ 10: \quad \bar{r}^{n+1} \leftarrow 2r^{n+1} - r^n$$

11: **end for**

306 Closed-form expressions for the update rules in Algorithm 1 can be easily
307 computed by applying the definition of proximal operator (see Appendix
308 A). The resulting expressions are reported here below, where \mathcal{P} denotes
309 the projection operation over a given real interval, i.e. values are clipped if
310 exceeding the interval limits. Details about the derivation of these expression
311 can be found in Appendix A.

$$\text{prox}_{\sigma D_1^*}(\tilde{p}) = \frac{\tilde{p} + \sigma \beta_{\lambda_1}(r)}{\max(1, \|\tilde{p} + \sigma \beta_{\lambda_1}(r)\|_2)} \quad (21)$$

$$\text{prox}_{\sigma D_2^*}(\tilde{q}) = \frac{\tilde{q} + \sigma \beta_{\lambda_2}(w)}{\max(1, \|\tilde{q} + \sigma \beta_{\lambda_2}(w)\|_2)} \quad (22)$$

$$\text{prox}_{\rho a(\bar{u}, \cdot) + \rho b(\bar{r}, \cdot) + \rho C}(\tilde{v}) = \begin{cases} \mathcal{P}_{[0,1]}(\tilde{v}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_2|\bar{r} - r_S|) & \text{if } \Phi_{i,j} = 1, \bar{u}_{i,j} \geq u_{S\ i,j} \\ \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_1(u_S - \bar{u}) - \rho\eta_2|\bar{r} - r_S|) & \text{if } \Phi_{i,j} = 1, \bar{u}_{i,j} < u_{S\ i,j} \end{cases} \quad (23)$$

$$\text{prox}_{\tau A + \tau a(\cdot, v)}(\tilde{u}) = \begin{cases} \mathcal{P}_{[u_m, u_M]}(\tilde{u}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[u_m, u_M]}(\tilde{u} - \tau\eta_1) & \text{if } \Phi_{i,j} = 1, \tilde{u}_{i,j} > u_{S i,j} + \tau\eta_1 \\ \mathcal{P}_{[u_m, u_M]}(\tilde{u} + v\tau\eta_1) & \text{if } \Phi_{i,j} = 1, \tilde{u}_{i,j} < u_{S i,j} - v\tau\eta_1 \\ \mathcal{P}_{[u_m, u_M]}(u_S) & \text{otherwise} \end{cases} \quad (24)$$

$$\text{prox}_{\tau B + \tau b(\cdot, v)}(\tilde{r}) = \begin{cases} \mathcal{P}_{[r_m, r_M]}(\tilde{r}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[r_m, r_M]}(\tilde{r} - v\tau\eta_2) & \text{if } \Phi_{i,j} = 1, \tilde{r}_{i,j} > r_{S i,j} + v\tau\eta_2 \\ \mathcal{P}_{[r_m, r_M]}(\tilde{r} + v\tau\eta_2) & \text{if } \Phi_{i,j} = 1, \tilde{r}_{i,j} < r_{S i,j} - v\tau\eta_2 \\ \mathcal{P}_{[r_m, r_M]}(r_S) & \text{otherwise} \end{cases} \quad (25)$$

312 The operations indicated in the proximal operators are pixel-wise, al-
 313 though the pixel coordinates have not been made explicit for clearer read-
 314 ing.

315 5. Experimental results

316 The algorithm presented in Section 4 is evaluated with a new data set
 317 acquired in an urban scenario by a Mobile Mapping System (MMS), com-
 318 posed of lidar measures and camera-originated images. With this data set,
 319 we provide a qualitative evaluation of our algorithm in comparison with other
 320 methods, by showing the reconstructed depth and reflectance images, and we
 321 assess the quality of the visibility estimation task, which is a crucial charac-
 322 teristic of our algorithm. Moreover, we also provide a quantitative analysis
 323 by means of a benchmark data set publicly available. Before showing results
 324 and comparisons, in Section 5.1 we motivate some critical choices in terms
 325 of model and algorithmic parameters.

326 5.1. Parameters of the algorithm and model choices

327 Our finally resulting joint inpainting model (7) consists of four terms:
 328 two data-fidelity terms, $F(u, v|u_S)$ and $G(r, v|r_S)$, a “removal” cost depend-
 329 ing solely on the variable v , $H(v|u_S, r_S)$, and the two-fold regularization
 330 term $R(u, r|w)$. As discussed in Section 3.1, for the data-fidelity terms we
 331 opt for an ℓ_1 measure of the error, in order to promote more contrasted so-
 332 lutions (Chan and Esedoglu, 2005). The visibility attribute v weights the
 333 data matching cost of each single pixel (data matching is more and more

334 relaxed, as v tends to zero, i.e. when that particular point is considered to
 335 be excluded). However, over-estimated depths ($u > u_S$) are not weighted by
 336 v but are fully penalized. These values relate to pixels where either there
 337 is noise on a visible point that is slightly corrected ($u - u_S$ is small), or the
 338 value u_S represents an outlier (e.g. it is due to a mobile object). At present,
 339 we do not have a way to handle the latter case.

340 In $H(v|u_S, r_S)$ (4), each point removal cost is the product between $(1 - v)$
 341 (the level of “invisibility” of the point) and a coefficient α depending on the
 342 local input depth and reflectance: $\alpha = k_1 u_S + k_2 r_S$. This choice has been
 343 made in order to balance all terms in (7) where v appears. Let us now
 344 observe the “complete” update rule for v (last case of (23), i.e. for points
 345 with under-estimated depth). According to it, we have that at each iteration
 346 v is incremented/decremented by a quantity $\Delta v = \rho(\alpha - \eta_1 \Delta u - \eta_2 \Delta r)$. Let
 347 us suppose that the fluctuations on depth are significantly larger than the
 348 fluctuations on reflectance (the appearance of a hidden point can cause a big
 349 “jump” in depth, while the reflectance values might still be similar. For the
 350 sake of simplicity we can then adjust the value of α only on the basis of the
 351 depth input value. The proposed simplified expression for α is then:

$$\alpha = k u_S. \quad (26)$$

352 With the assumptions made we therefore have $\Delta v \propto (k u_S - \eta_1 \Delta u)$. The
 353 attribute v for a certain pixel increases (it gets a higher confidence as a
 354 visible point) if $\frac{\Delta u}{u_S} < \frac{k}{\eta_1}$, i.e. if the relative depth deviation is below a certain
 355 threshold. k is an adimensional parameter that contributes determining this
 356 threshold. Conversely, v decreases for relative depth deviations exceeding
 357 the threshold. As for the update of v for points with over-estimated depths
 358 (second case of (23)), if we hypothesize that α , adjusted on depth, is large
 359 enough w.r.t. the reflectance deviation, we have that v progressively tends
 360 to one (unless large absolute reflectance deviations occur).

361 As for the regularization term $R(u, r|w)$, we proposed in Section 3.3 to
 362 combine two distinct coupled total variation terms: $\text{TV}_{\lambda_1}(u, r)$ (depth is
 363 individually coupled with reflectance) and $\text{TV}_{\lambda_2}(r, w)$ (reflectance is individ-
 364 ually coupled with the color image). By having two separate coupled TV
 365 terms, each one encoded by a dual variable that evolves independently from
 366 the other one, the reflectance gradient is constantly brought back to the refer-
 367 ence gradient of the color image. At the same time the “correct” gradient
 368 information is transferred to the depth via the second term. Figure 5 shows

369 an example of results obtained with the algorithm for the same test case as
 370 Figure 3.

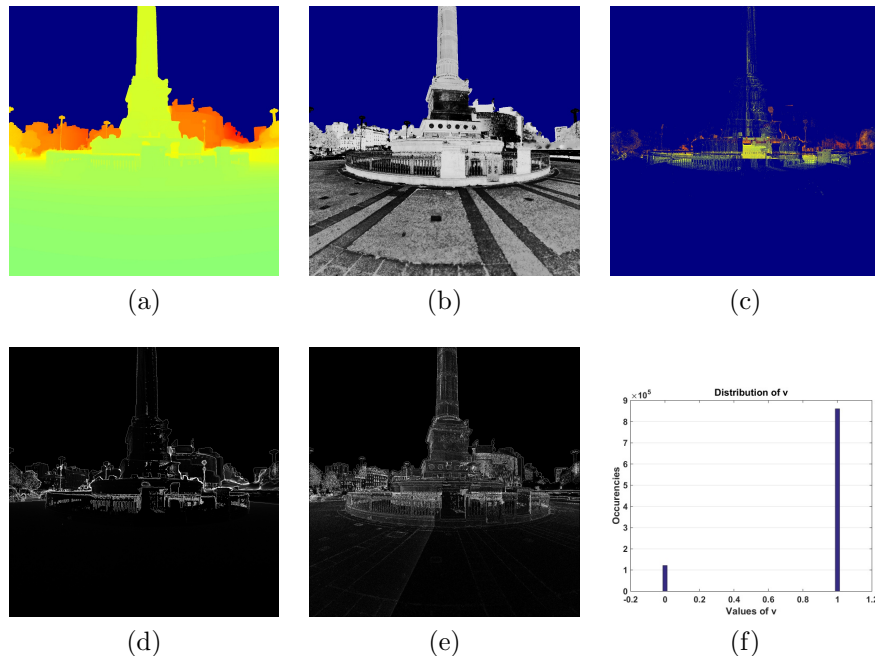


Figure 5: Output of the proposed algorithm for the image *Column1*: (a) Inpainted depth, (b) Inpainted reflectance, (c) Removed points ($v = 0$), (d) Final depth gradient, (e) Final reflectance gradient, (f) Final histogram of v .

371 For the example test of Figure 5, as well as for all the results reported
 372 hereinafter, the following parameters, found with multiple tests, have been
 373 used to characterize the model (7): $\eta_1 = 1.7$, $\eta_2 = 50$, $k = 0.05$ (the co-
 374 efficient determining α according to (26)), $\lambda_1 = 0.5$, $\lambda_2 = 1$. These values
 375 have been found empirically by letting them vary one by one and observing
 376 the obtained visual results. The two data terms $F(u, v|u_S)$ and $G(r, v|r_S)$
 377 are attributed different weights. The larger coefficient assigned to the re-
 378 flectance data term ($\eta_2 > \eta_1$) means that a greater data fidelity is imposed
 379 on reflectance. Depth values have instead a greater “freedom” in deviating
 380 from their original values. The two coupling parameters λ_1 and λ_2 being
 381 in the same order of magnitude, it shows that the two coupling terms have
 382 a similar importance. As for the parameters, inherent to the primal-dual
 383 optimization scheme (Algorithm 1), the following values have been set after
 384 testing: $\rho = 10$, $\tau = 0.004$, $\sigma = 14$.

385 If we observe the input sparse depth image of Figure 3, we see that
 386 the major problems come from the fact that depth values referring to the
 387 building behind the column appear mixed with foreground depths. With
 388 our algorithm we are able to resolve these conflicts, as we can see in the
 389 inpainted depth image (Figure 5a). Part of the input pixels have in fact
 390 been removed, i.e. classified as non-visible ($v = 0$). Figure 5c reports the
 391 locations of such points in the original depth image. From the histogram of
 392 the values of v (Figure 5f) it is evident that the algorithm produces a bi-
 393 partition of the points according to their visibility attribute. Figure 5 shows
 394 also the inpainted reflectance and the final depth and reflectance gradients.
 395 By comparing the latter to the original gradients (Figure 3), we can observe
 396 that they end up incorporating elements of the color image gradient, while
 397 removing erroneous edges. In the next section we will present more results
 398 obtained with our algorithm, also in comparison to other inpainting methods.

399 5.2. Results with urban data

400 We consider a data set acquired by a MMS system (Paparoditis et al.,
 401 2012) at *Place de la Bastille*, Paris, consisting of one lidar point cloud in the
 402 order of one billion of points and hundreds of optical images simultaneously
 403 acquired by 5 cameras mounted on the vehicle. Given a reference optical
 404 image, we project onto it the available lidar points to form the initial depth
 405 and reflectance incomplete images. Note that not all the points are effectively
 406 visible from the image view point. The incomplete depth and reflectance
 407 images, along with the reference color image chosen, represent the input of
 408 the algorithm (u_S , r_S , and w respectively).

409 Figures 6–9 present results for four images (cropped w.r.t. the full size)
 410 of the data set: *Column1*, *Column2*, *Buildings1*, *Buildings2*. For each refer-
 411 ence image, the input sparse depth and reflectance images, obtained via
 412 projection, are shown, as well as the inpainted depth and reflectance im-
 413 ages, obtained with four different methods. For the output depth images of
 414 Figure 8 and 9 we added some shading by modulating the color intensity
 415 of each pixel based on the zenith angle of the normal vector, to emphasize
 416 high-frequency changes. Moreover, for the inpainted depths, an alternative
 417 view of the resulting 3-D point cloud is proposed, where the coordinates of
 418 the points are retrieved thanks to the computed depths and color texture is
 419 applied to enrich the points. A color box is overlaid to the first of these 3-D
 420 views to highlight areas where the comparison between the different methods
 421 is particularly significant.

Our algorithm, presented in Section 4, gives as output the two inpainted images u and r . As for the produced depth image, our algorithm is visually compared with nearest neighbor (NN) interpolation, the anisotropic total generalized variation (ATGV) method of (Ferstl et al., 2013), and our previous depth inpainting method (Bevilacqua et al., 2016), which does not rely on reflectance information. We refer to the latter as Depth Inpainting with Visibility Estimation (DIVE). The optimization problem of DIVE is the following:

$$\min_{\substack{u \in [u_m, u_M] \\ v \in [0, 1]}} \eta \int_{\Omega_S} (\max(0, u - y))^2 dx_1 dx_2 + \eta \int_{\Omega_S} v (\max(0, y - u))^2 dx_1 dx_2 + \int_{\Omega_S} (ku_S)^2 (1 - v) dx_1 dx_2 + \text{TV}_\lambda(u, w) . \quad (27)$$

422 The DIVE problem can be related to our proposed model (7), if we consider
 423 in the latter $\eta_1 = \eta$, $\eta_2 = 0$, $\lambda_1 = \lambda$, and we suppress the coupled TV term
 424 related to the reflectance (depth is instead coupled directly with the color
 425 image). Moreover, in (27) we have a ℓ_2 -norm data fidelity term; as a conse-
 426 quence of that, the coefficient of the removal cost term follows a quadratic
 427 law (we have $\alpha = (ku_S)^2$, instead of $\alpha = ku_S$, as in (7)).

428 As for the produced reflectance image, our algorithm is compared with
 429 nearest neighbor (NN) interpolation, the ATGV method of (Ferstl et al.,
 430 2013) applied to reflectance, and a reduced version of our model (7) limited to
 431 reflectance. We refer to this method as Reflectance Inpainting with Visibility
 432 Estimation (RIVE). The RIVE method is derived from the solution of the
 433 following optimization problem:

$$\min_{\substack{r \in [r_m, r_M] \\ v \in [0, 1]}} \eta \int_{\Omega_S} v |r - r_S| dx_1 dx_2 + \int_{\Omega_S} (kr_S)(1 - v) dx_1 dx_2 + \text{TV}_\lambda(r, w) . \quad (28)$$

434 Also in this case we can derive the considered problem (RIVE) as a simplified
 435 version of our proposed model (7), where $\eta_1 = 0$, $\eta_2 = \eta$, $\lambda_2 = \lambda$, and the
 436 coupled TV term related to depth is suppressed. Moreover, the coefficient of
 437 the removal cost, while still following a linear law, here depends on the input
 438 reflectance r_S .

439 The four examples reported show the better performance of our algorithm
 440 in generating complete depth and reflectance images from real lidar measures.
 441 Results with the image *Column1*, reported in Figure 6, particularly prove the

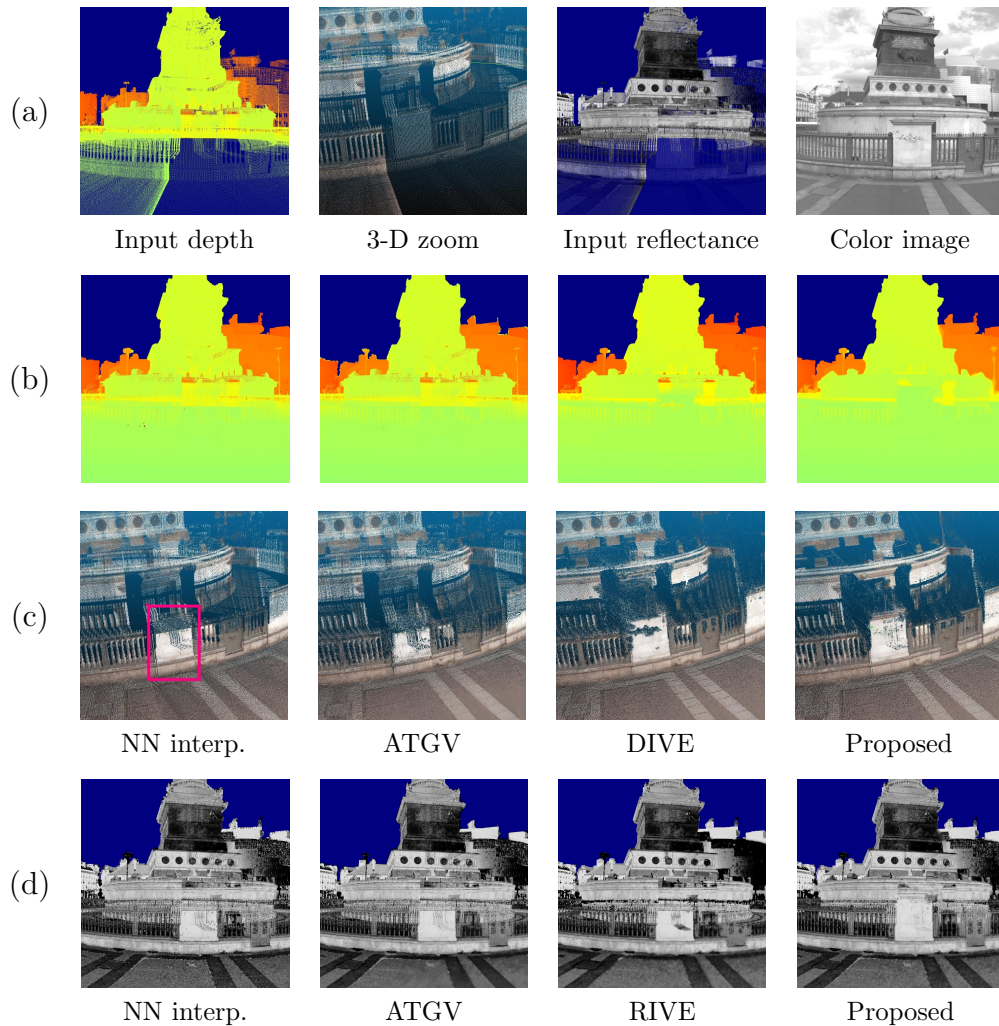


Figure 6: Visual results for the image *Column1*. Row (a) shows the related input images: depth (with a 3-D zoom), reflectance, and reference color image. Rows (b) and (c) report the results obtained in terms of inpainted depth images (with related 3-D zoomed-in view) with the algorithms indicated below. Row (d) shows the inpainted reflectance images obtained with different methods, our proposed method always reported as last.

442 effectiveness of our algorithm in detecting and removing hidden points ap-
 443 pearing in the front, thus producing inpainted images correct from the image
 444 view point. These points, in yellow/orange according to the color code used
 445 for depth, appear mixed to visible points belonging to the column and the
 446 fence. By looking at the depth images generated (row (b)), our algorithm

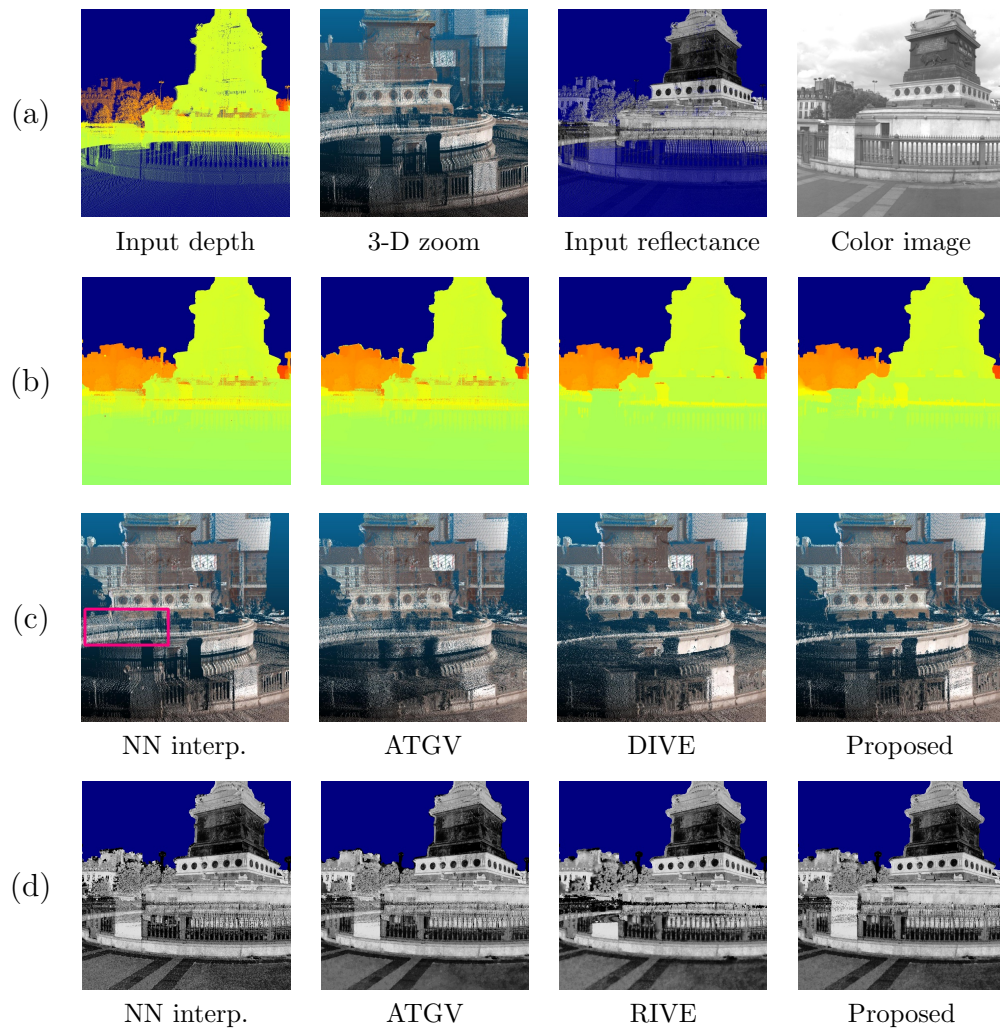


Figure 7: Visual results for the image *Column2*. Row (a) shows the related input images: depth (with a 3-D zoom), reflectance, and reference color image. Rows (b) and (c) report the results obtained in terms of inpainted depth images (with related 3-D zoomed-in view) with the algorithms indicated below. Row (d) shows the inpainted reflectance images obtained with different methods, our proposed method always reported as last.

447 is the only one which is able to remove the misleading points and correctly
 448 reconstruct the foreground depth plane. This is even more visible by ob-
 449 serving the main marble pole highlighted in the 3-D views (row (c)). While
 450 other methods are not able to reconstruct the pole, since “distracted” by the
 451 interfering background depths, the reconstruction is better performed in our

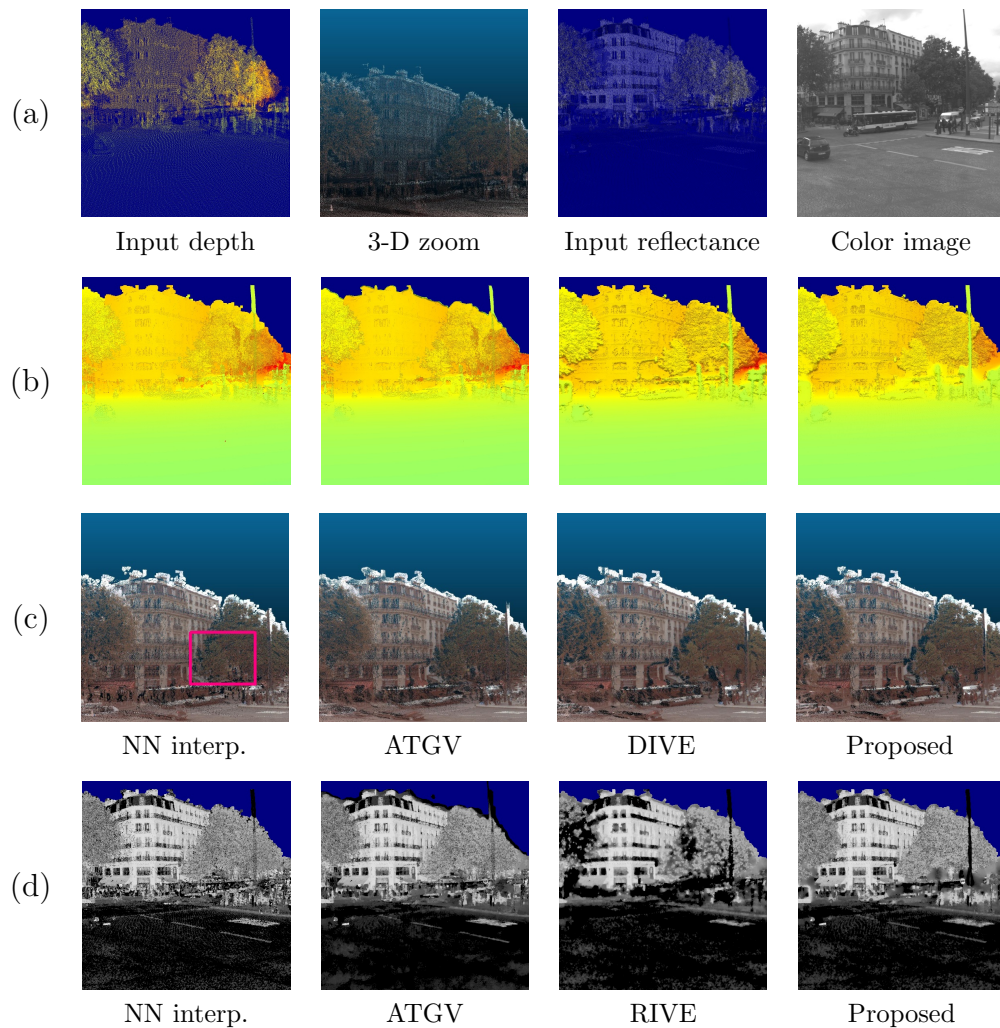


Figure 8: Visual results for the image *Buildings1*. Row (a) shows the related input images: depth (with a 3-D zoom), reflectance, and reference color image. Rows (b) and (c) report the results obtained in terms of inpainted depth images (with related 3-D zoomed-in view) with the algorithms indicated below. Row (d) shows the inpainted reflectance images obtained with different methods, our proposed method always reported as last.

452 case. Results on the reflectance image confirm the trend. By observing again
 453 the main marble pole, we clearly see that the reflectance is better inpainted.
 454 This is possible thanks to the joint use of depth information, which helps de-
 455 tecting hidden points by leveraging depth over- and under-estimations, and
 456 the coupling with the color image gradient, which helps correctly restoring

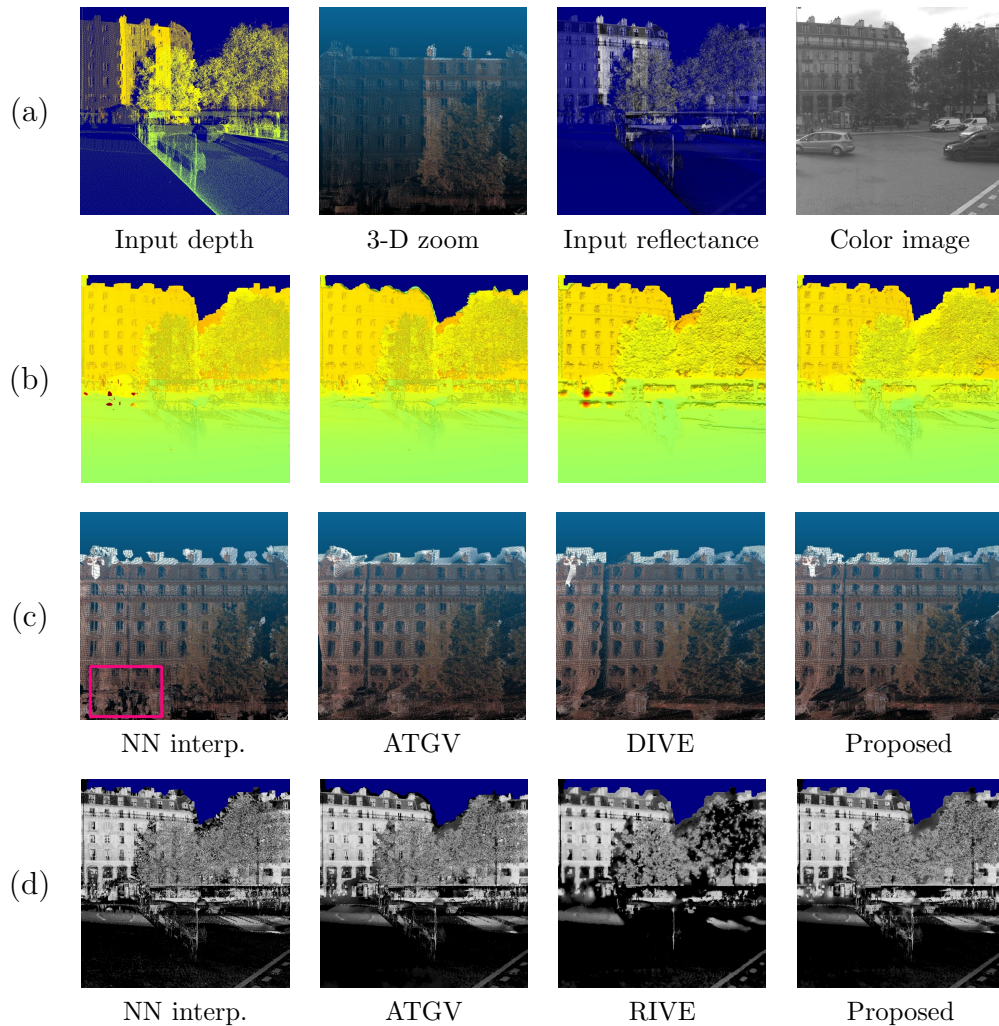


Figure 9: Visual results for the image *Buildings2*. Row (a) shows the related input images: depth (with a 3-D zoom), reflectance, and reference color image. Rows (b) and (c) report the results obtained in terms of inpainted depth images (with related 3-D zoomed-in view) with the algorithms indicated below. Row (d) shows the inpainted reflectance images obtained with different methods, our proposed method always reported as last.

457 the edges. Similar considerations can be made for the image *Column2* (visual
458 results are reported in Figure 7). Here the box overlaid on the 3-D views indi-
459 cates an area where points, non-visible from the reference image view point,
460 should be removed. The removal of these points, as well as the inpainting of
461 depth and reflectance, is performed more efficiently by our method.

462 Figures 8 and 9 show results w.r.t. two other images taken peripherally to
 463 the scene. For the image *Buildings1*, we can observe that with our algorithm
 464 the inpainted depth and reflectance images look more satisfactory, the pole on
 465 the left being completely unveiled as a foreground element. The box overlaid
 466 on the 3-D views highlights a part of the scene where the depth values of
 467 two trees interfere. Our proposed algorithm (as well as the DIVE method
 468 (Bevilacqua et al., 2016)) makes a correct distinction between the two depth
 469 layers. Figure 9, reporting results related to the image *Buildings2*, presents
 470 the problem of wrong lidar measures appearing in the front. Our method
 471 turns out to be the most effective in clearing out these points, as also shown
 472 in the area highlighted by the box.

473 5.3. Performance on visibility estimation

474 While in the previous section we evaluated the performance of the algo-
 475 rithm in terms of produced inpainted images u and r , we now want to assess
 476 the quality of the third output of the algorithm, i.e. v , the visibility attribute.

477 As visibility is estimated while performing the depth and reflectance es-
 478 timation, we can say that our algorithm fuses two problems: hidden point
 479 removal (HPR) and inpainting. Typically HPR is, instead, possibly per-
 480 formed as a preliminary operation. For HPR “stand-alone” the state of the
 481 art is represented by variations of (Katz et al., 2007) that relate the visible
 482 point set to the convex hull of a viewpoint-dependent transformation of it,
 483 discarding points based on a concavity threshold as seen from the view point.
 484 While this approach is effective, there is in general no globally satisfactory
 485 concavity threshold that would both correctly detect hidden surfaces and
 486 keep background points close to foreground silhouettes. To compare the two
 487 strategies for estimating visibility (the dedicated operation of (Katz et al.,
 488 2007) and our “soft” estimation), we show an example in Figure 10, related
 489 to the image *Column1*. In our case, we consider hidden points those depth
 490 values that are assigned $v = 0$ at the end of the algorithm. As for (Katz
 491 et al., 2007), a concavity parameter equal to 4 has been chosen after tuning.

492 The images obtained show that the “quality” of the visibility estimation
 493 process is comparable, if not higher with our method. If we observe closely
 494 the zoomed-in areas in Figure 10, in fact, we can see that the HPR method
 495 wrongly selects points around the silhouettes (see first patch), while some-
 496 times missing the detection of actual hidden points (see last two patches).

497 As a further test, we also compare our method (which jointly performs
 498 visibility estimation and inpainting), with a two-step approach, where visibil-

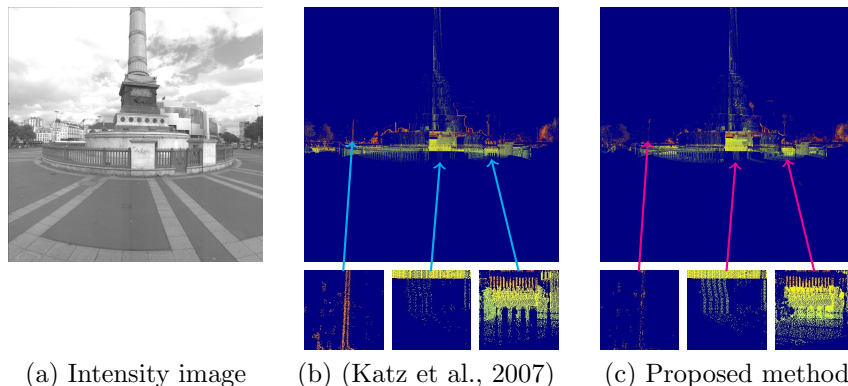


Figure 10: Detected hidden points in the case of the image *Column1*, by the state-of-the-art method of (Katz et al., 2007) and our method. The three patches below each image represent zoomed-in areas of the images themselves at same locations.

499 ity estimation (hidden point removal) is performed as a preliminary operation
 500 by the algorithm of (Katz et al., 2007). Depth is subsequently inpainted with
 501 the ATGV-based algorithm of (Ferstl et al., 2013). Figure 11 reports results
 502 for such comparison with two images, the two-step approach being denoted
 503 as “HPR + ATGV”.

504 In the two cases of Figure 11, we can observe a better outcome with our
 505 algorithm. For the image *Column1*, the preliminary point removal operation
 506 is not able to remove all the ambiguities in the central part of the image,
 507 where the depth values of the fence and the column are confused. For the
 508 image *Buildings2*, the HPR method of (Katz et al., 2007) exceeds in removing
 509 several points along the upper board of the image, causing blurred edges in
 510 the final reconstructed depth image. Besides the benefits observable in the
 511 qualitative assessment, the joint approach of our method has the advantage of
 512 not requiring an explicit parameter to be globally set (the concavity threshold
 513 in the case of (Katz et al., 2007)) to perform HPR. This is instead done in a
 514 “soft” way that adapts to the input image.

515 5.4. Quantitative evaluation with a benchmark data set

516 In this section we perform a quantitative analysis of our algorithm by
 517 using the publicly available KITTI Vision Benchmark Suite (Geiger et al.,
 518 2013). The KITTI suite includes data acquired by a MMS similar to the one
 519 considered for our personal data set in Section 5.2, and allows to evaluate al-
 520 gorithms on several applications. The vehicle used is a standard car equipped

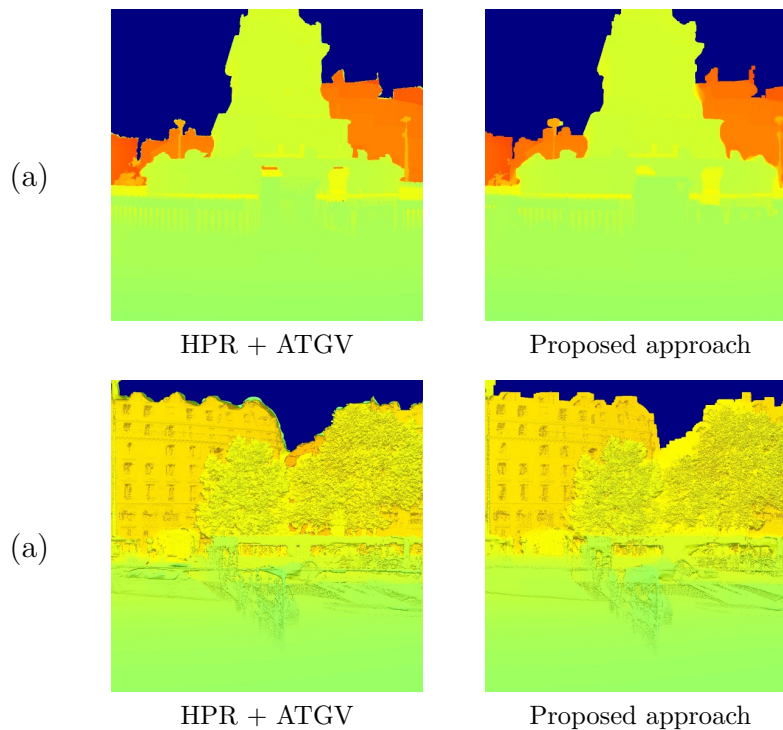


Figure 11: Comparison between our joint approach and a two-step approach, where visibility estimation and inpainting are performed separately, on the images *Column1* (a) and *Buldings2* (b).

521 with two high-resolution color and grayscale video cameras (enabling stereo
 522 vision), acquiring images with a pixel resolution equal to 1242×375 . Ac-
 523 curate 3-D measures are provided by a Velodyne lidar scanner and a GPS
 524 localization system. Thus, the lidar measures are generally used as ground
 525 truth for algorithm evaluations. In (Menze and Geiger, 2015) a novel data
 526 set is presented for stereo benchmarking, which considers also moving ob-
 527 jects. By making a special processing on the latter and manually remov-
 528 ing erroneous points due to occlusions, ground truth disparity maps are
 529 obtained. These maps appear “cleaner” and denser than the input depth
 530 images that can be obtained with the raw lidar data, and can therefore be
 531 used to evaluate algorithm estimating disparity. To exploit this possibility,
 532 as described in (Schneider et al., 2016, Sec. 4.3), we use the ground truth
 533 maps of this stereo benchmark data set to have a quantitative evaluation
 534 of our depth+reflectance inpainting algorithm. As done by the authors of

535 (Schneider et al., 2016), we identify 82 frames (provided ground truth dis-
 536 parity maps) for which we can find correspondences in the raw data set, i.e.
 537 a corresponding color image and related lidar point cloud. We then use the
 538 raw data lidar to compute an input depth (e.g., Figure 12a) and use the
 539 provided ground truth map to compute a Mean Absolute Error (MAE). The
 540 ground truth maps, although denser than the input maps, are sparse, i.e.
 541 they are not defined for all pixels (only about 19% of the pixels have values).
 542 Thus, the MAE is computed only for those pixels which are defined in the
 543 respective ground truth map.

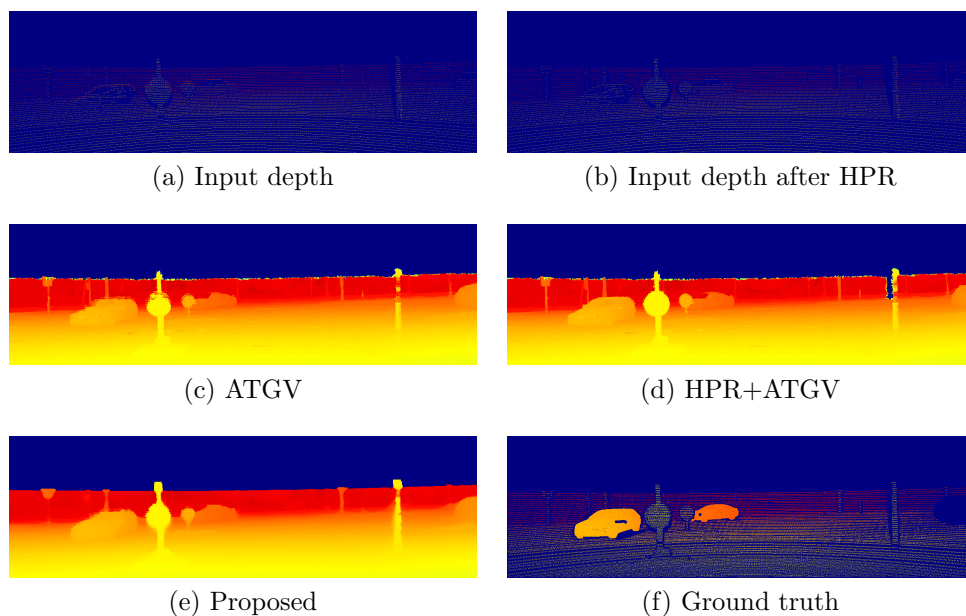


Figure 12: Case example from the 2015 KITTI stereo benchmark data set. For each input depth map (a), we have a ground truth disparity map available yet sparse (f), w.r.t. which it is possible to compute an error by only considering the pixels where it is defined. By applying a hidden point removal (HPR) algorithm to the input depth data it is possible to create a new input map where background hidden pixels have been removed (b). Results for different depth inpainting strategies are reported (c, d, e).

544 We computed the MAE for all 82 frames of the found correspondences, for
 545 our method and the ATGV-based algorithm of (Ferstl et al., 2013). As done
 546 in Section 5.3, we also compare with a two-step approach, where ATGV-
 547 based inpainting is preceded by a hidden point removal (HPR) operation,
 548 performed with the algorithm of (Katz et al., 2007). The resulting average
 549 MEAs, which are measured as the average pixel displacement between two

550 disparity maps, are reported in Table 1.

	ATGV	HPR+ATGV	Proposed
Average MAE (<i>px.</i>)	2.13	2.07	1.99

Table 1: Average Mean Absolute Error (MAE), i.e. average pixel displacement between ground truth and reconstructed disparity maps, obtained by averaging the results of 82 frames of the 2015 KITTI stereo benchmark data set.

551 When creating the ground truth maps, the authors of the KITTI bench-
552 mark data set have removed objects presenting particular issues in terms of
553 visibility. Other objects are instead manually handled (they are removed
554 from the scene and re-inserted after fitting a CAD model). Thus, the ground
555 truth maps basically consist of the latter and fixed parts of the scene (e.g.
556 streets and walls) that do not yield any ambiguity. Due to this relative
557 “simplicity” of the data set, the performance in terms of average MAE are
558 rather similar among the three methods (ATGV, HPR+ATGV, and proposed
559 method), with our method obtaining a slightly lower error. Nevertheless, we
560 can observe that the ATGV method of (Ferstl et al., 2013) produces more
561 artifacts (see, for example, the reconstructed pole on the left in Figure 12c,
562 in comparison to Figure 12e). Most of these artifacts can be removed by
563 performing a preliminary HPR step (see, in Figure 12b, an example of input
564 depth map cleaned out of ambiguous pixel). The combination of a HPR step
565 and the ATGV-based depth upsampling algorithm of (Ferstl et al., 2013)
566 yields inpainted depth maps with a visual quality comparable to the one of
567 our approach. However, as stated in Section 5.3, with our approach we keep
568 the advantage of having an all-in-one procedure performing jointly inpaint-
569 ing and “soft” visibility estimation (without the need of setting a per-image
570 global threshold as requested by the algorithm of (Katz et al., 2007)). We
571 also expect for our method a greater improvement of the MAE metric and
572 the visual outcome on more complex scenes.

573 6. Conclusion

574 In this paper we presented a novel strategy to jointly inpaint depth and
575 reflectance images with the guidance of a co-registered color image, and by
576 simultaneously estimating a visibility attribute for each pixel. The problem
577 studied and the proposed approach are particularly suited for data sets ac-
578 quired by Mobile Mapping Systems (MMS): vehicles that can easily image

579 urban scenes by means of optical cameras and lidar sensors. By projecting
580 the 3D lidar points onto a chosen reference image, we obtain depth and re-
581 flectance images, which suffer of practical issues due to the big diversity of
582 the lidar and optical sensor acquisitions. By estimating visibility, we aim at
583 solving one of these issues, i.e. the appearance (in depth and reflectance) of
584 parts of objects non-visible from the image view point, but captured by the
585 lidar sensor. Those points are meant to be detected by our algorithm and
586 thus discarded in the inpainting process. The proposed approach consists in
587 a variational optimization problem, where three variables (depth, reflectance,
588 and visibility) are simultaneously estimated. As a regularization term, a two-
589 fold coupled total variation (TV) term is proposed, where the gradients of
590 depth, reflectance and color image are matched two by two, by leveraging
591 the inherent correlation between them. The proposed algorithm is compared,
592 in terms of inpainted images, to other inpainting algorithms, which do not
593 take into account the simultaneous detection of possibly erroneous measures.
594 The clear superiority of the proposed method w.r.t. the latter proves that
595 the visibility estimation is a necessary step. Another comparison is made
596 with a simplified version of the algorithm, which accounts for visibility but
597 considers alternatively either depth or reflectance. The worse performance of
598 the simplified algorithm indicates that the joint exploitation of depth and re-
599 flectance is a key aspect for the success of the algorithm. The mutual benefit
600 comes from the fact that depth is particularly important for the visibility es-
601 timation task; in turn, reflectance is crucial in restoring the correct edges, via
602 coupling with the color image. Future work will continue in the direction of
603 solving practical issues with lidar-based images to inpaint. Notably, another
604 problem is related to disocclusions: the detection of mobile objects is in this
605 case necessary to prevent occlusions in the produced depth and reflectance
606 images. In this case, we could not only use each point measurement as a vote
607 that there is a surface at that point, but we could also exploit the informa-
608 tion that the line of sight from the sensor to that point is unobscured, e.g.
609 as done in (Xiao et al., 2015). Other work, concerning modifications to the
610 model presented in this paper, could consist in considering multiple depth
611 candidates per pixel when several points project into the same 2-D location
612 (instead of the Z-buffer approach currently used).

613 **Appendix A. Derivation of the proximal operators in Algorithm 1**

614 In this section we detail the derivation of the closed-form expressions of
 615 the proximal operators for the update of three primal variables (v , u , and r)
 616 in Algorithm 1, as listed in Section 4.2. Let $f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ be a closed
 617 proper convex function. The proximal operator or mapping $\text{prox}_f : \mathbb{R}^n \rightarrow \mathbb{R}^n$
 618 of f (Parikh and Boyd, 2013) is defined by:

$$\text{prox}_f(v) = \arg \min_{x \in \mathbb{R}^n} f(x) + \frac{1}{2} \|x - v\|_2^2 . \quad (\text{A.1})$$

619 Broadly speaking, the proximal operator of a function is a mathematical
 620 tool that allows to make an approximation to a certain value, while making
 621 a compromise between the accuracy of the approximation and a cost given
 622 by the function itself.

623 Given the general definition (A.1), we can derive the expressions for the
 624 proximal operators of the functions considered in our algorithm. We have
 625 that the operations involved are independent for each coordinate of the pro-
 626 cessed images. Therefore, the expressions reported below are to be intended
 627 per coordinate, although the spatial indices indicating a particular pixel lo-
 628 cation are not specified for brevity.

629 • $\boxed{\text{prox}_1 = \text{prox}_{\sigma D_1^*}(\tilde{p})}$

⇔

$$\begin{aligned} \text{prox}_1 &= \arg \min_p \frac{1}{2} (p - \tilde{p})^\top (p - \tilde{p}) - \sigma \beta^\top p + \delta_P(p) \\ &= \arg \min_p \frac{1}{2} p^\top p - \tilde{p}^\top p - \sigma \beta^\top p + K + \delta_P(p) \\ &= \arg \min_p \frac{1}{2} p^\top p - (\tilde{p} + \sigma \beta)^\top p + K + \delta_P(p) \\ &= \arg \min_p \frac{1}{2} (p - (\tilde{p} + \sigma \beta))^\top (p - (\tilde{p} + \sigma \beta)) + K' + \delta_P(p) \\ &= \frac{\tilde{p} + \sigma \beta}{\max(1, \|\tilde{p} + \sigma \beta\|_2)} \end{aligned} \quad (\text{A.2})$$

630 (The variable p , even considered for a single coordinate, is vector-
 631 valued.)

632 • $\boxed{\text{prox}_2 = \text{prox}_{\rho a(\bar{u}, \cdot) + \rho b(\bar{r}, \cdot) + \rho C}(\tilde{v})}$

⇔

$$\begin{aligned} \text{prox}_2 &= \arg \min_v \frac{1}{2}(v - \tilde{v})^2 + \rho\eta_1\Phi v \max(0, u_S - \bar{u}) \\ &\quad + \rho\eta_2\Phi v |\bar{r} - r_S| + \rho\alpha\Phi(1 - v) + \delta_{[0,1]}(v) \end{aligned} \quad (\text{A.3})$$

633

◆ If $\Phi_{i,j} = 0$ (point out of the sparse domain), we trivially have:

$$\begin{aligned} \text{prox}_2 &= \arg \min_v \frac{1}{2}(v - \tilde{v})^2 + \delta_{[0,1]}(v) \\ &= \mathcal{P}_{[0,1]}(\tilde{v}) . \end{aligned} \quad (\text{A.4})$$

634

◆ If $\Phi_{i,j} = 1$ and $\bar{u}_{i,j} \geq u_{S i,j}$, we have:

$$\begin{aligned} \text{prox}_2 &= \arg \min_v \frac{1}{2}(v - \tilde{v})^2 + \rho\eta_2v|\bar{r} - r_S| + \rho\alpha(1 - v) + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}v^2 - v\tilde{v} + \rho\eta_2v|\bar{r} - r_S| - \rho\alpha v + K + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}v^2 - v(\tilde{v} + \rho\alpha - \rho\eta_2|\bar{r} - r_S|) + K + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}[v - (\tilde{v} + \rho\alpha - \rho\eta_2|\bar{r} - r_S|)]^2 + K' + \delta_{[0,1]}(v) \\ &= \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_2|\bar{r} - r_S|) . \end{aligned} \quad (\text{A.5})$$

635

◆ If $\Phi_{i,j} = 1$ and $\bar{u}_{i,j} < u_{S i,j}$, we have:

$$\begin{aligned} \text{prox}_2 &= \arg \min_v \frac{1}{2}(v - \tilde{v})^2 + \rho\eta_1v(u_S - \bar{u}) + \rho\eta_2v|\bar{r} - r_S| \\ &\quad + \rho\alpha(1 - v) + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}v^2 - v\tilde{v} + \rho\eta_1v(u_S - \bar{u}) + \rho\eta_2v|\bar{r} - r_S| - \rho\alpha v \\ &\quad + K + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}v^2 - v(\tilde{v} + \rho\alpha - \rho\eta_1(u_S - \bar{u}) - \rho\eta_2|\bar{r} - r_S|) \\ &\quad + K + \delta_{[0,1]}(v) \\ &= \arg \min_v \frac{1}{2}[v - (\tilde{v} + \rho\alpha - \rho\eta_1(u_S - \bar{u}) - \rho\eta_2|\bar{r} - r_S|)]^2 \\ &\quad + K' + \delta_{[0,1]}(v) \\ &= \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_1(u_S - \bar{u}) - \rho\eta_2|\bar{r} - r_S|) . \end{aligned} \quad (\text{A.6})$$

636

◆ Summing up, we have:

$$\text{prox}_2 = \begin{cases} \mathcal{P}_{[0,1]}(\tilde{v}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_2|\bar{r} - r_S|) & \text{if } \Phi_{i,j} = 1, \bar{u}_{i,j} \leq u_{S i,j} \\ \mathcal{P}_{[0,1]}(\tilde{v} + \rho\alpha - \rho\eta_1(u_S - \bar{u}) - \rho\eta_2|\bar{r} - r_S|) & \text{if } \Phi_{i,j} = 1, \bar{u}_{i,j} > u_{S i,j} \end{cases} . \quad (\text{A.7})$$

637

- $\boxed{\text{prox}_3 = \text{prox}_{\tau A + \tau a(\cdot, v)}(\tilde{u})}$

⇔

$$\begin{aligned} \text{prox}_3 &= \arg \min_u \frac{1}{2}(u - \tilde{u})^2 + \tau\eta_1\Phi \max(0, u - u_S) \\ &\quad + \tau\eta_1\Phi v \max(0, u_S - u) + \delta_{[u_m, u_M]}(u) \end{aligned} \quad (\text{A.8})$$

638

◆ If $\Phi_{i,j} = 0$, we trivially have:

$$\begin{aligned} \text{prox}_3 &= \arg \min_u \frac{1}{2}(u - \tilde{u})^2 + \delta_{[u_m, u_M]}(u) \\ &= \mathcal{P}_{[u_m, u_M]}(\tilde{u}) . \end{aligned} \quad (\text{A.9})$$

639

◆ If $\Phi_{i,j} = 1$ and $u_{i,j} > u_{S i,j}$, we have:

$$\begin{aligned} \text{prox}_3 &= \arg \min_u \frac{1}{2}(u - \tilde{u})^2 + \tau\eta_1(u - u_S) + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}u^2 - u\tilde{u} + \tau\eta_1u + K + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}u^2 - u(\tilde{u} - \tau\eta_1) + K + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}[u - (\tilde{u} - \tau\eta_1)]^2 + K' + \delta_{[u_m, u_M]}(u) \\ &= \mathcal{P}_{[u_m, u_M]}(\tilde{u} - \tau\eta_1) . \end{aligned} \quad (\text{A.10})$$

By substituting the optimal value found for u in the splitting condition, we have:

$$u_{i,j} > u_{S i,j} \Rightarrow \tilde{u}_{i,j} > u_{S i,j} + \tau\eta_1 .$$

640

◆ If $\Phi_{i,j} = 1$ and $u_{i,j} < u_{S i,j}$, we have:

$$\begin{aligned} \text{prox}_3 &= \arg \min_u \frac{1}{2}(u - \tilde{u})^2 + \tau\eta_1v(u_S - u) + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}u^2 - u\tilde{u} - \tau\eta_1vu + K + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}u^2 - u(\tilde{u} + v\tau\eta_1) + K + \delta_{[u_m, u_M]}(u) \\ &= \arg \min_u \frac{1}{2}[u - (\tilde{u} + v\tau\eta_1)]^2 + K' + \delta_{[u_m, u_M]}(u) \\ &= \mathcal{P}_{[u_m, u_M]}(\tilde{u} + v\tau\eta_1) . \end{aligned} \quad (\text{A.11})$$

By substituting the optimal value found for u in the splitting condition, we have:

$$u_{i,j} < u_{S i,j} \Rightarrow \tilde{u}_{i,j} < u_{S i,j} - v\tau\eta_1 .$$

641 \blacklozenge The remaining case is: $\Phi_{i,j} = 1$ and $u_{i,j} = u_{S i,j}$. This directly
642 implies the solution for the proximal operator:

$$\text{prox}_2 = \mathcal{P}_{[u_m, u_M]}(u_S) . \quad (\text{A.12})$$

From the previous cases, we can derive the related validity condition on the calculation point $\tilde{u}_{i,j}$, i.e.:

$$-v\tau\eta_1 < \tilde{u}_{i,j} - u_{S i,j} < \tau\eta_1 .$$

643 \blacklozenge Summing up, we have:

$$\text{prox}_3 = \begin{cases} \mathcal{P}_{[u_m, u_M]}(\tilde{u}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[u_m, u_M]}(\tilde{u} - \tau\eta_1) & \text{if } \Phi_{i,j} = 1, \tilde{u}_{i,j} > u_{S i,j} + \tau\eta_1 \\ \mathcal{P}_{[u_m, u_M]}(\tilde{u} + v\tau\eta_1) & \text{if } \Phi_{i,j} = 1, \tilde{u}_{i,j} < u_{S i,j} - v\tau\eta_1 \\ \mathcal{P}_{[u_m, u_M]}(u_S) & \text{otherwise} \end{cases} \quad (\text{A.13})$$

644 \bullet $\boxed{\text{prox}_4 = \text{prox}_{\tau B + \tau b(\cdot, v)}(\tilde{r})}$

\Leftrightarrow

$$\text{prox}_4 = \arg \min_r \frac{1}{2}(r - \tilde{r})^2 + \tau\eta_2\Phi v|r - r_S| + \delta_{[r_m, r_M]}(r) \quad (\text{A.14})$$

645 \blacklozenge If $\Phi_{i,j} = 0$, we trivially have:

$$\begin{aligned} \text{prox}_4 &= \arg \min_r \frac{1}{2}(r - \tilde{r})^2 + \delta_{[r_m, r_M]}(r) \\ &= \mathcal{P}_{[r_m, r_M]}(\tilde{r}) . \end{aligned} \quad (\text{A.15})$$

646

◆ If $\Phi_{i,j} = 1$ and $r_{i,j} > r_{S i,j}$, we have:

$$\begin{aligned}
\text{prox}_4 &= \arg \min_r \frac{1}{2}(r - \tilde{r})^2 + \tau\eta_2 v(r - r_S) + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}r^2 - r\tilde{r} + v\tau\eta_2 r + K + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}r^2 - r(\tilde{r} - v\tau\eta_2) + K + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}[r - (\tilde{r} - v\tau\eta_2)]^2 + K' + \delta_{[r_m, r_M]}(r) \\
&= \mathcal{P}_{[r_m, r_M]}(\tilde{r} - v\tau\eta_2) .
\end{aligned} \tag{A.16}$$

By substituting the optimal value found for r in the splitting condition, we have:

$$r_{i,j} > r_{S i,j} \Rightarrow \tilde{r}_{i,j} > r_{S i,j} + v\tau\eta_1 .$$

647

◆ If $\Phi_{i,j} = 1$ and $r_{i,j} < r_{S i,j}$, we have:

$$\begin{aligned}
\text{prox}_4 &= \arg \min_r \frac{1}{2}(r - \tilde{r})^2 + \tau\eta_2 v(r_S - r) + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}r^2 - r\tilde{r} - v\tau\eta_2 r + K + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}r^2 - r(\tilde{r} + v\tau\eta_2) + K + \delta_{[r_m, r_M]}(r) \\
&= \arg \min_r \frac{1}{2}[r - (\tilde{r} + v\tau\eta_2)]^2 + K' + \delta_{[r_m, r_M]}(r) \\
&= \mathcal{P}_{[r_m, r_M]}(\tilde{r} + v\tau\eta_2) .
\end{aligned} \tag{A.17}$$

By substituting the optimal value found for r in the splitting condition, we have:

$$r_{i,j} < r_{S i,j} \Rightarrow \tilde{r}_{i,j} < r_{S i,j} - v\tau\eta_1 .$$

648

◆ The remaining case is: $\Phi_{i,j} = 1$ and $r_{i,j} = r_{S i,j}$. This directly implies the solution for the proximal operator:

649

$$\text{prox}_3 = \mathcal{P}_{[r_m, r_M]}(r_S) . \tag{A.18}$$

From the previous cases, we can derive the related validity condition on the calculation point $\tilde{r}_{i,j}$, i.e.:

$$|\tilde{r}_{i,j} - r_{S i,j}| < v\tau\eta_2 .$$

◆ Summing up, we have:

$$\text{prox}_3 = \begin{cases} \mathcal{P}_{[r_m, r_M]}(\tilde{r}) & \text{if } \Phi_{i,j} = 0 \\ \mathcal{P}_{[r_m, r_M]}(\tilde{r} - v\tau\eta_2) & \text{if } \Phi_{i,j} = 1, \tilde{r}_{i,j} > r_{S i,j} + v\tau\eta_2 \\ \mathcal{P}_{[r_m, r_M]}(\tilde{r} + v\tau\eta_2) & \text{if } \Phi_{i,j} = 1, \tilde{r}_{i,j} < r_{S i,j} - v\tau\eta_2 \\ \mathcal{P}_{[r_m, r_M]}(r_S) & \text{otherwise} \end{cases} \quad (\text{A.19})$$

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