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# Bending instability in galactic discs: advocacy of the linear theory

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## ABSTRACT

We demonstrate that in  $N$ -body simulations of isolated disc galaxies, there is numerical vertical heating which slowly increases the vertical velocity dispersion and the disc thickness. Even for models with over a million particles in a disc, this heating can be significant. Such an effect is just the same as in numerical experiments by Sellwood. We also show that in a stellar disc, outside a boxy/peanut bulge, if it presents, the saturation level of the bending instability is rather close to the value predicted by the linear theory. We pay attention to the fact that the bending instability develops and decays very fast, so it cannot play any role in secular vertical heating. However, the bending instability defines the minimal value of the ratio between the vertical and radial velocity dispersions  $\sigma_z/\sigma_R \approx 0.3$  (so indirectly the minimal thickness), which stellar discs in real galaxies may have. We demonstrate that observations confirm the last statement.

**Key words:** methods: numerical – galaxies: kinematics and dynamics – galaxies: structure.

## 1 INTRODUCTION

In this paper, we study the mechanisms of vertical heating of stellar discs in isolated disc galaxies by numerical simulations. Our main interest is to understand the role which the bending instability plays in real galaxies. We also will pay attention to the artificial numerical effects in  $N$ -body models of the disc galaxies.

The bending instability (or ‘firehose’ instability) for an infinitely thin stellar sheet with the non-zero velocity dispersion was studied for the first time by Toomre (1966). Based on the linear analysis, Toomre (1966) found from qualitative considerations that such a sheet would be unstable to bending perturbations if the ratio of the vertical to horizontal velocity dispersions,  $\sigma_z/\sigma_x$  (velocity anisotropy), does not exceed 0.3. In several subsequent studies (Kulsrud, Mark & Caruso 1971; Polyachenko & Shukhman 1977; Araki 1985), this criterion has been specified. For example, Polyachenko & Shukhman (1977) were the first who made an accurate analysis of the corresponding dispersion relation for a finite-thickness homogeneous layer and found the critical value of the velocity anisotropy to be  $\approx 0.37$  (below this value, the system is unstable). Araki (1985) revised such an analysis for a more realistic model. He took into account a non-uniform volume density distribution in the  $z$ -direction and anisotropic Gaussian velocity dispersion. Araki’s criterion gives  $\sigma_z/\sigma_x \approx 0.29$  that is very close to the value obtained by Toomre (see a discussion by Merritt & Sellwood 1994).

In our previous paper (Sotnikova & Rodionov 2003, hereafter SR03) we claimed that in numerical models of realistic stellar discs,

the saturation level of the bending instability is actually significantly higher than the value 0.3–0.37 predicted by the linear theory, i.e. the disc is unstable even though  $\sigma_z/\sigma_R > 0.3$ . Merritt & Sellwood (1994) and Khoperskov et al. (2010) came to the similar conclusion. For example, Merritt & Sellwood (1994) presented the results of  $N$ -body experiments with finite-thickness discs that confirmed instability to bending even when the velocity anisotropy was larger than the critical value for the instability in an infinite slab. In this paper, we demonstrate that such a conclusion was probably wrong, and the saturation level is actually very close to the linear criterion.

SR03 considered initially thin models and studied processes which cause the vertical heating (thickening) of the stellar disc of isolated disc galaxies. Four mechanisms of vertical heating have been revealed.

- (i) An initial bending instability which develops and decays rather fast (well within the first Gyr of the evolution).
- (ii) An axisymmetric bending instability in the central parts of hot, barless models, connected with the formation of the X-shaped structure in the central parts of models.
- (iii) A bending instability (buckling) of a bar, which also causes the formation of the X-shaped structures.
- (iv) Slow vertical heating which we explain as heating due to vertical inhomogeneities in the stellar matter.

Two of these mechanisms (ii and iii) cause the vertical heating of a stellar disc up to the value  $\sigma_z/\sigma_R \approx 0.8$ . Both mechanisms lead to the formation of boxy or peanut structures in the central part of the model. The backbone of these structures is stable families of periodic orbits which bifurcate from the planar  $x1$  family in the

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presence of vertical resonances (Patsis et al. 2002). The existence of vertical resonances is independent of  $m = 2$  perturbations in the disc plane so that the X-shaped structure may appear in almost axisymmetric cases (ii). It should be noted that this X-shaped structure develops asymmetrically with respect to the disc plane (see fig. 2 in SR03), so at initial stages it looks like bending of the disc.

Two points are of the essence. First, these two mechanisms are probably caused by some type of the orbit instability (Patsis et al. 2002; Skokos, Patsis & Athanassoula 2002), not by the bending instability. So, it is expected that this process is not subject to the linear criterion of the bending instability. Secondly, from the observational point of view, a boxy (or peanut) structure in the central part of a galaxy is a bulge (or a pseudo-bulge), and not a disc. So if we talk about the disc, we should consider regions outside this structure.

In this paper, we will demonstrate that, in the absence of the boxy/peanut structure, the bending instability heats the stellar disc up to the value of  $\sigma_z/\sigma_R$  not higher than  $\sim 0.3$ , which is predicted by the linear theory. In the presence of boxy/peanut structures, the above statement is correct only for regions outside these structures (in the case of models with a bar – outside the bar).

In SR03 we did not discuss regions outside the boxy/peanut structure (so outside a bar for models with bars, or outside the X-shaped structure for dynamically hot models). But from the data published in SR03, one can find that the value of  $\sigma_z/\sigma_R$  in a disc does not exceed  $\sim 0.35$  (see fig. 5 in SR03), i.e. the saturation level of the initial bending instability is close to the linear criterion. Here we confirm this result for models with a live halo.

We will also demonstrate that slow vertical heating (mechanism iv), which was explained in SR03 as heating due to vertical inhomogeneities in the stellar matter, is a purely numerical effect, and for sufficient number of particles there is no heating of this kind at all even in the presence of a live halo.

In Section 2 we describe our numerical models, in Section 3 we discuss the results of our numerical simulations and we conclude in Section 4.

## 2 THE NUMERICAL MODEL

We consider four families of models. Each family consists of three models with identical parameters but with a different number of particles (i.e. with different spatial resolution). All our models include a stellar disc and a dark halo. Two types of haloes are used: a relatively massive one (halo ‘A’) and a relatively light one (halo ‘B’).

For the first family, we model the potential of a halo as a static one (these models are analogous to the model 9\_1 from SR03). We denote this family as ‘AR’.

The second family is identical to the first family but all models have a live halo. We call this family ‘AL’.

The models from the third family (‘ATL’) are similar to ‘AL’ but they have an initially thick stellar disc in which we do not expect the bending instability to develop.

The fourth family (‘BL’) includes a live halo of type B.

In each family, we consider three models with different number of particles in the disc:  $2 \times 10^5$ ,  $1 \times 10^6$  and  $5 \times 10^6$ . The models with a live halo have  $5 \times 10^5$ ,  $2.5 \times 10^6$  and  $5 \times 10^6$  particles, respectively, in the halo.

We refer to the models under the name of their family, adding the number of particles in the disc. For example, AR.5M is the model from the family AR with 5 million particles in the disc.

While constructing the initial equilibrium models for our simulations, we use the iterative method presented in Rodionov & Sotnikova (2006) and Rodionov, Athanassoula & Sotnikova (2009).

We use the following density distribution in a stellar disc:

$$\rho_d(R, z) = \frac{M_d}{4\pi h^2 z_0} \exp\left(-\frac{R}{h}\right) \operatorname{sech}^2\left(\frac{z}{z_0}\right), \quad (1)$$

where  $M_d$  is the total disc mass,  $h$  is the disc scalelength,  $z_0$  is its scaleheight and  $R$  is the cylindrical radius. For all models discussed here,  $M_d = 5 \times 10^{10} M_\odot$  and  $h = 3$  kpc. The disc in all models except the family ALT has  $z_0 = 0.01$  kpc. The initial disc is extremely (unrealistically) thin because we are studying how it thickens due to the bending instability. For the ‘thick’ ALT models,  $z_0 = 0.1$  kpc.

For all models, we prepared the disc with the velocity dispersion profile  $\sigma_R$  as the input (see also Rodionov et al. 2009 for details). The radial profile of  $\sigma_R$  together with the density distribution in the disc defines the profile of the Toomre parameter  $Q_T$  (Toomre 1964). We choose the profile of  $\sigma_R$  so that the profile of  $Q_T$  has a wide minimum in the region of 2–8 kpc and the minimal value of  $Q_T \sim 1.5$ . This value is justified by the results of numerical experiments by Khoperskov, Zasov & Tyurina (2003). Modelling marginally stable stellar discs with various input data, they demonstrated that  $Q_T$  varies along the radius passing through a minimum  $Q_T = 1.2$ – $1.6$  just beyond the region at the galactocentric distance of  $(1-2) \cdot h$ , independently of the model choice. This value provides marginal in-plane stability of the disc and barless models. Starting with such a profile of the Toomre parameter, we can suppress the bar formation only in the case of a rigid halo (as for the AR models). We cannot avoid the bar formation in the case of a live halo (as for the AL models).

For the AR and AL models, we set

$$\sigma_R(R) = 80 \exp(-R/9) \text{ km s}^{-1}.$$

For the BL models, we chose

$$\sigma_R(R) = 100 \exp(-R/9) \text{ km s}^{-1}.$$

For the ATL models, we have imposed additional constrains. We have chosen the profile  $\sigma_R$  and the value of  $z_0$  so that the ratio  $\sigma_z/\sigma_R \approx 0.33$  across the disc:

$$\sigma_R(R) = 100 \exp(-R/6) \text{ km s}^{-1}.$$

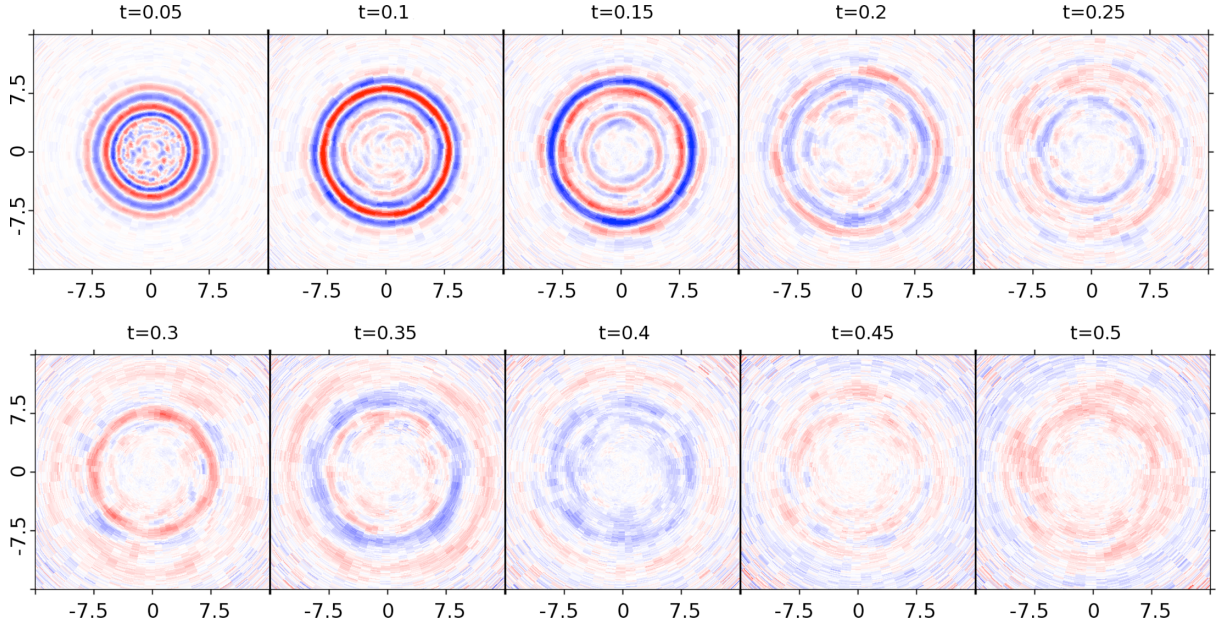
The dark halo is modelled as a truncated NFW halo (Navarro, Frenk & White 1996, 1997):

$$\rho_h(r) = \frac{C_h \cdot T(r/r_t)}{(r/r_h)(1 + r/r_h)^2}, \quad (2)$$

with  $T(x) = 1/(\cosh(x) + 1/\cosh(x))$ , where  $r_h$  is the halo scalelength,  $C_h$  is a parameter defining the mass of the halo and  $r_t$  is a truncated radius. For all models,  $r_h = 10$  kpc and  $r_t = 15$  kpc.

For halo ‘A’,  $C_h = 0.00491212$ , which gives the total mass of the halo  $4.083 \times 10^{11} M_\odot$ . Inside the region of four disc scalelengths (inside 12 kpc), the mass of the halo is approximately three times greater than the mass of the disc. For halo ‘B’,  $C_h = 0.00163737$ , which gives the total mass of the halo  $1.361 \times 10^{11} M_\odot$ , so inside a sphere of the radius 12 kpc the mass of the halo is approximately equal to the mass of the disc.

We use the following unit system: the unit of length is  $u_l = 1$  kpc, the unit of the velocity is  $u_v = 1$  km s<sup>-1</sup>, the unit of mass is  $u_m = 10^{10} M_\odot$  and, consequently, the unit of time is  $u_t \approx 0.98$  Gyr. For simplicity, while converting this time unit into giga-years we assume that  $u_t = 1$  Gyr.



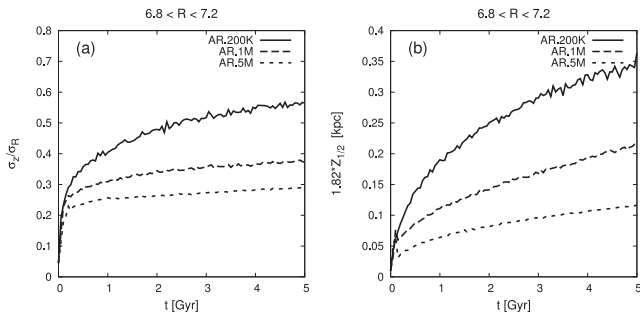
**Figure 1.** Initial stages of the bending instability for the model AR.5M – the highest resolution model from the family AR,  $t$  in Gyr. The colour intensities correspond to the different values of  $\bar{z}$  (the mean value of the  $z$  coordinate of particles) in a given area in the disc. Red colour means positive values of  $\bar{z}$ , and blue colour means negative values of  $\bar{z}$ . The range of  $\bar{z}$  is from  $-0.05$  to  $0.05$  kpc (the most intense red colour corresponds to  $\bar{z} = 0.05$  kpc, and the most intense blue colour corresponds to  $\bar{z} = -0.05$  kpc).

To simulate the evolution of our models, we use the fast  $N$ -body code GYRFALCON (Dehnen 2000, 2002). The softening length was chosen as  $\epsilon = 0.005$  kpc. We varied the integration time step according to the rule  $0.1\sqrt{\epsilon/|a|}$ , where  $a$  is the gravitation acceleration. The energy conservation for all our models except AL.200K and BL.200K (the two lowest resolution models with a live halo) is better than 0.5 per cent. For AL.200K and BL.200K, the energy conservation is about 3 per cent. We performed additional experiments for these models with a smaller time step, and so with the better energy conservation, and we did not find any significant difference in the final results.

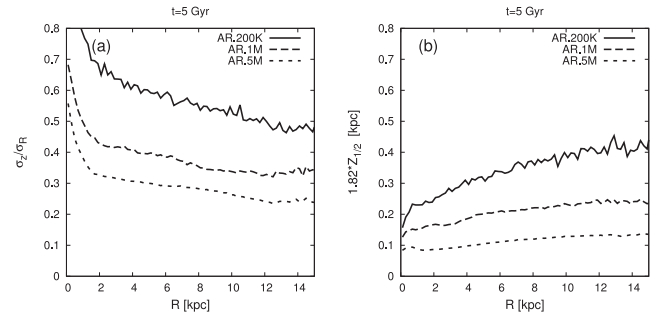
### 3 RESULTS

First, we examined the models from the family AR. These models have a rather massive rigid halo. Inside the region of four disc scalelengths, the halo mass is three times greater than the disc mass.

In Fig. 1, we show the evolution of initial bending for the highest resolution model from this family ( $N = 5 \times 10^6$  for a disc). In Fig. 2,



**Figure 2.** The evolution of the ratio  $\sigma_z/\sigma_R$  (left-hand panel) and of the disc thickness (right-hand panel) for the models from the family AR. All values are calculated in an annulus within  $6.8 < R < 7.2$ .



**Figure 3.** The radial profiles of the ratio  $\sigma_z/\sigma_R$  and of the disc thickness for the models from the family AR at the moment  $t = 5$  Gyr.

we show the evolution of the ratio  $\sigma_z/\sigma_R$  and of the disc thickness in the area close to 7 kpc.<sup>1</sup> Fig. 3 demonstrates the radial profiles of these parameters at  $t = 5$  Gyr for all models from the family. As a measure of the thickness, we use the quantity  $1.82 \cdot z_{1/2}$ , where  $z_{1/2}$  is the median of  $|z|$ . This quantity is a good estimation of the value of  $z_0$  for the density distribution (1) and is more robust than the commonly used dispersion of  $z$  (see Sotnikova & Rodionov 2006, hereafter SR06). For example, for the model AR.5M, the thickness of the model calculated via dispersion of  $z$  is almost twice overestimated because of a small number of particles with large values of  $z$ , i.e. because of a long tail in the distribution of particles over  $z$  (SR06).

The most striking feature in Figs 2 and 3 is the difference between the plots for the same models with different numbers of particles. These models were initially very thin. Therefore, the initial evolution of these models is rather predictable: disc starts bending (see Fig. 1). The main mode is a ‘bell’ mode ( $m = 0$ ). It propagates

<sup>1</sup> We will use the same radius for all our models, and a radius 7 kpc was chosen in order to be outside the bar for the models with a live halo.

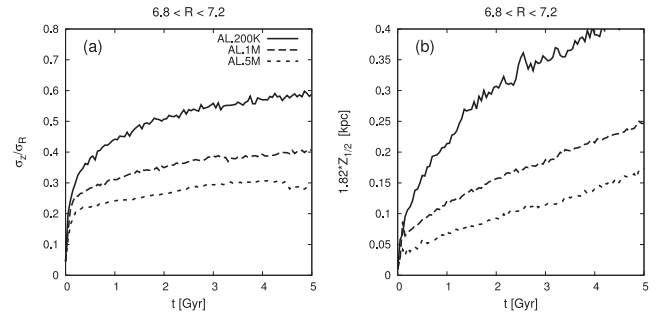


outwards. After approximately 150 Myr, the amplitude of bending reaches its maximum and starts decaying. After 1 Gyr, bending is almost fully decayed. The thickness and the vertical velocity dispersion are growing while bending develops (see Fig. 2, the very beginning of the evolution). We already described this process in SR03 and called it the ‘initial bending’. This initial bending heats up the disc and increases the ratio  $\sigma_z/\sigma_R$  only to a value which does not exceed the value given by the linear criterion of the bending instability. Actually, in the case of the AR models, the final value of  $\sigma_z/\sigma_R$  is even smaller than 0.3 (see Fig. 2a for the model AR.5M).

During the initial stages of bending, the evolution of the AR models with different numbers of particles is similar, but later they start diverging. The thickness and the vertical dispersion of the models increase with different rates. At the moment  $t = 5$  Gyr, the final models have very different thickness profiles (see Fig. 3). Even for the models with  $N = 10^6$  and  $5 \times 10^6$  in a disc, the difference in the thickness is about a factor of 2. We have already described this slow vertical heating in SR03 and tried to explain it as heating due to vertical inhomogeneities of the matter. But this explanation was, at least partly, wrong. Despite having an initial disc resistant to bar-like instability ( $Q_{T,\min} \approx 1.5$ , rigid halo; see Khoperskov et al. 2003), we did not avoid the formation of transient small-scale spirals due to noise amplification in our low-resolution simulations (see fig. 8 in SR03). The spiral activity increases basically the in-plane velocity dispersion. Such a heating effect was noticed for the first time by Sellwood & Carlberg (1984). But some authors are worried about another effect: discs thicken as spiral activity increases (for example, Walker, MihosCh & Hernquist 1996; McMillan & Dehnen 2007). Such a relation is rather strange. The enhanced in-plane motions due to transient spirals could be redirected into vertical motions only by some real or artificial agent. It could be giant molecular clouds (GMCs) in real galaxies or a separate population of heavy particles in a disc, which was not included in numerical models under discussion. Actually, such an agent does arise in some numerical models. In the model 9\_1 from SR03, we noticed that the disc thickness is not the same in different parts of the model. The disc thickness in the regions where the spiral arms are located is smaller than the disc thickness in the interarm space. In other words, inhomogeneities in the in-plane distribution of stars produce inhomogeneities in the vertical distribution of matter, and our hypothesis was that such features could ‘transfer’ in-plane velocity dispersion into the vertical one making a disc thicker.

Now we have another explanation. We are inclined to think that at the late stages of the disc evolution, slow vertical heating is a pure numerical effect (a numerical two-body relaxation). As one can see from Figs 2 and 3, the greater the number of particles, the weaker is the vertical heating effect. But we failed to eliminate completely this numerical effect because of the limited number of particles. Even for the model with 5 million particles, this numerical heating is noticeable. Sellwood (2013) has recently resumed the old debate about collisional relaxation in very flattened systems. He reminded the theoretical arguments by Rybicki (1971) and presented numerical results which confirm that two-body scattering in discs is much more rapid than that expected in spherical models with the same  $N$ . As a result, the outcome of simulations will depend on the number of particles used in numerical experiments. In the experiments by Sellwood (2013), spiral activity lasted too long and increases slowly the in-plane dispersion even for  $N = 4 \times 10^6$ . At the same time, while increasing the number of particles up to several millions, the disc thickening abruptly lessens.

There is another argument against our old hypothesis. In our low-resolution models, the spiral activity gradually lessens and  $\sigma_R$

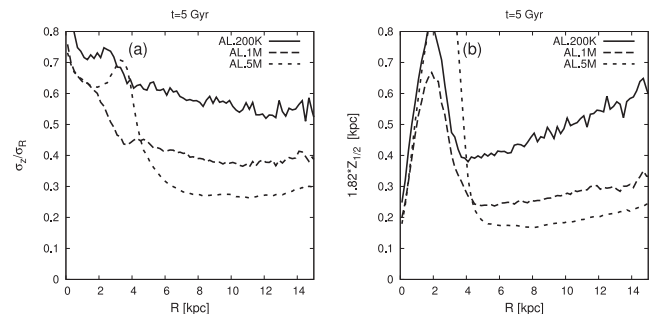


**Figure 4.** The same as in Fig. 2, but for the family AL.

is almost constant at the late stages of evolution (see fig. 3 in SR03). In contrast to this in the simulations with  $N > 10^6$  in a disc, the in-plane velocity dispersion  $\sigma_R$  continues to slowly grow, probably, due to a much clear manifestation of spirals. Such self-excitation behaviour has been recently discussed by Henriksen (2012). So one could expect more efficient vertical heating in high-resolution models, which is actually opposite to observed results (see Figs 2 and 3). Thus, vertical heating in our simulations is mainly due to the numerical relaxation, especially for models with moderate numbers of particles. We cannot completely exclude the possibility of out-plane heating due to vertical inhomogeneities but the efficiency of such an effect is small.

Despite the artificial heating revealed in our experiments, the final value of the ratio  $\sigma_z/\sigma_R$  for the highest resolution model A.5M is almost everywhere less than 0.3 (see Fig. 3a). Only in the very central region (inside 1 kpc),  $\sigma_z/\sigma_R$  is relatively large. But in this very central region, the model forms a subtle X-shaped structure, which is a very efficient source of vertical heating (SR03).

Now let us consider the models from the family AL, which are completely identical to the models from the family AR, but they have a live halo. Figs 4 and 5 demonstrate the evolution of the ratio  $\sigma_z/\sigma_R$  and of the disc thickness in the area close to 7 kpc for the models AL. In Fig. 5, the radial profiles of these quantities at  $t = 5$  Gyr are shown. In these models, in contrast with the models comprising a rigid halo, there is a bar, which was quite expected (Athanasoula 2002). In all AL models, a bar appears after approximately a half Gyr of the evolution. The bar grows and then slows down due to the angular momentum exchange (Athanasoula 2003). While growing, a flat bar starts buckling (i.e. the bending instability of the bar develops; see Raha et al. 1991). It results in the formation of the boxy/peanut structure. This structure can be easily recognized in the profiles of the disc thickness (see Fig. 5b, inside  $R < 6$  kpc). As we discussed in SR03, buckling of the bar leads to an increase of the thickness and of the ratio  $\sigma_z/\sigma_R$  in the region of the bar. Outside the bar, in the region of the disc, the evolution of the AL models is



**Figure 5.** The same as in Fig. 3, but for the family AL.

similar to the evolution of AR models, with the proviso that the AL models are a little thicker and have a slightly larger value of  $\sigma_z/\sigma_R$  (Figs 4 and 5).

To summarize:

- (i) the bending instability heats the disc up to the value predicted by the linear criterion;
- (ii) there is artificial vertical heating which depends on the number of particles, and even for the models with 1 and 5 million particles in the disc, the difference in the final thickness is significant.

The dependence of the effect on the number of particles employed supports the idea about two-body relaxation in flattened systems (Sellwood 2013).

All our models with a live halo are thicker than the models with a rigid halo in the region outside a bar (see, for example, the high-resolution models AR.M5 and AL.M5 in Figs 2–5). It could be due to physical or numerical effects. For example, some ‘physical’ (not just numerical) interaction between the disc and the halo could result in disc thickening. Athanassoula (2002) showed that a live halo can stimulate bar growth by absorbing angular momentum, which a rigid halo cannot. McMillan & Dehnen (2007) were inclined to such an explanation while analysing the greater increase in disc thickness seen in the simulations with a live halo compared to that with a rigid halo. The bar also can influence the areas outside itself and make them heat up in the vertical direction. Another explanation is more efficient numerical heating in the case of a live halo. For example, in our highest resolution experiments, the mass of a particle in the halo is rather large and eight times greater than the mass of a particle in the disc. So, halo particles can efficiently scatter disc particles. Probably, if we took a sufficiently large number of particles in a halo, the model with a live halo would behave similarly to the model with a rigid halo in the region outside the bar. Now we cannot say which version is right, because we even cannot claim that the models with a rigid halo converge while the number of particles increases. But we want to underline that the answer to this question does not alter our main conclusions about the initial heating due to the bending instability.

We carried out additional numerical experiments to prove our conclusion about numerical heating. We considered the evolution of initially relatively thick models (ATL) in which we did not expect the bending instability to develop. In these models, the initial ratio  $\sigma_z/\sigma_R \approx 0.33$  throughout the disc. Fig. 6 demonstrates the initial evolution of the ratio  $\sigma_z/\sigma_R$  and of the disc thickness in the area close to 7 kpc. The initial conditions for these models were well beyond the unstable region of the bending instability, and, as a result, these models did not have the stage of initial bending. Fig. 6 shows pure artificial numerical heating, which decreases with increasing

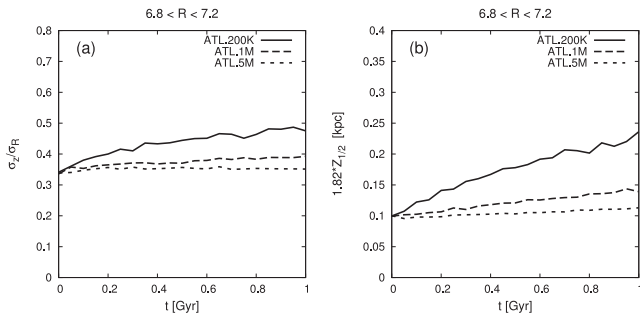


Figure 6. The same as in Fig. 2, but for the family ATL.

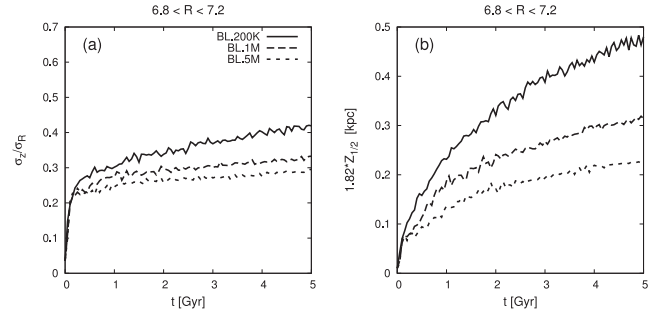


Figure 7. The same as in Fig. 2, but for the family BL.

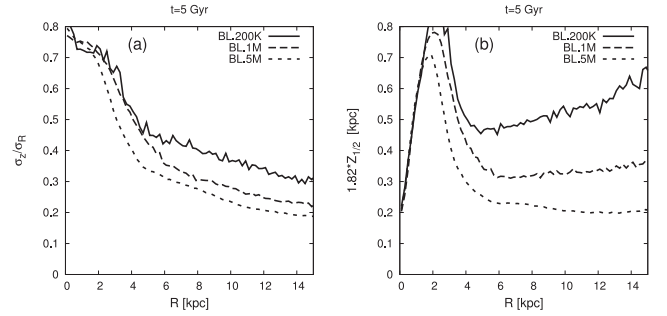


Figure 8. The same as in Fig. 3, but for the family BL.

number of particles. Starting from almost the same initial conditions ( $\sigma_z/\sigma_R \approx 0.31$  at  $t = 0$ ), Sellwood (2013) came to similar results: only for  $N = 4 \times 10^6$ , the ratio  $\sigma_z/\sigma_R$  did not change with time at a reference radius.

The models which we have just considered had a relatively massive dark halo. The models from the family ‘BL’ have a less massive halo. The halo mass inside the region of four disc scalelengths is barely equal to the total disc mass. Fig. 7 demonstrates the evolution of the ratio  $\sigma_z/\sigma_R$  and of the disc thickness inside the thin ring with radius equal to 7 kpc. In Fig. 8, we show the evolution of radial profiles of these quantities at  $t = 5$  Gyr. The evolution of the BL models is similar to the evolution of the AL models. The initial bending instability develops. It results in very rapid heating of the disc up to the level predicted by the linear criterion. This stage is accompanied by the formation of a bar with its subsequent buckling and the appearance of the boxy/peanut structure. After 5 Gyr of the evolution in the area outside the bar, the BL models are thicker than the AL models, but they have similar values of  $\sigma_z/\sigma_R$ , because the halo of the BL models is less massive. The disc thickness can be estimated via the vertical equilibrium condition for an isothermal slab (Spitzer 1942):

$$\sigma_z^2 = \pi G \Sigma_d z_0, \quad (3)$$

where  $\Sigma_d$  is the surface density of the slab. This relation gives the upper limit for the disc thickness. If there is a massive dark halo, the disc thickness will be lower for the same value of  $\sigma_z$  [see equation 4 in SR06 and equation (4) in this paper].

Again we can see that the models with different numbers of particles are significantly different. So, there is numerical heating; however, in the case of the BL family, the difference between low- and high-resolution models is less dramatic. The values of  $\sigma_z/\sigma_R$  for the models BL.1M and BL.5M are close (see Figs 7a and 8a), but the thicknesses are more visibly different (see Figs 7b and 8b).

Similar to previous results, the highest resolution model in the BL family (BL5M model) outside the bar has  $\sigma_z/\sigma_R$ , which corresponds to the linear criterion of the bending instability.

#### 4 SUMMARY

Modelling how the bending instability develops in  $N$ -body simulations, a few authors, for example Merritt & Sellwood (1994), SR03 and Khoperskov et al. (2010), proceeded from the assumption that the saturation level of the instability was much higher than that given by the linear criterion. Merritt & Sellwood (1994) found unstable long-wavelength modes ( $m = 0, 1, 2$ ) in thin finite-thickness discs with realistic density profiles even when the ratio  $\sigma_z/\sigma_R$  is much less extreme than the critical value for the instability in an infinite slab (see their fig. 3). Based on the results of a large series of numerical experiments, SR03 concluded that the saturation level of large-scale bending perturbations is a factor of 2 or 3 higher than the linear one. Khoperskov et al. (2010) have been studying the non-linear dynamics of the bending instability and the vertical structure of a stellar disc embedded into a spherical halo. They considered the development of the bending instability to be the main factor of the disc thickness increase.

Slow gradual heating of a disc in the vertical direction was usually thought as real and triggered by the bending instability and not by the numerical relaxation because in such experiments the increase of  $\sigma_R$  eventually ceases when the activity of transient small-scale spirals decays. Moreover, suggesting the galaxies to be marginally stable against bending perturbations, Zasov, Makarov & Mikailhova (1991), Zasov et al. (2002) and Khoperskov et al. (2010) tried to estimate the contribution of a dark halo to the galaxy mass model based only on the value of the disc thickness  $z_0/h$  for edge-on galaxies (see equation 4). The critical value for the ratio  $\sigma_z/\sigma_R$  was taken from the results of  $N$ -body simulations and it was much larger than  $\sigma_z/\sigma_R = 0.3$  (see fig. 1 in Zasov et al. 1991, fig. 5 in Zasov et al. 2002 and fig. 12 in Khoperskov et al. 2010). In SR06 we pointed out that the inclusion of a compact bulge in the model makes the connection between the relative thickness and the halo mass ambiguous (see fig. 6 in SR06). We presented arguments that in simulations with  $N < 10^6$  the value of the ratio  $\sigma_z/\sigma_R$  was strongly overestimated, while simulations were started from ‘thin’ initial conditions. As Sellwood (2013) has demonstrated, such an overestimation is due to two-body relaxation in flattened rotating systems with insufficient numbers of particles employed.

So, the slow increase of the thickness and of the vertical velocity dispersion, which we observed in the model 9\_1 in SR03, was completely due to the numerical effect. For a relatively small number of particles in the disc ( $N = 2 \times 10^5$ ), this numerical heating is drastic. Even for the models with  $5 \times 10^6$  particles in the disc, this effect cannot be eliminated. In future, we should be very careful while interpreting the results of numerical simulations concerning disc thickening or the increase of the vertical velocity dispersion, especially while starting from the very thin models.

In SR03 we showed that if we start from a sufficiently thin stellar disc, the bending instability develops at the initial stage of the disc evolution. This instability develops and decays rather fast and results in the increase of the disc thickness and of the ratio  $\sigma_z/\sigma_R$ . Numerical experiments with a larger number of particles have demonstrated that during the initial bending, the ratio  $\sigma_z/\sigma_R$  increases till the level corresponding to the linear criterion of the bending instability. It gives an argument that this initial bending is caused by the real bending instability described by Toomre (1966).

Now we can conclude that for our models of stellar discs in isolated galaxies in the absence of a separate agent causing out-plane scattering (GMCs, nearby external galaxies), there are three mechanisms of vertical heating.

(i) The initial bending instability, which develops and decays within the first Gyr of the evolution. It heats the disc up to the level corresponding to the linear criterion of the bending instability.

(ii) The formation of an X-shaped (boxy/peanut) structure in the central parts of some barless models.

(iii) The bending instability of the bar (buckling of the bar), which also causes the formation of boxy/peanut structures.

It is possible that the last two mechanisms are not associated with the bending instability but are related to the orbital instability (Patsis et al. 2002; Skokos et al. 2002). Both mechanisms operate in the central parts of the galaxy and are responsible for the formation of boxy/peanut structures, which perhaps correspond to a pseudo-bulge in real galaxies. As follows from our simulations, the presence of boxy/peanut structures does not affect areas outside them. Outside the bar, the value of  $\sigma_z/\sigma_R$  is not greater than that predicted by the linear theory of the bending instability, and the stellar disc remains very thin.

If we talk about the disc itself (not about the central regions of a pseudo-bulge), then in our models there is nothing more than the classical ‘linear’ bending instability which heats the disc up to the value  $\sigma_z/\sigma_R \approx 0.3$ . This value is rather small, and Toomre (1966) doubted about the role of the bending instability in disc heating. Now, we share his doubts. But the most important point is that this instability is rather fast and does not affect the secular evolution of the stellar disc. It develops and decays in less than a half Gyr. It cannot produce any long-lived out-plane structures in a disc. The only role which it could play in real galaxies is to define the minimal value of  $\sigma_z/\sigma_R$ . So, in the absence of other mechanisms of vertical heating, real galaxies would be marginally stable against the bending instability (would have  $\sigma_z/\sigma_R \approx 0.3$ ).

One interesting question arises: are there real galaxies, which are marginally stable against the bending instability, or are they all completely overheated in the vertical direction due to scattering of stars by GMCs or because of an interaction with nearby satellites?

The value of  $\sigma_z/\sigma_R$  given by the linear criterion of the bending instability is rather small, and Toomre (1966) emphasized this fact. The value of  $\sigma_z/\sigma_R$  in the solar neighbourhood is about 0.5 (for example, Dehnen & Binney 1998; Orlov et al. 2006) and depends weakly on the spectral types of stars. Only for stars of spectral classes A and B, this ratio is about 0.4 (Mignard 2000, table 6). As to external galaxies, they are rather thick: the typical value of  $h/z_0$  is less than 5, and for the thinnest of them this ratio is less than 10 (Kregel, van der Kruit & de Grijs 2002; Mosenkov, Sotnikova & Reshetnikov 2010). By combining the equation of equilibrium in the vertical direction (equation 3) and the condition of stability in the disc plane (Toomre 1964), and neglecting the bulge contribution in the mass model, we can derive the expression

$$\frac{z_0}{h} \approx 0.62 \frac{\sigma_z^2}{\sigma_R^2} \frac{M_d(4h)}{M_d(4h) + M_h(4h)} Q_T^2, \quad (4)$$

where  $Q_T$  is the Toomre parameter,  $M_d(4h)$  is the disc mass inside the sphere with the radius of four exponential scalelengths and  $M_h(4h)$  is the halo mass inside  $4h$  (see the derivation of this equation in SR06). If we assume that a galaxy has  $\sigma_z/\sigma_R = 0.3$ ,  $Q_T = 1.4$  and does not have significant amount of dark matter, then the value of the ratio  $h/z_0$  for such a galaxy would be  $\approx 9$ . So, the galaxy, without dark matter, would be as thin as the thinnest galaxies. But with

a dark halo the galaxy would be thinner than observed galaxies. However, if we increase the value of the Toomre parameter, the resulting value of  $h/z_0$  will decrease. For example, a galaxy with  $\sigma_z/\sigma_R = 0.3$ ,  $Q_T = 2$  and  $M_d(4h) = M_h(4h)$  would, again, have  $h/z_0 \approx 9$ . So we can conclude that at least the thinnest galaxies could be marginally stable against the bending instability. On the other hand, we expect the galaxies with  $h/z_0 = 5$  to be on average overheated and have  $\sigma_z/\sigma_R > 0.3$ .

Until recently, no reliable data about the shape of the velocity ellipsoid ( $\sigma_z/\sigma_R$ ) have been available for external galaxies. At present, this ratio is measured directly in several galaxies (Gerssen, Kuijken & Merrifield 1997, 2000; Shapiro, Gerssen & van der Marel 2003; Gerssen & Shapiro Griffin 2012). There is a clear trend: the later the type of a galaxy, the smaller is the ratio  $\sigma_z/\sigma_R$  (see fig. 5 in Gerssen & Shapiro Griffin 2012). For the early-type galaxies,  $\sigma_z/\sigma_R$  is significantly greater than 0.3 (for example,  $\sigma_z/\sigma_R = 0.75 \pm 0.09$  for Sab galaxy NGC 2985). Thus, there are strong arguments that real early-type galaxies are overheated with respect to the bending instability because they have  $\sigma_z/\sigma_R$  much greater than 0.3. It means that in early-type galaxies, some other mechanisms of vertical heating operate and they are not included in our simple models of isolated disc galaxies. There are at least two candidates for the role of such mechanisms: heating by GMCs and interaction with external galaxies or satellites (see, for example, Sellwood 2010 and the discussion in the introduction in SR03).

Surprisingly, for the late-type galaxies, the ratio  $\sigma_z/\sigma_R$  is close to 0.3 ( $0.25 \pm 0.20$  for NGC 2280 and  $0.29 \pm 0.12$  for NGC 3810; Gerssen & Shapiro Griffin 2012). It means that these galaxies are marginally stable against the bending instability. This result is a bit shocking, because the late-type galaxies possess a large amount of gas including the gas in the molecular form. For example, for NGC 3810, CO emission is detected (Mao et al. 2010). But for unknown reasons, the scattering of stars on molecular clouds is ineffective in such galaxies.

Another intriguing result is the absence of any trend of  $\sigma_z$  and  $\sigma_z/\sigma_R$  with the surface density  $\Sigma_{H_2}$  of molecular gas, which is usually thought as a ‘three-dimensional heating agent’. At the same time, there is a correlation between  $\sigma_R$  and  $\Sigma_{H_2}$ . If it is real, this fact poses the question about the role of GMCs in in-plane and out-plane scattering of stars.

Summarizing, we can conclude that the answer to the question ‘does the bending instability play any role in the galaxy evolution?’ is most likely affirmative. The bending instability defines the minimal value of  $\sigma_z/\sigma_R$  for real galaxies, and there are arguments that galaxies, which are marginally stable against the bending instability, do exist and have  $\sigma_z/\sigma_R \approx 0.3$ .

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