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► **To cite this version:**

Jean-Baptiste Doc, Christophe Vergez, Philippe Guillemain, Jean Kergomard. Sound production on a "coaxial saxophone". *Journal of the Acoustical Society of America*, 2016, 140 (5), pp.3917-3924. 10.1121/1.4967368 . hal-01430850

**HAL Id: hal-01430850**

**<https://hal.science/hal-01430850v1>**

Submitted on 21 Feb 2017

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# Sound production on a “coaxial saxophone”

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Sound production on a “coaxial saxophone” is investigated experimentally. The coaxial saxophone is a variant of the cylindrical saxophone made up of two tubes mounted in parallel, which can be seen as a low-frequency analogy of a truncated conical resonator with a mouthpiece. Initially developed for the purposes of theoretical analysis, an experimental verification of the analogy between conical and cylindrical saxophones has never been reported. The present paper explains why the volume of the cylindrical saxophone mouthpiece limits the achievement of a good playability. To limit the mouthpiece volume, a coaxial alignment of pipes is proposed and a prototype of coaxial saxophone is built. An impedance model of coaxial resonator is proposed and validated by comparison with experimental data. Sound production is also studied through experiments with a blowing machine. The playability of the prototype is then assessed and proven for several values of the blowing pressure, of the embouchure parameter, and of the instrument’s geometrical parameters.

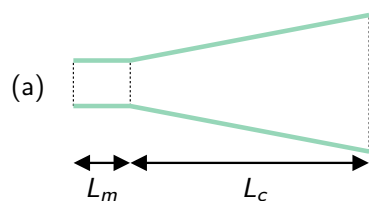
## 1 Introduction

In the family of the single-reed instruments, the main difference between the clarinet and the saxophone is the shape of the resonator of the instrument. The resonance frequencies of these instruments exhibit different behaviors: cylindrical resonators produce mainly odd harmonics while conical resonators produce even and odd harmonics (when a mouthpiece with an adequate volume is used).

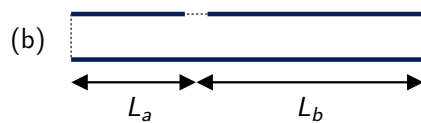
In the literature [1, 2, 3], an analogy has been made and reveals that it is possible to produce even harmonics with resonators constructed with cylindrical tubes. This analogy was first suggested by Benade [4]. Based on a low frequency approximation, this analogy consists in representing a conical resonator by two cylindrical pipes with equal cross section areas mounted in parallel. A first tube is equivalent to a truncated cone (the resonator part), and the other tube represents the missing part of the truncated cone (the volume of which is usually equal to that of the mouthpiece). Note that this analogy corresponds also to a particular case of a series of stepped cones [5]. Initially, this analogy was developed for the purpose of theoretical analysis, taking advantage of the fact that in the time domain the reflection functions are simpler for cylinders than for cones. However, in this analogy, the volume of the mouthpiece is considered to be null. This is why in [1] the authors make the following remark: “...cylindrical saxophones cannot be built easily for experimental observations (because of the finite volume of a real mouthpiece)...”. This would explain why, to our knowledge, no playable example of cylindrical saxophone has emerged.

The aim of the present paper is to show how it is yet possible to produce sounds experimentally with a modified cylindrical saxophone - called coaxial saxophone in this paper. The first section recalls the analogy between conical and cylindrical saxophones and explains how the volume of the mouthpiece is harmful for the playability of the instrument with this geometry. The coaxial saxophone geometry is then introduced in Sec-

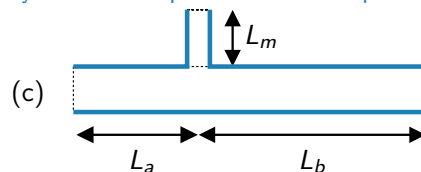
Conical resonator with mouthpiece



Cylindrical saxophone without mouthpiece



Cylindrical saxophone with mouthpiece



**Fig. 1:** Schematics of: (a) a conical resonator with mouthpiece, (b) a cylindrical saxophone without mouthpiece and (c) a cylindrical saxophone with mouthpiece.

tion 3: the coaxial alignment of the two cylindrical pipes makes it possible to minimize the damaging effect of the mouthpiece volume. Simulations and measurements on a prototype are studied in order to understand the modal behavior of a coaxial saxophone. Finally, the prototype is played with an artificial mouth and the sound produced is analyzed. Particular attention is paid to the nature of the oscillation regimes - periodic or not - which are produced by this instrument according to the control parameters imposed by the musician.

## 2 Problem statement

This section recalls the analogy between conical and cylindrical saxophones in order to highlight the main

limitation of this analogy for the sound production. In [1], it is shown that, at low frequencies, a conical resonator can be analogous to two cylindrical tubes mounted in parallel. For a truncated cone with a cylindrical mouthpiece whose volume is the same as the missing volume of the cone (see Fig. 1(a)), the input impedance is:

$$Z_{tc} = jZ_c \left( \frac{1}{kL_m} - \frac{kL_m}{3} + \cot kL_c \right)^{-1}, \quad (1)$$

where  $Z_c = \rho c/S$ .  $S$  is the cross section area of the small end of the truncated cone,  $\rho$  is the air density, and  $c$  is the speed of sound.  $L_m$  is the length of the mouthpiece,  $L_c$  is the length of the truncated cone and  $k$  is the wavenumber. For cylindrical resonators mounted in parallel (see Fig. 1(b)), the input impedance is:

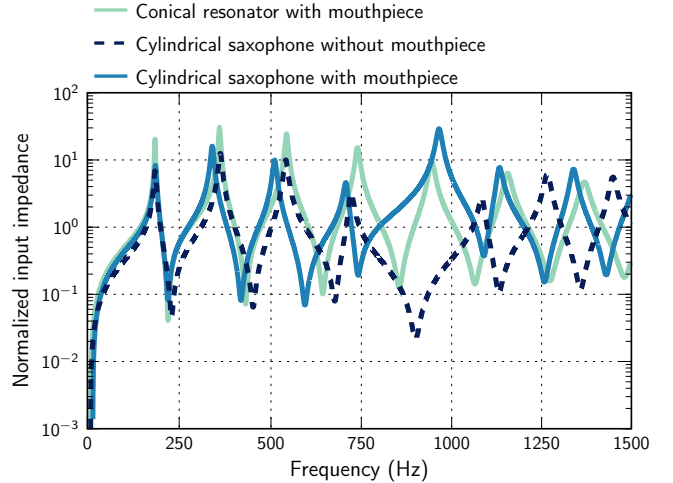
$$Z_{ts} = jZ_c (\cot kL_a + \cot kL_b)^{-1}, \quad (2)$$

where  $L_a$  and  $L_b$  are the lengths of the two cylinders. Within the low frequency approximation, Equation (1) is an expansion of Eq. (2) at the third order provided that  $L_b = L_c$  and  $L_a = L_m$ . But the analogy between these two equations has no practical application since the mouthpiece is not taken into account in Eq. (2). From Eq. (2) it can be deduced that the resonance frequencies of two cylindrical resonators mounted in parallel are perfectly harmonic. Adding a mouthpiece to this cylindrical saxophone (sketched in Fig. 1(c)) results in an alteration of the harmonicity of the impedance peaks of the instrument.

The impedance curves plotted in Fig. 2 are obtained through simulations for the three resonator geometries presented in Fig. 1. The parameters used for these simulations are detailed as follows. The conical resonator has an input radius  $r_c = 7.5 \cdot 10^{-3}m$ , the apex angle is  $\theta = 0.035rad$ , the length of the conical tube is  $L_c = 0.72m$  and the length of the mouthpiece is  $L_m = 0.08m$ . The cylindrical saxophone has a radius  $r = 7.5 \cdot 10^{-3}m$ , the lengths of the two mounted cylinders are  $L_b = 0.75m$  and  $L_a = 0.185m$ , respectively. The mouthpiece has a radius  $r = 7.5 \cdot 10^{-3}m$ , and its length is determined by the value of the inner volume,  $V_m = 1.5 \cdot 10^{-5}m^3$  (standard values for tenor saxophone mouthpieces). The mouthpiece is treated as a cylindrical tube plugged in series with the resonator.

These input impedances illustrate that at low frequencies the analogy between a conical and a cylindrical saxophone is basically valid in the absence of a mouthpiece. The low frequency limit is represented by the anti-resonance localized near 900 Hz (depending on the geometrical dimensions of the cylindrical saxophone [1]). Below this limit the values of resonance frequencies are similar for the first three peaks. Differences between peak amplitudes are explained by larger visco-thermal losses in the cylindrical saxophone compared with a conical saxophone.

The addition of the mouthpiece strongly modifies the input impedance of the cylindrical saxophone. Indeed, the addition of a mouthpiece results in two defects. The first one is a significant alteration of the harmonicity of the resonance frequencies. For instance, focusing on the two lowest resonance frequencies  $f_1$  and  $f_2$ , the



**Fig. 2:** Simulated input impedance for: a conical resonator with mouthpiece, a cylindrical saxophone (dashed line) and a cylindrical saxophone with mouthpiece. The input radius and the length of the conical resonator are  $r_c = 7.5 \cdot 10^{-3}m$  and  $L_c = 0.72m$  respectively. The cylindrical saxophone has a radius  $r = 7.5 \cdot 10^{-3}m$ , the lengths of the two mounted cylinders are  $L_b = 0.75m$  and  $L_a = 0.185m$  respectively. The mouthpiece volume is  $V_m = 1.5 \cdot 10^{-5}m^3$ .

ratio  $f_2/f_1 = 2$  without mouthpiece and  $f_2/f_1 = 1.9$  when the mouthpiece is added. As studied in previous work [6], inharmonicity favors the bifurcation towards non-conventional oscillation regimes, such as multiphonic sounds and squeaks. The second effect of the mouthpiece is to generate an additional resonance, around 950 Hz. This resonance is due to the discontinuity between the mouthpiece cross-section and the connection of the two cylindrical tubes (the effective cross-section is doubled at the connection). Note that the frequency of this additional resonance is related to the length of the mouthpiece (quarter wave resonance). Furthermore, it can be observed that the amplitude of this resonance is greater than that of the other resonance amplitudes. This hinders the emergence of self-sustained oscillations corresponding to the two lower registers.

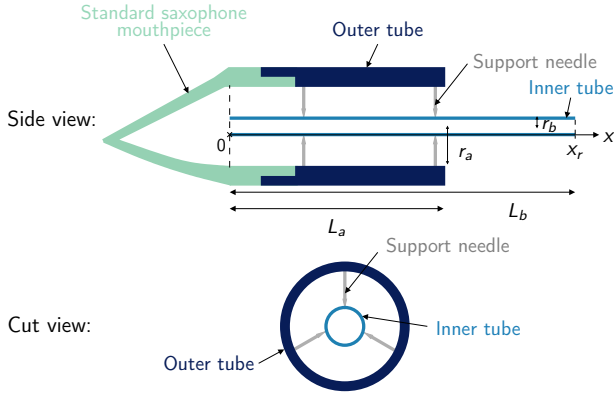
These two modifications of the input impedance caused by the addition of a mouthpiece suggest some trouble with the instrument playability. To ensure the ease of playing, the effect of the mouthpiece needs to be minimized, and a suitable geometry for the cylindrical saxophone has to be sought.

## 3 Coaxial resonator

### 3.1 Description of the coaxial geometry

In order to minimize the effect of the mouthpiece volume, a coaxial geometry is chosen, replacing the original alignment of the pipes. The geometry proposed is sketched in Fig. 3. The construction of the coaxial geometry ensures that the cross-section area at the end of the mouthpiece is equal to the sum of the cross-section areas of the two cylinders: no discontinuity occurs and thus the emergence of any additional resonance is avoided.

A prototype is constructed according to the geometrical dimensions presented in Table 1. For the design of this prototype, simulations of input impedances have been carried out, as detailed in Section 3.2.1. The prototype



**Fig. 3:** Schematic of the coaxial geometry. No discontinuity is present at the connection between the mouthpiece and the resonator. The cylindrical part of the mouthpiece is extended by the outer tube inserted therein.

	$r_b$	$e$	$L_b$ (max)
Inner tube	5.5mm	0.975mm	0.5m
	$r_a$	$L_a$ (max)	
Outer tube	8.5mm	0.163m	

**Table 1:** Dimensions of the prototype of coaxial resonator.  $r_a$  and  $L_a$  are the radius and the length of the outer tube,  $r_b$  and  $L_b$  those of the inner tube.  $e$  is the wall thickness of the inner tube.

outer and inner tubes are made of plexiglass and copper, respectively. The inner tube is centered inside the outer tube by means of two rows of support needles (see Fig. 3). Several lengths of copper tubes are used to test different playing frequencies. In order to ensure the analogy with a conical resonator, the diameter and the thickness of the inner tube are chosen so that its cross-section area is equal to that of the annular gap between the two cylinders. A standard tenor saxophone mouthpiece is used (Yamaha 4C model). The bore of this mouthpiece is cylindrical. In order to minimize the effective mouthpiece volume, the cylindrical part of the mouthpiece is associated with the outer tube. To achieve this, the inner cylinder is inserted inside the mouthpiece up to  $x = 0$  (see Fig. 3).

## 3.2 Input impedance of a coaxial resonator

The coaxial saxophone can be viewed as a particular case of the cylindrical saxophone. Therefore, it is necessary to verify how a classical model of cylindrical saxophone can be modified to correctly represent the coaxial alignment of the tubes. An experimental and numerical study of the input impedance of the coaxial resonator is then carried out in order to highlight the effect of the geometrical parameters.

### 3.2.1 Computation of input impedances

Input impedances of coaxial saxophones are computed in the same way as those of cylindrical saxophones (see Section 2). The visco-thermal losses at the walls are greater in the annular region than in a simple cylindrical pipe. A crude approximation is made: visco-thermal losses are assumed to be proportional to the wall area of

the annular region (on the interval  $x = [0, L_a]$ ). For further discussion about this assumption, see [8, 9]. Therefore, the ratio  $\mu$  between the surfaces of the inner and outer walls of the annular guide is defined:

$$\mu = \frac{2\pi L_a(r_a + r_b)}{2\pi L_a r_b}. \quad (3)$$

To ensure the analogy with the cylindrical saxophone, the cross-sectional area of the annular gap must be equal to that of the inner tube ( $S_a = S_b$ ), thus  $r_a = \sqrt{2}r_b$  leads to  $\mu = 1 + \sqrt{2}$ . Notice that if the coefficient  $\mu$  was ignored, the height of the impedance peaks would be overestimated, but the overall shape of the input impedances would be preserved.

The input impedance of the coaxial saxophone is given by:

$$Z_{cs} = jZ_c (\cot k_a L_a + \cot k_b L_b)^{-1}, \quad (4)$$

where the wavenumbers  $k_b$  and  $k_a$  are expressed [7] with the ratio  $\mu$  as:

$$k_b = k_0 \left( 1 + \frac{1.044\sqrt{-2j}}{r_{vb}} \right) \quad (5)$$

and

$$k_a = k_0 \left( 1 + \mu \frac{1.044\sqrt{-2j}}{r_{va}} \right). \quad (6)$$

The Stokes numbers for the inner and the outer tubes are expressed as  $r_{va} = r_a \sqrt{k_0/l_v}$  and  $r_{vb} = r_b \sqrt{k_0/l_v}$ , where  $k_0 = \omega/c$  and  $l_v$  is the viscous characteristic length.

It remains to use an approximation for the radiation of the outer tube. Despite to the annular bore between the outer and the inner tubes, it is assumed to radiate like a cylindrical tube. Therefore, the radiation impedance for both tubes is that of a tube without flange  $Z_c (j0.61kr_b + \frac{1}{4}(kr_b)^2)$ . At low frequencies, this approximation is rigorous concerning the real part of the impedance (because of the energy conservation) whereas it is crude for the imaginary part of the radiation impedance.

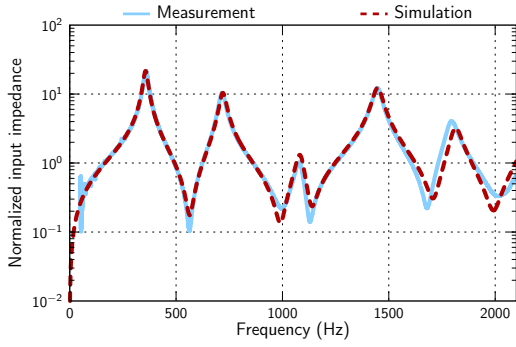
### 3.2.2 Validation of the impedance model

To validate the impedance model of coaxial saxophone, experiments are carried out for various values of length  $L_a$  and  $L_b$ . First, simulated and measured input impedances for different lengths  $L_b$  are compared in Fig. 4. The apparatus developed in [10] is used for the measurement of the input impedances. No mouthpiece is considered for the impedance measurement, the impedance sensor is plugged at position  $x = 0$  (see the Fig. 3).

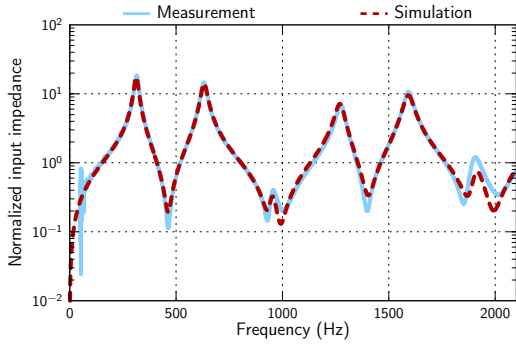
A qualitative agreement is observed between measurements and simulations (see Fig. 4). The multiplicative coefficient  $\mu$  models correctly the visco-thermal losses in the annular region, since the heights of the resonance peaks are well estimated. Thus, the model of coaxial saxophone presented previously can be used for the design of more advanced prototypes.

### 3.2.3 Influence of the mouthpiece volume

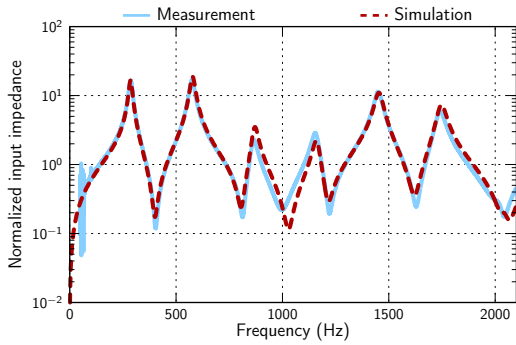
The influence of the mouthpiece volume is now studied experimentally, focussing on the inharmonicity of the



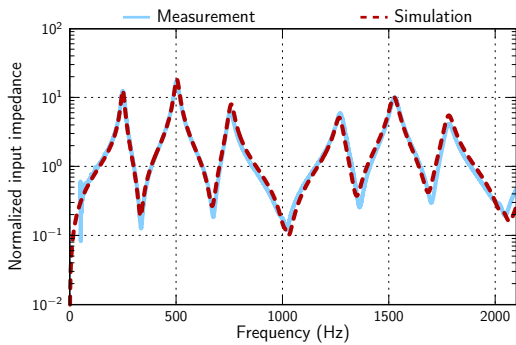
(a)  $L_b = 0.294$



(b)  $L_b = 0.359$

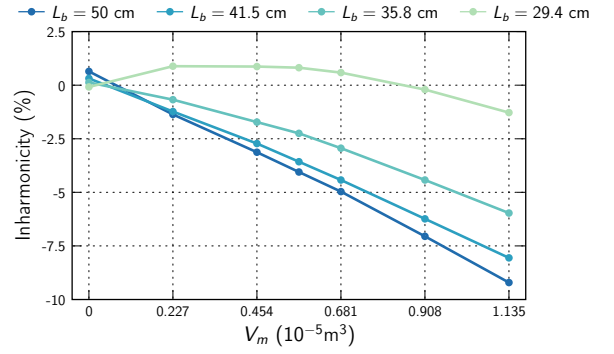


(c)  $L_b = 0.415$



(d)  $L_b = 0.5$

**Fig. 4:** Comparison between the measured input impedance for the coaxial prototype and the simulated impedance for a cylindrical saxophone without mouthpiece (see Section 3.2.1),  $L_a = 16.3\text{cm}$  and  $L_b$  is varying.



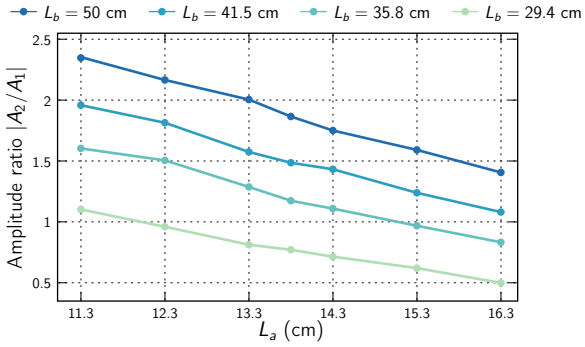
**Fig. 5:** Measured inharmonicity between the two first impedance peaks as a function of the equivalent volume of the mouthpiece.

two lowest resonance frequencies. In order to modify experimentally the mouthpiece volume on the prototype, the inner tube is pulled from  $x = 0$  to  $x = x^* > 0$  (see Fig. 3). The reduction of the outer tube length (from  $L_a$  to  $L_a - x^*$ ) allows easy change of the mouthpiece volume to an arbitrary value. Inharmonicity arising from this modification of  $L_a$  is much lower than that brought by the addition of an equivalent mouthpiece volume (this fact was verified according to simulations). For the inharmonicity estimation, resonance frequencies are obtained from the zero crossing of the input impedance phase. The inharmonicity between the first two resonance frequencies is defined as  $\eta = f_2/(2f_1) - 1$ . Fig. 5 shows that the larger the mouthpiece volume, the greater the inharmonicity. The most relevant piece of information is that a high range of inharmonicity is obtained with standard values of mouthpiece volume. For most resonator lengths  $L_b$ , the inharmonicity is greater than 2.5%, which may limit the stability of the first periodic oscillation regimes [11] (ideally the inharmonicity should be zero). This result highlights why a mouthpiece with a short chamber should be preferred for playing a saxophone with coaxial geometry. Indeed, the volume of the mouthpiece mostly brings a negative inharmonicity and thus the global inharmonicity of the coaxial resonator is deteriorated - which does not happen with a truncated cone.

### 3.2.4 Influence of the length of the outer tube

Other experiments are carried out to study the influence of the outer tube on the input impedance of the coaxial saxophone. Fig. 6 presents the amplitude ratio  $A_2/A_1$  between the two first resonance peaks as a function of the length  $L_a$  of the outer tube. The equivalent mouthpiece volume is modified when  $L_a$  is shifted, but this has a negligible influence on the amplitude ratio compared to the influence of the increase in  $L_b$ . This has been confirmed by simulations.

Results reveal that the shorter the outer length  $L_a$ , the higher the amplitude ratio  $A_2/A_1$ . This is consistent with what is happening with a conical saxophone. For the shorter outer tubes tested, the amplitude of the second resonance becomes larger than twice the first one. Furthermore, Fig. 6 shows that the overall evolution of the relative amplitude of the two first resonance frequencies can be different from one resonator length  $L_b$  to another. In particular, it can be observed that for



**Fig. 6:** Measured amplitude ratio between the two first resonance peaks as a function of the length of the outer tube.

a short resonator tube ( $L_b = 29.4\text{cm}$ ) the amplitude of the first impedance peak is almost always higher than that of the second peak, even for a short outer tube ( $L_a = 12.3\text{cm}$ ).

Similar effects are observed with conical saxophones when the resonator conicity increases. Indeed, in the context of the analogy with the conical saxophone, modifying  $L_a$  can be seen as a way to adjust the conicity of the instrument [12]. In practice, a simple change in the length of the external cylinder is expected to modify the tone color regardless of the fingering played. This is exemplified in the next section.

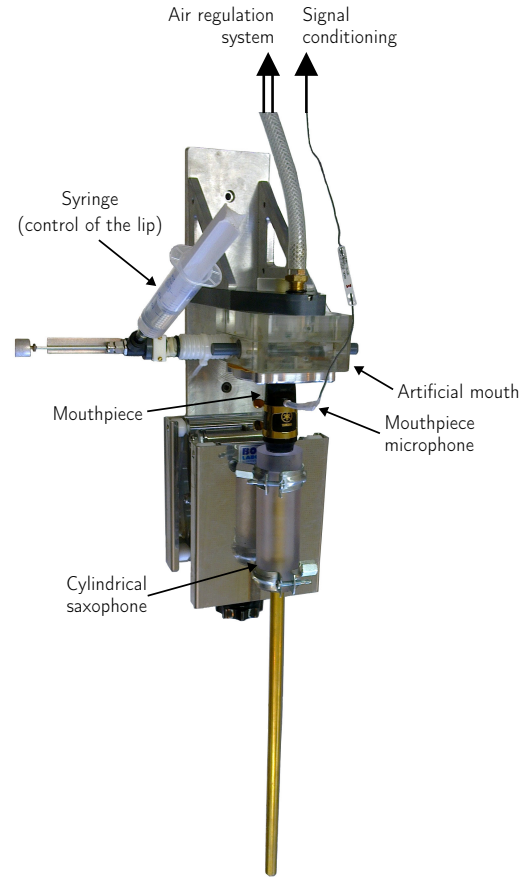
## 4 Sound production on a coaxial saxophone

In order to study the sound production, an artificial mouth is used to play the prototype of coaxial saxophone in controlled conditions.

### 4.1 Experimental setup

A photo of the experimental device is shown in Fig. 7. The coaxial saxophone is connected to the artificial mouth at the level of the mouthpiece (played with a Hemke premium reed with force 4). Inside the artificial mouth, an artificial lip is pressed against the reed and brings the required damping to avoid squeaks when playing the instrument [13]. The artificial lip is made with a latex tube filled with water through a syringe. The bearing force of the lip on the reed is controlled by the water pressure imposed by the syringe. The artificial lip is placed around 1cm downstream from the tip of the reed. The experimental device used in this study has been used for a previous study [6] where details concerning its functioning are given.

In order to measure the acoustic signal produced by the instrument, an Endevco pressure microphone (8507C-5 model) is inserted into the mouthpiece. Another similar sensor is used to measure the static pressure in the artificial mouth to control in real-time the servo-valve regulating the supply pressure through a closed-loop system (technical details are available in [14]). In addition, a Burkert flowmeter (8701 model) is used to measure the volume flow entering the instrument. A National Instrument acquisition module (NI9215) uses a



**Fig. 7:** Picture of the experimental device and the prototype of coaxial saxophone.

sampling frequency of 44kHz to digitize the experimental signals, which are then recorded on a computer.

The control parameters imposed by the artificial mouth are estimated by recording the air flow entering the instrument and the static pressure [6]. The embouchure parameter  $\zeta$  and the blowing pressure  $\gamma$  are considered to be independent. These dimensionless parameters are estimated experimentally from the measurement of the so-called nonlinear characteristics of the exciter [15] (noted NLC hereafter).

After each modification of the bearing force of the artificial lip on the reed (by acting on the syringe), the NLC is measured. By fitting the measurement with the NLC model [15], the maximum airflow  $U_{\max}$  and the corresponding pressure difference  $P_{\max}$  are estimated. The embouchure parameter  $\zeta$  is then obtained through the following relation [15]:

$$\zeta = \frac{\sqrt{3} U_{\max}}{2 P_{\max}} Z_c. \quad (7)$$

Likewise, the dimensionless blowing pressure is estimated from the following relationship:

$$\gamma = \frac{P_m}{3P_{\max}}, \quad (8)$$

with  $P_m$  the static supply pressure. More details about the estimation of the control parameters  $\gamma$  and  $\zeta$  can be found in [6].

Note that the control parameters  $\gamma$  and  $\zeta$  are dimensionless. The minimum pressure required for the

complete closure of the reed channel corresponds to  $\gamma = 1$ . When the bearing force of the lip on the reed is large enough to completely close the reed channel, the parameter  $\zeta$  vanishes.

## 4.2 Mouthpiece pressure waveform

One way to describe sound production on a coaxial saxophone consists in studying the mouthpiece pressure waveform.

Figs. 8(a) shows the mouthpiece pressure signal. It is similar to the classical Helmholtz motion obtained for conical reed instruments [3, 12], with a v-shaped episode followed by a quasi-plateau. Furthermore, Fig. 8(b) shows that the sound contains both even and odd harmonics. Therefore the analogy between conical and coaxial saxophones is verified experimentally.

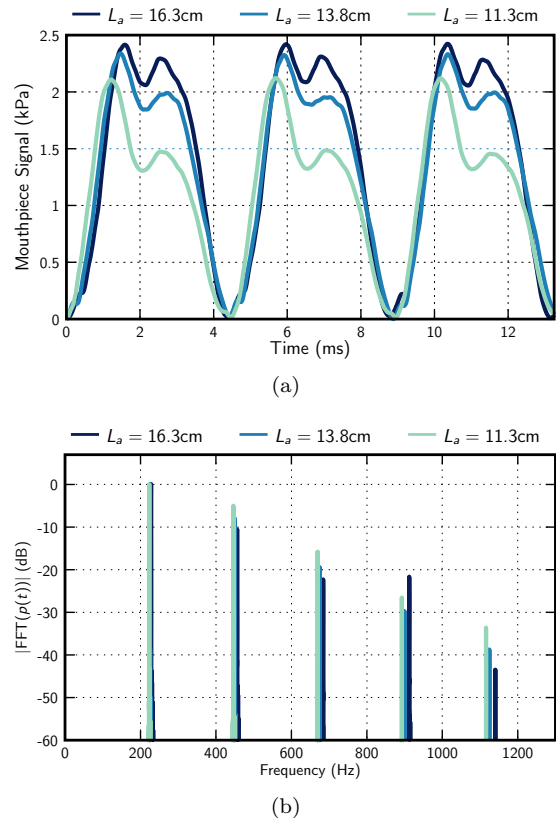
In Figs. 8, it can also be observed that the length  $L_a$  of the outer tube, which corresponds to the truncated part of the cone, modifies the internal waveform. The length of the outer tube can be viewed as the parameter that determines an equivalent conicity (apex angle of the cone) [12].

The spectral content of sounds produced by the coaxial saxophone may be modified by the adjustment of the length  $L_a$ . As shown in Fig. 8(b), the shorter the outer tube, the greater the second and third harmonic (compared to the first one). Therefore, the tone color of a coaxial saxophone can be easily modified just by adjusting (or changing) the length of outer tube.

Likewise, Fig. 8(b) shows that the playing frequency also depends on the length of the outer tube. This is particularly useful in practice for the tuning of the instrument.

## 4.3 Maps of the oscillation regimes in the control parameter space

A general overview of the sound production can be obtained by deriving maps of oscillation regimes with respect to the control parameters  $\gamma$  and  $\zeta$ . For this purpose, the methodology detailed in a previous work [6] is applied to the prototype of coaxial saxophone. For a linearly increasing ramp of the blowing pressure, the parameters space is spanned by horizontal lines corresponding to the constant embouchure setting  $\zeta$ . For each value of  $\zeta$ , a spectrogram of the mouthpiece pressure signal is computed. This is relevant because the pressure ramps used during the experiments are very slow (about one minute from the minimum to maximum pressure value). The same qualitative results are obtained with upward and downward ramps. For each value of  $\zeta$  obtained experimentally, instantaneous frequencies and amplitudes are estimated on the spectrogram by the Matlab function *findpeaks*. An algorithm is used to classify these data into different oscillation regimes. Three types of regimes are identified: a static regime (no sound), a quasi-periodic regime (multiphonic sound, noted QP in Fig. 9) and a periodic regime. Different playing frequencies of periodic regimes are possible and are represented by a register index. In Fig. 9, R1 and R2 correspond to the first and second registers, respectively.



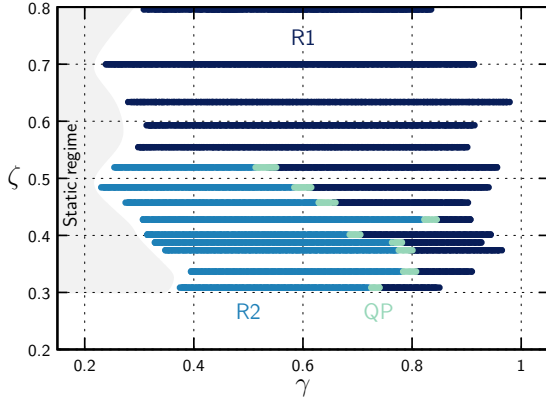
**Fig. 8:** (a) Mouthpiece pressure signal and (b) its spectrum amplitude measured for different lengths of the outer tube  $L_a$ . The resonator length is constant  $L_b = 35.8\text{cm}$ . The control parameters are  $\gamma = 0.31$  and  $\zeta = 0.44$ .

Fig. 9 shows maps for four different lengths of the resonator  $L_b$ , which can be seen as different fingerings, while the length of the outer tube is kept constant. Each regime is symbolized by a particular color.

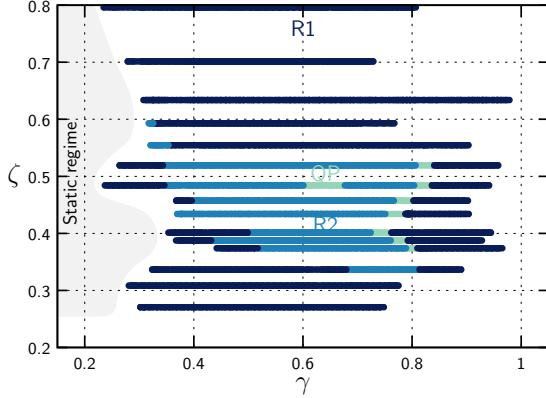
The main periodic regimes of self-sustained oscillations produced by the coaxial saxophone are associated with the two first resonance frequencies of the instrument. This result was expected since, as shown in Fig. 4, the amplitudes of these two resonance peaks are higher than those of the other resonances peaks. Consequently, the bifurcation to these registers is favored. Overall, the range of control parameters available to play the different registers of the instrument is rather wide. Although quantitative, these results indicate the good playability of the instrument, since registers can be stable even for large variations of  $\gamma$ .

Furthermore, quasi-periodic oscillations also appear on the maps in Fig. 9. This is related, at least partly, to the inharmonicity due to the mouthpiece (as detailed in Sections 2 and 3). For the two following resonator lengths,  $L_b = 29.4\text{cm}$  and  $L_b = 35.8\text{cm}$  (see Figs. 9(a) and 9(b)), quasi-periodicity is very rare and appears only as a transition between the two periodic registers. Thus, quasi-periodicity does not appear here as a defect that could alter the playability of the instrument.

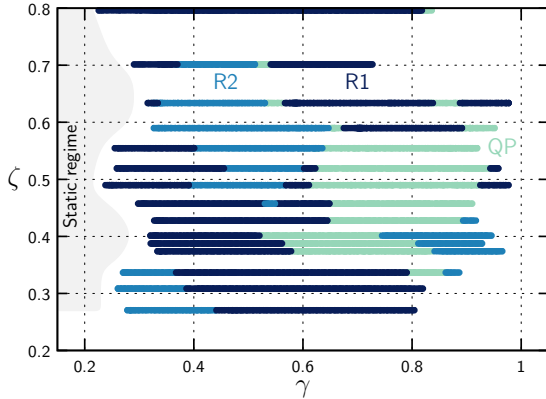
However for the resonator lengths  $L_b = 41.5\text{cm}$  and  $L_b = 50\text{cm}$ , quasi-periodicity is more present. Indeed, as shown in Figs. 5 and 6, the inharmonicity and the relative amplitude of the second resonance are greater for longer resonators. This feature of the coaxial resonator is an aggravating factor that favors the production of



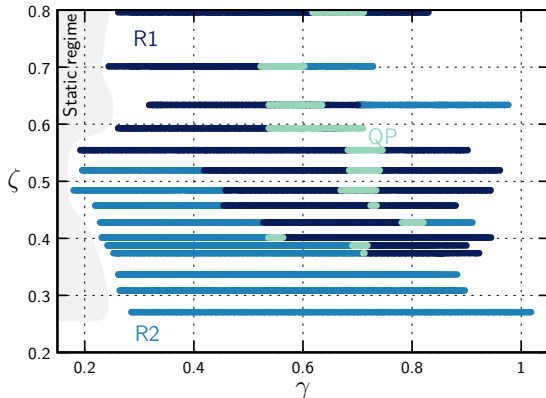
(a)  $L_b = 29.4$  cm



(b)  $L_b = 35.8$  cm



(c)  $L_b = 41.5$  cm



(d)  $L_b = 50$  cm

**Fig. 9:** Mapping of the oscillation regimes for different lengths of the internal tube  $L_b$  in the space of the control parameters  $(\gamma, \zeta)$ . The length of the outer tube is set at  $L_a = 16.3$  cm. R1 and R2 correspond to the first and second registers, respectively, while QP means quasi-periodicity.

quasi-periodic regimes [11]. This explains why periodic regimes are less widespread for Figs. 9(c) and 9(d).

In summary, the maps in Fig. 9 show the variability of oscillation regimes playable with a coaxial saxophone. Extensive ranges of periodic regimes are playable, which illustrates the good playability of the coaxial saxophone. However, in the design of a more complete instrument, some properties concerning resonances of the instrument (such as inharmonicity) are to be avoided in order to provide a good stability in periodic sound production. Moreover, this is important to avoid strong adjustments in control parameters that the musician may have to impose between two fingerings.

## 5 Conclusion

The results presented above show that a coaxial geometry makes it possible to experimentally verify the cylindrical saxophone analogy. The prototype of coaxial saxophone presented in this paper can produce periodic oscillation with a waveform very similar to that of conical resonators. The measurement of maps in the control parameter space illustrates the playability of the first two registers of the instrument. The use of an impedance model proves that the mouthpiece volume must be minimized to favor the playability of the instrument. Similarly, it is shown that the coaxial alignment of the two tubes prevents a parasitic resonance limiting the stability of periodic regimes.

As shown by the patent recently published about original pipe structures for wind instruments [16, 17], there is an industrial interest for radically new types of acoustic resonators. With an aim towards realistic musical instruments, current research could be applied to the coaxial saxophone. Numerical optimization could be applied to help the instrument maker to design a complete instrument [18] (resonator geometry, position of the side holes, etc). Finally, the study of bifurcation mechanisms, associated with the analysis of musicians' playing strategies, would make it possible to anticipate the playability and the stability of a complete instrument.

### Acknowledgements

This work was done in the frameworks of Labex MEC (ANR-10-LABX-0092) and of the A\*MIDEX project (ANR-11-IDEX-0001-02), funded by the French National Research Agency (ANR). It was also supported by the CAGIMA project financed by the French National Research Agency (project ANR-11-BS09-022).

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