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# Knee ligaments mechanics

## From experiments to FE simulations

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*ABSTRACT. Acting as track rod for joints, knee ligaments are viscoelastic complex structures which are loaded up to damage and failure during trauma situations. This paper presents 10 years of joint research at the Laboratory of Biomechanics and Application and the Laboratory of Acoustics and Mechanics in investigating the mechanical behaviour of such structures. Starting from clinical knowledge, we show, how experimental, histological, theoretical and FE simulation approaches have been performed to investigate such the behaviour of such structures.*

*RÉSUMÉ. Sortes de biellettes pour les systèmes articulaires, les ligaments du genou sont des structures viscoélastiques complexes qui lors de traumatismes des systèmes articulaires sont très fortement sollicités jusqu'à la rupture. Cet article présente de façon rétrospective 10 années de recherche conjointe entre le Laboratoire de biomécanique appliquée et le Laboratoire de mécanique et d'acoustiques pour décrire le comportement de telles structures. Il s'agit de montrer, à partir des données cliniques, comment les volets expérimentaux, histologiques, théoriques et enfin de simulation par éléments finis ont été utilisés pour aborder le comportement de ces structures.*

*KEYWORDS: knee ligaments, failure, damage, behavior law.*

*MOTS-CLÉS : ligaments du genou, rupture, endommagement, modèle de comportement.*

## **1. Introduction. General comments**

Ligaments are complex structures that bind bones together and support organs. These fibrous connective tissues have to manage two opposite functions which are to ensure joint stability and joint mobility. From this specificity these tissues can be summarized as anisotropic, inhomogeneous, quasi incompressible material which work under large strain levels and exhibit viscoelastic properties. The understanding of mechanical properties and contribution of ligaments in joints mechanics was then largely studied from low strain rate loading to evaluate ligaments contribution during physiological kinematics up more recent research concerning high strain rate loadings corresponding to sports trauma and transportation crash situations (car occupant, pedestrian, cyclists...).

After a summarized overview of ligaments properties (from structure, clinical features up to mechanical understanding), this paper report research performed in partnership in order to investigate mainly damage behaviour under dynamic loading from experiments to numerical approaches. Four knee ligaments are studied here: the anterior cruciate ligament (ACL), the posterior cruciate ligament (PCL), the fibular collateral ligament (FCL) and the medial collateral ligament (MCL).

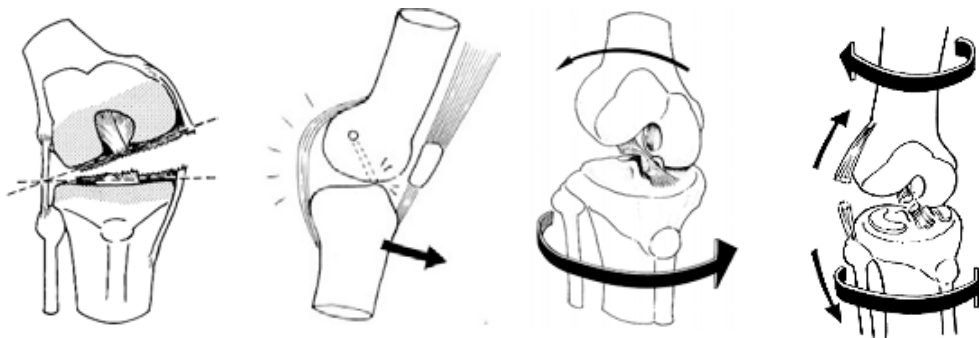
### **1.1. *Clinical knowledge***

Under crash situation (car occupant, pedestrian, cyclist, motocyclists) or sports trauma, knee ligaments can be considered as cut-out structure. Many clinical studies were performed in order to identify injury level, frequency, the associated injuries and provide assumption in term of injury mechanisms (Mansat 1989, Liorzou 1990, Zuinen 1991, Laporte 1999, Teresinski 2003). These works show that the most frequently injured ligament is the anterior cruciate ligament. Moreover, in most of cases a cruciate ligament injury is associated with others injuries of neighbouring structure such as surrounding lateral ligaments, meniscus or bone structures. From medical analysis the main injury mechanisms could be induced (Liorzou 1990, Zuinen 1991) as pure varus or valgus movements, internal or external rotation or hyperextension. In most of case combination of these two loading cases are postulated with flexion valgus and external rotation, flexion valgus and internal rotation. Note that antagonist muscles contributions (in flexion) are also responsible of anterior cruciate ligaments failure. The main injury cases are summarised in figure 1.

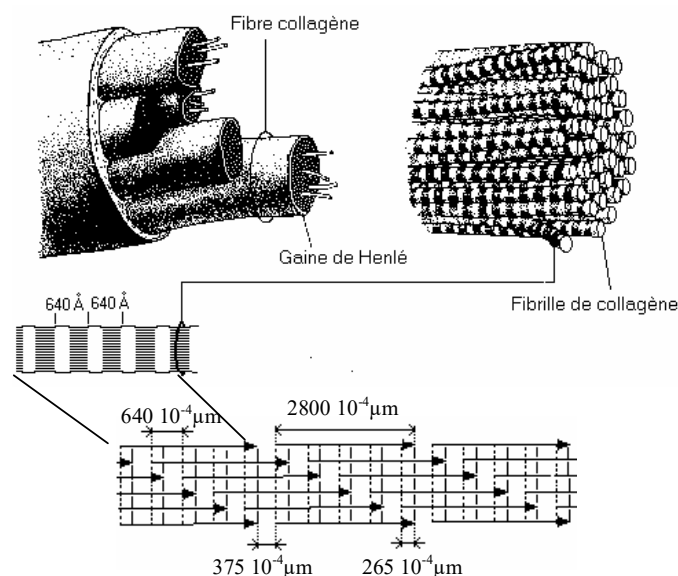
### **1.2. *The tissue structure***

Because of their constituents and their geometrical assembly, ligaments are highly complex structures which can be viewed as a composite material. From the geometrical point of view, ligaments structure assembly can be defined from the

tropocollagen structure (Angstrom level) up to the fibre level (centimetre) (cf. figure 2). It is therefore essential to understand the contribution of each level in the hierarchical structure to the mechanical properties of the material as a whole. The main deformation mechanisms can occur at four different levels. At the lowest level; in the tropocollagen molecule and the microfibril (which length is 2800 Angstrom), elongation mechanisms induce a molecule profile setting (Sasaki 1996, Mosler 1989, Folkhard 1987). Comminou (1976), Maes (1999) have proposed mechanical models based on the waviness of the microfibrils. At the fibrils scale (which length is 10  $\mu\text{m}$  to 100  $\mu\text{m}$ ), the arrangement of overlapping and gap regions has been described on the basis of the Hodge Petruska model (Hodge 1962, Sasaki 1996). Next, the fibre is a geometrical assembly of disjointed fibrils embedded in an amorphous substance. When fibres are loaded, the pressure applied causes fluid to exude from the tissue, as the result of which, the material shows viscous behaviour (Lanir 1983, Humphrey 1987). Lastly, the fibre-ligament or fibre-bone assembly cannot be precisely described in terms of fibre interactions. In addition, the amorphous substance can be accounted for an incompressible fluid.



**Figure 1.** *Injury mechanisms assumptions: Valgus, internal rotation*



**Figure 2.** *Overview of ligaments hierarchical structure from bound of fibres to tropocollagen*

Ligaments material is 55% to 65% to water. Its other components are the elastin (10-15% of dry weight) which ensures tissue capacity to exhibit high strain level, collagen (70-80% of dry weight) which is responsible of tissues strength, proteoglycal matrix (1-3% dry weight) which induces viscous response through exudation process (Martin 1998). Ligaments insertions can be summarized as fibro cartilage and hydroxyapathite discontinuous gradient which have a strong influence on failure properties of the structure. This point will be discussed in detail hereafter.

### **1.3. About experimental studies**

Many of the tensile traction tests performed have used quasi-static loading (Woo 1982, 1990, 1991, Mommerteeg 1995, 1996, Butler 1992). For cyclic loading, the strain rates used to achieve dynamic loading are commonly below  $10^2\%s^{-1}$ . These loading conditions used to validate viscoelastic models consist in applying forces at strain rate lower than this generally observed during dynamic loadings. In crash situation, structures are submitted to very high strain rate ( $10^4\%s^{-1}$ ). With such strain rate, the determined time necessary for wave propagation in the whole structure is in the same range of value as those observed experimentally (Liao 1999, Hurschler 1997, Arnoux 2002, Subit 2004).

The methods used to test the tensile strength of soft tissues can be divided into three categories, depending on the goal of the authors.

- Testing of a ligament alone: both ends of the ligament are blocked in resin with a moulding or chucking device (Woo 1982, Viidick 1973). It enables to test the ligament only, without any other surrounding tissues, but the mounting of the ligament is not an easily reproducible method and the mounting procedure may damage the ligament.

- Testing of the bone-ligament-bone complex: the bone ends are embedded in a very stiff resin (PMMA or epoxy) (Mommerteeg 1995, 1996, Pioletti 1997, Arnoux 2000, 2002, Subit 2004). Besides the samples include the insertion sites and make it possible to study the failure mechanisms of the whole ligamentous structure. The sample can be considered as a material which makes it possible, to some extents, to develop stress-strain behaviour laws. One of the drawbacks of this method is the difficulty to correlate the orientation of the insertion sites on the sample and on the complete knee joint (under physiologic conditions).

- Testing the ligament and the distal part of the femur and the proximal part of the tibia in a mechanical system (Frisen 1969, Noyes 1975, Woo 1990, 1991, Race 1994, Subit 2004). The ligament can be tested in physiological positions, since the orientation of the insertion sites is then linked to the angle of knee flexion. But the cut of samples requires sacrificing some ligaments, often 3, to study one particular ligament.

Besides the difficulty in measuring dynamic tensile strength in such materials, data reported in the literature have shown wide variability, a common observation for living tissues. While such a wide data dispersion is biological (“we are all different”), some parameters can be recognised as having a specific influence on tissue behaviour. Diament (1972) and Noyes (1976) showed that age strongly affects mechanical properties of human tissue with a subsequent effect on experimental results. Woo (1991) suggested that they should be obtained for three age groups (22-35, 40-50, 60-90 yr). As for the strain-rate effects, two contradictory hypotheses have been put forward: according to Fung (1973), ligament behaviour is strain-rate independent, whereas for Holden (1994), and Pioletti (1997) it is strain-rate dependent. Maes (1989) and Lam (1990) showed the effects of temperature and proposed a thermoelastic model for ligament microstructure. These authors noted that ligament collagen exhibits a physical reaction to high temperature (around 50°C), contracting before gradually desegregating. The orientation of the fibres during loading, also affects the mechanical properties observed with structure and failure models (Woo 1993, Mommerteeg 1996, Subit 2004). In order to obtain reproducible behaviour for cyclic loading, Fung (1993) subjected samples to cyclic tensile tests before running experiments. Mass impact has been used in most of the dynamic loading experiments. The major drawback of such protocols used for dynamic and cyclic loading is that they do not allow quasi-static loading.

#### ***1.4. Mechanical properties and behaviour laws***

From histological consideration it appears that ligaments are strongly inhomogeneous and anisotropic materials, which exhibit viscoelastic behaviour under large strain level. Note that ligaments are usually assumed as homogenous and isotropic materials. Main mechanical properties of knee ligaments are summarized in table 1.

Concerning behaviour laws, many advances have been made in characterising the mechanical behaviour of connective tissues, using thermodynamic (Dehoff 1978, Bingham 1979, Pioletti 1997, Arnoux 2002), structural (Lanir 1980, Maes 1989, Sasaki 1996, Viidick 1973) or phenomenological formulation (Fung 1973, Viidick 1973). However, most of the previous models have dealt only with the mechanical behaviour of these tissues under low strain rates or focusing on the elastic response of the structure. Few models for soft tissues taking into account damage were developed (Kwan 1989, Hurschler 1997, Liao 1999). These models were based on a structural formulation taken into account fibres bundles with different initial lengths and different ultimate forces at failure with, for some authors a statistic distribution of these properties. Arnoux *et al.* (2002) focus on a macroscopic formulation coupling viscoelasticity to damage.

**Table 1. Summary of main ligaments properties**

		Young Modulus (MPa)	Max Strain (%)	Max stress (MPa)	Max Force (N)
Anterior cruciate Ligament	Viidick, 1973	30	60	200	6000
	Arnoux, 2000	20-115	18-24		75.5-605
Posterior cruciate Ligament	Viidick, 1973	35	60	100	6000
	Noyes, 1976y	111 ± 26	48.5 ± 11.9	13.3 ± 5.0	0.734 ± 0.266
	Noyes, 1976o	65.3 ± 24.0	60.25 ± 6.78	37.8 ± 9.3	1.73 ± 0.66
	Race, 1994	248 ± 119	18 ± 5.3	35.9 ± 15.2	1620 ± 500
	Arnoux, 2000	24-207	18-24		158-505
Medial colateral ligament	Viidick, 1973	15	40	400	3000
	Arnoux, 2000	11.5-51	21-38		160-350
Tibioal colateral ligament	Viidick, 1973	15	30	400	3000
	Arnoux, 2000	21.5-32.5	21-38		155-400

## 2. Experimental investigations

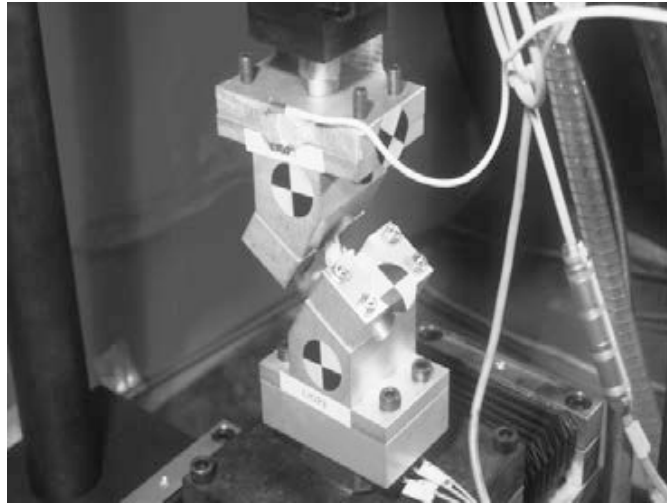
From an experimental point of view, with a first set of experiments on a bone insertion-ligament-bone insertion complex, it was possible to study ligaments behaviour in main fibre directions and to identify failure properties to provide data to validate behaviour laws under high strain rate levels. The second set of experiments focus on ligaments insertions mechanics and histology, but also on ligaments behaviour under physiological loading conditions focussing on effects of joint positions and various strain rates. The experimental investigation reported below followed the chronology of our investigations

### 2.1. Bone ligament bone complex under high speed loading

During a pedestrian impact with vehicle impact velocities ranged between 20 to 40km/h, it has been shown that knee ligaments were subjected to high strain rate level and then severely injured. In order to investigate mechanical behaviour of ligaments under such high speed loading and then there failure properties, a new experimental device was designed (Arnoux 2000, 2002) with the following three specifications:

- the sample is composed of a bone-ligament-bone complex. The bone ends are embedded in resin and mounted in a specific set up using spherical joint in order to put chose ligaments orientation before each test;

- both static and dynamic solicitation are performed using a hydraulic actuator in order to study tensile properties of ligaments as well in static condition as with cyclic loading, as with dynamic loading up to failure;
- solicitations can be obtained for all realistic various orientations of the ligaments (fibre direction, anatomical axis).

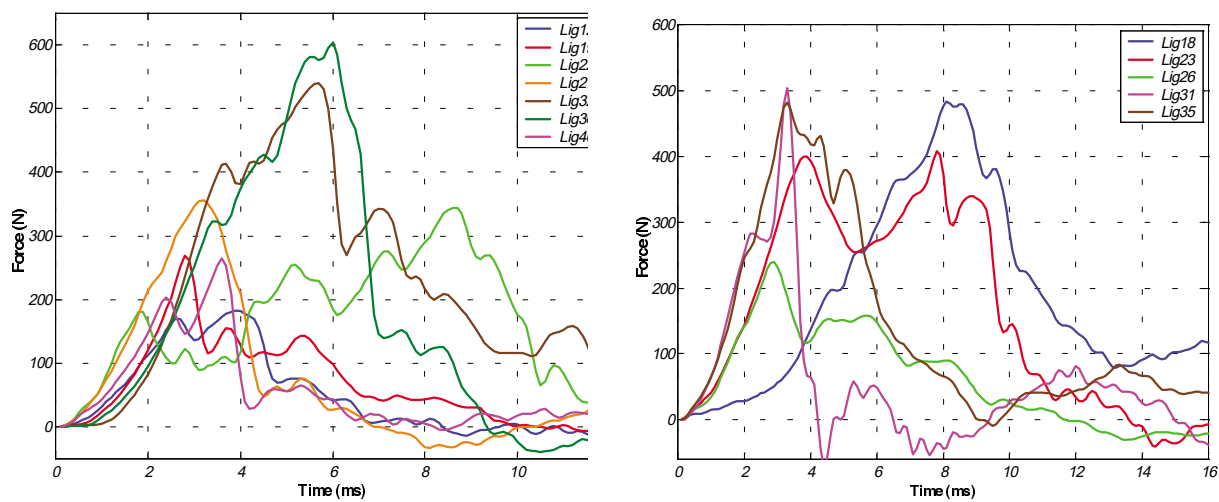


**Figure 3.** *The orientation and locking system*

In the first set of experiment the objective was to perform tensile tests in the direction of the fibres axes at a velocity of 1.98m/s. Knee ligaments (ACL, PCL, FCL and MCL) from 4 human cadavers (an 88-year-old man and 3 women aged 58, 81 and 89 years) were tested. The cadavers (obtained from the Marseilles University Faculty of Medicine) were treated with Winckler solution (Winckler 1974) to ensure proper preservation of soft tissues, then held at +3°C until testing. Prior to testing, the knees were thawed to room temperature for 3 hours, then x-rayed to check bone integrity. The ligaments were cut off with bone ends from PMHS knees.. Before extracting the bone-ligament-bone complex, the articulation was locked in full extension with a Hauffmann external fixator using a pin in the distal part of the femur and the proximal part of the tibia. The four bone-ligament-bone complexes were then dissected. The dimensions of each ligament were recorded before and after removing. In accordance with work by Noyes and Grood (1976), the ligament was assumed to have an elliptic cross section. The two axes of this elliptic section were determined at the tibia insertion, at the medial portion of the ligament and at the femoral insertion. The length of the ligament was calculated from three measurements taken by placing the calliper on the ligament insertions. Each bone insertion was then embedded in a truncated spherical mass of resin (F1-Excel-Euroresin). After terminating sample preparation, the entire sample was plunged into a Winckler solution until tensile testing.

In the subsequent force versus displacement curves (Figure 4), the zero force at test onset was translated to the initial force value (*i. e.* initial pre-load minus ligament relaxation before the tensile test).





**Figure 4.** Forces versus time curves for the anterior and posterior cruciate ligaments

*For the anterior (ACL) and posterior (PCL) cruciate ligaments*

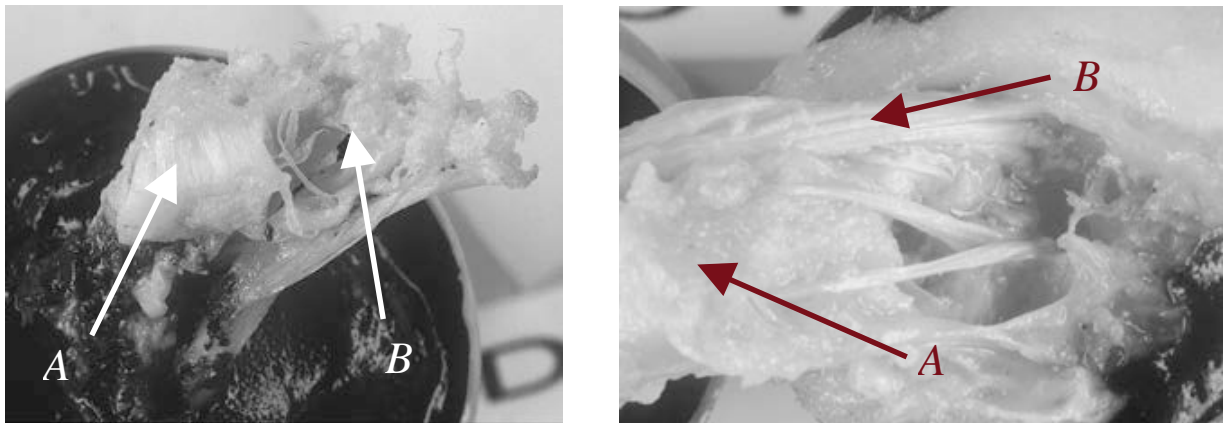
As was showed by Fung (1973, 1993) and Vidiick (1973), the force-time curve (Fig. 4) exhibits three phases:

- In the first non-linear phase, the force increased exponentially with time, describing the physiological range of solicitation. This first phase was strongly dependent on the initial force applied to the structure. Since the initial pre-load was between 5 and 20N and the strain-rate level was very high, this non linear elastic phase was not as prominent as observed in low strain level tensile testing (Pioletti 1997, Fung 1993, Mommerteeg 1996);

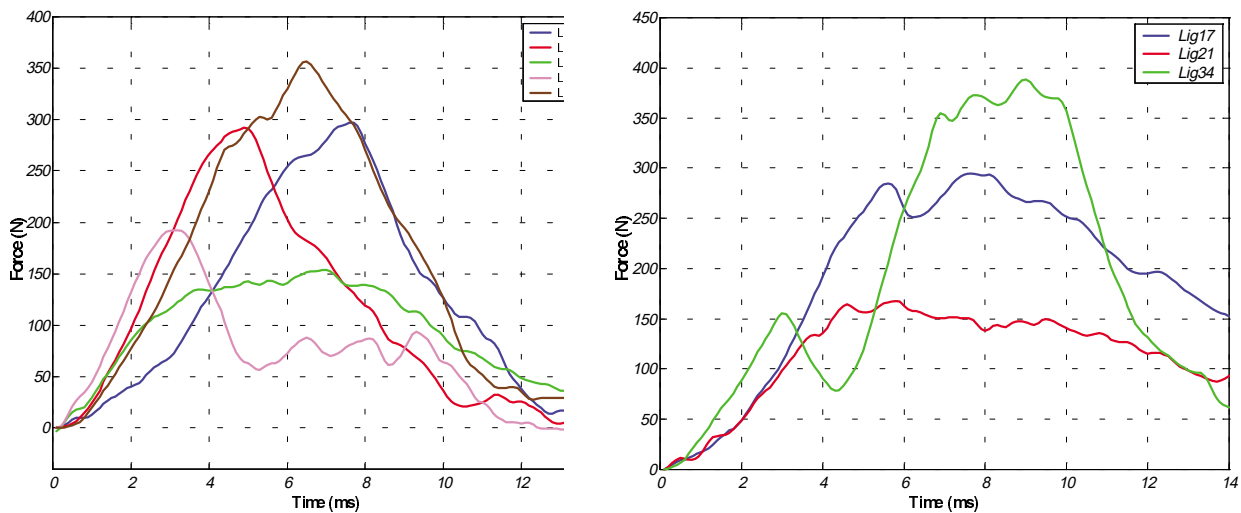
- The second phase was fairly elastic. The calculated stiffness of the structure ranged between 50 and 145 N/mm for the anterior cruciate ligaments and between 60 and 180 N/m for the posterior cruciate ligaments. According to the assumption made on ligament cross section and measurement accuracy, the corresponding Young Modulus were between 25 and 210 MPa and between 11 and 51 MPa for the cruciate and collateral ligaments respectively;

- The final phase was a region of brittle damage leading to failure. Two peaks were observed. These peaks corresponded to the respective failures of the two main fibre bundles of the structure (Fig 4 & 5). Each cruciate ligament being constituted by two fibre bundles with different length and orientation, loading was not identical, leading to a two-step failure pattern. The ultimate force at failure differed from one test to another, but the ultimate deformation always occurred between 18 and 24 % (according to the initial force for both the anterior and posterior cruciate ligaments). All cruciate ligaments tore off the tibia insertion. The failure pattern was not the same for the two bundles of fibres. The anterior bundle peeled off its insertion, while the posterior one pulled out of the thickness of the cortical bone (Fig. 5). The observed behaviour (Fig. 4 & 5) was associated with the failure peaks of the two fibres bundles. In light of bone behaviour, bone avulsion could correspond to the

more brittle peak in the force-time curve whereas peelings would correspond to the more ductile failure peak. Therefore, while failure was observed at the ligament insertions, the entire structure was assumed to have been damaged since swelling effects were observed after failure.



**Figure 5.** ACL and PCL failure profile. A was for the bone avulsion and B was for the peeling failure



**Figure 6.** Forces versus time curves for the medial and fibular collateral ligament

*For the medial (MCL) and fibular (FCL) collateral ligaments*

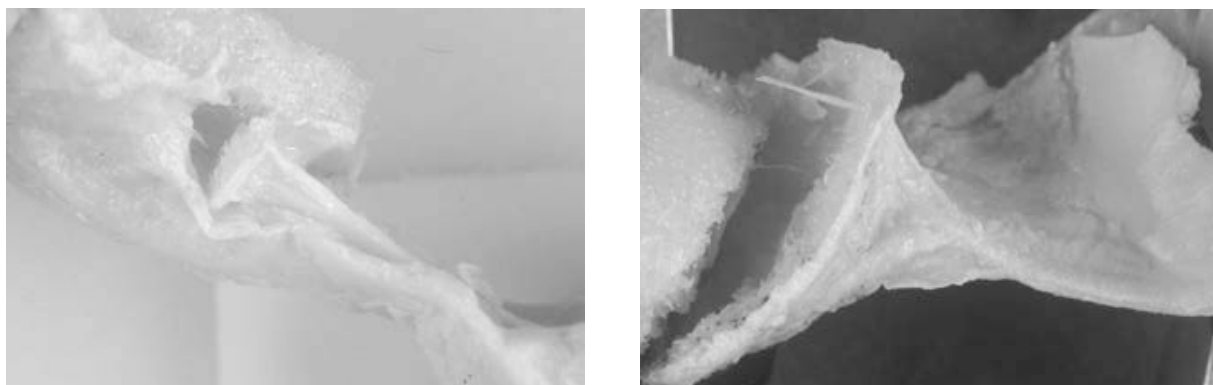
As observed for the cruciate ligaments, the force-time curve of the collateral ligaments could be divided into in three phases: a stretching phase, a quasi-linear elastic and damage phase (Fig. 6).

– Despite the initial force applied to the collateral ligaments, the toe region was clearly observed for initial forces ranged from 9 to 28 N (Fig. 6).

– The elastic phase stiffness was ranged from 33 to 53 N/mm for the MCL and from 28 to 78 for the FCL, which was less than observed for the cruciate ligaments. The corresponding Young moduli (according to the assumed cross section and

measurement accuracy) were ranged from 12 to 51 MPa and 19 to 32 MPa for the MCL and FCL respectively.

– Since collateral ligaments are composed of only one fibre bundle, the damage phase described only one peak on the force-time curve. Compared with the cruciate ligaments, this was a very ductile peak. Depending on the initial force applied to the structure, deformation failure occurred at 25 to 38%. The failure occurred at the femoral insertion in more than 80% of the samples and exhibited a bone avulsion pattern (Fig. 7). As cortical bone thickness, and thus resistance, at the insertion of the collateral ligaments was very thin, the failure mode (probably in mode I) was not comparable with the failure modes of the cruciate ligaments (combined mode I and mode II). This observation demonstrated that the structural properties of the ligament insertions are not the same for the cruciate and collateral ligaments. Assuming the insertion is a ligament-to-bone hydroxyapatite gradient, this gradient is sharper for the collateral ligaments. In addition, the ligament-bone interface was very thin and can be considered as a membrane which was pulled out during the damage phase. We also observed damage throughout the structure, with swelling and peeling effects (Fig. 8).



**Figure 7.** *MCL and FCL failure profile*



**Figure 8.** *Swelling profile of the structure*

### *On this first set of experiments*

Although we used very high strain rates, results for all ligaments studied were qualitatively the same as those presented in the literature. A quantitative comparison with data reported earlier is not exactly appropriate because of the preservation method and elderly age group used.

The accuracy of the structure's Young modulus is highly dependent on the accuracy of the ligament section measurement. It is therefore more relevant to evaluate the (1D) stiffness of the structure. The stiffness measurements we obtained showed a wide spread and were less than those presented by others authors (Race 1994, Woo 1991, Mommersteeg 1996). These differences may be induced by strain rate, the cadaver preservation method, or the age and sex groups of the study population (3/4 were old women) and more generally to the variability of living tissues.

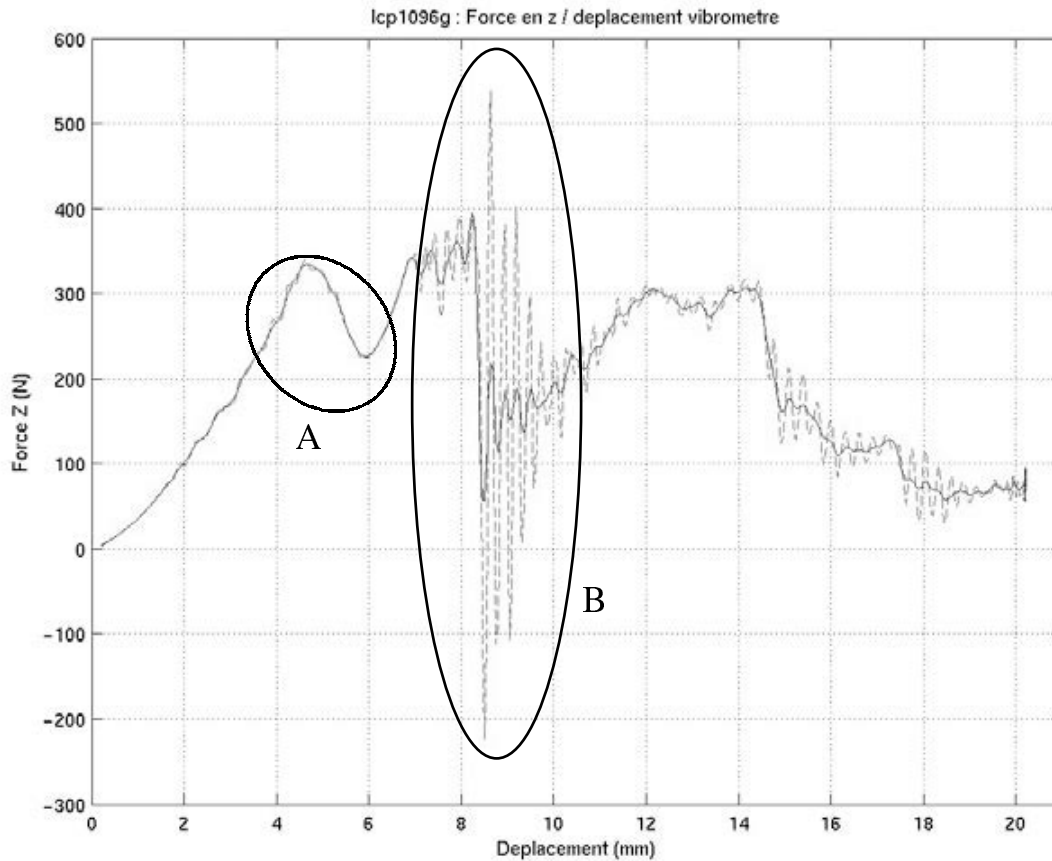
The failure occurred rapidly for all ligaments tested. While the failure force showed a strong dispersion (between 170 and 500 N for the cruciate ligaments and between 150 and 300N for the collateral ligaments), the corresponding ultimate deformations (according to the initial force applied) were between 18 and 24% for the cruciate ligaments and between 25 and 38% for the collateral ligaments. These percentages could be considered as a criterion for deformation failure. As the recruitment of cruciate or collateral ligament fibres occurs depending on their direction, the damage localisation could be explained by the orientation of the ligament insertion, at least in part. The failure patterns were reproducible with the four type of ligaments tested and demonstrated the influence of bone insertion structure and of the bone-ligament hydroxyapatite gradient. This gradient could account for the brittle or ductile failure types observed for cruciate and collateral ligaments respectively. Peeling failure under traction force is interpreted as the decohesion of fibres which are linked laterally and longitudinally. This failure could then be understood as a combination of mode I and mode II failure. Either for bone avulsion in the cortical layer or bone avulsion of the ligament insertion, failure was interpreted as mode I failure.

As all tensile tests were performed with the same velocity and as initial forces which reduced viscous effects were applied, it was difficult to conclude on the effect of strain rate in this work. Nevertheless it would be reasonable to assume that the structure was subjected to dynamic effects. The celerity calculation for the structure showed that the duration of time propagation of a traction wave was the same as the duration for the tensile test.

## ***2.2. Bone ligament bone complex under various loading velocity***

Given the results of the dynamic tests, new tensile tests were performed for a wide range of velocity, with the view to describing the ligament failure depending on the velocity. The protocol was improved and updated before this second series of

tests (Subit, 2004): (1) a system was added to the tensile device to avoid rotation of the hydraulic jack during the tests, (2) the geometry of each ligament was acquired using CT-scan pictures (Bidal, 2003) to make possible the measurement of their dimensions, (3) the ligaments were preconditioned (Fung, 1993), and (4) pre-stressed prior to the test. The preconditioning was 10 cycles of a triangular loading and unloading from 0 to 1 % strain, for a strain rate of 0.5%/s. Failure tests were performed for velocities of 20 mm/s and 1m/s.



**Figure 9.** Load-displacement curve of a PCL tested up to failure (velocity: 1m/s). The three processes of damage and failure are visible. Dashed line: unfiltered signal; regular line: filtered signal (cut-off frequency: 3.5 kHz; sampling rate: 32 kHz)

The results reported here are limited to two cruciate ligaments. As presented in 2.1, various processes are responsible of the ligament damage and failure. The qualitative and quantitative results are in line with those described in 2.1. The objective of the tests was to link the changes in the microstructure, in particular when damage occurs, to the macroscopic behaviour. The quasi-static tests showed how failure occurs at the insertion sites, in particular the failure of fibres one after another, bringing about brittle failure. The dynamic tests show other processes: the loss of cohesion between fibres, and the failure at the insertion site (figure 9). The figure 9 is the load-displacement curve of a PCL tested up to failure (velocity: 1m/s). The quasi-static and dynamic results lead to the following trends:

– the sequence of loading and unloading, of about 10 N-amplitude, seems to be linked to the loss of cohesion between the ligament fibres and the superficial part of the cortical layer, and occurs for quasi-static loading,

– a smoother failure, the load decrease of which is ranged from 50 to 100 N seems to be linked to the loss of cohesion between fibres or bundles of fibres in the ligament itself. This loss of cohesion modifies the stress field and induces the decrease of the load, while the bundles of fibres are not loaded again (part A on the curve),

– a sharp failure (load variation of 100 to 200 N) seems to be relevant of bone failure. The bone is known to be a brittle material. In the case, the unfiltered signal (dash curve) has big oscillations (part B), showing that energy is dissipated suddenly, which is coherent with the bone failure.

These three damage mechanisms can occur successively or simultaneously. It is evident that the failure process is linked to the velocity. But these results do not allow concluding of the role of the orientation of the insertion site in the failure process.

### **3. Modelling knee ligaments behaviour**

From these experimental data, several way were chosen to describe knee ligaments mechanical behaviour. Structural approaches such as elastic cohesive fibrous FE model at the fibril level were introduced in order to describe failure process (Arnoux 2002). At a macroscopic level, FE simulation of adhesive properties of ligaments insertions were evaluated in order to focus on the effects of ligaments insertions to the damage properties of the whole structure (Subit 2004) by assuming ligament as elastic. Lastly at a macroscopic level a viscoelastic model with damage using a thermodynamic formalism was studied (Arnoux 2002).

#### **3.1. Structural modelling of fibre mechanics**

From histological analysis, it might be conjectured that, due to the fibrous structure of ligaments (Arnoux 2002), the failure process might be caused by successive decohesion events occurring at different scales and aggregating so as to generate micro-voids or micro-cracks, well up to aggregate themselves generating voids or cracks at larger scale. It remains the possibility of exploring some assembly of model-fibres linked together with appropriately distributed cohesive forces. Given the fibrous structure modelled by elastic brick elements, the ability to bear tensile stress is due to two classes of cohesive forces: forces exerted by the fore end of fibres on the aft end of the previous fibre (head to tail or longitudinal cohesive force); forces exerted from flank to flank between neighbouring fibres (flank to flank or transversal cohesive force). In this study, tensile tests in the fibre directions have

to be taken into account. The head to tail forces were mainly tensile forces, while the flank to flank forces were mainly shear forces. The constitutive laws were therefore developed based on a fibrous model subjected to tensile traction applied in the direction of the fibre axes (this model could be a fibril). The material was assumed to be homogeneous and periodic. Because of the dynamic kinematics involved, the damage was assumed to be preponderant, and the viscosity was neglected. The following fibrous model consists of an assembly of fibres (which could be microfibrils, fibrils and fibres), in which joints were formed laterally and longitudinally between the fibres.

In this model, the interactions between bricks were described using unilateral constraint and frictional cohesive contact laws. The joint model was given by a cohesive Mohr Coulomb law between nodes candidates to contact which corresponds to a vertical displacement of the Coulomb cone. With such a cohesive law between bricks, three main situations are possible at the joints; (i) inside the cone (A for instance in Fig. 10) the bricks were stuck together, (ii) on the cone border (C for instance in Fig. 10), the stick status is lost and friction occurs between the bricks which corresponds to damage (i. e. the joints are broken between the fibre components), (iii) no contact is possible between bricks (the contact is lost). With the Mohr Coulomb law, the formulation for the adhesive unilateral is described in relation (1) with the corresponding friction law with adhesion defined by the threshold  $\mu l$  is given in relations (1 to 4):

$$q_N \geq 0 ; R_N \geq -1 ; q_N \cdot (R_N + 1) = 0 \quad [1]$$

$$U_T = 0 \Rightarrow R_T \in ]-\mu R_N - \mu l ; \mu R_N + \mu l[ \quad [2]$$

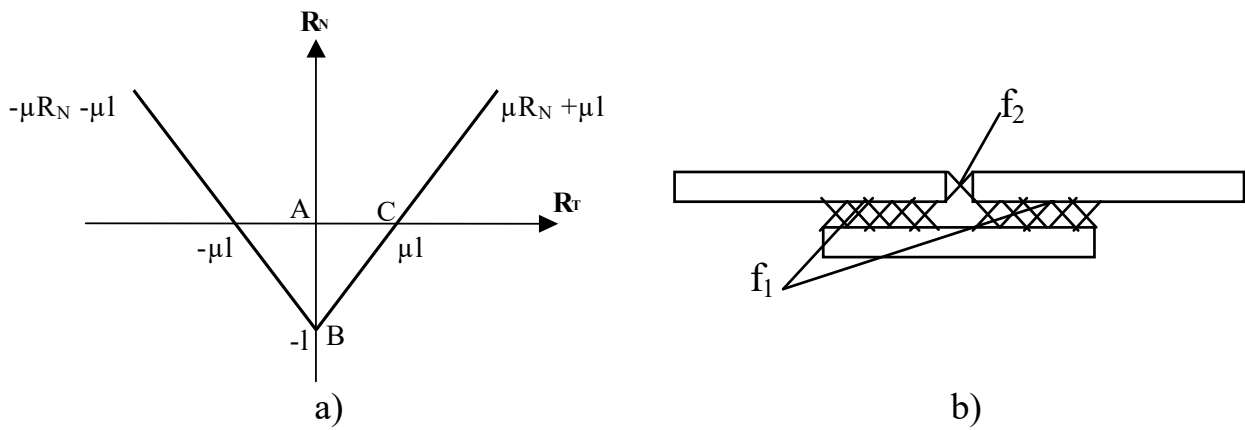
$$U_T < 0 \Rightarrow R_T = -\mu R_N - \mu l \quad [3]$$

$$U_T > 0 \Rightarrow R_T = \mu R_N + \mu l \quad [4]$$

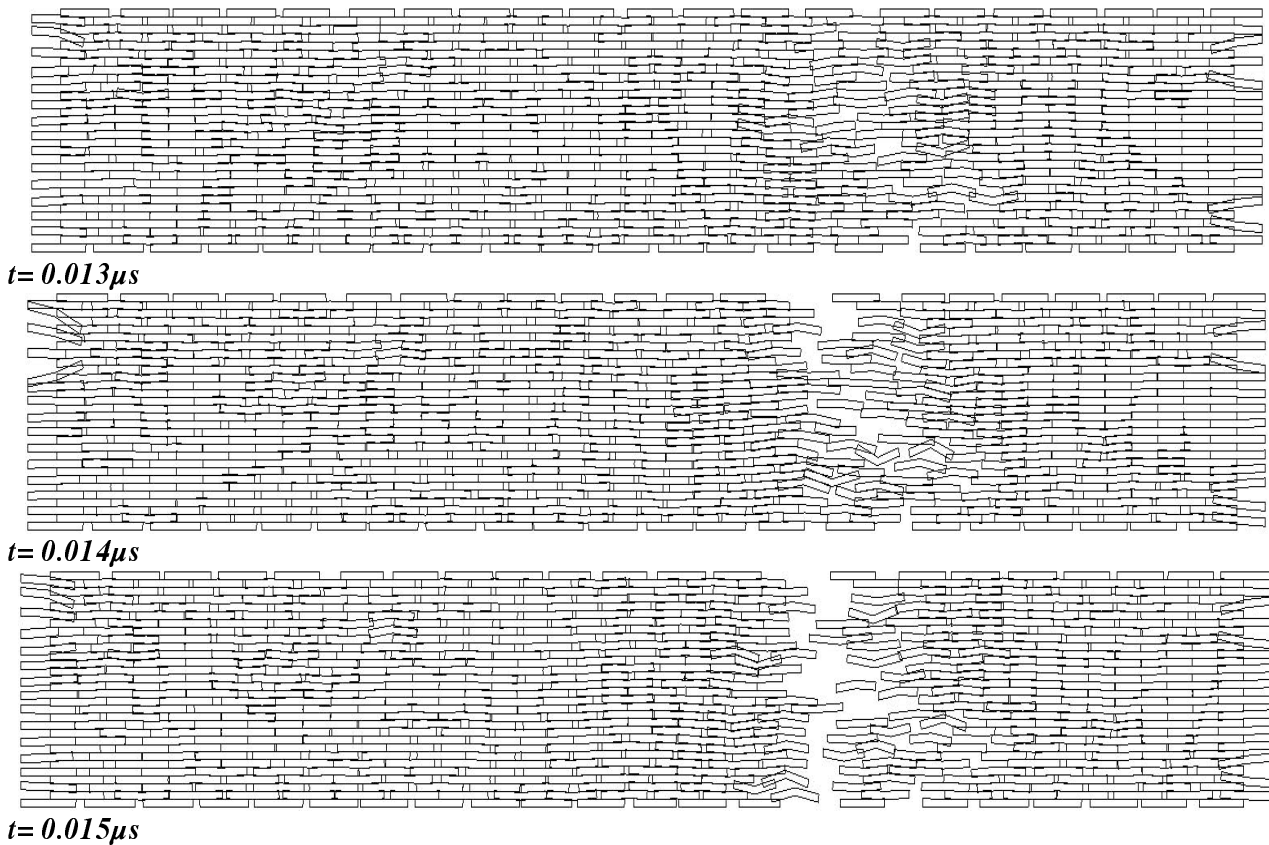
The static sliding coefficient  $\mu$  is defined by  $\mu = \text{tg}(\alpha) = f_2 / f_1$  (cf. Fig. 10).

The cohesive thresholds differ between longitudinal ( $f_1$ ) and transversal ( $f_2$ ) joints and are related to maximal head to tail and flank to flank cohesive forces (Fig. 10). With this model, damage results from brick decohesion mechanisms which was assumed to occur at around 25% of strain levels in that of ligaments. One way of introducing heterogeneity into the material is to randomise failure thresholds.

The representative volume is an assembly of 21 fibres longitudinally in 30 successive layers. As boundary conditions on the lateral sides, the displacements were taken to be zero for all left hand side nodes ( $U_x = 0$ ) and a velocity of 5 m/s was prescribed to every right hand side nodes velocities of 5m/s were applied. A periodic boundary condition may be viewed as the best way to fit the influence of material surrounding the sample in a tensile experiment. For this purpose the upper and lower sides were prescribed a stress  $\sigma_{22} = 0$ . Besides the displacements of the upper and lower sides are the same.



**Figure 10.** a) Mohr coulomb graph on microfibril adhesive properties; b) illustration of cohesive coefficients.  $f_1$  is the cohesive coefficient of the shear between two bricks layers (representative of an plane assembly of three microfibrils).  $f_2$  is the tensile cohesive coefficient for the bricks cohesion on the same layer



**Figure 11.** Three steps of the failure process in ligaments microstructure

This cohesive model provides a physical description of the damage resulting from joint decohesion processes (figure 11). The ratio between the failure stress between longitudinal and transversal joints (contacting the head and flank of a fibre) do not affect the behaviour of the structure, whereas the scattering and the friction coefficient after failure can be used to control the behaviour and the evolution of damage in the model. The qualitative results obtained were quite in line with those obtained at the macroscopic level in the literature. Periodic boundary conditions



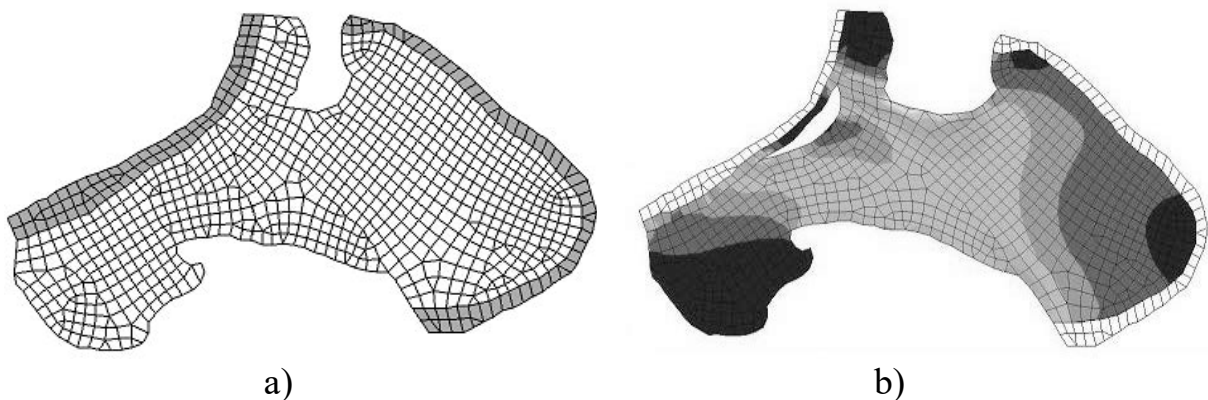
were applied. These models are helpful to understand the failure mechanism and the parameters which influence failure in fibrous structure. These parameters are pertinent to study failure at the microstructure and therefore at the macrostructure level. The main difficulty in such approaches remains the ability to describe homogenisation process and to correlate microscopic parameters to macroscopic one at the level of phenomenological model.

### 3.2. Adhesive properties on ligaments to bone insertions

The histological studies of the ligament-to-bone attachment show that the transition from the ligament to the bone is sharp, and that it may thus be considered as an interface (Subit 2004). This interface must resist to normal and tangential loadings. The first model tested in the same as this presented above. The idea was to evaluate the capability of the Mohr-Coulomb model to describe the various types of ligament failure observed experimentally, since the normal and tangential thresholds enable to take into account the failure due to loadings in both directions, that is to say between fibres (tangential) and between the bone and the ligament.

The 2D-geometry of a PCL was meshed from a CT-scan slice, using quadrangular mainly and triangular elements (linear interpolation) (Figure 12 a). The problem is solved assuming plane strains, for prescribed displacement, in quasi-static. All the materials are supposed to be elastic, linear and isotropic. The ligament is white ( $E = 9 \text{ MPa}$ ,  $\nu = 0.3$ ) and the cortical bone is grey ( $E = 18000 \text{ MPa}$ ,  $\nu = 0.3$ ). The lamellar was not modelised. The left part is the tibial end, and the right one is the femoral end. The right end is blocked; a displacement of 20 mm is prescribed on the other end in the ligament direction (horizontal here). The model was implemented using the user subroutine “*uinter*” of Abaqus. The friction coefficient was 0.2 and the cohesion threshold was 5 MPa. The figure 12b shows the failure at the tibial insertion.

This finite elements analysis is a good starting point for further investigation, namely the work on a 3D geometry, as well as the identification of the parameters of the Mohr-Coulomb law to modelise ligament failure, depending on the velocity (failure at the insertion site or in the ligament).



**Figure 12.** a) Mesh of the 2D-geometry of a PCL; b) deformed shape

### 3.3. Macroscopic description of tissue mechanics

In order to define a behaviour law which can be further implemented at a macroscopic level we developed a thermodynamic visco hyperelastic model with finite transformation damage, by combining a viscoelastic model (Pioletti 1997) and a damage model (Andrieux, 1996).

As ligaments are assumed to be submitted to large displacements, the current strain tensor considered, is the Cauchy Green tensor. The density is assumed to be time independent. The material satisfies the balance conditions (mass and movement conservation) and the two thermodynamics principles.

Based on the local action principle, the principle of material frame-indifference and the principle of fading memory, the stresses occurring in the material were divided into an elastic stress response, a viscous stress and a long-term memory stress. Regarding loading conditions (dynamic up to failure) and since the fading memory contribution is negligible compared to the elastic and viscous contribution and to the damage effects the stresses we consider in this study will be reduced to the elastic and viscous contributions.

*Free energy and viscous potential* : In the case of an elastic deformation, it has been shown (Germain 1986) using the second thermodynamic principle, that the elastic contribution ( $S_e$ ) to the response of the model can be derivated from a hyperelastic potentiel.

$$S_e = 2\rho_0 \frac{\partial W_e}{\partial C} \quad [5]$$

In the same way the viscous response ( $S_v$ ) was introduced using a convex dissipative pseudo potentiel (Germain 1986).

$$S_v = S - 2\rho_0 \frac{\partial W_e}{\partial C} = 2\rho_0 \frac{\partial W_v}{\partial \dot{C}} \quad [6]$$

The ligament is assumed to be homogenous and isotropic. With the representation theorem, the free energy  $W_e$  is written as a function of the three invariants of  $C$ . From (5), the isotropic stress  $S_e$  is obtained by deriving  $W_e$  with respect to  $C$  :

$$S_e = 2\rho_0 \left[ \frac{\partial W_e}{\partial I_1} Id + \frac{\partial W_e}{\partial I_2} (trCId - C) + \frac{\partial W_e}{\partial I_3} I_3 C^{-1} \right] \quad [7]$$

For an incompressible material the viscoelastic behaviour law of the ligament becomes :

$$S = -pC^{-1} + 2\rho_0 \left[ -\frac{\partial W_e}{\partial I_2} C + \frac{\partial W_v}{\partial \dot{C}} + \left( \frac{\partial W_e}{\partial I_1} + \text{tr}C \frac{\partial W_e}{\partial I_2} \right) \right] \quad [8]$$

Damage evolution law is introduced with a scalar ranging from 0 to 1 and based on a Lemaitre and Chaboche formulation (Kachanov 1958, Chaboche 1987). The thermodynamic pairs of variables become  $(C, S)$  and  $(D, Y)$ , where  $Y \in \mathbf{R}$  is the thermodynamic force associated with the damage  $D$ . In this study, damage is assumed to be time independent, not to vary after the ultimate deformation and to affect both the viscous and the elastic properties of the material. The damage energy in the material is written as follows :

$$W(C, \dot{C}, D) = (1 - D)(W_e + W_v) \quad [9]$$

The associated thermodynamic force  $Y$  ( $Y \in \mathbf{R}$ ) is expressed as:

$$Y = -\frac{\partial W}{\partial D} = W_e + W_v \geq 0 \quad [10]$$

The evolution damage law is written using a pseudo potential  $\Omega_D(Y)$  which is a convex, positive closed function of  $Y$ . As damage is assumed to be time independent,  $\Omega_D(Y)$  is replaced by an indicatrice function of a non damage convex,  $f_D(Y, D) \leq 0$  (Andrieux 1996). With the normality rule, this becomes :

$$\dot{f}_D = \frac{\partial f_D}{\partial D} \dot{D} + \frac{\partial f_D}{\partial Y} \dot{Y} = 0 \quad [11]$$

As  $Y$  depends on the left Cauchy Green tensor  $C$ ,  $\dot{D}$  is expressed by:

$$\dot{D} = \begin{cases} 0 & \text{if } f_D < 0 \\ \dot{\delta} \frac{\partial f_D}{\partial Y} & \text{if } f_D = 0 \end{cases} \quad [12]$$

With  $\dot{\delta} = \left\langle \frac{\partial f_D}{\partial Y} \frac{\partial Y}{\partial C} : \dot{C} \right\rangle$ , and  $\langle \dots \rangle$  the positive part.

The parameters of an isotropic incompressible viscoelastic model accounting for damage model of the knee ligaments are calculated on the basis of tensile traction tests with a constant velocity of 1.98m/s in the fibre directions. Considering  $l_0$  the initial length,  $l$  the current length and  $v$  the velocity (which is assumed to be constant), the elongation of the ligament is defined by  $\lambda = l/l_0$  and  $l = l_0 + vt$ . The tangent linear application is also given by  $F_{ij} = 0$  if  $i \neq j$ ,  $F_{11} = \lambda$  and  $F_{22} = 1/\sqrt{\lambda}$

Here the free energy function  $W_e$  (Demiray 1972), the viscous pseudo-potential  $W_v$  [20] and the damage  $f_D$  function are expressed respectively as :

$$W_e = \alpha \exp[\beta(I_1 - 3)] + \gamma(I_2 - 3) \quad [13]$$

$$W_v = \frac{\eta}{4} \text{tr}(\dot{C})^2 (I_1 - 3) \quad [14]$$

$$f_D = Y - Q\sqrt{D + D_0} \leq 0 \quad [15]$$

Initially the ligament is assumed to be stress free  $S(I_d)=0$ . The experimental traction tests were performed without applying any lateral forces to the specimen so that  $S_{22}=0$ . Under these conditions, the constitutive equations become :

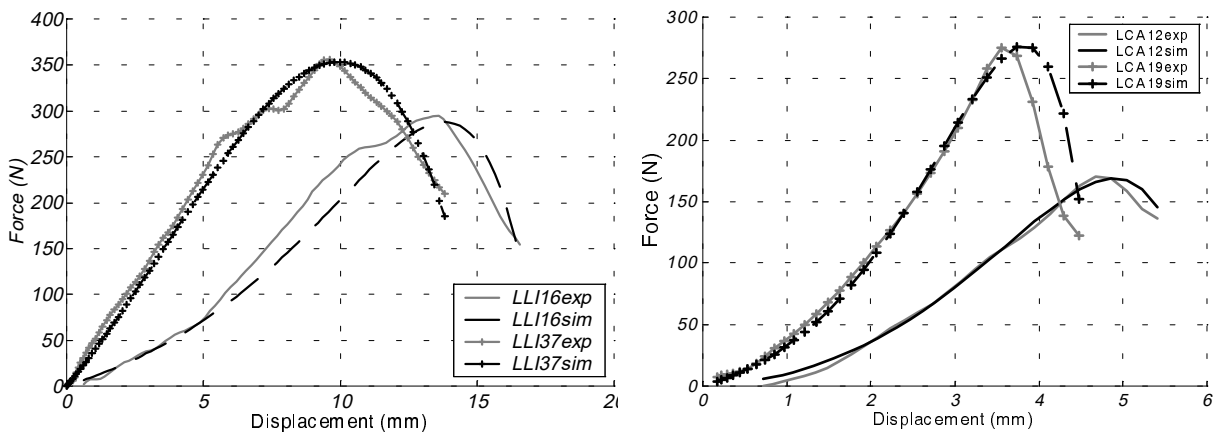
$$S_{11} = -p \frac{1}{\lambda} + (1 - D)\eta K \lambda^2 + 2\alpha\beta(1 - D) \left[ \exp(\beta K) - \frac{1}{\lambda} \right] \quad [16]$$

$$\text{With } p = +2\alpha\beta C_{22}(1 - D)\exp(\beta K) - \alpha\beta C_{22}(1 - D) \left( \lambda^2 - \frac{1}{\lambda} \right) - (1 - D)\eta K \frac{\lambda^2}{2\lambda^4} \quad [17]$$

$$\text{and } \gamma = -\frac{\alpha\beta}{2} \text{ and } K = \left( \lambda^2 + \frac{2}{\lambda} - 3 \right) \quad [18]$$

$$\text{If } f_D=0, \text{ then: } \dot{D} = \frac{n(D + D_0)\dot{\lambda}}{Q\sqrt{D + D_0}} \left[ -\frac{\alpha\beta}{\lambda} + \alpha\beta \exp\left( \beta \left( \lambda^2 + \frac{2}{\lambda} - 3 \right) \right) \right] \quad [19]$$

$$\text{If } f_D < 0, \text{ then } \dot{D} = 0 \quad [20]$$



**Figure 13.** Identification of the behaviour in the case of two medial collateral ligaments and two anterior cruciate ligaments

The complete viscoelastic model with damage introduced above, is characterised by six parameters, two accounting for the elastic behaviour ( $\alpha$ ,  $\beta$ ), one for the viscosity ( $\eta$ ) and three for the damage ( $n$ ,  $Q$ ,  $D_0$ ). The stress and the hydrostatic pressure are expressed as a function of the time variable and parameters ( $\alpha$ ,  $\beta$ ,  $\eta$ ,  $n$ ,  $Q$ ,  $D_0$ ) after analytically integrating the differential equation [19]. Identification of these parameters is performed by minimizing the distance between the simulated and experimental force/displacement curves. Minimization is performed using a conjugate gradient algorithm (cf. figure 13).

Usually, identification of the viscous component cannot be performed with only one type of experimental test. It is necessary to perform tensile tests for different strain rates or to perform relaxation tests. As we focused on the damage behaviour in the present study, cyclic loading before traction up to failure can damage the material and might therefore affect the mechanical behaviour. One solution to this problem might consist of performing further tensile tests specifically for the purpose of studying the viscous behaviour. Since experimentally the prescribed initial forces have attenuated the viscous effects and since from a numerical point of view the value of  $\eta$  did not significantly affect the results,  $\eta$  was taken to be small. The behaviour law is written as a relation between the stress and the strain, whereas the identification process is performed on the force displacement relations in order to be in agreement with the experimental measurements. Ligaments are a highly non-homogeneous material in which it is very difficult to determine the deformation and stress because of the ligaments length and section.

#### 4. Discussion. Conclusion

The biomechanical studies are a challenge for the ‘classical’ engineering mechanics. The tissues are living materials and their mechanical properties may change during their life, or during an experiment. The biological variability is another bias for the description of the mechanical behaviour. These points raise a problem for the experimental data processing, in particular to evaluate if the wide range of the results is due to the mechanical behaviour, which is the goal of the measurements, or due to the protocol that could be not reproducible. Indeed the experiment in biomechanics requires the design of new devices that are used for few samples, because of the availability of PMHS. Besides, the border between *material* and *structure* is hard to set. It is rarely possible to define a part in a human tissue in which it is realistic to assume the homogeneity of stress and strain field, even if the sample is of small size. This problematic is of main interest in the studies we performed, and explains the use of various protocols: any is perfect, as we described above, but each is useful to describe a particular aspect of the ligament behaviour. It led us to perform histological studies to describe the microstructure of these tissues to provide macroscopic models, based upon micro structural changes responsible of the failure for instance.

Lucid concerning the limitations of the results provided, these experimental data, and the models established after, are interested to lead simultaneously finite elements analysis. This numerical work is a good complement to the experimental one, since it enables to test several times the same sample for several velocities for example, or various boundary conditions. The work performed by the Laboratoire de Mécanique et d'Acoustique and the Laboratoire de Biomécanique Appliquée aims at integrating experimental, histological and numerical works to develop human body model capable to predict failure and injuries. Simultaneously to the work performed on ligaments, others aspect of the human tissues behaviours are studied: failure model (Jundt, 2004), muscle tonicity in finite elements models to improve the description of the muscle behaviour in case of car crash (Behr, 2004), validation of a finite element model of a pedestrian leg to predict pedestrian injuries (Cardot, 2003).

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