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Ultrasonic characterization of orthotropic elastic bovine bones

P. Lasaygues *, M. Pithioux

Laboratoire de Mécanique et Acoustique, CNRS, 31 Chemin Joseph Aiguier, 13402 Marseille Cedex 20, France

The aim of the present study was to determine the mechanical properties of bovine bones. An ultrasonic method was used to determine acoustical parameters such as the longitudinal and transverse velocities in the longitudinal and two radial directions of compact bone, i.e., in all directions of the plane. Waves propagating through bovine femoral bones were studied using an ultrasonic scanner for linear and sectorial scanning. The mechanical parameters of compact bone, such the Young's modulus and Poisson's ratio in the orthotropic case, were then determined from the measured velocities. The results are in line with those in the literature.

Keywords: Ultrasound; Compact bovine bone characterization; Orthotropic stiffness tensor; Elastic constants; Longitudinal velocity; Transversal velocity

1. Introduction

In biomechanics, one of the principal descriptors of the mechanical properties of a bone is its elastic properties. More recently, the elastic constants were used in conjunction with (micro-)mechanical modeling to elucidate details of bone (micro-)structure, which governs failure or remodeling of bones, for example. We now construct a numerical model of the microscopic behaviour of compact bone to analyse damage and failure of compact bones. We need accurate knowledge of the elastic constants of compact bone for this model. The elastic characteristics of bones are conventionally measured by mechanical tests (compression, tension, flexion, and torsion). The damage sustained by the sample with these techniques makes it difficult to test it in other directions or to use it again for other measurements. In this study, the elastic characteristics of compact bone were measured by an ultrasonic method. The theory of elastic waves is essential because it relates the elastic

moduli of a material to the velocity of propagation of these waves along arbitrary directions in a solid.

In this paper, we describe the inversion of ultrasonic wavespeed measurements on a bovine bone to determine its orthotropy and the appropriate number of elastic constants needed to represent its elastic properties. Similar methods of this kind have been used to study compact as well as porous bones [1–11]. The method developed is classic because many authors have tried to apply the ultrasonic method. In the bibliography, a few authors measured the longitudinal velocities only. Others measured the longitudinal and transverse velocities, but in this case, transducers are in contact with the samples, and the samples are cubes with parallel faces that are difficult to prepare in a bone. One advantage of our study is that it yields rapid and accurate results, and does not require the use of samples with precise dimensions and perfectly parallel faces: rough preparations of the specimen are sufficient. The area under investigation must have interfaces, which are approximately parallel (focus of the transducers). Another important feature of the method is that it is non-destructive. Furthermore, this study is reproducible because we consistently used the same experimental procedure on bone samples from animals of the same age (about five years), sex (female), and weight (about 400 kg).

* Corresponding author. Tel.: +33-49116-4277; fax: +33-49122-8248.

E-mail address: lasaygues@lma.cnrs-mrs.fr (P. Lasaygues).

Waves propagating both longitudinally and transversally through bovine femoral bones in all planar directions were used here, and the experimental device developed at the Laboratoire de Mécanique et d'Acoustique (LMA) for ultrasonic medical imaging [12,13] [29,30] was extended to include the measurement of their mechanical properties. The acoustic device used allows various degrees of freedom, since the position of both the target and the transducers can be adjusted. In particular, one can apply rotation to the target, and move the transducer receiver laterally. This makes it possible to monitor the wave propagation occurring in a system, which obeys the Snell–Descartes laws, and also measure shear waves and determine their velocities in planar directions. The latter point is original in ultrasonic methods.

The bench was designed for performing both reflection and transmission measurements.

A brief summary follows of the basic theory and its application in the conventional approach for determining elastic moduli in materials. The mechanical parameters of bovine bone were then determined experimentally at various points on different specimens.

2. Conventional approaches

For a linear, anisotropic material, Hooke's law translates the wave equation by the following equation:

$$\rho \ddot{u}_i = C_{ijkl} \varepsilon_{kl,j} = C_{ijkl} u_{k,lj} \quad i = 1, 2, 3 \quad (1)$$

where ρ is the density, \ddot{u}_i the i th component of the acceleration of the volume, and ε_{kl} is the component of the strain infinitesimal tensor. C_{ijkl} is the fourth-rank symmetric stiffness tensor of the material. If a positive-definite strain energy function is postulated for the material, C_{ijkl} will have only 21 independent components.

For the cortical bovine bone microstructure, a hexagonal symmetry is identified, and only nine independent elements were determined.

In many cases, the elastic wave measurements will rely on plane waves. A plane wave solution to the equation of motion is expressed in terms of the planar displacements specified by $\vec{u}(\vec{x}, t)$, which are propagating in the direction specified by the unit vector, the wave normal \vec{p} . Hence

$$\vec{u}(\vec{x}, t) = \vec{A} \exp \left(i \omega t - \frac{i \vec{x} \cdot \vec{p}}{v} \right) \quad (2)$$

where $\vec{A} = A \vec{d}$ and A is the wave amplitude and \vec{d} the polarization vector. \vec{x} is the position vector. $\omega = v k$ the angular frequency of the wave and k the wave number.

By replacing (2) in (1), we obtain an equation with ρv^2 the eigenvalues. The resolution of this eigenvalue problem is classic today and well-optimized [14–16]. The aim of our work is not the resolution of the method but, more prosaically, to apply the method to the mechanical characterization of bovine bones which have a very complex structure. In this study, bones are considered orthotropic elastic material. The paragraphs which follow rapidly explain the algorithm we use.

By introducing the Kelvin–Christoffel stiffnesses, $\Gamma_{ik} \equiv C_{ijkl} p_j p_l$, the eigenvalues are the solution of the characteristic equation $\det |\Gamma_{ik} - \rho v^2 \delta_{ik}| = 0$, where v is the phase velocity of the wave.

The useful bone is orthotropic and the specific fibre direction is x_3 -plane. In terms of abbreviated Voigt notation, the corresponding Kelvin–Christoffel stiffnesses are

$$\begin{aligned} \Gamma_{11} &= p_1^2 C_{11} + p_2^2 C_{66} + p_3^2 C_{55} & \Gamma_{23} &= p_2 p_3 (C_{23} + C_{44}) \\ \Gamma_{22} &= p_1^2 C_{66} + p_2^2 C_{22} + p_3^2 C_{44} & \Gamma_{13} &= p_1 p_3 (C_{13} + C_{55}) \\ \Gamma_{33} &= p_1^2 C_{55} + p_2^2 C_{44} + p_3^2 C_{33} & \Gamma_{12} &= p_1 p_2 (C_{12} + C_{66}) \end{aligned} \quad (3)$$

If v_L , respectively v_T , indicate the longitudinal velocity, respectively the transversal velocity, along the x_1 -direction, we deduce coefficients in which the plane propagates along the $p_1 = 1$ direction (i.e. $p_2 = p_3 = 0$) of the material: $C_{11} = \rho v_L^2$ and $C_{66} = \rho v_T^2$.

Similarly, if v_L and v_T are the corresponding wave velocities, respectively in pure longitudinal mode and in pure transverse mode in the x_2 -plane, the C_{22} coefficient is directly obtained by a normal incidence measurement in the x_2 -plane: $C_{22} = \rho v_L^2$ and $C_{44} = \rho v_T^2$.

The coefficient C_{12} , respectively C_{23} , are obtained for waves sent along the direction at 45° to the x_1 - and x_2 -axes (i.e. $p_1 = p_2 = 1/\sqrt{2}$, $p_3 = 0$), respectively to the x_2 - and x_3 -axes (i.e. $p_1 = 0$, $p_2 = p_3 = 1/\sqrt{2}$).

In the $x_1 x_3$ -plane with the fibres, and for a given direction, $\vec{p} = (\cos \theta, 0, \sin \theta)$ where θ is the refraction angle, we solve the characteristic equation and find:

$$\begin{vmatrix} C_{11} \cos^2 \theta + C_{55} \sin^2 \theta - \eta & 0 & (C_{13} + C_{55}) \cos \theta \sin \theta \\ 0 & C_{66} \cos^2 \theta + C_{44} \sin^2 \theta - \eta & 0 \\ (C_{13} + C_{55}) \cos \theta \sin \theta & 0 & C_{55} \cos^2 \theta + C_{33} \sin^2 \theta - \eta \end{vmatrix} = 0 \quad (4)$$

where $\eta = \rho v = \rho C_b^2$, the eigenvalues to be determined. The resolution of this determinant is presented in Appendix A.

3. Measurement system and experimental testing configuration

It should be clear from the foregoing that wavespeed data should be collected over a broad range of arbitrary directions in a specimen in order to recover the whole set of elastic constants. A number of ultrasonic measurement techniques have been developed in which the transducer is in contact with the specimen [17,18]. In our case the specimen is immersed in a water bath and the speeds of propagation of waves at various incident angles correspondingly refracted into the specimen are measured.

Accordingly, the general architecture (Fig. 1) of the mechanical system is composed of a main symmetric arm that holds two transverse arms with which two transducers can be moved in parallel angular scanning is carried out by rotating either the main arm or the object holder carries out. The transducers can be positioned and oriented accurately, permitting both linear and sectorial scanning. All the movements are produced by six stepping motors sequentially driven by a programmable translator-indexer device fitted with a power multiplexer. The translator-indexer device and the power multiplexer are integrated in a control rack that also includes other remote controls, such as that adjusting the distance traveled by the transverse arms, or the out-of-water setting automatism. The increments are multi-

ples of 0.75×10^{-2} mm for translations and of $(1 \times 10^{-2})^\circ$ for rotations [13].

To obtain accurate results by taking bone heterogeneity into account, we used a higher frequency (1 MHz) and focused transducers. Focused transducers made it possible to target small surfaces and perform measurements at different points on the bone sample.

For each point, we carry out a sweep of 5 mm. This sweep of focused transducers synthesizes an antenna of several elements. The front of the synthetic wave course medium following a slice of height corresponds to the vertical resolution of the transducers. Consequently, the “wave plane” and 2D diagram hypothesis are respected.

4. Identification of bovine bone elastic constants

4.1. In vitro wavespeed measurements

Bovine bones were assumed to be elastically orthotropic. x_3 denotes the axis parallel to the fibres of the bone (Fig. 2), and x_2 the axis to be parallel to the thickness of the compact area of the bone.

Four fresh bovine femoral bones were studied. The bones were frozen prior to the experiments. The epiphyses were cut so that we could focus only on compact bone. Two test-pieces were obtained by first cutting bones in the axial direction and then removing the marrow from each part (Fig. 2a and b). Then, to investigate this orthotropic specimen, we cut out small samples (Fig. 2c–e) and set them in water at room temperature. The robot held each sample in either the horizontal or the vertical position, depending on the type of experiment.

We studied the density of bovine bone samples which are very compact. To do this, we used two significant measurements. The first, based on the Archimedes principle, gives the average density of each sample. The second, using X-ray tomodensitometry, gives a local density in the plane perpendicular to the wave front [30]. Analysis of the results shows that the overall measurement corresponds to the arithmetic mean of the values recorded by X-ray tomodensitometry. With X-ray tomodensitometry, we obtain a narrow variation of bovine compact bone density. This can be explained by the lamellar structure of bovine bones, so that the fibres constituting the bone are all oriented in the same direction. The compact zone of the bovine bones is then considered homogeneous or weakly heterogeneous. We can also take ultrasonic measurements of various points, relatively brought closer together, allowing us, strictly speaking, to take account of this slight heterogeneity of the medium. Consequently, for each sample and each measurement point, the density used for calculations is the total value average (Tables 1 and 2).

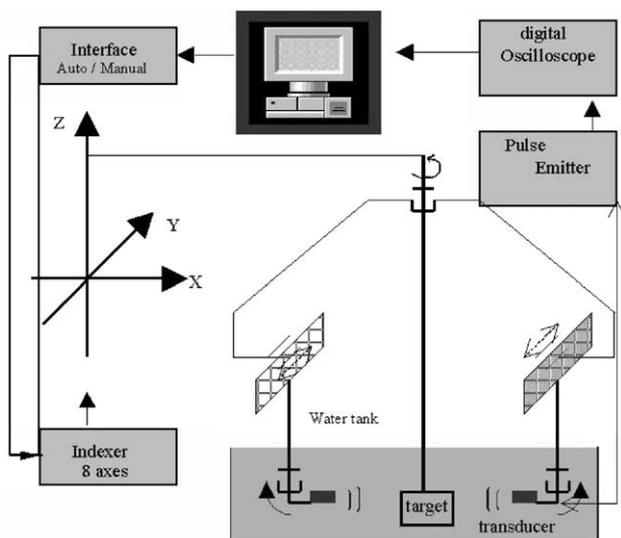


Fig. 1. Experimental measurement system.

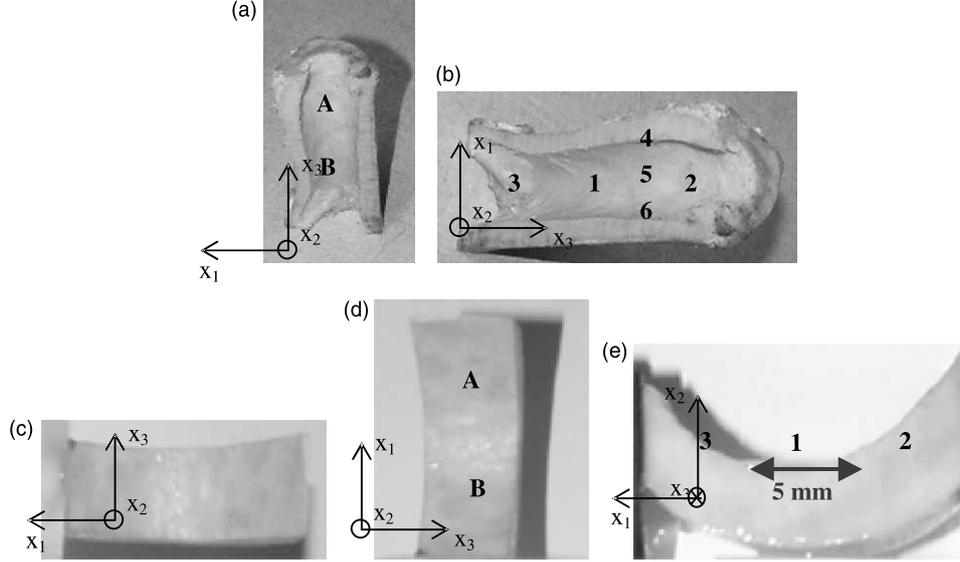


Fig. 2. Direction of observation of bovine bone samples: (a) vertical position, (b) horizontal position, (c) radial direction (vertical position), (d) radial direction (horizontal position), (e) longitudinal direction (axis $0x_3$).

Table 1
Longitudinal (v_L) and transverse (v_T) velocities in the ($0x_2$) direction

Experiments	v_L (m/s)	v_T (m/s)
<i>Bone 1</i>		
Horizontal position	3383 ± 20	2048 ± 45
Point 1	3315 ± 20	2002 ± 34
Point 2	3655 ± 23	1888 ± 28
Point 3	3191 ± 22	2026 ± 38
Vertical position	3247 ± 15	1713 ± 30
Point A	3055 ± 18	1767 ± 29
Point B	3460 ± 21	1773 ± 25
<i>Bone 2</i>		
Point A	3414 ± 16	1822 ± 25
Point B	3430 ± 16	1833 ± 26
Point 1	3352 ± 16	1953 ± 24
Point 2	3347 ± 16	1953 ± 50
Point 3	3317 ± 16	1953 ± 41
Point 4	3487 ± 21	2067 ± 44
Point 5	3352 ± 20	1949 ± 40
Point 6	3583 ± 21	1997 ± 36

The density of the two samples is $1.8 \times 10^3 \text{ kg/m}^3$.

Table 2
Longitudinal and transverse measurements in different directions (orthotropic)

Experiments	v_L (m/s)	v_T (m/s)	ρ (kg/m^3)
Bone 3 (longitudinal direction, axis $0x_3$)	4271 ± 20	2065 ± 50	2.1×10^3
Bone 3 (radial direction, vertical position, plane $0x_2x_3$)	3235 ± 15	1686 ± 20	2.1×10^3
Bone 3 (radial direction, horizontal position, plane $0x_1x_2$)	3321 ± 15	1981 ± 31	2.1×10^3
Bone 3 (radial direction, axis $0x_1$)	3515 ± 16	2082 ± 40	2.1×10^3
Bone 4 (longitudinal direction, axis $0x_3$):	4027 ± 13	1981 ± 35	1.9×10^3
Bone 4 (radial direction, vertical position, plane $0x_2x_3$)	3349 ± 11	1730 ± 20	1.9×10^3
Bone 4 (radial direction, horizontal position, plane $0x_1x_2$):	3350 ± 11	1976 ± 37	1.9×10^3
Bone 4 (radial direction (axis $0x_1$))	3472 ± 11	1973 ± 40	1.9×10^3

We then determined the acoustical thickness e by

$$e = d - v_w \left(\frac{t_1 + t_2}{2} \right) \quad (5)$$

where d is the distance between the two transducers, t_1 and t_2 are the time taken by the reflected echo to travel between each of the bone surfaces taken by the wave system, and v_w is the water velocity (Fig. 3a).

We finally determined the longitudinal and transverse celerity v

$$v = \frac{v_w}{\sqrt{1 + \frac{v_w \Delta t}{e} \left(\frac{v_w \Delta t}{e} - 2 \cos \theta \right)}} \quad (6)$$

where Δt is obtained by cross correlation between the reference signal without the sample and the signal obtained at the angle of incidence with the sample (Fig. 4), and θ is the angle of incidence of the emitted wave (Fig. 3b).

Using the isotropic hypothesis, assumed by many authors, the transverse velocity is expressed as half the

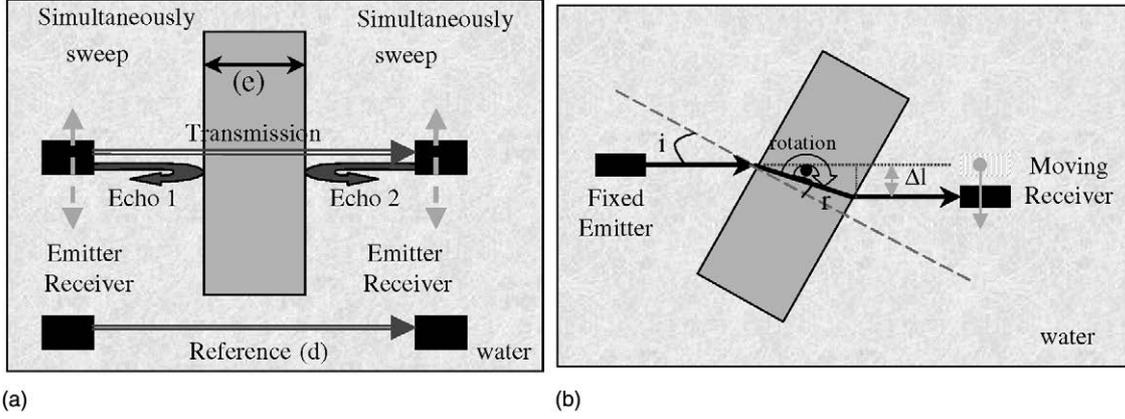


Fig. 3. Principle of wave recording in (a) echo mode and (b) transmission mode.

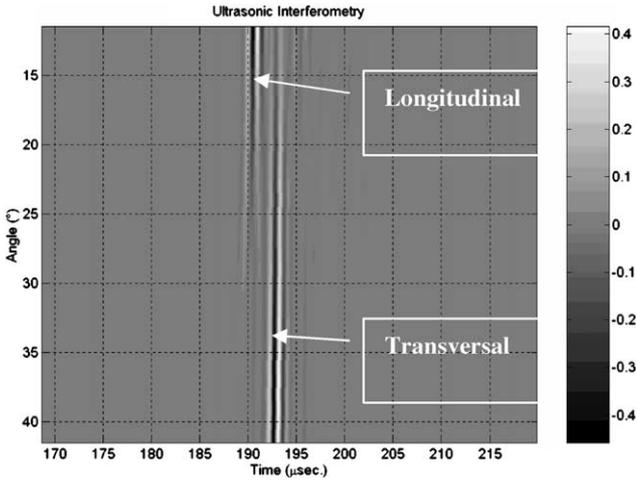


Fig. 4. Set of transmission signals versus incidence angle obtained with one sample at 1 MHz.

longitudinal velocity: $v_T = v_L/2$. This leads to a constant Poisson's ratio $\nu = 0.33$.

In our case, to observe transverse and longitudinal waves separately and because the critical θ_c was between $25^\circ \leq \theta_c \leq 40^\circ$, we applied a 60° rotation to the bone samples. To accurately describe the bone heterogeneity, measurements were carried out at several points (Fig. 2a and b) with samples in the vertical and horizontal positions (Table 1).

4.2. Recovery of elastic constants

Both v_L and v_T were measured in the radial planes and in the longitudinal axis of the bone. The ultrasonic method was used to measure longitudinal and transverse velocities in the radial and fibre (longitudinal) directions. Their thickness ranged between 1 and 2 cm (Fig. 2).

The results for celerity are in given Table 2 and the matrix rigidity values in Table 3. To validate our results,

Table 3
Elastic constants of bovine bone (orthotropic)

	Bone 3	Bone 4
C_{11} (GPa)	23.5	22
C_{22} (GPa)	26	23.5
C_{12} (GPa)	6.55	7.6
C_{13} (GPa)	8.35	7.5
C_{23} (GPa)	8.2	7.7
C_{33} (GPa)	34.6	31.7
C_{44} (GPa)	9.2	7.6
C_{55} (GPa)	6	5.6
C_{66} (GPa)	6.05	5.8
E_1 (GPa)	20.6	18.7
E_2 (GPa)	23.4	20
E_3 (GPa)	30.2	28
G_{12} (GPa)	3	2.9
G_{13} (GPa)	3	2.8
G_{23} (GPa)	4.6	3.7
ν_{12}	0.12	0.26
ν_{13}	0.2	0.17
ν_{21}	0.21	0.28
ν_{23}	0.18	0.17
ν_{31}	0.29	0.26
ν_{32}	0.24	0.25

we compared them with the literature [1,3,5,6,10,19–28]. A fairly wide dispersion was observed in the Poisson's ratio, Young's modulus and longitudinal velocities obtained on compact bone. For example, Poisson's ratio ranged between 0.16 and 0.35. In the few studies in which Poisson's ratio has been determined, the values ranged between 0.1 and 0.5. In the literature, the longitudinal velocity values varies between 2700 and 4200 m/s; in our studies they ranged between 2900 and 4500 m/s. Elastic constants were also compared with figures in the literature. Our results are quite in line with the literature.

5. Conclusion

The ultrasonic method proposed here is an original approach to the study of bone characteristics because

the bone is free to rotate around the clamp axis. This makes it possible to monitor wave propagation, to measure shear waves, and to determine the velocities of these waves (C_T) in all the longitudinal and radial directions and for all angles of rotation of the samples. The latter point is an original feature that conventional ultrasonic methods do not provide.

The longitudinal velocity and Young's modulus obtained here are in line with the figures in the literature. Basic assumption is that bone is orthotropic. Here, the values of the longitudinal velocity in the fibre direction (4000–4400 m/s) were greater than those in the radial direction (3000–3600 m/s). Young's modulus therefore ranged from 14 to 21 GPa in the radial direction and 20 to 25 GPa in the longitudinal direction. Poisson's ratio ranged from 0.16 to 0.32 in the radial direction and 0.34 to 0.35 in the longitudinal direction. For the future, as only a few measurement points were used in the present experiments, it was difficult to finely analyze the distribution of the bone characteristics in different regions of the samples. One approach consists in establishing the velocity map of the sample.

Moreover, the viscosity in compact bones should be investigated to determine whether any dispersion occurs. This study should be performed using a method to determine the velocities and attenuation at various frequencies. We also recommend applying these results to burnt quantitative images of compact, cancellous bones and osteoporosis bones [29]. For human bones we could then take the age, weight, etc. of the person into account. Finally, the results of this study can be used in a numerical model of bones to analyze structural failure [30].

Appendix A. Resolution of the characteristic equation

This appendix presents the various steps in the resolution of characteristic equation Eq. (4).

The resolution of the determinant gives the following equation:

$$(\beta - \rho C_b^2)(\rho^2 C_b^4 - \rho C_b^2 A + B) = 0 \quad (\text{A.1})$$

where

$$\begin{cases} A = C_{11} \cos^2 \theta + C_{33} \sin^2 \theta + C_{55} \\ B = (C_{11} \cos^2 \theta + C_{55} \sin^2 \theta)(C_{55} \cos^2 \theta + C_{33} \sin^2 \theta) \\ \quad - (C_{13} + C_{55})^2 \cos^2 \theta \sin^2 \theta \\ \beta = C_{66} \cos^2 \theta + C_{44} \sin^2 \theta \end{cases}$$

Finally, the system to solve is

$$\begin{cases} \beta - \rho C_b^2 = 0 \\ (\rho^2 C_b^4 - \rho C_b^2 A + B) = 0 \end{cases} \quad (\text{A.2})$$

The first equation gives the first eigenvalue β . The corresponding unit eigenvector component is $[0, 1, 0]$, and

this vector is orthogonal to the excitation plane ($x_1, 0, x_3$). This is the transverse mode not exciting in fluid.

The second equation gives the other two eigenvalues

$$2\rho C_b^2 = A \pm \sqrt{A^2 - 4B} \quad (\text{A.3})$$

The plus sign corresponds to the quasi-longitudinal wave and the minus sign corresponds to the quasi-transverse wave.

To determine coefficients C_{33} , C_{55} , and C_{13} , let us assume $Y_1 = C_{33}$, $Y_2 = C_{55}$, and $Y_3 = C_{13}$, and $a = C_{11}$ (known).

Let $\alpha_k = \cos^2 \theta_k$ and $b_k = \rho C_{bk}$ be the result of a k -measurement, with θ_k the refraction angle and C_{bk} the celerity of the longitudinal or transverse wave. We found the following equation:

$$\begin{aligned} f_k(Y_1, Y_2, Y_3) = & A_1^k Y_1 + A_2^k Y_2 + A_3^k Y_3^2 + A_4^k Y_1 Y_2 \\ & + 2A_3^k Y_2 Y_3 + A_5^k \end{aligned} \quad (\text{A.4})$$

where

$$\begin{cases} A_1^k = (1 - \alpha_k)(a\alpha_k - b_k) \\ A_2^k = a\alpha_k^2 - b_k \\ A_3^k = -\alpha_k(1 - \alpha_k) \\ A_4^k = (1 - \alpha_k)^2 \\ A_5^k = -b_k(a\alpha_k - b_k) \end{cases} \quad (\text{A.5})$$

and $k \in \{1, N\}$ with N the total number of measurements. Eq. (A.4) gives rise to an N -equations system with three unknowns.

To find Y_1 , Y_2 , and Y_3 , we define the Euclidean norm and the functional $F(Y) = \sum_k \beta_k (f_k(X))^2$ where β_k ($0 \leq \beta_k \leq 1$) is a weighting factor that can be used to increase or decrease the influence of measure k . X is the vector of the unknown, which is minimized using a Newton method. The X_i determined are good approximations of the zeros of the functional F . From all these calculations, we obtain:

$$\max_{i \leq 3} |F(X_i)| \leq 10^{-11} \quad (\text{A.6})$$

It is well known that the first-arrival signal detected in an ultrasonic waveform is usually most easily identified in a waveform, provided the source and receiver separation is not too great. The question then is what relationship exists between a material's elastic constants and this longitudinal wave signal [14,15]. To determine this, the Kelvin-Christoffel stiffnesses are separated into an isotropic and a small anisotropic component. The functional dependence between the wavespeed and the elastic constants is similar for phase and group velocity values [15]. For an orthotropic solid, there are particularly simple relations between the ultrasonic phase velocities measured along particular directions and the elastic constants [14,15]. Hence, measuring all the wavespeeds, longitudinal and shear, along each of the three principal orthogonal acoustic axes of the material, makes it possible to determine six of nine elastic moduli

needed to completely characterize an orthotropic material. These results are tabulated in [16].

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