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Corrigendum to "a compactly generated pseudogroup which is not realizable"

There is an error in [1], Section 3, on page 1208, at the place "it is easily verified that". The identity that follows this phrase does not hold for $C \neq 1$.

The problem takes place in the construction of two germs α, β at 0 of smooth (C^∞) diffeomorphisms of the real halfline $[0, +\infty)$, infinitely tangential to the identity, which generate a free nonabelian group.

The alleged method cannot work, since if it did, it would apply as well to Thurston's Example 2 in [3], making this example smooth, contrarily to Thurston's Stability Theorem.

Instead, there is a correct (and classical) construction of such α and β, as follows.

After White [5], the two homeomorphisms $A : x \mapsto x + 1$ and $B : x \mapsto x^3$ of the real line generate a free group. Hence, so do A and $B^{-1} \circ A \circ B$. By analyticity, so do the germs of A and $B^{-1} \circ A \circ B$ at $+\infty$. After a conjugation by $x \mapsto 1/x$, we find that the germs at 0+ of the two real analytic diffeomorphisms of the line

$$f : t \mapsto t(1 + t)^{-1}$$

$$g : t \mapsto t(1 + t^3)^{-1/3}$$

generate a free group. Now, remember the Muller-Tsuboi conjugacy:

Lemma ([2,4]). *Let h be the germ at 0 of a smooth diffeomorphism of $[0, +\infty)$, such that $h'(0) = 1$. Consider*

$$\phi : t \mapsto e^{-1/t}.$$

Then, $\phi^{-1} \circ h \circ \phi$ is smooth and infinitely tangential to the identity at 0.

Hence, $\alpha := \phi^{-1} \circ f \circ \phi$ and $\beta := \phi^{-1} \circ g \circ \phi$ work.

The rest of the paper [1] is not changed, nor its results.

I'm grateful to Sam Nariman for pointing to me this error.

References

[1] G. Meigniez, *A compactly generated pseudogroup which is not realizable,* J. Math. Soc. Japan, vol. 62, no. 4 (2010), 1205-1218.

[2] M.-P. Muller, *Sur l'approximation et l'instabilité des feuilletages,* preprint IRMA (Strasbourg 1982).

[3] Thurston, W. *A generalization of the Reeb stability theorem.* Topology 13 (1974), 347–352.

[4] T. Tsuboi, "Γ_1-structures avec une seule feuille," Transversal structure of foliations (Toulouse, 1982), Astérisque, tome 116 (1984), p. 222–234

[5] S. White, *The group generated by $x \mapsto x + 1$ and $x \mapsto x^p$ is free,* Journal of Algebra 118 (1988), 408–422.

Université de Bretagne-Sud, L.M.A.M., BP 573, F-56017 Vannes, France
Email address: Gael.Meigniez@univ-ubs.fr