

Spectral methods plasticity modeling for fatigue damage estimation: experimental application – uniaxial case

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Abstract – In this paper, we present techniques for fatigue damage evaluation using spectral methods and a model taking into account confined elasto-plastic behavior. The model is associated with a local fatigue approach, covering the whole endurance domain (low cycle and high cycle fatigue). It uses Neuber’s method and is valid for limited plasticity. To validate this modeling, we perform a correlation between spectral methods, modified spectral methods and experimental tests. Results presented here are focused on the uniaxial loading case.

Key words: Spectral methods / plasticity / random process / Rainflow Count / fatigue damage

1 Introduction

The phenomenon of fatigue has become a major design criterion in the engineering industry, requiring fatigue methods that better model real physical phenomena. Conventional fatigue methods are defined in time domain because their loading input data is usually a stress or strain time history. Algorithms such as Rainflow [1, 2] extract stress cycles from the time history to obtain a stress distribution. To determine the lifetime generated by the stress distribution, we can use a fatigue law such as Basquin equation and a damage accumulation rule such as Palmgren-Miner law. Nevertheless, these methods require knowing the stress time history or the envelope of this time history.

In the case of random vibrations, the mechanical structure may have a dynamic response. It is difficult to know the stress time history and to make the classical fatigue analysis. In time domain, it is only calculated the damage associated with one realization of the random vibrations process. If one wants the damage caused by random processes (in this paper, only stationary ergodic random processes are considered), it is necessary to calculate damages on several different realizations of the same stochastic process. This approach is called Monte Carlo simulation. The main drawback of Monte Carlo simulations is the computation time [3].

To overcome these problems it is suitable to work in the frequency domain. In addition, we model the random vibrations as a gaussian stochastic process, characterized by its PSD. Fatigue spectral methods are entirely defined in the frequency domain. The average damage is then calculated from the PSD of stress. The stress cycles distribution is not calculated but is estimated with a probability density function. This probability density depends on parameters of the PSD and the selected spectral method. Spectral methods have shown their reliability in fatigue life predictions, however, their theory requires several assumptions about the structure response (Fig. 1a) that limit their applications.

Our proposal extends the range of SM to low cycle fatigue, by modeling confined plastic behavior of materials and we use a fatigue law that take into account this plasticity. Figure 1b defines the framework in which we will expand the use of spectral methods. In the presence of non-linearity, the response of the structure is a stationary ergodic non-gaussian stochastic process. Although the assumption of zero mean is no longer true, we make the assumption that the response of the structure with a non-linearity is zero mean because the non-linearity (plasticity) is confined.

To allow comparison between the proposed method and the methods of bibliography, we conducted vibration fatigue tests which will serve as a reference. Tests will be performed in the case of a nearly uniaxial response to be consistent with the definition of the methods studied.

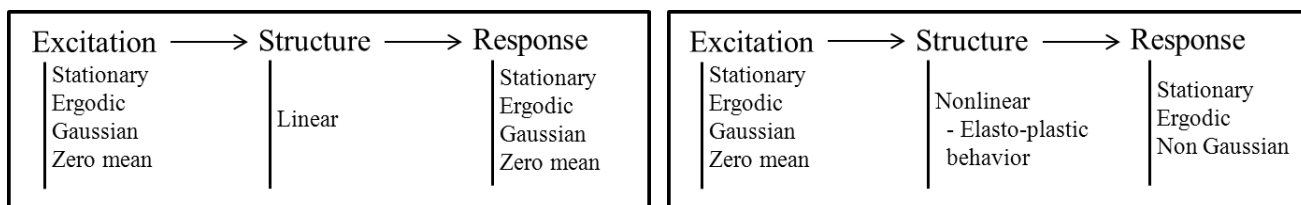
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Nomenclature

$E[\bullet]$	Mathematical expectation
f	Frequency (Hz)
$G_{xx}(f)$	Power spectral density of $x(t)$
$R_{xx}(\tau)$	Autocorrelation function of $x(t)$
λ_m	Spectral moment of order m
ν_0	Number of zero-upcrossings
ν_a	Average number of maxima
γ	Irregularity factor
N	number of cycles (lifetime)
σ	Stress amplitude
σ_m	mean stress
ε	Strain amplitude
C, k	Material parameters of Basquin model
E	Young's modulus
ε_f	Fatigue ductility coefficient
σ_f	Fatigue strength coefficient
b, c	Material parameters of Manson-Coffin-Basquin model
$h(P, V)$	Joint probability density function of peak and valley
$P_a(\sigma)$	Amplitude probability density function of $\sigma(t)$
d	Damage at a given amplitude σ
$E[d]$	Average damage in the interval σ and $\sigma + d\sigma$
$E[D]$	Average damage
T_f	Life time (seconds, hours or days)
$X(t)$	Gaussian stochastic process
$Z(t)$	Non-Gaussian stochastic process
$G(\bullet)$	Direct transformation
$G^{-1}(\bullet)$	Inverse transformation
MOY_\bullet	Average of stationary ergodic stochastic process $\bullet(t)$
STD_\bullet	Standard deviation of the stationary ergodic stochastic process $\bullet(t)$
S	Skewness
K	Kurtosis
h_3, h_4	Parameters of Hermite model
k'	Cyclic strength coefficient
n'	Cyclic strain hardening exponent
$\varepsilon_{e_i} \sigma_{e_i}$	Strain and stress obtained by elastic behavior for class i
$\varepsilon_{r_i} \sigma_{r_i}$	Strain and stress obtained by Neuber's model for class i
$H(\bullet)$	Cycles distribution with linear behavior
$H_{ep}(\bullet)$	Cycles distribution with elasto-plastic behavior

Acronyms

PSD	Power Spectral Density
PDF	Probability Density Function
RMS	Root Mean Square
SM	Spectral Method
SMM	Spectral Method Modified
HM	Hermite Model + spectral method



(a) Assumptions necessary for using spectral methods

(b) Assumptions of our case study

Fig. 1. Definition of required assumptions in spectral methods and positioning of our framework.

2 General purpose

2.1 Properties of stochastic processes

Power spectral density

In the case of a stationary Gaussian stochastic process, the PSD contains information defining this process [4]. Moreover, if the process is ergodic, the PSD can be calculated from the autocorrelation function of a realization of the random process $x(t)$ as follows, equation (1) :

$$G_{xx}(f) = \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-i2\pi f\tau} d\tau \quad (1)$$

where $R(\tau) = E[x(t)x(t+\tau)]$. This equation (1) is known as the Wiener-Khintchine theorem.

Spectral moments and signal parameters

The spectral moments are important characteristics of a random process, they are calculated by equation (2).

$$\lambda_m = \int_{-\infty}^{+\infty} G_{xx}(f) |f|^m df \quad (2)$$

They are related to the amount of information contained in a random process in the time domain. If for example $x(t)$ is a displacement and a random process with zero mean, λ_0 is the variance of the displacement, λ_2 is the variance of the velocity and λ_4 is the variance of the acceleration.

Spectral moments determine signal parameters of random processes. These results were demonstrated by Rice [5]. The number of zero-upcrossings is defined in equation (3).

$$\nu_0 = \sqrt{\frac{\lambda_2}{\lambda_0}} \quad (3)$$

We define in equation (4) the number of maxima as:

$$\nu_a = \sqrt{\frac{\lambda_4}{\lambda_2}} \quad (4)$$

Another useful parameter is the irregularity factor. It represents the bandwidth of the PSD. It is calculated from the number of maxima and the number of zero-upcrossings, equation (5).

$$\gamma = \frac{\nu_0}{\nu_a} = \sqrt{\frac{\lambda_2}{\lambda_0 \lambda_4}} \quad (5)$$

2.2 Fatigue law

Spectral methods commonly use the Basquin fatigue model [6]. This gives a power law relation between stress amplitude and life. This model leads to a linear Stress-Life curve in a log-log scale. It is defined in equation (6)

$$N\sigma^k = C \quad (6)$$

In the case of the proposed approach, the fatigue model used is Basquin-Manson-Coffin's formulation [6]. This is a non linear model that determines total strain amplitude as a function of life. It is governed by equation (7):

$$\varepsilon = \frac{\sigma'_f}{\mathbf{E}} (2N)^b + \varepsilon'_f (2N)^c \quad (7)$$

2.3 Spectral methods

In a classical approach of damage, counting methods define each stress cycle extracted by a peak of level P and valley of level V or amplitude σ and mean value σ_m . Each extracted cycle is a random event that has its own probability of occurrence. Consider $h(P, V)$ the joint probability density of cycles extracted functions of peaks P and valley V. By simple variable change, the joint distribution of cycles is obtained in terms of amplitude σ and mean value σ_m , equation (8):

$$P_{a,m}(\sigma, \sigma_m) = 2h(\sigma_m + \sigma, \sigma_m - \sigma) \quad (8)$$

The amplitude probability density function (PDF) is then, equation (9) :

$$P_a(\sigma) = \int_{-\infty}^{+\infty} P_{a,m}(\sigma, \sigma_m) d\sigma_m \quad (9)$$

Damage due to each stress amplitude σ , according to Basquin equation is equation (10) :

$$d = \frac{1}{N} = C^{-1} \sigma^k \quad (10)$$

The expectation of damage contribution of cycles whose amplitude is between σ and $\sigma + d\sigma$, equation (11) :

$$E[d] = C^{-1} \nu_a \sigma^k P_a(\sigma) d\sigma \quad (11)$$

where $\nu_a P_a(\sigma) d\sigma$ is the number of maxima per unit time in the interval $[\sigma; \sigma + d\sigma]$.

The damage expectation per unit time for the law of Palmgren-Miner is equation (12).

$$E[D] = C^{-1} \nu_a \int_0^{+\infty} \sigma^k P_a(\sigma) d\sigma \quad (12)$$

$P_a(\sigma)$ is the amplitude probability density function and depends on the formulation made by each author. In our case, we use the probability density function (PDF) defined by Lalanne (Rice theory) [7] and Dirlik (Rainflow approach) [8].

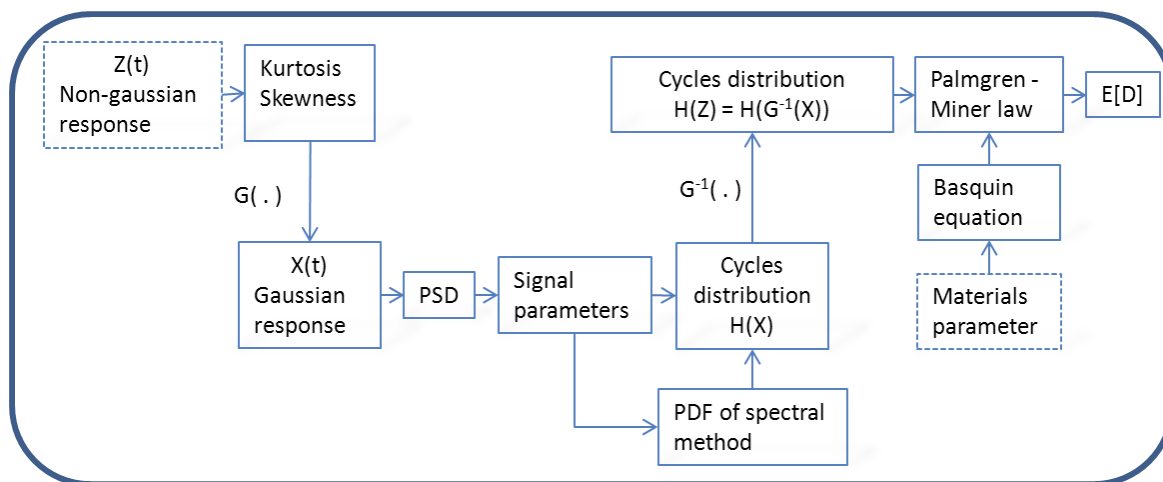


Fig. 2. Methodology diagram for calculating fatigue for non-gaussian process with a reversible transformation of the hermite model type.

1 Lalanne's method

2 Lalanne [7] defines the amplitude probability density
3 function from the work of Rice [5], given in equation (13).

$$P_a^{Lal}(\sigma) = \sqrt{\frac{1-\gamma^2}{2\pi\lambda_0}} \exp\left(\frac{-\sigma^2}{2\lambda_0(1-\gamma^2)}\right) + \frac{\gamma\sigma}{2\lambda_0} \exp\left(-\frac{\sigma^2}{2\lambda_0}\right) \times \left[1 + \text{Erf}\left(\frac{\gamma\sigma}{\sqrt{2\lambda_0(1-\gamma^2)}}\right)\right] \quad (13)$$

4 where Erf(x) is the error function, defined by Erf(x) =
5 $\frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha$. The average damage can be calculated
6 from the equation (14).

$$E[D^{Lal}] = C^{-1} \nu_a \int_0^{+\infty} \sigma^k P_a^{Lal}(\sigma) d\sigma \quad (14)$$

7 Dirlik's method

8 In 1985, Dirlik [9] proposed to determine directly from
9 the PSD the amplitude density probability based on the
10 definition of a rainflow cycle. He determines the rainflow
11 cycles probability density empirically, from a large num-
12 ber of Monte Carlo simulations with different forms of
13 PSD and irregularity factors. The probability density by
14 Dirlik is given in equation (15).

$$P_a^{Dir}(\sigma) = \frac{1}{\sqrt{\lambda_0}} \left[\frac{D_1}{Q_D} \exp\left(-\frac{\varphi}{Q_D}\right) + \frac{D_2\varphi}{R^2} \exp\left(-\frac{\varphi^2}{2R^2}\right) + D_3\varphi \exp\left(-\frac{\varphi^2}{2}\right) \right] \quad (15)$$

where

$$\varphi = \frac{\sigma}{\sqrt{\lambda_0}}; \quad x_m = \frac{\lambda_1}{\lambda_0} \sqrt{\frac{\lambda_2}{\lambda_4}}; \quad D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2}$$

$$R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 - D_1^2}; \quad D_2 = \frac{1 - \gamma - D_1 - D_1^2}{1 - R} \quad (16)$$

$$D_3 = 1 - D_1 - D_2; \quad Q_D = \frac{1.25(\gamma - D_3 - (D_2R))}{D_1}$$

Equation (17) indicates the average damage with the cy-
cles probability density by Dirlik.

$$E[D^{Dir}] = C^{-1} \nu_a \int_0^{+\infty} \sigma^k P_a^{Dir}(\sigma) d\sigma \quad (17)$$

2.4 Hermite's model

In case the structure response is a stationary ergodic
non-Gaussian stochastic process, there are ways to per-
form the calculation of the fatigue damage. Hermite's
model [10, 11] is a reversible transformation that can
transform a non-Gaussian processes in Gaussian process.
Combined with spectral methods, this allows fatigue life
calculation of a non-Gaussian stochastic process [12, 13].
The global methodology is described in Figure 2. In our
case, the non-Gaussian processes can be generated by a
structure with a non-linear behavior. Here we use the Her-
mite model because it is easy to implement but there are
other models such as Ochi and Ahn [14, 15] and Power-law
model [16].

Equation (18) represents the $G(\bullet)$ transformation [17]
applied to our problem where X_0 is the reduced centered
variable ($X_0 = \frac{X - MOY_x}{STD_x}$).

$$X_0 = [\sqrt{\xi^2(Z) + \eta_3 + \xi(Z)}]^{1/3} - [\sqrt{\xi^2(Z) + \eta_3 - \xi(Z)}]^{1/3} - \eta_1 \quad (18)$$

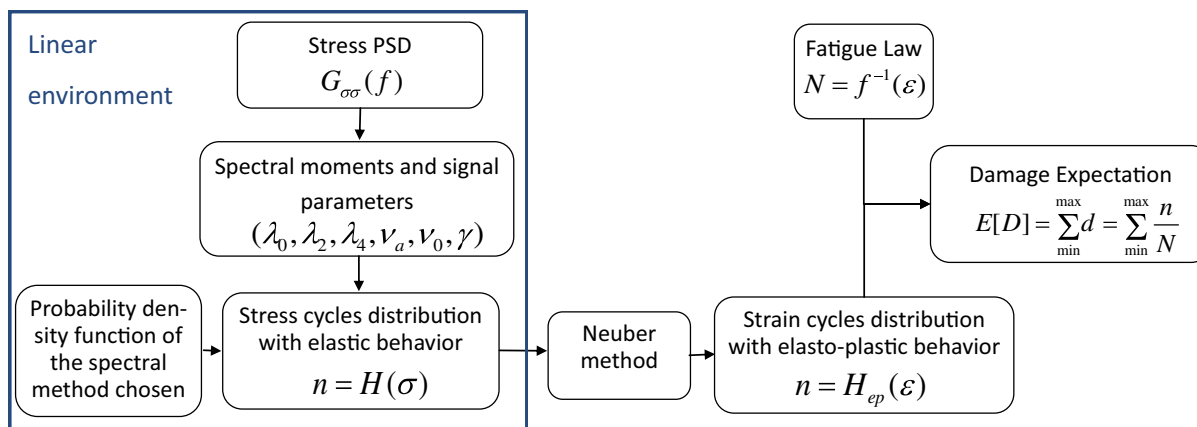


Fig. 3. Proposed fatigue process in the spectral methods.

1 Where

$$\begin{aligned} \xi(Z) &= 1.5\eta_2 \left(\eta_1 + \frac{Z - MOY_z}{STD_z} \right) - \eta_1^3, & \eta_1 &= \frac{h_3}{3h_4}, \\ \eta_2 &= \frac{1}{3h_4}, & \eta_3 &= (\eta_2 - 1 - \eta_1^2)^3, & h_3 &= \frac{S}{6}, \\ h_4 &= \frac{(K - 3)}{24} \end{aligned} \quad (19)$$

2 Equation (20) represents the inverse transformation of G
3 (G^{-1}):

$$Z = MOY_z + STD_z[X_0 + h_3((X_0^2 - 1) + h_4((X_0^3 - 3X_0))] \quad (20)$$

4 3 Modeling of plasticity in spectral methods

5 3.1 Modelization in spectral methods

6 To take into account elasto-plastic behavior, the spec-
7 tral methods need two essential elements to provide a
8 good estimation of the damage. The first element is a
9 fatigue law that takes into account the plastic behavior.
10 The second element is a model giving the elasto-plastic
11 local responses due to the loading. These two elements are
12 taken into account by using the Basquin-Manson-Coffin
13 model and the rule of Neuber [18, 19].

14 Manson-Coffin-Basquin’s law can take into account
15 the plastic behavior of the material during the accumu-
16 lation of damage. Neuber’s method defines a distribution
17 that takes into account the elasto-plastic behavior from a
18 linear calculation respecting the assumptions of spectral
19 methods (Fig. 1). In spectral domain, it is difficult to ac-
20 cess to the real strain time history, but for each strain level
21 it is possible to obtain the cycles number corresponding,
22 this is the distribution of cycles number (H function). The
23 H function is generally used for the discretized calculus

24 Figure 4 shows how we apply for each stress increment
25 the Neuber’s method to obtain the real strain. To repre-
26 sent the present phenomenon, we use to apply the correc-
27 tion of Ramberg-Osgood equation and Neuber equation,

see equation (21).

$$\varepsilon_{ri} \begin{cases} \varepsilon_{ri}\sigma_{ri} = \frac{\sigma_{ei}\sigma_{ei}}{\mathbf{E}} \leftarrow \text{Neuber} \\ \varepsilon_{ri} = \frac{\sigma_{ri}}{\mathbf{E}} + \left(\frac{\sigma_{ri}}{k'} \right)^{1/n'} \leftarrow \text{Ramberg-Osgood} \end{cases} \quad (21)$$

We must add the assumption that the total number of
cycles from distribution remains unchanged regardless of
the materials behavior [20]. The distribution is defined
for a given time interval. In this time interval, the cycles
number is given by the system frequency and not by the
strain or stress amplitudes. We work only on the alloca-
tion of the cycles number on the space of possible real
strain (Fig. 4). This means that the area under the curve
remains the same before and after correction, only the cy-
cles modification on the high strain modifies the damage
[21]. This process is integrated in the process of spectral
methods as shown in Figure 3. The assumptions of spec-
tral methods are respected since it applies in the linear
part of the process.

4 Tests and results

4.1 Specimen description

The specimen in Figure 5 is designed to meet the
assumption of the method described previously (con-
fined plasticity). It is made from quenched & tempered
30NiCrMo8 steel, whose general properties are summa-
rized in Table 1. The specimen consists of two parts (cylin-
drical portion and inertia block), crimped together by a
shrink fitting process to allow transfer of forces. In Fig-
ure 5c, we show in blue the clamping area of the specimen.
The flat part prevents any rotation of the specimen dur-
ing the test. The cylindrical portion of the clamping area
allows for a recessed well defined and reproducible on all
specimens. Table 2 gives the experimental fatigue data
for the laws of Basquin and Coffin-Manson-Basquin. The
experimental value for the Basquin slope k is 12.03. To
avoid dispersion due to choice of damage laws, we chose

Table 1. Mechanical properties of 30NICRMO8.

Young’s modulus	Poisson’s coefficient	Yield strength	Tensile strength
195 GPa	0.33	945 MPa	1135 MPa

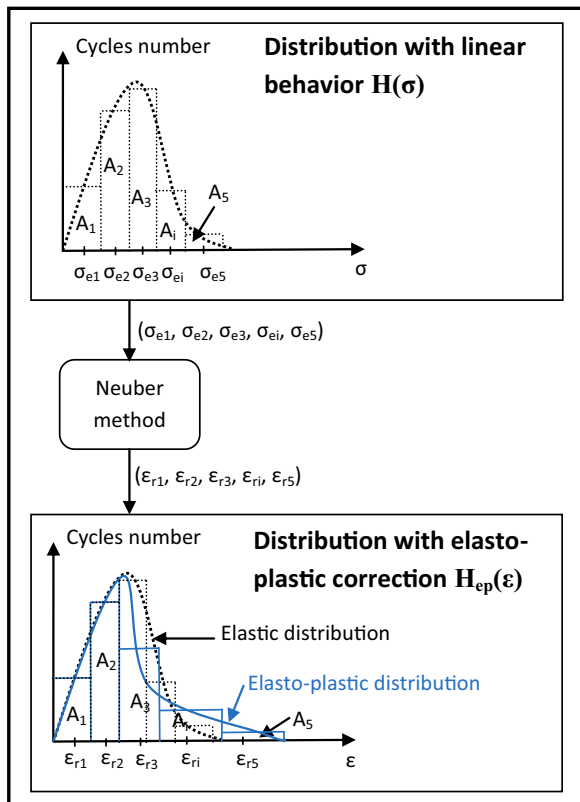


Fig. 4. Correction method for the distribution of cycles number in the spectral methods.

- 1 to adjust it to the slope of the elastic line from Basquin-
- 2 Manson-Coffin ($1/0.085 = 11.764$).

3 4.2 Test bench description

4 Each test is composed of three specimens (Fig. 6) to
 5 save time and set repetitive. The specimens are encased in
 6 a clamping system. The excitation system is a electrody-
 7 namic vibrator which is controlled by several parameters
 8 (RMS value, leveling the PSD). The tests are performed
 9 on the second eigenmode of the specimen in order to have
 10 a uniaxial loading. The PSD excitation is a PSD band
 11 90 Hz–120 Hz, centered around the eigenmode. Table 5
 12 in first column presents the different levels of PSD selected
 13 for our tests. We have 9 specimens for each level, requir-
 14 ing 3 tests. Accelerometers are placed on the specimen in
 15 order to detect crack initiation.

16 The lifetime is defined as the time between the start
 17 of the test and the initiation of the crack. The crack ini-
 18 tiation is detected by the variation of the frequency and
 19 amplitude of the eigenmode.

Table 2. Fatigue properties of 30NiCrMo8 steel.

S N parameter		E N parameter			
k	C	b	c	ϵ'_f	σ'_f (MPa)
11.764	$3.659 \times 10e36$	-0.085	-0.752	1.741	1750.000

4.3 Numerical model and modal analysis

20 To compare the experimental lifetimes for different nu-
 21 merical methods, we modeled the specimen using a finite
 22 element code (ABAQUS). This allows retrieving the stress
 23 frequency response function which is the input data for
 24 calculation methods in fatigue.
 25

26 Numerical model was refined thanks to correlations
 27 with experimental data (density, Young modulus and
 28 damping). Table 3 shows the experimental eigenfrequen-
 29 cies from average of measurements performed on 10 test
 30 pieces (shock hammer and shaking table), we can see that
 31 the gap with the frequency of the numerical model is
 32 less than 1% in the case of our study. Figure 7 shows
 33 each modal shape with excitation along \vec{Z} , we note that
 34 the second eigenmode is orthogonal to the first and third
 35 eigenmodes.

36 A convergence study of the mesh was performed in
 37 order to ensure good estimation of stresses and strains
 38 in the critical area. We refined the mesh until the values
 39 of stress and strain does not vary between two refined
 40 meshing. We obtained that 41% of the degrees of freedom
 41 are located in the notch so that it represents only 5% of
 42 the total volume of the specimen.

43 We also performed a damping factor analysis. This pa-
 44 rameter must be correctly quantified because it strongly
 45 affects the values of stress and strain. In our case, the
 46 damping is low, the slightest error in estimating it in-
 47 volves lifetimes completely wrong. To determine a damp-
 48 ing value consistent across all tests, we calculate the
 49 damping of each test during the first two hours of testing
 50 where the specimen is healthy. We calculate the damp-
 51 ing means on all tests. We chose the damping mean as
 52 damping value for our calculations. This value is 0.0016
 53 or 0.16% damping.

4.4 Results and observations

54 The presented results are in the context of an exci-
 55 tation of the second eigenmode which generates a nearly
 56 uniaxial response. Table 4 summarizes the mean and stan-
 57 dard deviation of the experimental life times by level of
 58 excitation. Means and standard deviations were calcu-
 59 lated on a set of 36 specimens (9 specimens per level).
 60 Data are calculated in seconds and hours. The last col-
 61 umn of the table indicates the ratio between mean and
 62

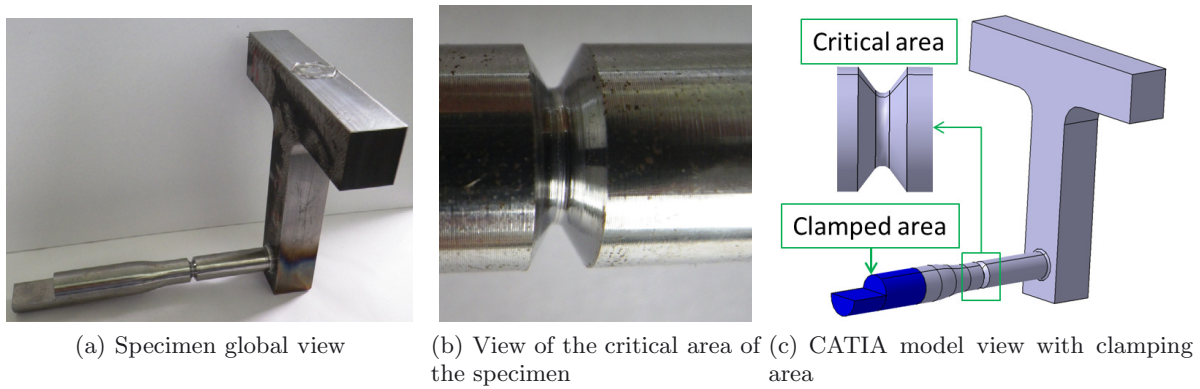


Fig. 5. Vibration fatigue test specimen.

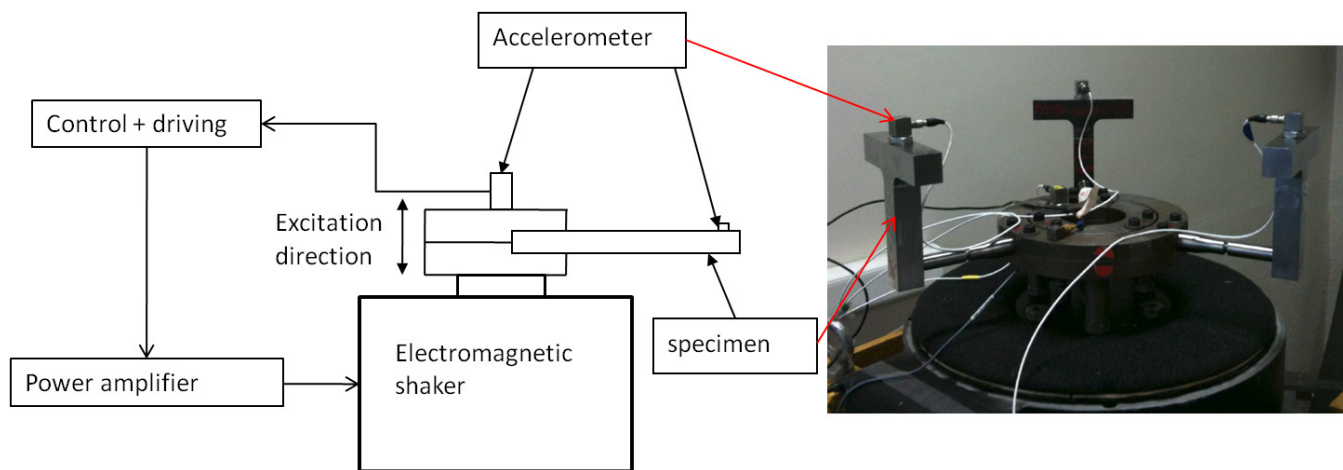


Fig. 6. Schema of test bench with the different elements that compose it and a picture of the test bench with specimens.

Table 3. Dynamic parameters (experimental and numerical data).

Eigenmode	Experimental eigenfrequency (Hz)	Numerical eigenfrequency (Hz)	Damping (%)	Loading type	Loading state (in critical area)
1st	94.5	94.2 (-0.31%)	0.11	Bending and Torsion	multiaxial
2nd	102.7	102.1 (-0.58%)	0.16	Bending	uniaxial
3rd	248.7	256.5 (+3.14%)	0.48	Torsion and Bending	multiaxial

Table 4. Experimental average lifetime by excitation level well as the associated standard deviation.

Level of PSD acceleration (g^2/Hz)	Lifetime (initiation)	Average	Standard deviation	Standard deviation/Average
0.0040	Hours	4.4	0.8	0.175
	Seconds	1.6+E04	2.8+E03	
0.0034	Hours	9.6	1.2	0.120
	Seconds	3.5E+04	4.2E+03	
0.0030	Heures	17.6	2.2	0.124
	Secondes	6.3E+04	7.8E+03	
0.0026	Heures	29.7	5.0	0.164
	Secondes	1.1E+05	1.8E+04	

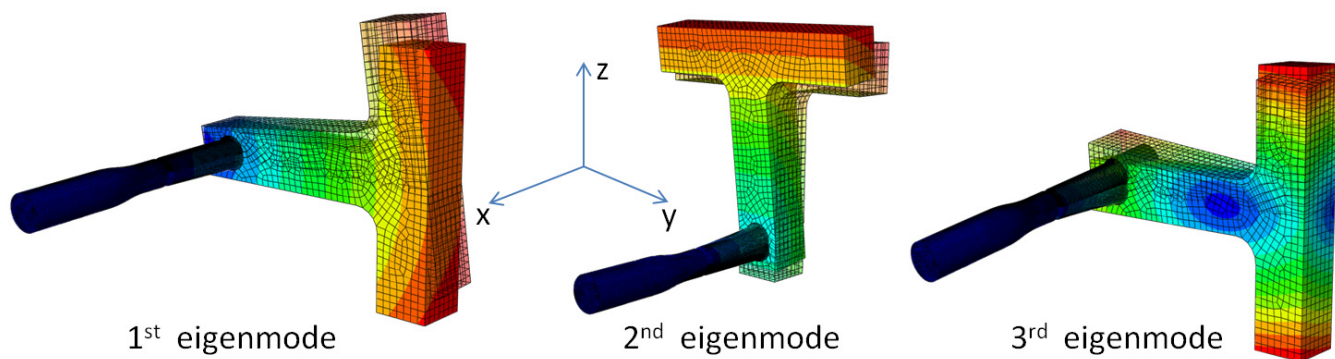


Fig. 7. The deformations of each eigenmode with in hard, the specimen at rest and in transparent, the specimen deformed.

Table 5. Comparaision between the experimental lifetimes and the lifetimes of the different methods. in parenthesis, the relative error from the experimental lifetime.

Excitation level (g ² /Hz)	Experi-mental life (s)	RMS (MPa)	Numerical data					
			SMM (s)		SM (s)		Hermite (HM) (s)	
			Dirlik	Lalanne	Dirlik	Lalanne	Dirlik	Lalanne
0.0040	1.6E+04	229	1.2E+04 (-25%)	1.4E+04	1.6E+02 (-99.1%)	1.1E+02	3.1E+03 (-80.6%)	2.9E+03
0.0034	3.5E+04	211	3.4E+04 (-2.9%)	3.2E+04	1.6E+03 (-95.9%)	1.5E+03	1.4E+04 (-60.0%)	3.5E+04
0.0030	6.3E+04	197	5.7E+04 (-9.5%)	5.4E+04	9.2E+03 (-85.3%)	8.7E+03	3.8E+04 (-39.6%)	6.3E+04
0.0026	1.1E+05	175	1.1E+05 (+3.6%)	1.0E+05	7.6E+04 (-30.9%)	7.2E+04	9.5E+05 (-13.6%)	1.1E+05

1 standard deviation. In our case, the ratio is quite low re-
 2 gardless of the level of excitation. It appears that there is
 3 no relationship between this ratio and the level of excita-
 4 tion. This gives a good level of confidence in the experi-
 5 mental results.

6 In Table 5, we compare experimental and numerical
 7 lifetimes for Lalanne and Dirlik models. In the second col-
 8 umn, we reported the RMS value obtained by calculating
 9 the PSD stress. This value is the same regardless of the
 10 method selected because this is an input data for fatigue
 11 calculation. Third column gives the experimental lifetime
 12 average per excitation level. The lifetime average per level
 13 is obtained from nine specimens of three tests. The other
 14 columns are the lifetimes obtained from the numerical
 15 methods. The average lifetimes are calculated from the
 16 probability density function (PDF) of Lalanne and Dirlik.
 17 In case of Dirlik PDF, in parentheses, the values of rela-
 18 tive errors compared to experimental lifetimes. The rela-
 19 tive error is defined in equation (22).

$$\text{Relative error(\%)} = \frac{\text{calculated lifetime} - \text{experimental lifetime}}{\text{experimental lifetime}} \times 100 \quad (22)$$

20 For the proposed model (SMM), the relative errors com-
 21 pared to experimental lifetimes are low; it shows a good
 22 accuracy of the method. In contrast for classical spectral
 23 methods (SM), the maximum relative error is 99.1%, it is
 24 obtained between the experimental lifetime and the life-
 25 time with classical spectral method at the level 1. The re-
 26 sult is consistent because the lower the calculated lifetime,
 27 the more elastoplastic the material behavior is. Since the
 28 spectral methods ignore the elasto-plastic behavior, this
 29 implies skewed results. Conversely, in the case of level 4,

the loads are small and the material works global in elas-
 30 tic behavior. In reference [22], for the steel 30NiCrMo8,
 31 an RMS value of less than 180 MPa involves elastic be-
 32 havior during the fatigue test. This explains good corre-
 33 lation between classical spectral methods predictions and
 34 experimental results.

35 Table 5 results are illustrated in Figure 8. As X-
 36 axis there are the lifetimes obtained with the different
 37 numerical methods. As Y-axis, there are the lifetimes ob-
 38 tained experimentally. A point on the graph form the
 39 combination of a life of a numerical method associated
 40 with the experimental life corresponding thereto. A point
 41 on the median line means a simulation fitting perfectly
 42 experimental data. This allows to see that the main in-
 43 fluence is due to material nonlinearity; the probability
 44 density function selected (Lalanne or Dirlik) has little ef-
 45 fect. This highlights that it is the material nonlinearity
 46 that are in default in classical spectral methods.

47 We also observe that the Hermite model associated
 48 with spectral methods tend to give better results at any
 49 level than classical spectral methods. Nevertheless, this
 50 model diverges when the nonlinearity increases and moves
 51 away from the experimental results. The proposed model
 52 gives a good estimate of the lifetime of the specimen at
 53 any level. This shows the influence of the elasto-plastic
 54 behavior on the fatigue life.

5 Conculsion and perspectives

56 The proposed method allows a better estimation of
 57 the lifetime by including the influence of the plastic be-
 58 havior of the material on the fatigue phenomenon. To
 59

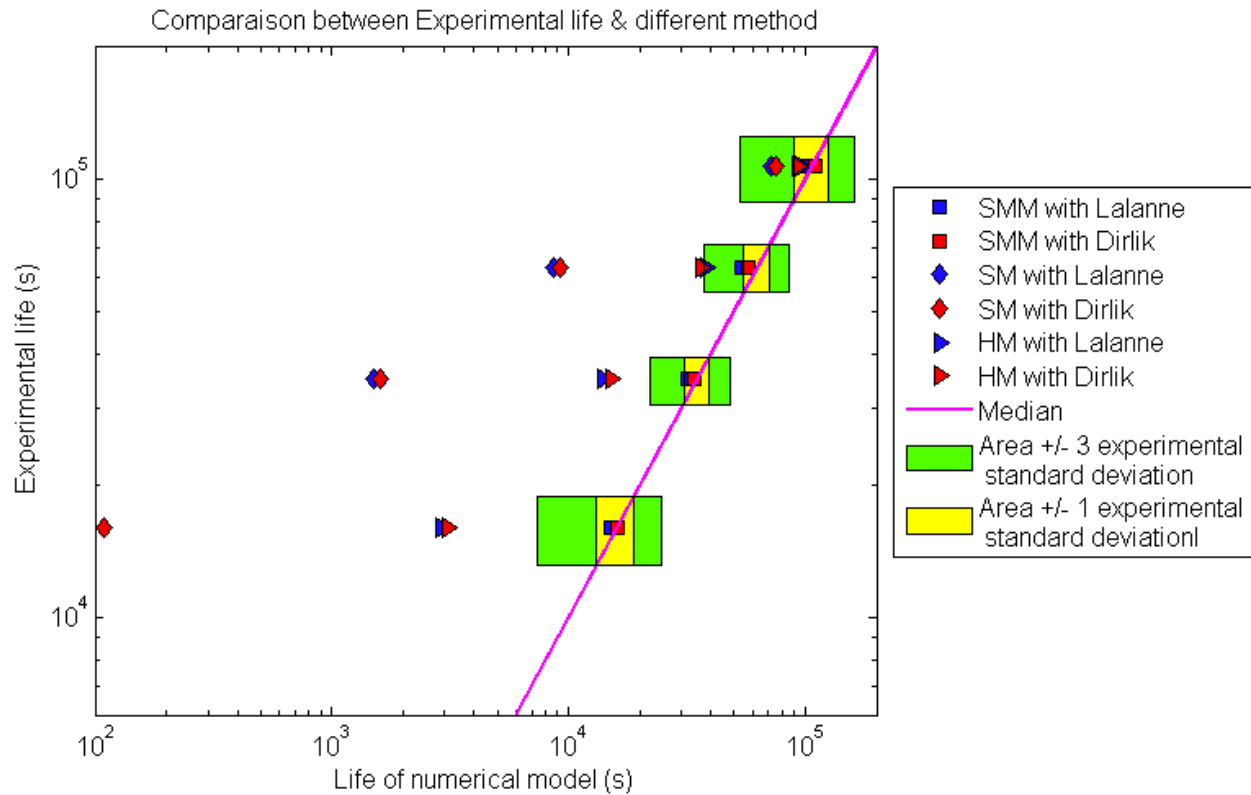


Fig. 8. Synthesis of different experimental and numerical lifetimes for the specimen excited on the second eigenmode.

1 consolidate the results obtained, other tests will be per-
 2 formed on different levels and different type of sollicitation.

3 We found that the Hermite model gives good re-
 4 sults but tend to diverge when the nonlinearity increased.
 5 There may be two reasons for this. The first is that the
 6 method does not model directly the nonlinearity. The sec-
 7 ond is that the fatigue law used is a S-N law that does
 8 not take into account the plasticity.

9 We have shown that if the classical spectral methods
 10 give incorrect results; this was due to the non-respect of
 11 assumptions, in our case, the linearity of the material be-
 12 havior.

13 For the following, we have several axes of
 14 improvement:

15 – We study the impact of experimental and numerical
 16 scatter on the different lifetimes results. This involves
 17 both the variability of experimental lifetimes and of
 18 numerical lifetimes. We also study the impact of the
 19 damping scatter on the calculated lifetimes.

20
 21 – We will study and model the inclusion of non re-
 22 versed strains (non-zero mean of the process studied).
 23 And, we will focus on the modeling of hysteresis phe-
 24 nomenon of material in the fatigue calculation.

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