HAL
open science

Effect of an attached end mass in the dynamics of uncertainty nonlinear continuous random system

Americo Cunha Jr, Rubens Sampaio

To cite this version:

Americo Cunha Jr, Rubens Sampaio. Effect of an attached end mass in the dynamics of uncertainty nonlinear continuous random system. Mecánica Computacional, 2012, XXXI, pp.2673-2683. hal-01438656

HAL Id: hal-01438656

https://hal.science/hal-01438656v1

Submitted on 17 Jan 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Copyright

Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXI, págs. 2673-2683 (artículo completo)
Alberto Cardona, Paul H. Kohan, Ricardo D. Quinteros, Mario A. Storti (Eds.)
Salta, Argentina, 13-16 Noviembre 2012

EFFECT OF AN ATTACHED END MASS IN THE DYNAMICS OF UNCERTAINTY NONLINEAR CONTINUOUS RANDOM SYSTEM

Americo Barbosa da Cunha Jr and Rubens Sampaio

Department of Mechanical Engineering, PUC–Rio
Rua Marquês de São Vicente, 225, Gávea, Rio de Janeiro - RJ, Brazil

Keywords: nonlinear dynamics, stochastic modeling, uncertainty quantification, Monte Carlo method

Abstract.

This work studies the dynamics of a one dimensional elastic bar with random elastic modulus and prescribed boundary conditions, say, fixed at one end, and attached to a lumped mass and two springs (one linear and another nonlinear) on the other extreme. The system analysis assumes that the elastic modulus has gamma probability distribution and uses Monte Carlo simulations to compute the propagation of uncertainty in this continuous–discrete system. After describing the deterministic and the stochastic modeling of the system, some configurations of the model are analyzed in order to characterize the effect of the lumped mass in the overall behavior of this dynamical system.

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

1 INTRODUCTION

The dynamics of a mechanical system depends on some parameters such as physical and geometrical properties, constraints, external and internal loading, initial and boundary conditions. Most of the theoretical models used to describe the behavior of a mechanical system assume nominal values for these parameters, such that the model gives one response for a given particular input. In this case the system is *deterministic* and its behavior is described by a single set of differential equations. However, in real systems they do not have a fixed value since they are subjected to uncertainties of measurement, imperfections in manufacturing processes, change of properties, etc. This variability in the set of system parameters leads to a large number of possible system responses for a given particular input. Now the system is *stochastic* and there is a family of differential equations sets (one for each realization of the random system) associated to it.

This work aims to study the propagation of uncertainty in the dynamics of a nonlinear continuos random system with a discrete element attached to it. In this sense, this work considers a one dimensional elastic bar, with random elastic modulus, fixed on the left extreme and with a lumped mass and two springs (one linear and another nonlinear) on the right extreme (fixed-mass-spring bar).

This paper is organized as follows. In section 2 is presented the deterministic modeling of the problem, the discretization procedure and the algorithm used to solve the equation of interest. The stochastic modeling of the problem is shown in section 3, as well as the construction of a probability distribution for the elastic modulus, using the maximum entropy principle, and a brief discussion on the Monte Carlo method. In section 4, some configurations of the model are analyzed in order to characterize the effect of lumped mass in the system dynamical behavior. Finally, in section 5, the main conclusions are emphasized and some directions for future work outlined.

2 DETERMINISTIC APPROACH

The continuous system of interest is the one-dimensional fixed-mass-spring bar shown in Figure 1.

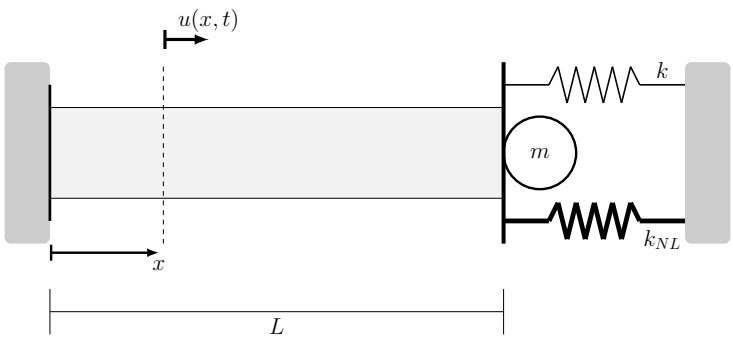


Figure 1: Sketch of a bar fixed at one and attached to two springs and a lumped mass on the other extreme.

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

2.1 Strong Formulation

The displacement of this system u evolves according to the following partial differential equation

$$\rho A \frac{\partial^2 u}{\partial t^2}(x,t) + c\frac{\partial u}{\partial t}(x,t) = \frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}(x,t)\right) + f(x,t), \tag{1}$$

which is valid for $0 < x < L$ and $0 < t < T$, being L the bar unstretched length and T a finite instant of time. In this equation ρ is the mass density, E is the elastic modulus, A is the circular cross section area, c is the damping coefficient, and $f(x,t)$ is an external force depending on position x and instant t.

The left side of the bar is fixed at a rigid wall while the right side is attached to a lumped mass m and two springs fixed to a rigid wall. The first spring (of stiffness k) is linear and exerts a restoring force proportional to the stretching on the bar. The second spring (of stiffness k_{NL}) is nonlinear and its restoring force is proportional to the cube of the stretching. The force which the lumped mass exerts on the bar is proportional to acceleration. These boundary conditions read as

$$u(0,t) = 0 \qquad \text{and} \qquad EA\frac{\partial u}{\partial x}(L,t) = -ku(L,t) - k_{NL}\left[u(L,t)\right]^3 - m\frac{\partial^2 u}{\partial t^2}(L,t). \tag{2}$$

Initially, any point x of the bar presents displacement and a velocity respectively equal to

$$u(x,0) = u_0(x) \qquad \text{and} \qquad \frac{\partial u}{\partial t}(x,0) = \dot{u}_0(x), \tag{3}$$

for $0 \leq x \leq L$. In these equations u_0 and $\dot{u}_0$ are given functions of position x.

Moreover, it is noteworthy that u is assumed to be as regular as needed for the initial–boundary value problem of Eqs.(1), (2), and (3) to be well posed.

2.2 Variational Formulation

Let $\mathcal{U}_t$ be the class of (time dependent) basis functions and $\mathcal{W}$ be the class of weight functions. These sets are chosen as the space of functions with square integrable spatial derivative, which satisfy the essential boundary condition defined by Eq.(2).

The variational formulation of the problem under study says that one wants to find $u \in \mathcal{U}_t$ that satisfy, for all $w \in \mathcal{W}$, the weak equation of motion given by

$$\mathcal{M}(\ddot{u},w) + \mathcal{C}(\dot{u},w) + \mathcal{K}(u,w) = \mathcal{F}(w) + \mathcal{F}_{NL}(u,w), \tag{4}$$

where $\mathcal{M}$ is the mass operator, $\mathcal{C}$ is the damping operator, $\mathcal{K}$ is the stiffness operator, $\mathcal{F}$ is the external force operator, and $\mathcal{F}_{NL}$ is the nonlinear force operator. These operators are, respectively, defined as

$$\mathcal{M}(\ddot{u},w) = \int_0^L \left(\rho A\ddot{u}(x,t)w(x)\right)dx + m\ddot{u}(L,t)w(L), \tag{5}$$

$$\mathcal{C}(\dot{u},w) = \int_0^L c\dot{u}(x,t)w(x)dx, \tag{6}$$

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

$$\mathcal{K}(u,w) = \int_0^L \left(EAu'(x,t)w'(x)\right) dx + ku(L,t)w(L), \tag{7}$$

$$\mathcal{F}(w) = \int_0^L f(x,t)w(x)dx, \tag{8}$$

$$\mathcal{F}_{NL}(u,w) = -k_{NL}\left[u(L,t)\right]^3 w(L), \tag{9}$$

where $\dot{}$ is an abbreviation for temporal derivative and $'$ is an abbreviation for spatial derivative.

The variational formulations for the initial conditions of Eq.(3), which are valid for all $w \in \mathcal{W}$, are respectively given by

$$\widetilde{\mathcal{M}}(u(\cdot,0),w) = \widetilde{\mathcal{M}}(u_0,w), \tag{10}$$

and

$$\widetilde{\mathcal{M}}(\dot{u}(\cdot,0),w) = \widetilde{\mathcal{M}}(\dot{u}_0,w), \tag{11}$$

where $\widetilde{\mathcal{M}}$ is the associated mass operator, defined as

$$\widetilde{\mathcal{M}}(u,w) = \int_0^L \rho A u(x,t)w(x)dx. \tag{12}$$

2.3 An Eigenvalue Problem

Now consider the following generalized eigenvalue problem associated to Eq.(4),

$$-\nu^2\mathcal{M}(\phi,w) + \mathcal{K}(\phi,w) = 0, \tag{13}$$

where ν is a natural frequency and ϕ is an associated mode shape.

In order to solve Eq.(13), the technique of separation of variables is employed, which leads to a Sturm-Liouville problem (Al Gwaiz, 2007), with denumerable number of solutions. Therefore, this generalized eigenvalue problem has a denumerable number of solutions, all of then such as the following eigenpair (ν_n^2, ϕ_n), where ν_n is the n-th bar natural frequency and ϕ_n is the n-th bar mode shape.

It is important to observe that, the eigenfunctions $\{\phi_n\}_{n=1}^{+\infty}$ span the space of functions which contains the solution of the Eq.(13) (Brezis, 2010). As can be seen in Hagedorn and DasGupta (2007), these eigenfunctions satisfy, for all $m \neq n$, the orthogonality relations given by

$$\mathcal{M}(\phi_n,\phi_m) = 0, \tag{14}$$

and

$$\mathcal{K}(\phi_n,\phi_m) = 0, \tag{15}$$

which made then good choices for the basis function when a weighted residual procedure (Finlayson and Scriven, 1966) is used to approximate the solution of a nonlinear variational equation, such as Eq.(4).

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

2.4 Mode Shapes and Natural Frequencies

According to Blevins (1993), a fixed-mass-spring bar has its natural frequencies and the corresponding orthogonal modes shape given by

$$\nu_n = \lambda_n \frac{\bar{c}}{L}, \tag{16}$$

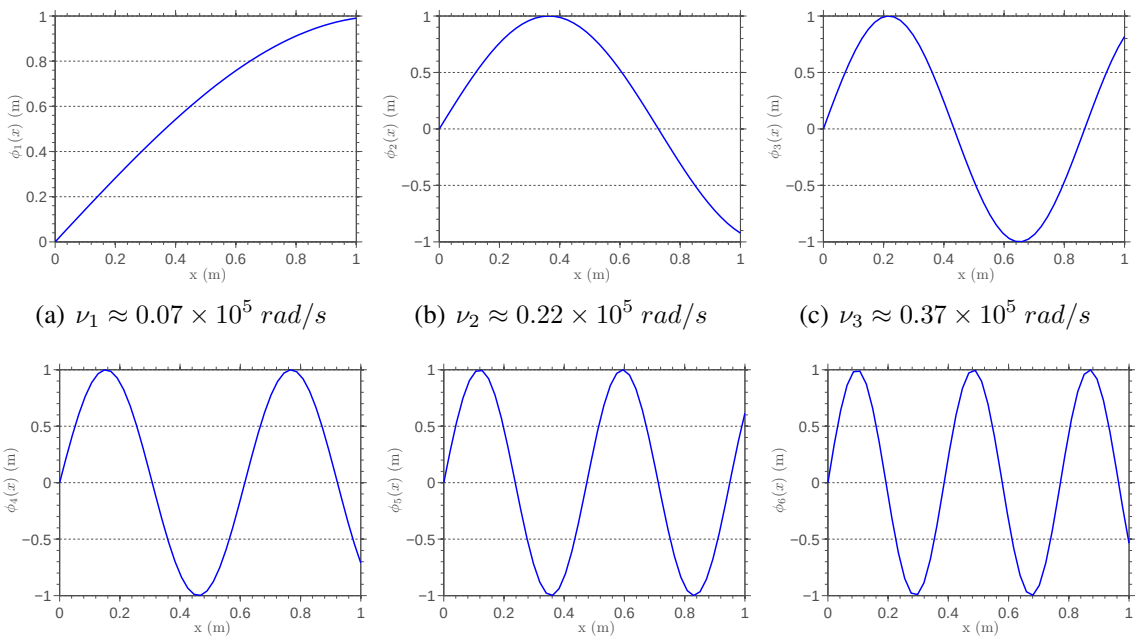
and

$$\phi_n(x) = \sin\left(\lambda_n \frac{x}{L}\right), \tag{17}$$

where $\bar{c} = \sqrt{E/\rho}$ is the wave speed, and the λ_n are the solutions of

$$\cot(\lambda_n) + \left(\frac{kL}{AE}\right)\frac{1}{\lambda_n} - \left(\frac{m}{\rho AL}\right)\lambda_n = 0. \tag{18}$$

The first six orthogonal modes shape of the fixed-mass-spring bar with $m = 1.5\ kg$, whose the other parameters are presented in the beginning of section 4, are illustrated in Figure 2. In this figure each sub-caption indicates the approximated natural frequency associated with the corresponding mode.



(a) $\nu_1 \approx 0.07 \times 10^5\ rad/s$ (b) $\nu_2 \approx 0.22 \times 10^5\ rad/s$ (c) $\nu_3 \approx 0.37 \times 10^5\ rad/s$

(d) $\nu_4 \approx 0.52 \times 10^5\ rad/s$ (e) $\nu_5 \approx 0.67 \times 10^5\ rad/s$ (f) $\nu_6 \approx 0.83 \times 10^5\ rad/s$

Figure 2: The first six orthogonal modes shape and the corresponding (approximated) natural frequencies of a fixed-mass-spring bar with $m = 1.5\ kg$.

2.5 Galerkin Formulation

In order of approximate the solution of Eqs.(4), (10) and (11) the Galerkin method (Hughes, 2000) is employed. Therefore, the displacement field u is approximated by a linear combination

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

of the form

$$u^N(x,t) = \sum_{n=1}^{N} u_n(t)\phi_n(x), \tag{19}$$

where the basis functions ϕ_n are the orthogonal modes shape of the fixed-mass-spring bar, exemplified in the end of section 2.4, and the coefficients u_n are time-dependent functions. For a reason that will be clear soon, define $\mathbf{u}(t)$ of $\mathbb{R}^N$ as the vector in which the n-th component is $u_n(t)$.

Since u^N is not a solution of Eq.(4), when the field u is approximated by u^N a residual function is obtained. This residual function is orthogonally projected into the vector space spanned by the functions $\{\phi_n\}_{n=1}^N$ in order to minimize the error incurred by the approximation (Hughes, 2000). This procedure results in the following $N \times N$ set of nonlinear ordinary differential equations

$$[M]\,\ddot{\mathbf{u}}(t) + [C]\,\dot{\mathbf{u}}(t) + [K]\,\mathbf{u}(t) = \mathbf{f}(t) + \mathbf{f}_{NL}\left(\mathbf{u}(t)\right), \tag{20}$$

supplemented by the following pair of initial conditions

$$\mathbf{u}(0) = \mathbf{u}_0 \qquad \text{and} \qquad \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0. \tag{21}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, and the upper dot again denotes the time derivative. Also, $\mathbf{f}(t)$, $\mathbf{f}_{NL}\left(\mathbf{u}(t)\right)$, $\mathbf{u}_0$, and $\dot{\mathbf{u}}_0$ are vectors of $\mathbb{R}^N$, which respectively represent the external force, the nonlinear force, the initial position, and the initial velocity.

The initial value problem of Eqs.(20) and (21) has its solution approximated by Newmark method (Newmark, 1959). The reader interested in details about this integration scheme is encouraged to see Hughes (2000).

3 STOCHASTIC APPROACH

3.1 Probabilistic Model

Consider a probability space $(\Omega, \mathbb{A}, \mathbb{P})$, where Ω is sample space, $\mathbb{A}$ is a σ-field over Ω and $\mathbb{P}$ is a probability measure. In this probabilistic space, the elastic modulus is assumed to be a random variable $E : \Omega \to \mathbb{R}$ that associates to each event $\omega \in \Omega$ a real number $E(\omega)$. Consequently, the displacement of the bar is the random field $U : [0, L] \times [0, T] \times \Omega \to \mathbb{R}$, which evolves according the following stochastic partial differential equation

$$\rho A \frac{\partial^2 U}{\partial t^2}(x,t,\omega) + c\frac{\partial U}{\partial t}(x,t,\omega) = \frac{\partial}{\partial x}\left(E(\omega)A\frac{\partial U}{\partial x}(x,t,\omega)\right) + f(x,t), \tag{22}$$

being the partial derivatives now defined in the mean square sense (Papoulis and Pillai, 2002). This problem has boundary and initial conditions similar to those defined in Eqs.(2) and (3), by changing u for U only.

3.2 Elastic Modulus Distribution

The elastic modulus cannot be negative, so it is reasonable to assume the support of random variable E as the interval $(0, +\infty)$. Therefore, the probability density function (PDF) of E is a

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

nonnegative function $p_E : (0, +\infty) \to \mathbb{R}$, which respects the following normalization condition

$$\int_0^{+\infty} p_E(\xi) d\xi = 1. \tag{23}$$

Also, the mean value of E is known real number μ_E, i.e.,

$$\mathbb{E}[E] = \mu_E, \tag{24}$$

where the expected value operator of E is defined as

$$\mathbb{E}[E] = \int_0^{+\infty} E(\xi) p_E(\xi) d\xi \tag{25}$$

Finally, one also wants that E has finite variance, i.e.,

$$\mathbb{E}\left[(E - \mu_E)^2\right] < +\infty, \tag{26}$$

which is possible (Soize, 2000), for example, if

$$\mathbb{E}\left[\ln(E)\right] < +\infty. \tag{27}$$

Following the suggestion of Soize (2000), the maximum entropy principle (Shannon, 1948; Jaynes, 1957a,b) is employed in order to consistently specify p_E. This methodology chooses for E the PDF which maximizes the differential entropy function, defined by

$$\mathbb{S}[p_E] = -\int_0^{+\infty} p_E(\xi) \ln\left[p_E(\xi)\right] d\xi, \tag{28}$$

subjected to (23), (24), and (27), the restrictions that effectively define the known information about E.

Respecting the constraints imposed by (23), (24), and (27), the PDF that maximizes Eq.(28) is given by

$$p_E(\xi) = \mathbb{1}_{(0,+\infty)} \frac{1}{\mu_E} \left(\frac{1}{\delta_E^2}\right)^{\left(\frac{1}{\delta_E^2}\right)} \frac{1}{\Gamma(1/\delta_E^2)} \left(\frac{\xi}{\mu_E}\right)^{\left(\frac{1}{\delta_E^2} - 1\right)} \exp\left(-\frac{\xi}{\delta_E^2 \mu_E}\right), \tag{29}$$

where $\mathbb{1}_{(0,+\infty)}$ denotes the indicator function of the interval $(0, +\infty)$, δ_E is the dispersion factor of E, and Γ indicates the gamma function. This PDF is a gamma distribution.

3.3 Stochastic Solver: Monte Carlo Method

Uncertainty propagation in the stochastic dynamics of the continuous–discrete system under study is computed by Monte Carlo (MC) method (Metropolis and Ulam, 1949). This stochastic solver uses a Mersenne twister pseudorandom number generator (Matsumoto and Nishimura, 1998), to obtain many realizations of the random variable E. Each one of these realizations defines a new Eq.(4), so that a new variational problem is obtained. After that, these new variational problems are solved deterministically, such as in section 2.5. All the MC simulations reported in this work use 4^5 samples to access the random system. Further details about MC method can be seen in Liu (2001); Shonkwiler and Mendivil (2009); Robert and Casella (2010).

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

4 NUMERICAL EXPERIMENTS

The numerical experiments presented in this section adopt the following deterministic parameters for the studied system: $\rho = 7900\ kg/m^3$, $c = 10\ kNs/m$, $A = 625\pi\ mm^2$, $k = 650\ N/m$, $k_{NL} = 650 \times 10^{13}\ N/m^3$, $L = 1\ m$, and $T = 8\ ms$. Besides that, four values for the lumped mass are considered: $m = 1.5,\ 7.5,\ 15,$ and $75\ kg$. The random variable E, is characterized by $\mu_E = 203\ GPa$ and $\delta_E = 10\%$.

The initial conditions for displacement and velocity are respectively given by

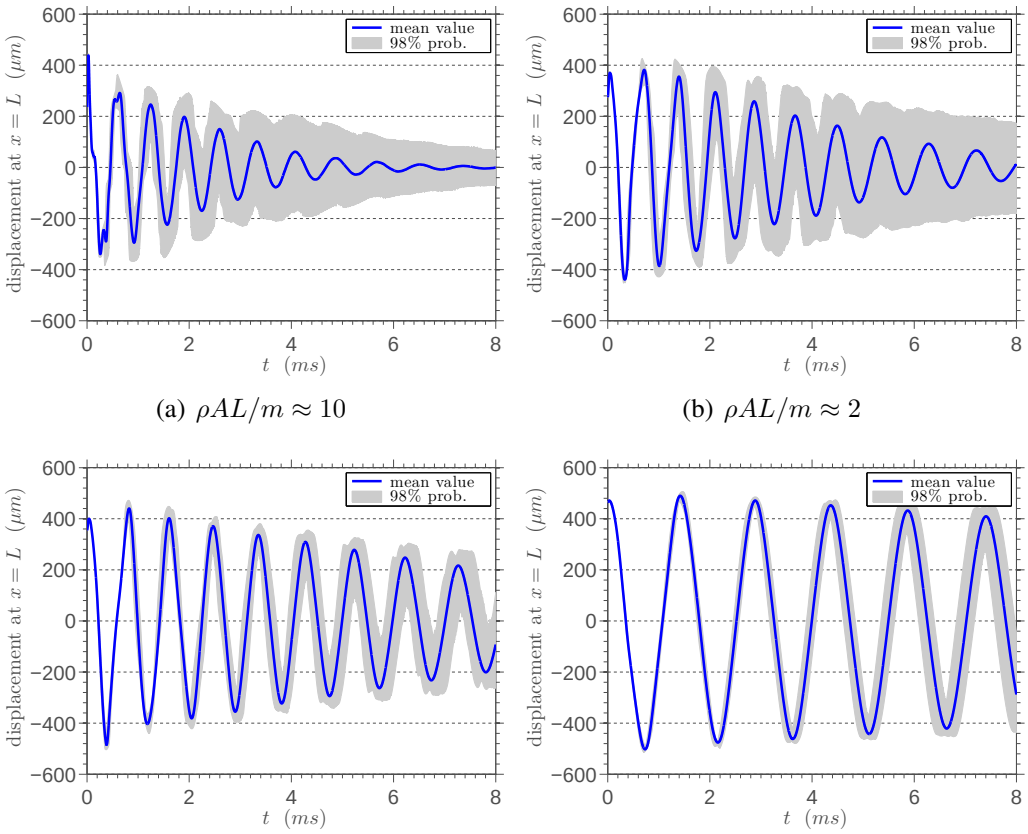
$$u_0 = \alpha_1 \phi_3(x) + \alpha_2 x, \qquad \text{and} \qquad \dot{u}_0 = 0, \tag{30}$$

where $\alpha_1 = 0.1\ mm$ and $\alpha_2 = 0.5 \times 10^{-3}$. Note that u_0 reaches the maximum value at $x = L$. This function is used to “activate" the spring cubic nonlinearity, which depends on the displacement at $x = L$.

The time-dependent external force acting on the system has the form of a sine wave with circular frequency equal to the first natural frequency

$$f(x,t) = \sigma \phi_1(x) \sin(\nu_1 t), \tag{31}$$

where the external force amplitude is $\sigma = 1\ N$.



(a) $\rho AL/m \approx 10$ (b) $\rho AL/m \approx 2$

(c) $\rho AL/m \approx 1$ (d) $\rho AL/m \approx 0.2$

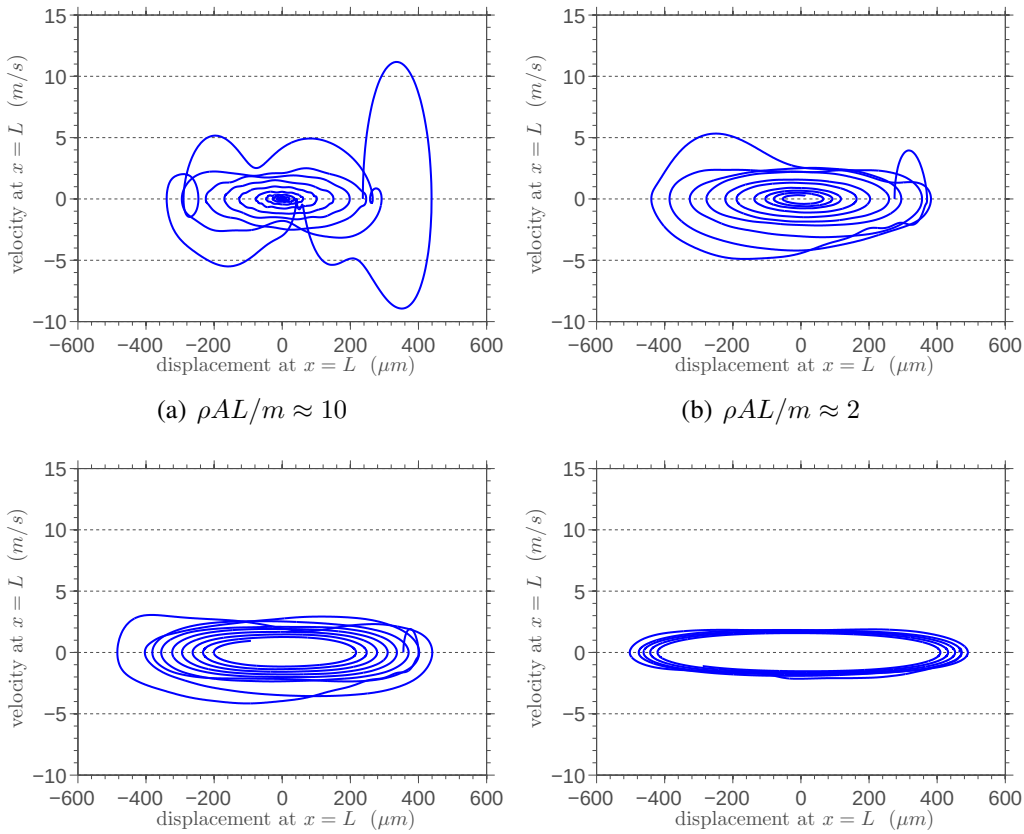
Figure 3: This figure illustrates the mean value (blue line) and a 98% of probability interval of confidence (grey shadow) for the random process $U(L,\cdot,\cdot)$, for several values of the continuous–discrete mass ratio.

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

4.1 Analysis of Random System Envelope of Reliability and Phase Space

The mean value of $U(L,\cdot,\cdot)$ and an envelope of reliability, wherein a realization of the stochastic system has 98% of probability of being contained, are shown, for different values of the continuous–discrete mass ratio $\rho AL/m$, in Figure 3. By observing this figure one can note that, as the value of lumped mass increases, the decay of the system displacement amplitude decreases significantly. This indicates that this continuous–discrete system is not much influenced by damping for small values of the continuous–discrete mass ratio.

The mean phase space of the fixed-mass-spring bar at $x = L$ is shown, for different values of the continuous–discrete mass ratio, in Figure 4. The observation made in the previous paragraph can be confirmed by analyzing this figure, since the system mean orbit tends from a stable focus to an ellipse as the continuous–discrete mass ratio decreases. In other words the limiting behavior of the system when $\rho AL/m \to 0^+$ is a mass-spring system.



(a) $\rho AL/m \approx 10$ (b) $\rho AL/m \approx 2$

(c) $\rho AL/m \approx 1$ (d) $\rho AL/m \approx 0.2$

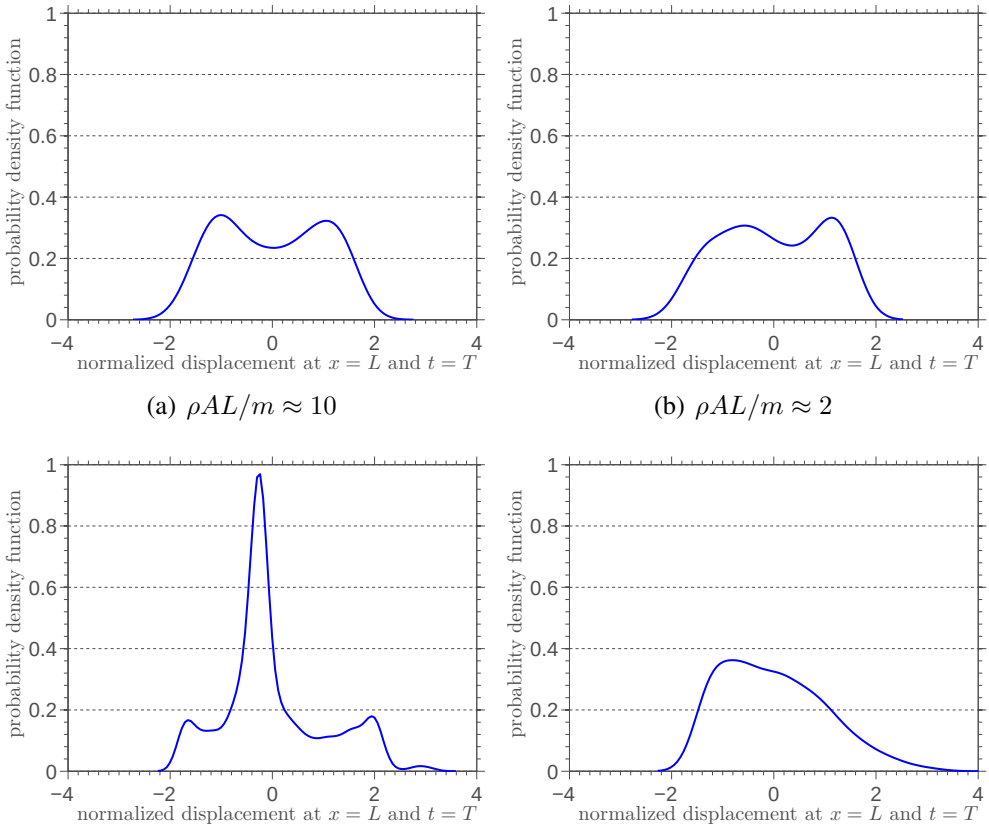
Figure 4: This figure illustrates the mean value of the fixed-mass-spring bar phase space at $x = L$, for several values of the continuous–discrete mass ratio.

4.2 Analysis of Random System PDF

The difference between the system dynamical behavior, for different values of m, is even clearer if one looks to the PDF estimations of the (normalized) random variable $U(L,T,\cdot)$, which are presented in Figure 5. For large values of the continuous–discrete mass ratio, the PDF of $U(L,T,\cdot)$ displays bimodal shape, which tends to a unimodal shape as the lumped mass grows, i.e., the continuous–discrete mass ratio decreases. Furthermore, it can be noted

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

that when $\rho AL/m \approx 1$ the greatest probability occurs around the mean value of $U(L, T, \cdot)$.



(a) $\rho AL/m \approx 10$ (b) $\rho AL/m \approx 2$

(c) $\rho AL/m \approx 1$ (d) $\rho AL/m \approx 0.2$

Figure 5: This figure illustrates estimations to the PDF of the (normalized) random variable $U(L, T, \cdot)$, for several values of the continuous–discrete mass ratio.

5 CONCLUDING REMARKS

A model to describe the dynamics of fixed-mass-spring bar with a random elastic modulus is presented. The aleatory parameter is modeled as a random variable with gamma distribution, being the probability distribution of this parameter obtained by the principle of maximum entropy. The paper analyzes some configurations of the model to order to characterize the effect of the lumped mass in the overall behavior of this dynamical system. This analysis shows that the dynamics of the random system is significantly altered when the values of the lumped mass are varied.

6 ACKNOWLEDGMENTS

The authors are indebted to Brazilian Council for Scientific and Technological Development (CNPq), Coordination of Improvement of Higher Education Personnel (CAPES), and Foundation for Research Support in Rio de Janeiro State (FAPERJ) for the financial support.

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar

REFERENCES

Al Gwaiz M.A. *Sturm-Liouville Theory and its Applications*. Springer, New York, 2007.

Blevins R.D. *Formulas for Natural Frequency and Mode Shape*. Krieger Publishing Company, Malabar, 1993.

Brezis H. *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Springer, New York, 2010.

Finlayson B. and Scriven L. The method of weighted residuals- A review. *Applied Mechanics Reviews*, 19:735–748, 1966.

Hagedorn P. and DasGupta A. *Vibrations and Waves in Continuous Mechanical Systems*. Wiley, Chichester, 2007.

Hughes T.J.R. *The Finite Element Method*. Dover Publications, New York, 2000.

Jaynes E.T. Information theory and statistical mechanics. *Physical Review Series II*, 106:620–630, 1957a. doi:10.1103/PhysRev.106.620.

Jaynes E.T. Information theory and statistical mechanics II. *Physical Review Series II*, 108:171–190, 1957b. doi:10.1103/PhysRev.108.171.

Liu J.S. *Monte Carlo Strategies in Scientific Computing*. Springer, New York, 2001.

Matsumoto M. and Nishimura T. Mersenne twister: a 623-dimensionally equidistributed uniform pseudo-random number generator. *ACM Transactions on Modeling and Computer Simulation*, 8:3–30, 1998. doi:10.1145/272991.272995.

Metropolis N. and Ulam S. The Monte Carlo Method. *Journal of the American Statistical Association*, 44:335–341, 1949. doi:10.2307/2280232.

Newmark N. A method of computation for structural dynamics. *Journal of the Engineering Mechanics Division*, 85:67–94, 1959.

Papoulis A. and Pillai S.U. *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, New- York, 4th edition, 2002.

Robert C.P. and Casella G. *Monte Carlo Statistical Methods*. Springer, New York, 2010.

Shannon C. A mathematical theory of communication. *Bell System Technical Journal*, 27:379–423, 1948.

Shonkwiler R.W. and Mendivil F. *Explorations in Monte Carlo Methods*. Springer, New York, 2009.

Soize C. A nonparametric model of random uncertainties for reduced matrix models in structural dynamics. *Probabilistic Engineering Mechanics*, 15:277 – 294, 2000. doi: 10.1016/S0266-8920(99)00028-4.

Copyright © 2012 Asociación Argentina de Mecánica Computacional http://www.amcaonline.org.ar