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Improving the Speed Delivery for Robotic Warehouses^{*}

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Abstract: This paper studied a new order fulfilling system using mobile robots instead of manual picking at online retailers' distribution centers. This new generation of sustainable green warehouse systems can improve productivity and flexibility. We measured the performance of system, and provided design rules for velocity of robots. We build open queue models for the new order fulfilling system and calculated throughput time of this system given the number of robots. This is one of earliest papers to introduce this new material handling system.

Keywords: Mobile Fulfilling System; Speed Delivery; Order Picking; Robotic warehouse; Warehouse Management; Optimization of facility layout; Open Queue theory

1. INTRODUCTION

Recently online retailer Zappos had installed a new Kiva Mobile Fulfilling System (MFS) at its distribution center in Shepherdsville, Kentucky in 2008. This new system significantly makes contribution to developing a supply chain to speed delivery (Marks et al., 2011). Robotic warehouse techniques are researched before, but until 2010, Kiva MFS has been applied to warehouses like Zappos, Walgreens, Staples, Diapers and Gap. Business Week report "Kiva Systems are revolutionizing companies' supply-chain and distribution networks" (Scanlon, 2009). In 2012, Amazon bought Kiva system for 775 million USD and implemented a number of Kiva MFS. In August 2015, the company officially changed its name from Kiva Systems to Amazon Robotics (Nussbaum, 2015). Some other material handling companies and robot companies are developing different robotic picking systems. For example, RMT Robotics, a high-tech company in Toronto, Canada is developing robotic picking systems and robotic case picking systems (see RMTrobotics.com). ADAM (autonomous delivery and manipulation) robots of RMT companies are similar to Kiva's robots. Jingdong (see jd.com), one of the largest online retailers with revenue of 18.51 billion USD and sales of 41.9 billion USD in 2014, has built a MFS and implementing robotic picking in a warehouse of 5,000 m^2 in Shanghai.

Kiva MFS is a "parts-to-picker" warehouse systems. Compared with other order fulfillment systems, Kiva MFS can boost flexibility since it can adapt to changes of product types which is not easy in traditional "AS/DS" equipments. Another advantage of Kiva MFS is in scalability: While traditional materials handling technology need large initial investment on equipments like Mini-load AS/DS (about 750,000 USD per aisle, 3-10 aisles are common in a warehouse) and carousels (about 125,000 USD per

carousel, 3-10 carousels are common in a warehouse), Kiva MFS only needs to buy appropriate number of shelves and robots (about 5,000 USD per robot) that fit to their current demand volume (Kopytoff, 2012). When demand scale increases, Kiva MFS simply needs to add robots and shelves incrementally. Kiva MFS is regarded as a new generation of sustainable green warehouse systems since it can cut carbon footprint (traditional systems need to keep on in most time whether they are needed or not. Kiva MFS robots only run when they are needed), has less noise (silent robots compared with noisy fork trucks and conveyor belts), reduce injuries to warehouse employees since it is more economically designed (in a traditional warehouse, workers need to pick, lift, carry by hands or fork trucks, which bring hazards. Now workers only need to monitor stations), reduce lighting since robots can work with almost no lighting.

Our research is related to two research topics of AGV (Automated guided vehicle system): the fixed guided paths design and vehicle application in warehousing. In guided paths design with a fixed route, an AGV simply moves along the path determined in advance (Ventura and Lee, 2001), with the objective to minimize transport time including AGV's travel time and waiting time. However, other AGVs use a non-oriented path guided system (Singh and Tiwari, 2002), prefer softwares to hardwares in path management, and can relatively easily change their paths (Tompkins et al., 2010). Our research is closer to unfixed guided paths. The selection of vehicle routing and scheduling influences the performance of AGV systems (Toth and Vigo, 2001). One of our objectives of the routing is to minimize throughput time of the warehouse. Hu and Egbelu (2000) studied different scheduling strategies for idle vehicles discussing existing idle AGVs when service requests arrive. Lee et al. (1998) proposed the algorithm of sequential path generation to heuristic can generate collision-free shortest path for multiple AGVs. Qiu et al.

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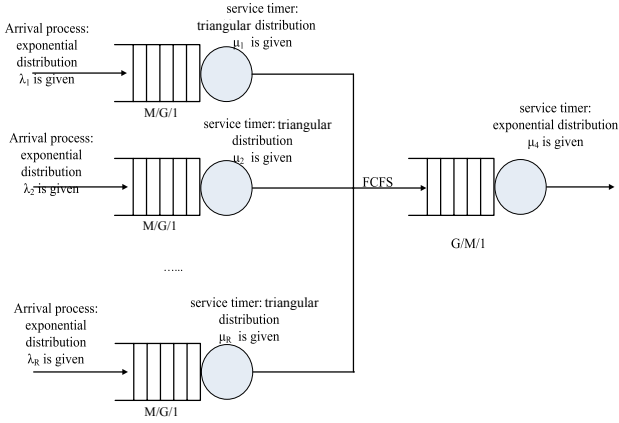


Fig. 1. A queue networks model for Kiva system

(2002) developed efficient algorithms for AGV's scheduling and routing problems. The focus strategy of our paper is on a random selection strategy and the minimize throughput time selection strategy.

2. MODEL AND ANALYSIS

We use the following notations:

R : the number of robots.

N : the number of pickers.

v : the velocity of robots.

w : the width of aisles.

W : the width of the warehouse.

L : the length of aisles.

Tr : throughput time.

We consider two protocols: (1) R robots only serve for one order picker, (2) serve for multiple order pickers.

2.1 Pickers do not share robots

In this section, we consider the situation where R robots serve for 1 order picker. After receiving an order request, a robot follows the protocol:(1) leave order picker station, (2) go to storage grid to pick a shelf which includes requested item , (3) return to a picker station it is serving, (4) after the worker pick the item the robot will return the shelf to its original storage position and the robot will also return the picker station.

The system is a centralized system and the order arrival process to robots are Poisson processes. We assume the storage position follows uniform distribution.

The serve protocol of the robot is as follows: After orders arrive, each robot, originally staying with the order picker, will leave the picker and go to storage positions to pick one shelf which can provide items to that order. Then the robot will return to the picker it is serving.

LEMMA 1 If robot r serves the leftmost or rightmost picker, a single robot serve system can be described as a $M/Tr/1$ queue on condition that $W = L$.

PROOF The arrival process is Poisson. We mainly need to show the inter-serve time follows uniform distribution. We assume the beginning position of robot r is the leftmost picker's position. Note a robot receives their orders information wirelessly and use cameras to read navigational barcode stickers on the warehouse floor, and robots will travel in a straight lines. Supposed a robot receives an order with a storage position $X_r \sim U[0, W]$ and $Y_r \sim U[0, L]$, let $Z = X + Y$ and its density function is $h(z)$.

R robots will carry shelves to the picker. R departure processes merge into an arrival process to the picker. The inter-service time of the picker follows exponential distribution. So the picker service system can be described as a $G/M/1$ system. We describe the system by Figure 1.

(1)Analysis of robots

We use ED to denote the expected delay time, and ES to denote the expected service time. The expected waiting time $EW = ED + ES$. From Wolff (1989)(Equation 154,Page 280), we have $EW = \frac{\lambda E(S^2)}{2(1-\rho)} + \frac{1}{\mu}$, where $ED = \frac{\lambda E(S^2)}{2(1-\rho)}$ is the delay time and $\frac{1}{\mu}$ is the service time. For robot r as a server in a $M/G/1$ queue, we have $E(S^2) = \frac{7(W+L)^2}{6v^2}$, $\mu = \frac{v}{W+L}$, $\rho = \frac{\lambda_r(W+L)}{v}$. Substituting $E(S^2)$, μ , ρ into EW , we obtain the expected waiting time at robot r is,

$$EW_r = \frac{7\lambda_r(W+L)^2}{12v^2(1 - \frac{\lambda_r(W+L)}{v})} + \frac{W+L}{v}. \quad (1)$$

The equation 1 can be used to design the velocity of robots, we have the following design rule,

DESIGN RULE 1 There exists a minimal acceptable velocity of robots such that expected waiting time at robots is less than ω_R .

PROOF Since $EW_r(v)$ is decreasing function of v . Let the upper bound of EW_r (a measure for service level)at robots be ω_R , the minimal acceptable velocity of robots \underline{v} is given by

$$\frac{7\lambda_r(W+L)^2}{12\underline{v}^2(1 - \frac{\lambda_r(W+L)}{\underline{v}})} + \frac{W+L}{\underline{v}} = \omega_R. \quad (2)$$

Or if the velocity of robots is given by mechanical reasons, the maximal throughput to guarantee a service level ω_R is $\Sigma_1^R \lambda_r$, and λ_r can obtain from equation 2. We have a bound of throughput

$$THR_R = \Sigma_1^R \frac{12v(\omega_R - (W+L))}{(W+L)(12v\omega_R - 5(W+L))}. \quad (3)$$

We substitute the squared coefficient of variation of an inter-arrival time of robots $c_{aR}^2 = 1, \rho = \frac{\lambda_r(W+L)}{v}$, the squared coefficient of variation of an inter-service time of robots $c_{sR}^2 = \frac{1}{3}$, $\mu = \frac{v}{W+L}$, EW_r from equation 1, into the equation $c_{dR}^2 = c_{aR}^2 + 2\rho^2 c_{sR}^2 - 2\rho(1-\rho)\mu EW$ from Whitt (1983)(Equation 37,Page 2799),and obtain the squared coefficient of variation of an inter-departure time,

$$c_{aR}^2 = 1 + \frac{2\lambda_r^2(W+L)^2}{3v^2} - 2\lambda_r\left(1 - \frac{\lambda_r(W+L)}{v}\right)\left[\frac{7\lambda_r(W+L)^2}{12v^2(1 - \frac{\lambda_r(W+L)}{v})} + \frac{W+L}{v}\right].$$

(2) Superposition of departure streams from robots

The departure processes are merged into one process, which is also arrival process of the picker. We use methods from Whitt (1983) (see page 2797-2794).

$$\lambda_{aP} = \sum_1^R \lambda_r \quad (4)$$

With the asymptotic method, the squared coefficient of variation of an inter-arrival time of the picker is $c_{aP}^2 = \sum_{r=1}^R \frac{\lambda_r}{\sum_{r=1}^R \lambda_r} \left\{ 1 + \frac{2\lambda_r^2(W+L)^2}{3v^2} - 2\lambda_r\left(1 - \frac{\lambda_r(W+L)}{v}\right)\left[\frac{7\lambda_r(W+L)^2}{12v^2(1 - \frac{\lambda_r(W+L)}{v})} + \frac{W+L}{v}\right] \right\}$.

(3) Analysis of pickers

The expected delay time in a picker

$$ED_P = \frac{\rho(c_{aP}^2 + c_{sP}^2)}{2\mu(1-\rho)} g(\rho, c_{aP}^2, c_{sP}^2) \quad (5)$$

is given by Whitt (1983) (see page 2802). $g(\rho, c_{aP}^2, c_{sP}^2)$ is defined as $\exp[-\frac{2(1-\rho)(1-c_{aP}^2)^2}{3\rho(c_{aP}^2+c_{sP}^2)}]$ when $c_{aP}^2 < 1$, and 1 when $c_{aP}^2 \geq 1$. We substitute $c_{sP}^2 = 1$, and $\lambda_{aP} = \sum_1^R \lambda_r$ into ED_P , and obtain $ED_P = \frac{\rho(c_{aP}^2+1)}{2\mu_P(1-\rho)} g(\rho, c_{aP}^2, c_{sP}^2) = \frac{\sum_1^R \lambda_r (c_{aP}^2+1)}{2\mu_P(\mu_P - \sum_1^R \lambda_r)} g(\rho, c_{aP}^2, c_{sP}^2)$.

We consider the mean number of jobs in queue L_q . This is also the number of robots who are carrying shelves and waiting before the picker.

$$L_q = \frac{(\sum_1^R \lambda_r)^2 (c_{aP}^2 + 1)}{2\mu_P(\mu_P - \sum_1^R \lambda_r)} g(\rho, c_{aP}^2, c_{sP}^2) \quad (6)$$

In this system, there is an upper bound \bar{l} on L_q . We show we can find a design rule for velocity of robot.

DESIGN RULE 2 There exists a minimal acceptable velocity of robots such that the number of robots who are carrying shelves and waiting before the picker L_q is less than space limitation \bar{l} .

PROOF c_{aP}^2 is a decreasing function of v . $g(\rho, c_{aR}^2, c_{sR}^2)$ is decreasing function of v when $c_{aR}^2 < 1$, and 1 when $c_{aR}^2 \geq 1$. So L_q is a decreasing function of v . By solving the following function,

$$L_q(v_2) = \frac{(\sum_1^R \lambda_r)^2 (c_{aP}^2(v_2) + 1)}{2\mu_P(\mu_P - \sum_1^R \lambda_r)} g(\rho, c_{aP}^2(v_2), c_{sP}^2) = \bar{l}. \quad (7)$$

We can get a minimal acceptable velocity of robots, \underline{v}_2 .

(4) Analysis of stability of systems

For robots, $\lambda_r < \mu_R = \frac{v}{W+L}$, $r = 1, \dots, R$.

For pickers' workstation, $\sum_1^R \lambda_r < \mu_P$.

(5) Throughput time

We estimate the average customer throughput time of orders, which is a sum of components: the delay time that an order spends in the queue at storage grids waiting for being carried by robots ED_r , the average service time of a robot $\frac{1}{\mu_R}$, the delay time that order spends in the

queue at storage grids waiting for being handled by pickers' workstation ED_P , and expected picking time by order pickers $\frac{1}{\mu_P}$. From $Tr = ED_r + \frac{1}{\mu_R} + ED_P + \frac{1}{\mu_P}$, we have

$$Tr = \frac{7\lambda_r(W+L)^2}{12v^2(1 - \frac{\lambda_r(W+L)}{v})} + \frac{W+L}{v} + \frac{1}{\mu_R} + \frac{\sum_1^R \lambda_r (c_{aP}^2+1)}{2\mu_P(\mu_P - \sum_1^R \lambda_r)} g(\rho, c_{aP}^2, c_{sP}^2) + \frac{1}{\mu_P}.$$

(6) Optimal robot number

We make two assumptions: 1) We assume the work is equally allocated to R robots, and the arrival rate of the R th a robot is $\lambda_r = \lambda/R$.

2) We assume λ_r are equal, then $\sum_1^R \lambda_r = \lambda$.

We obtain the function of $Tr(R)$,

$$Tr(R) = \frac{7\frac{\lambda}{R}(W+L)^2}{12v^2(1 - \frac{\lambda(W+L)}{Rv})} + \frac{1}{\mu_R} + \frac{\lambda(c_{aP}^2+1)}{2\mu_P(\mu_P - \lambda)} g(\rho, c_{aP}^2, c_{sP}^2) + \frac{1}{\mu_P}.$$

$$Tr(R) = \frac{7\frac{\lambda}{R}(W+L)^2}{12v^2(1 - \frac{\lambda(W+L)}{Rv})} + \frac{1}{\mu_R} + \frac{\lambda(c_{aP}^2+1)}{2\mu_P(\mu_P - \lambda)} g(\rho, c_{aP}^2, c_{sP}^2) + \frac{1}{\mu_P}. \quad (8)$$

LEMMA 2 $ED_r(R)$ is a decreasing function of R .

PROOF We can observe from

$$ED_r(R) = \frac{7\frac{\lambda}{R}(W+L)^2}{12v^2(1 - \frac{\lambda(W+L)}{Rv})} = \frac{7\lambda(W+L)^2}{12v^2(R - \frac{\lambda(W+L)}{v})}. \quad (9)$$

LEMMA 3 $ED_P(R)$ is a nonlinear function of R . When $R \geq \frac{3\lambda(W+L)}{2v}$, $ED_P(R)$ is an increasing function.

PROOF The expected delay time of the pickers' workstation

$$ED_P(R) = \frac{\lambda(c_{aP}^2+1)}{2\mu_P(\mu_P - \lambda)} g(\rho, c_{aP}^2, c_{sP}^2). \quad (10)$$

The monotony of $ED_P(R)$ is only related to $c_{aP}^2(R)$.

We substitute $\sum_{r=1}^R \lambda_r = \lambda$ into $c_{aP}^2(R)$ and obtain,

$$c_{aP}^2(R) = 1 + \frac{3\lambda^2(W+L)^2}{2R^2v^2} - \frac{2\lambda(W+L)}{Rv} = \frac{1}{3} + \frac{3}{2}\left[\frac{\lambda(W+L)}{Rv} - \frac{2}{3}\right]^2.$$

As R increases, the function $ED_P(R)$ is nonlinear. It first decreases and then increases after reaching the critical point. When $0 < R \leq \frac{3\lambda(W+L)}{2v}$, the function $ED_P(R)$ decreases, when $R \geq \frac{3\lambda(W+L)}{2v}$, $ED_P(R)$ increases. By solve the function, we can get the optimal robot number.

LEMMA 4 When $0 < R \leq \frac{3\lambda(W+L)}{2v}$, $Tr(R)$ is a decreasing function, we have a R limited by other constraints. When $R \geq \frac{3\lambda(W+L)}{2v}$, $Tr(R)$ is a convex function.

PROOF Let $\frac{\partial Tr(R)}{\partial R} = 0$, we can obtain R_0 from the function $R_0^2 = \frac{6\lambda^2}{\mu_P(\mu_P - \lambda)} g(\rho, c_{aR}^2, c_{sR}^2) [R_0 - \frac{\lambda(W+L)}{v}]^2 [\frac{v}{2\lambda(W+L)} - \frac{1}{R_0}]$. If $R > \frac{3\lambda(W+L)}{2v}$, the optimal number of robots $R^* = R_0$.

If $R \leq \frac{3\lambda(W+L)}{2v}$, the optimal number of robots $R^* = \frac{3\lambda(W+L)}{2v}$.

2.2 Pickers share robots

In this section, we consider the situation where one picker and R robots.

Assumptions:

Orders arrive following a Poisson process with the rate of λ . Orders are assigned to available robots following *FCFS*. The service rate of the picker processing an order is μ_p and the service time follows a general distribution. The picker handles robots in a *FCFS*. A robot puts back a shelf to the closed available spot after the picker finishes the shelf. A robot does not go back to the picker after returning a shelf, but stay there under the shelf until getting the instruction for the next assigned order. Two queuing systems that are connected in the queue network. *M/G/R/FCFS* for the first queuing. The service time μ_1 depends on the waiting time and processing time of the second queuing system. *M/G/1/FICFS* for the second queuing. The arrival rate depends on the first queue but it should be λ .

(1) Analysis of robots

The expected delay time in a robot

$$ED_r = \phi(\rho, c_{aR}^2, c_{sR}^2, R) \left(\frac{c_{aR}^2 + c_{sR}^2}{2} \right) ED_r(M/M/R). \quad (11)$$

$\phi(\rho, c_{aR}^2, c_{sR}^2, R)$ is given by equation(2.25) in Whitt (1993).

$ED_r(M/M/R) = \frac{1}{\mu R(1-\rho)} \left[\frac{(R\rho)^R}{R!(1-\rho)} \right] \zeta$ and ζ is given by equation (2.4) in Whitt (1993).

The departure process is also arrival process of the picker. We use methods from Whitt (1983) (see page 2799). The squared coefficient of variation of an inter-arrival time of the picker is $c_{aP}^2 = c_{aR}^2 = 1 + (1-\rho^2)(c_{aR}^2 - 1) + \frac{\rho^2}{\sqrt{R}}(c_{sR}^2 - 1) = 1 + \frac{\rho^2}{\sqrt{R}}(c_{sR}^2 - 1)$.

(3) Analysis of pickers

The expected delay time in a picker

$$ED_P = \phi(\rho, c_{aP}^2, c_{sP}^2, 1) \left(\frac{c_{aP}^2 + c_{sP}^2}{2} \right) ED_P(M/M/1). \quad (12)$$

where c_{aP}^2 is given by equation, $c_{sP}^2 = 1$, $\rho = \lambda/(\mu_P)$, $EW_r(M/M/1) = \frac{1}{\mu(1-\rho)} \left[\frac{\rho}{(1-\rho)} \right] \zeta$ and ζ is given by equation (2.25) in Whitt (1993).

We consider the mean number of jobs in queue L_q . This is also the number of robots who are carrying shelves and waiting before the picker.

$$L_q = \lambda \phi(\rho, c_{aP}^2, c_{sP}^2, 1) \left(\frac{c_{aP}^2 + c_{sP}^2}{2} \right) ED_P(M/M/1) \quad (13)$$

In this system, congestion near pickers' workstation is most serious since aisle width is limited here. There is an upper bound \bar{t} on L_q . We show we can find a design rule for velocity of robot.

(4) Analysis of stability of systems

For robots, $\lambda < R\mu_R$. For pickers' workstation, $\lambda < \mu_P$.

3. NUMERICAL EXPERIMENTS

This section shows how the mobile picking systems practices, especially to optimize the number of robots. An important objective for Kiva warehouse automation operations is to determine the optimal number of robots for "Pickers do not share robots" model.

While increasing the number of robots can improve the throughput time, it will also increase the robots fixed cost and robots waiting cost. While increasing the velocity of robots can improve the throughput time, it will also increase the technology cost of the robots. Our objective is to minimize the total operational cost, including the fixed cost of robots, the robots waiting cost and velocity increasing cost. The objective function is the sum of the fixed cost of robots, the waiting cost of robots and velocity increasing cost. The robots waiting cost $\sum_{r=1}^R c_r \lambda_r EW_r$, where c_R is the marginal cost of a robot, c_r is the expected waiting cost per robot, EW_r is the total expected waiting time, $c_r EW_r$ is the total expected waiting cost. The speed increasing cost $c_v \Delta v$, where c_v is the unit speed increasing cost, and Δv is the increment of the velocity of robots,

we have $\min c_R R + \sum_{r=1}^R c_r EW_r + c_v \Delta v$.

Since v_0 is the initial velocity of robots, and v_r is the velocity of r robot,

we have $\min c_R R + \sum_{r=1}^R c_r EW_r + c_v (v_r - v_0)$.

3.1 Determine the number of robots

We presented $\lambda = 0.4$, the width of the warehouse $W = 40$ and the length of aisles $L = 50$. Here we ignored other constraints that may affect the optimal number of robots. When $v=10$ m/min, by solving the three-dimensional equation(8), $R_0 = 2.33, 3.27 + 3.74i$ or $3.27 - 3.74i$. As R is an integer, we deleted the imaginary. Since $R_0 \leq \frac{2\lambda(W+L)}{v}$, we obtained the optimal number of robots $R^* = \frac{2\lambda(W+L)}{v} = 7$. Similarly, by solving the three-dimensional equation(8), when $v=20$ m/min, $R_0 = 1.12, 1.43 + 1.93i$ or $1.43 - 1.93i$, we obtained the optimal number of robots $R^* = 4$. When $v=30$ m/min, $R_0 = 0.74, 0.91 + 1.29i$ or $0.91 - 1.29i$, we obtained the optimal number of robots $R^* = 2$. When $v=40$ m/min, $R_0 = 0.55, 0.67 + 0.96i$ or $0.67 - 0.96i$, we obtained the optimal number of robots $R^* = 1$. When $v=50$ m/min, $R_0 = 0.44, 0.53 + 0.78i$ or $0.53 - 0.78i$, we obtained the optimal number of robots $R^* = 1$.

Figure 2 shows that with the increase of the number of robots, the throughput time non-linear increases. Since R is an integer, when $v=10$ m/min, the number of robots is less than seven, the trend of throughput time declines, and then begin to increase. When $v=20$ m/min, the number of robots is less than four, the trend of throughput time declines, and then begin to increase. When $v=30$ m/min, the number of robots is less than two, the trend of throughput time declines, and then begin to increase. When $v=40$ m/min, the number of robots is less than one, the trend of throughput time declines, and then begin to increase. When $v=50$ m/min, the number of robots is less

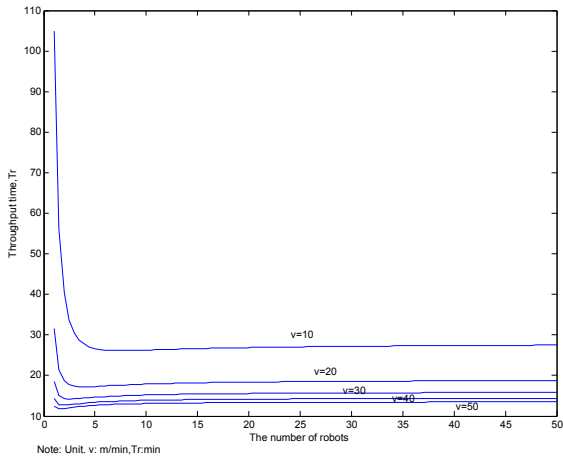


Fig. 2. Throughput time with the change of R

than one, the trend of throughput time declines, and then begin to increase. When the number of robots is larger than 30, the throughput time have no great changes.

3.2 Determine the velocity of robots

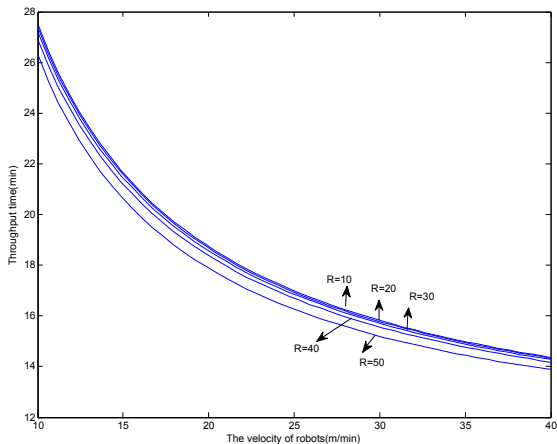


Fig. 3. Throughput time with the change of v

Figure 3 shows that as the velocity of robots increases, the throughput time decreases. Due to technical capacity, aisle width and other constraints, the velocity of robots has an upper bound.

4. CONCLUDING

This paper studies a new order fulfilling system featured by a number of robots transporting movable shelves to order pickers. This new generation of sustainable green warehouse systems can improve productivity and flexibility simultaneously. We measure the performance of system, and provide design rules for warehouses. We compare two protocols: To share or no to share robots. The models in this paper can be used to determine the number of robots, determine the velocity of robots, and compute throughput given service level. This is one of earliest papers to introduce this new material handling system.

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