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# Quantitative 3D electromagnetic field determination of 1D nanostructures from single projection

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## Abstract

One-dimensional (1D) nanostructures have been regarded as the most promising building blocks for nanoelectronics and nanocomposite material systems as well as for alternative energy applications. Although they result in confinement of a material, their properties and interactions with other nanostructures are still very much three-dimensional (3D) in nature. In this work, we present a novel method for quantitative determination of the 3D electromagnetic fields in and around 1D nanostructures using a single electron wave phase image, thereby eliminating the cumbersome acquisition of tomographic data. Using symmetry arguments, we have reconstructed the 3D magnetic field of a nickel nanowire as well as the 3D electric field around a carbon nanotube field emitter, from one single projection. The accuracy of quantitative values determined here is shown to be a better fit to the physics at play than the value obtained by conventional analysis. Moreover the 3D reconstructions can then directly be visualized and used in the design of functional 3D

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architectures built using 1D nanostructures.

*Keywords:* 3D electromagnetic fields, 1D nanostructures, phase reconstruction, transmission electron microscopy

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## 1 **1. Introduction**

2       Nanoscale structures, such as ultra-thin films, nanoparticles, and nanowires,  
3 have been intensively investigated over the past decade as a result of their  
4 novel size-dependant behavior. Amongst them, one dimensional (1D) nanos-  
5 tructures such as nanowires and nanotubes are of significant importance as  
6 they exhibit molecular scale properties and behavior such as density of states  
7 singularities, and emergent charge transport properties [1, 2, 3]. These struc-  
8 tures also have direct applications as functional nanostructures in nanoelec-  
9 tronics and nanophotonics. The recent progress in fabrication methods have  
10 enabled 1D nanostructures to be made from a wide variety of materials in-  
11 cluding ferromagnetic [4], ferroelectric [5], thermoelectric [6] and semicon-  
12 ductor [7]. In all these 1D nanostructures, it is necessary to quantitatively  
13 measure the local properties so that their behavior can be better under-  
14 stood. Moreover, enhancement of some properties such as local electromag-  
15 netic fields in 1D nanostructures needs to be quantified, so that it can be  
16 controlled for required applications.

17       In this work we focus on two particular 1D nanostructures: (1) ferromag-  
18 netic nanowires, and (2) electrostatically charged carbon nanotips. Ferro-  
19 magnetic nanowires form an integral part of existing and novel applications  
20 in magnetic storage [8] and sensing [9]. Magnetic nanowires with circular  
21 cross-section, are of utmost importance from theoretical and technological

22 aspects. Their high aspect ratio and cylindrical symmetry leads to novel  
23 domain wall behavior such as suppression of Walker breakdown, which is of  
24 interest in magnetic logic circuits and domain wall memory [10, 11]. They  
25 also have potential applications in single cell bio-sensing [12, 13] as well as  
26 in nanocomposites for exchange spring magnets [14]. They can be fabricated  
27 using a variety of methods such as lithography based templating [15], wet  
28 chemical methods and electro deposition [16]. 1D carbon-based nanostruc-  
29 tures such as carbon nanotubes are amongst the best candidates for field  
30 emission displays [17] and new high-brightness electron sources [18]. Since  
31 their discovery by Iijima [19], carbon nanotubes have been extensively stud-  
32 ied and are now used in a wide variety applications [20, 21]. One such ap-  
33 plication is field emitters because they provide a unique combination of high  
34 conductivity, high aspect ratio and robustness [22]. A carbon cone nanotip  
35 (CCnT) was recently successfully used in a high-voltage electron gun [23].

36 The confinement effects in 1D nanostructures can alter their properties  
37 and subsequently their behavior significantly. Hence it is necessary to under-  
38 stand the strong effect of their size on their three-dimensional (3D) properties  
39 such as the magnetic and electric fields associated with nanowires and nan-  
40 otubes completely before they can be used in applications. There are a vari-  
41 ety of methods to study electromagnetic fields at the nanometer length scale  
42 such as scanning tunneling microscopy [4], scanning electron microscopy with  
43 polarization analysis (SEMPA) [24], and transmission electron microscopy  
44 (TEM) [25, 26, 27, 28]. TEM offers the best approach because it can be  
45 used to determine their physical/crystal structure, composition as well as to  
46 image the electromagnetic field at a high spatial resolution. In particular for

47 Lorentz TEM, the electromagnetic information of the sample is encoded in  
 48 the phase shift of the electron wave,  $\phi$ , and can be written as a combination  
 49 of projections of the 3D magnetic vector potential,  $\mathbf{A}$ , and 3D electrostatic  
 50 potential,  $V$ , given as [29]:

$$\begin{aligned} \phi(x, y) &= \phi_e(x, y) - \phi_m(x, y) \\ &= C_E \int_{-\infty}^{+\infty} V(x, y, z) dz - C_B \int_{-\infty}^{+\infty} A_z(x, y, z) dz \end{aligned} \quad (1)$$

51 where  $\phi_e$  is the electrostatic phase shift,  $\phi_m$  is the magnetic phase shift,  
 52  $C_E = \pi/E\lambda$ ,  $E$ , the relativistic electron scattering potential and  $\lambda$ , the  
 53 wavelength of electrons,  $C_B = \pi/\Phi_0$ ,  $\Phi_0$ , the magnetic flux quantum and  
 54 the direction of propagation of the electrons is assumed to be along  $z$ . The  
 55 phase shift can be recovered experimentally using various techniques such as  
 56 transport-of-intensity based methods or off-axis electron holography. How-  
 57 ever, from the equations it can be seen that the recovered phase shifts are  
 58 an integration of the signal over the electron path that yield only projected  
 59 2D information. This means that the information from the sample itself is  
 60 integrated along with the stray field, thereby leading to inaccurate measure-  
 61 ments of electromagnetic fields. There are currently very few methods, which  
 62 have the capability to visualize the complete 3D vector fields associated with  
 63 nanowires. Nominally, determining the 3D magnetic field requires recording  
 64 a series of phase shift images as the sample is tilted about its axis. This  
 65 becomes a tedious and cumbersome process with various sources of error  
 66 or requires specialized electron microscopes [30, 31]. In a 1D nanostructure  
 67 such as a nanocylinder that is uniformly magnetized along its long axis, the  
 68 magnetic field possesses cylindrical symmetry with respect to the long axis.

69 In cases, where the 1D nanostructure does not possess cylindrical symmetry,  
70 the usual tilt based tomography must be performed to reconstruct the 3D  
71 magnetic field [32]. Similarly, carbon nanotubes under applied bias exhibit  
72 a cylindrically symmetric potential and electric field. Here we show that  
73 the cylindrical symmetry can be exploited to enhance the quantification of  
74 electromagnetic properties by recovering the entire 3D vectorial field in and  
75 around nanowires and nanotubes from a single image.

## 76 2. 3D magnetic field

77 The magnetic information from the sample is contained in the second term  
78 of equation (1),  $\phi_m$ , which can be separated using time reversal symmetry,  
79 i.e. by recording the phase shift additionally with the sample turned over by  
80  $180^\circ$  and computing the difference between the two phase shifts [33]. Taking  
81 the derivative of the magnetic phase shift with respect to  $x$ , and using the  
82 relation between vector potential and magnetic induction ( $\mathbf{B} = \nabla \times \mathbf{A}$ ) gives:

$$\frac{\partial}{\partial x} \phi(x, y) = C_B \int_{-\infty}^{\infty} B_y(x, y, z) dz. \quad (2)$$

83 This equation correlates the projection of one component of the magnetic  
84 induction with the magnetic phase shift image. When there is a cylindrically-  
85 symmetric magnetic induction present, using the inverse Abel transform [34],  
86 the  $y$  component of the magnetic induction,  $B_y$ , can be reconstructed from  
87 the phase shift image as:

$$B_y(\rho, y) = -\frac{1}{C_B \pi} \int_{\rho}^{\infty} \frac{\partial^2 \phi(x, y)}{\partial x^2} \frac{dx}{\sqrt{x^2 - \rho^2}}, \quad (3)$$

88 where  $\rho^2 = x^2 + z^2$ . A detailed derivation of the inverse Abel transform  
 89 is presented in the supporting information. It should be noted that using  
 90 the above equation provides the  $y$  component of all magnetization vectors in  
 91 3D. Furthermore, using the divergenceless condition  $\nabla \cdot \mathbf{B} = 0$  in cylindrical  
 92 coordinates,  $B_\rho$  can be calculated as:

$$B_\rho(\rho, y) = -\frac{1}{\rho} \int_{\rho}^{\infty} \rho' \frac{\partial B_y(\rho', y)}{\partial y} d\rho', \quad (4)$$

93 the  $x$  and  $z$  components of the magnetic induction being obtained as  $(B_x, B_z) =$   
 94  $(B_\rho \cos \theta, B_\rho \sin \theta)$ .

95 The numerical implementation of the reconstruction procedure was done  
 96 using the Interactive Data Language (IDL) and was tested by reconstructing  
 97 the magnetic induction of a uniformly magnetized spherical nanoparticle as  
 98 the analytical expression for this case is well known. The simulations were  
 99 performed using a  $256 \times 256$  pixel grid with a resolution of 1 nm per pixel.  
 100 The magnetic induction of such a sphere has cylindrical symmetry about the  
 101 axis of the magnetization vector [35], which here is assumed to be lying in the  
 102  $x - y$  plane. The computations were performed for a uniformly magnetized  
 103 sphere of radius,  $R = 16$  nm,  $B_0 = 1.0$  T, and magnetization unit vector  
 104 given by  $[0, 1, 0]$ . The magnetic phase shift of such a sphere was numerically  
 105 computed using the analytical expression given in the paper by De Graef  
 106 et. al. [36]. Figure 1(a) shows the simulated magnetic phase shift image of  
 107 the spherical nanoparticle. Figure 1(b) shows the derivative of the magnetic  
 108 phase shift with respect to the  $x$  axis (horizontal axis of the image) required  
 109 for reconstruction (equation (2)). The symmetry about the  $y$  axis of the  
 110 image (vertical axis) in the derivative image can be clearly observed. This

111 image was then used for numerical implementation of equation (3) to recon-  
112 struct the complete 3D magnetic induction inside and outside the spherical  
113 nanoparticle shown in Figure 1(c). The inverse Abel transform was com-  
114 puted using the modified Fourier-Hankel algorithm described by S. Ma *et al.*  
115 [37]. The  $B_\rho$  component was then calculated by numerically implementing  
116 equation (4) in polar coordinates. The numerical integration was performed  
117 using the 5-point Newton-Cotes integration formula. The vector quantities,  
118  $B_y$  and  $B_\rho$  as well as the scalar potential,  $V$  were then interpolated from po-  
119 lar coordinates to Cartesian coordinates. The color of the arrows indicates  
120 the magnitude of the  $y$  component of the magnetic induction with respect  
121 to the color bar, in units of T. The main advantage of this method, as illus-  
122 trated in Figure 1, is that it can be directly used to quantitatively map the  
123 3D magnetic induction, which leads to a better quantification of magnetic  
124 induction within the nanoparticle by eliminating the effect of surrounding  
125 stray fields. This can yield a more thorough understanding of the strength  
126 of magnetic interactions between spherical magnetic nanoparticles that can  
127 be useful for the self-assembly process. Additional comparisons as well as  
128 systematic errors analysis can be found in the supplementary material.

### 129 **3. 3D electric field**

130 The electrostatic information from the sample is similarly contained in  
131 the first term of equation (1),  $\phi_e$ . This term contains the contribution due  
132 to the charge density in the sample resulting from the applied bias as well as  
133 the mean inner potential (MIP) of the sample. The MIP can be assumed to  
134 be constant inside the sample, and hence its contribution to the electrostatic



135 phase shift depends only on the sample's thickness variation. This contri-  
 136 bution can be eliminated by computing the difference between phase shifts  
 137 of unbiased and biased samples. Resulting phase shift,  $\phi_{e,q}$ , due to only the  
 138 charge density can then be written as:

$$\phi_{e,q}(x, y) = C_E \int_{-\infty}^{\infty} V_q(x, y, z) dz, \quad (5)$$

139 where  $V_q$  represents the potential due to the charge density in the biased  
 140 sample. For a sample with cylindrically-symmetric potential, and assuming  
 141 that the sample is oriented such that the axis of symmetry is along the  $y$   
 142 axis, inverse Abel transform can be used to reconstruct the 3D electrostatic  
 143 potential as:

$$V_q(\rho, y) = -\frac{1}{C_E \pi} \int_{\rho}^{\infty} \left( \frac{\partial \phi_{e,q}(x, y)}{\partial x} \right) \frac{dx}{\sqrt{x^2 - \rho^2}}. \quad (6)$$

144 The electric field of the sample can then be computed numerically using the  
 145 relation  $\mathbf{E}(\mathbf{r}) = -\vec{\nabla} V$ , where  $\vec{\nabla}$  is the gradient operator in 3D.

#### 146 4. Results

147 **Magnetic nanowire.** Magnetic nanowires with circular cross section  
 148 which are uniformly magnetized along their long axis also possess cylindri-  
 149 cally symmetric magnetic induction. Similar to the case of the uniformly  
 150 magnetized nanosphere, it is possible to compute the complete 3D magnetic  
 151 field of such a nanowire using a single phase image. Experiments were per-  
 152 formed on a polycrystalline nickel (Ni) nanowire of 75 nm diameter [16]. The  
 153 nickel nanowires were grown by electro deposition in commercial 6  $\mu\text{m}$  thick

154 polycarbonate membranes. The membranes were then dissolved in  $\text{CH}_2\text{Cl}_2$ ,  
155 and the Ni nanowires were recovered on a carbon foil for TEM imaging.  
156 Only the region near the extremity of the nanowire was used for reconstruc-  
157 tion such that information about the magnetic field within the nanowire can  
158 be acquired along with the stray fields.

159 Figure 2(a) and (b) shows the experimental electrostatic and magnetic  
160 phase shifts of the nanowire. The phase shift was recovered using off-axis  
161 electron holography in a FEI Tecnai F20 TEM operating at 200 kV using  
162 the first transfer lens of the Cs-corrector as a Lorentz lens. The magnetic  
163 phase shift was obtained from the Ni nanowire by recording the phase shift  
164 images with the sample as-is and rotating it by  $180^\circ$  about its axis and  
165 then subtracting the two phase shifts. The mean inner potential (MIP)  
166 contribution to the phase shift gives an overview of the shape of the wire.  
167 Moreover, a weak electric field in vacuum can be seen due to charging effect  
168 on the nanowire giving us high confidence into the separation of the two phase  
169 shifts. The magnetic phase image was then used to perform the numerical  
170 reconstruction of the 3D magnetic field. The experimental reconstructions  
171 were performed on a  $256^3$  grid. Figure 2(c) shows the conventional approach  
172 of estimating magnetic induction in the nanowire using the relation,  $B_{proj} =$   
173  $\nabla\phi_m/C_B t$ , computed from the gradient of the phase shift and dividing by the  
174 thickness,  $t$  of the sample, derived from the MIP. With the same phase data,  
175 now we can reconstruct the entire 3D magnetic field. A vectorial view of the  
176 reconstruction can thus be displayed(Figure 2(e)) to emphasis the vectorial  
177 3D nature of the new dataset. Moreover we can visualize and measure the  
178 magnetic induction in the  $x - y$  plane in the middle of the nanowire as shown

179 in Figure 2(d).

180 Such slicing capability is the main advantage of our method as we can  
181 obtain more precise estimate of the magnetic saturation in the nanowire  
182 as shown in Figure 3. The conventional integrative approach (Figure 2(c))  
183 typically does not account for the integration of the stray fields outside the  
184 nanowire leading to an inaccurate quantification of the magnetic saturation  
185 of the material. This is schematically shown in Figure 3(a) where the stray  
186 fields are shown in the plane above the nanowire, the true magnetic induction  
187 of the sample is measured in the mid-plane of the nanowire, and the measured  
188 magnetic induction in the projected plane is the integration of these values.  
189 The  $\mu_0.M_s$  is now measured as  $0.7 \pm 0.1$  T as compared to the  $0.5 \pm 0.1$  T  
190 value obtained from the conventional phase gradient method as shown by  
191 the lineplots in Figure 3(b). It worth noting that this new experimental  
192 value for magnetic saturation of Ni was the one that was found to best fit  
193 micromagnetic simulations performed for domain wall studies in Ni nanowires  
194 [16]. The underestimation due to stray field contribution is eliminated in our  
195 measurements. It is however limited to the extent of the reconstruction  
196 volume. The strong decrease of the field away from the nanowire ( $1/r^3$ )  
197 versus the size of the reconstruction volume ( $\sim 286 \text{ nm}^3$ ) is sufficient to  
198 assume the complete removal of the stray field versus our sensitivity. The  
199 only remaining source of error in the quantification comes from the perturbed  
200 reference wave used for performing holographic phase reconstruction. The  
201 experimental induction map suffers slightly from a noisy phase reconstruction  
202 which is caused due to the strong diffraction contrast from the polycrystalline  
203 Ni nanowire. A Savitzky-Golay differentiation filter [38] was used to minimize

204 the effect of this noise.

205 **Charged Nanotube** There is intense interest in exploring carbon nanos-  
206 tructures for high efficiency field emitter applications: in particular carbon  
207 nanotubes and their derivatives. When an electrical bias is applied to a  
208 carbon nanotube, charge builds up on the surface, which is proportional to  
209 the applied bias until field emission occurs. The cylindrical geometry of the  
210 nanotubes results in a cylindrically-symmetric charge distribution and po-  
211 tential. It is possible to measure the effect of such a potential by measuring  
212 the resulting phase shift using off-axis electron holography. Here we have  
213 used the single image method developed in this work to determine the 3D  
214 potential around the charge distribution as well as to obtain the 3D electric  
215 field around the carbon nanotube.

216 Experiments were performed on carbon cone nanotip (CCnT) using an  
217 in-situ TEM holder (Nanofactory Instruments), in which the CCnT was ex-  
218 posed to an applied bias and the phase shift was measured using off-axis  
219 holography [39] (Figure 4(a)). The carbon cone nanotips (CCnTs) were pre-  
220 pared by deposition of pyrolytic carbon onto a multi-wall carbon nanotube.  
221 The CCnTs were then mounted on a tungsten tip using a dual Focused Ion  
222 Beam/FEG Scanning Electron Microscope (FIB/SEM). Figure 4(b) shows  
223 the phase image of the CCnT under an applied bias of 60 V. The contours  
224 are drawn every  $\pi$  radians. Figure 4(c) shows the 3D distribution of the re-  
225 constructed potential along with contours drawn every 10 V associated with  
226 the CCnT (shown in gray). The color bar shows the magnitude of the poten-  
227 tial in volts. The reconstructed 3D potential from the CCnT was then used  
228 to compute the 3D electric field as shown in Figure 5.

229 Figure 5(a) shows the conventional approach for estimating the electric  
230 field from the phase shift, similar to that for magnetic induction by com-  
231 puting the gradient of the phase shift. This method again represents an  
232 integrated measurement of the electric field that leads in that case to an  
233 overestimation of the measurement due to stray fields. Figure 5(b) shows  
234 the reconstructed electric field in the  $x - y$  plane in the middle of the CCnT  
235 obtained using our method. Figure 5(c) shows the quantitative comparison  
236 of the electric field as measured from the projection and in the middle of the  
237 CCnT, with the vertical dashed line showing the edge of the CCnT. Similar  
238 to the magnetic saturation measurement, here too, we can see that we have  
239 more accurate estimate of the electric field at the apex of the CCnT since  
240 we are removing the effect of field outside the CCnT. Figure 5(d) shows the  
241 3D vector field plot of the electric field in and around the CCnT, with the  
242 arrows colored according to the magnitude of the electric field in units of  
243 V/nm as shown by the colorbar. The quantitative analysis of such a biased  
244 CCnT field emission process is rather easy as a direct local electric field can  
245 be instantly measured with better accuracy. We were thus able to measure  
246 an electric field of  $2 \pm 0.5 \text{ V.nm}^{-1}$  at the extremity of the CCnT. Accurate  
247 measurement of this value is of importance as it enables the estimation of the  
248 field enhancement factor [40] that is used for Fowler-Nordheim plot analysis  
249 and work function estimation of carbon nanotubes. Knowing this value as  
250 well as the applied electrical bias and anode-CCnT distance, one can esti-  
251 mate a field enhancement value of  $\gamma = 23 \pm 6$  which is consistent with the one  
252 estimated using finite element modelling [39]. The estimation here is once  
253 again only limited by the disturbed reference wave that causes a lowering of

254 the measured electric potential in the image.

## 255 5. Discussion

256 The primary requirement for this method is that the field should be cylin-  
257 drically symmetric. This necessitates that the axis of symmetry in the phase  
258 image is accurately determined prior to the 3D reconstruction of the field.  
259 However, experimental limitations, such as low spatial resolution or a mag-  
260 netization distribution that is not completely symmetric, may prevent the  
261 accurate determination of the axis of symmetry. Furthermore, other experi-  
262 mental limitations such as the biprism orientation being not exactly perpen-  
263 dicular to the long axis of the 1D nanostructure can also introduce error in  
264 preserving cylindrical symmetry in the reconstruction. It is also true for the  
265 determination of the 3D electric potential and field. For example, the mag-  
266 netization of a uniformly magnetized nanosphere can have a  $z$ -component  
267 but depending on the direction of projection, the phase shift may appear  
268 to be cylindrically symmetric. Another possibility is that the magnetization  
269 can be in a ‘S’ or ‘C’ state such that at the edge of the particle, it rotates  
270 in plane, thus leading to incomplete cylindrical symmetry. In order to as-  
271 sess the effect of variation in symmetry and possible errors in determining  
272 the symmetric axis, the magnetization of the spherical nanoparticle was ro-  
273 tated by an angle  $\theta$  varying from  $0^\circ$  to  $20^\circ$  about the  $y$  axis in the  $x - y$   
274 plane. The phase shift was then computed for the corresponding magneti-  
275 zation. However, while implementing and performing the 3D reconstruction  
276 of the magnetic induction, the symmetric axis was assumed to be along  $y$   
277 axis. The 3D magnetic induction was then reconstructed using the single

278 image method and compared with the true 3D magnetic induction calcu-  
279 lated analytically. Of particular importance and relevance is the axial field  
280 magnitude ( $B_y$ ) that was compared numerically: for example the theoretical  
281 and reconstructed value of axial field at the tip of the sphere is 0.66 T and  
282 0.63 T for  $\theta = 0$  and decreases to 0.62 T and 0.59 T for  $\theta = 20^\circ$  respectively.  
283 This corresponds to only about 5% error. Similarly the error in the angle of  
284 the magnetization vector was also compared. The comparison showed that  
285 the error in determination of the angle of the magnetization vector is smaller  
286 inside the nanosphere than outside. This can be related to the fact that  
287 outside the nanosphere, the magnetization vector changes direction rapidly  
288 as compared to the magnetization vector inside the sphere. Additional de-  
289 tails about the calculation of errors and a rigorous quantitative comparison  
290 of errors is given in the supporting information. [The final source of error in](#)  
291 [the quantitative measurement of the electromagnetic fields using our method](#)  
292 [is due to the perturbed reference wave. However, it has been shown previ-](#)  
293 [ously that the reference wave perturbation at the length scale of the overlap](#)  
294 [area is not significant with error below 10%.](#)[39, 41]. This demonstrates that  
295 although the condition of cylindrical symmetry is a fairly strict criterion for  
296 applicability of this method, for experimental purposes, this method can still  
297 reliably give quantitative values of magnetic induction or electric field in and  
298 around the nanostructures in 3D.

299 [Finally, other approaches that quantitatively measure electromagnetic](#)  
300 [fields using phase reconstructed electron microscopy apart from tomographic](#)  
301 [methods, rely on building models of the sample either through finite ele-](#)  
302 [ment simulations or analytical models and then compare the experimental](#)

303 2D measurements with the results obtained from the models. Recent work by  
304 Beleggia *et. al.* have used this approach to show that using off-axis holog-  
305 raphy combined with theoretical modeling, it is possible to quantitatively  
306 measure the electric field around the tip of a biased needle and accurately  
307 take into account the shape of the tip [42]. However, the method presented in  
308 this work is unique in that it relies on experimental measurements to directly  
309 reconstruct the 3D electromagnetic fields.

## 310 **6. Conclusion**

311 Here we have successfully demonstrated that by exploiting the cylindrical  
312 symmetry of 1D nanostructures and the associated electromagnetic fields, it  
313 is indeed possible to reconstruct the entire 3D electromagnetic field from a  
314 single TEM phase image. This significantly reduces the time required to  
315 acquire multiple tilt series images as well as reducing the electron dose to the  
316 sample that is conventionally required for 3D reconstruction. The method  
317 was numerically implemented to reconstruct the 3D magnetic induction in  
318 and around a ferromagnetic nickel nanowire as well as the electric field in and  
319 around a carbon cone nanotip under applied bias. In both the cases, we have  
320 shown that using the information obtained in 3D, we can get more precise  
321 measurement of relevant physical quantities such as magnetic saturation, or  
322 field enhancement factors as compared to conventional approaches. This  
323 work opens new paths for quantitative analysis of the 3D electromagnetic  
324 interactions for technologically important one-dimensional nanostructures.



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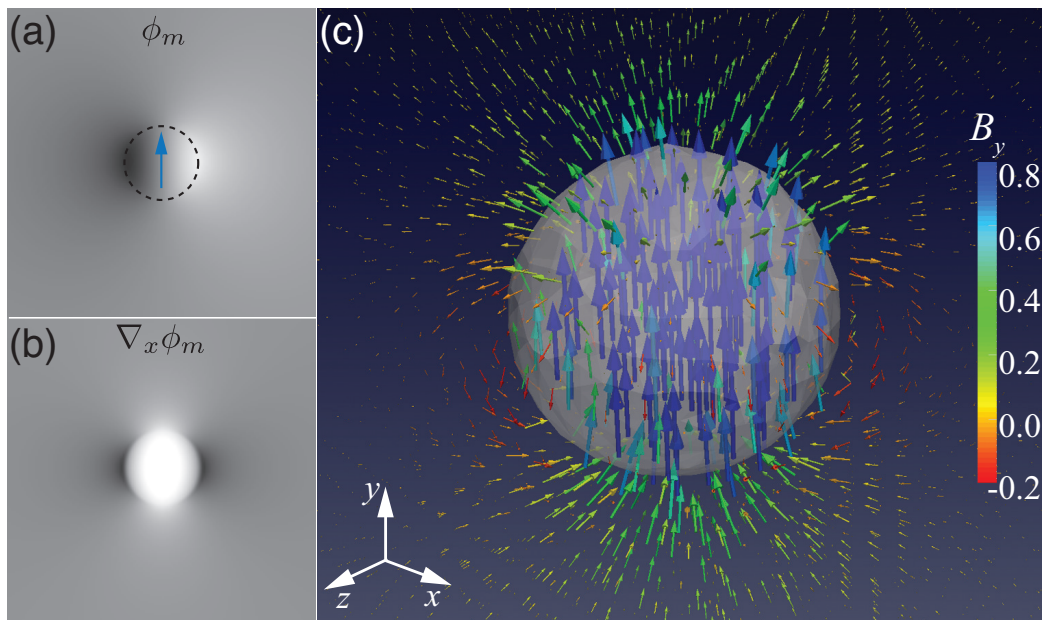


Figure 1: (a) shows the magnetic phase shift image of a uniformly magnetized spherical nanoparticle, indicated by the dashed line, and magnetization vector is indicated by the blue arrow, (b) the derivative of the magnetic phase shift with respect to  $x$  showing the symmetry about the vertical axis of the image, and (c) the 3D reconstructed magnetic induction inside and outside the spherical particle. The color bar shows the magnitude of the  $y$  component of the magnetic induction,  $B_y$ , in T.

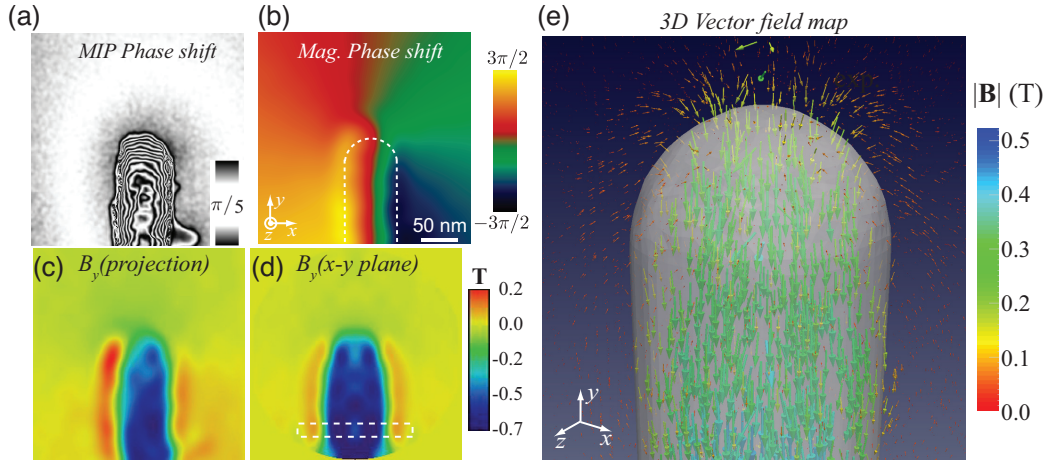


Figure 2: (a) shows the electrostatic contribution to the phase shift displayed as  $\cos(5\Delta\phi)$  of the nickel nanowire. The associated scale displays the amount of  $\phi_e$  between dark lines (b) shows the magnetic contribution to the phase shift ( $\phi_m$ ). (c) shows the  $B_y$  component calculated from the gradient of magnetic phase shift, and (d) shows the  $B_y$  component in the  $x - y$  plane in the middle of the nanowire. The color represents the strength of magnetic induction indicated by the colorbar. (e) shows the 3D vector plot of the reconstructed magnetic induction from the nanowire. The vectors are colored according to the  $y$  component of the induction indicated by the color bar (in units of T).

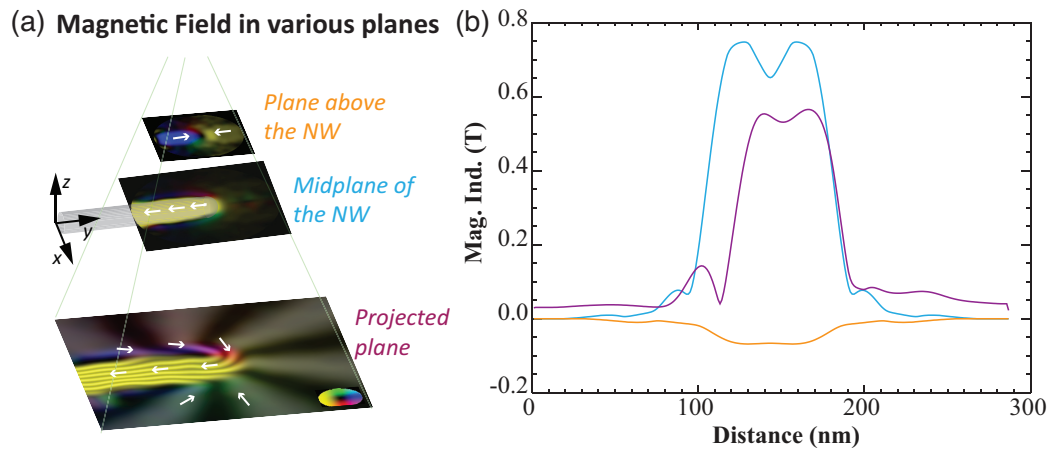


Figure 3: (a) shows a schematic to illustrate the advantage of our method by showing the magnetic induction that can be measured at various locations in 3D as opposed to the projected magnetic induction, and (b) shows the quantitative comparison of the magnetic induction at various locations represented in (a) from the white dashed region shown in Figure 2(d).

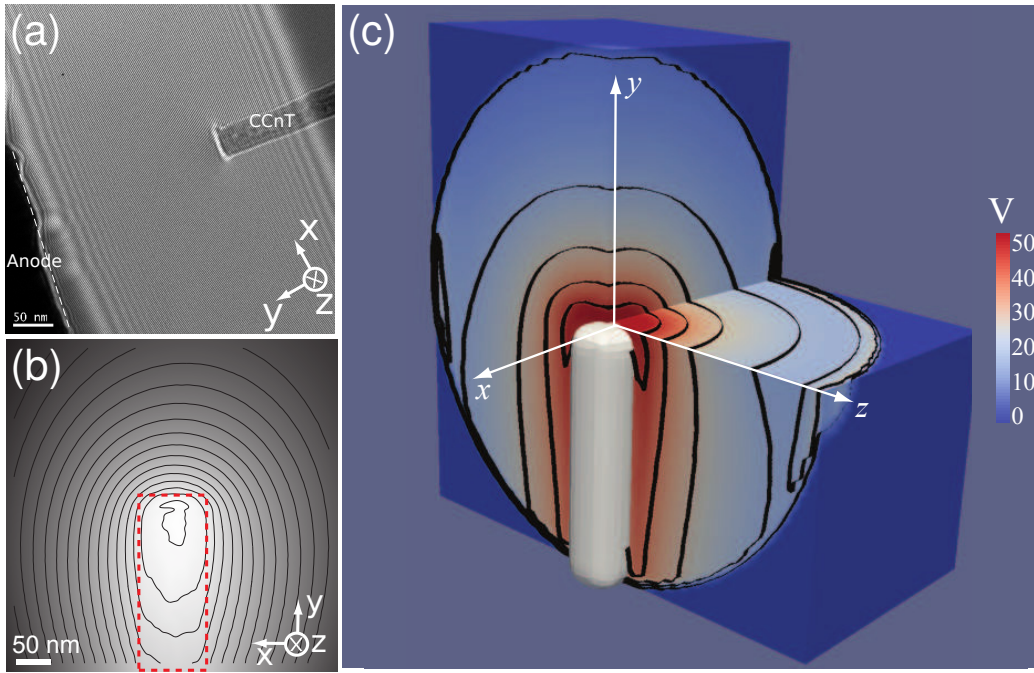


Figure 4: (a) shows the as acquired interference hologram, which shows the positions of the CCnT and the anode. (b) shows the experimentally measured phase shift from a carbon cone nanotip (CCnT) under an applied bias of 60 V, (c) shows the reconstructed 3D distribution of potential (in V) around the CCnT along with contours drawn every 10 V.

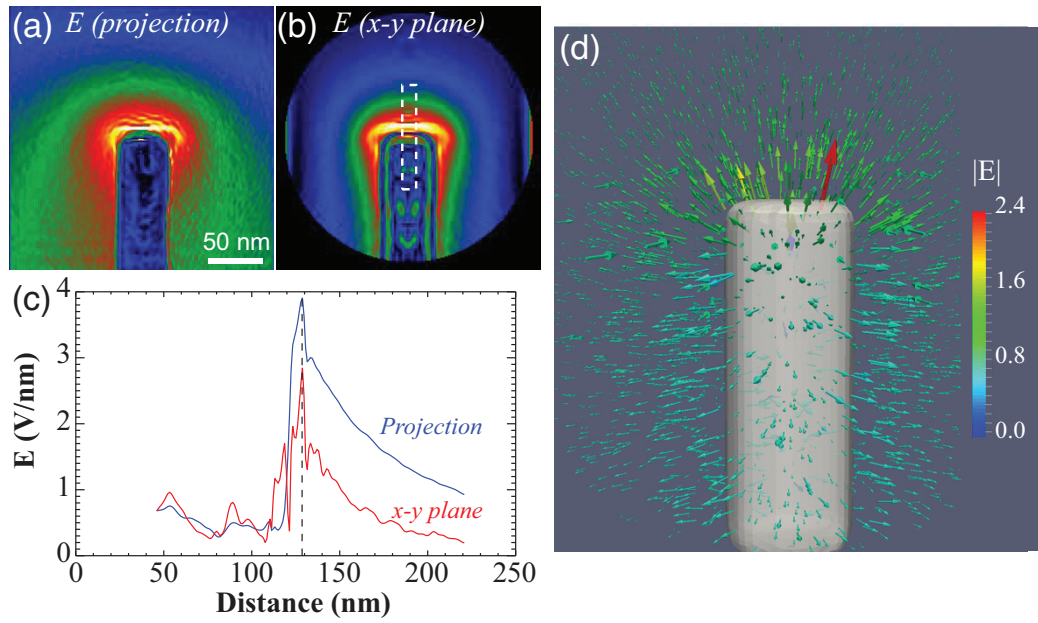


Figure 5: (a) shows reconstructed electric field in the projection plane computed from gradient of the phase shift, and (b) shows the reconstructed electric field in the  $x - y$  plane in the middle of the CCnT. (c) shows the quantitative comparison of the reconstructed electric field in the projection plane and the  $x - y$  plane in the middle of the CCnT. (d) shows the 3D vectorial view of the reconstructed electric field (in V/nm) around the CCnT.