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# A Noise-Robust Method with Smoothed $\ell_1/\ell_2$ Regularization for Sparse Moving-Source Mapping

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## Abstract

The method described here performs blind deconvolution of the beamforming output in the frequency domain. To provide accurate blind deconvolution, sparsity priors are introduced with a smoothed  $\ell_1/\ell_2$  regularization term. As the mean of the noise in the power spectrum domain depends on its variance in the time domain, the proposed method includes a variance estimation step, which allows more robust blind deconvolution. Validation of the method on both simulated and real data, and of its performance, are compared with two well-known methods from the literature: the deconvolution approach for the mapping of acoustic sources, and sound density modeling.

*Keywords:* Smoothed  $\ell_1/\ell_2$  regularization, Sparse representation, Proximal forward-backward, Acoustic moving-source localization, Beamforming blind deconvolution, Robustness algorithms.

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## 1. Introduction

Blind deconvolution has a central role in the field of signal and image processing. It has many applications in communications [1], nondestructive testing [2], image processing [3, 4, 5], medical imaging processing [6], and in acoustics [7]. Moreover, in underwater acoustics, blind deconvolution methods have already been proposed to estimate simultaneously the transfer function of the environment and the unknown source signal in multipath underwater sound channels [8, 9, 10]. In many realistic scenarios, the blurring kernel (or the system) is imprecise or not known. Thus, the deconvolution problem becomes blind and underdetermined, and often requires additional hypotheses.

One possible additional hypothesis is the sparsity of the signal, which is an extensively studied topic in signal processing. The main idea is to find the

most compact representation of a signal that consists of only a few nonzero elements. In acoustic signal processing, sparsity can be introduced either in the system or the signal domain (input). In the experimental context of this paper, the goal is to perform source mapping in a moving-source context<sup>1</sup>. The measurements are the pressures recorded on a horizontal line array during the pass-by experiment, and the signal of interest is the source locations inside the global vehicle. The positions of sources can be considered as sparsely distributed on a calculation grid. The question is then which measure can be used to evaluate the sparsity of a signal? In [11], Pereira used  $\ell_2^2$ -norm as a penalty to stabilize inverse problem solutions, which can be achieved using an adapted Tikhonov regularization method. However this penalty is not adapted for the considered case of sparse source positions. An  $\ell_1$ -norm is popular to restore the sparsity of the solution, as proposed in [12, 13]. However, in [14], Benichoux *et al.* showed that the use of the  $\ell_1$  norm suffers from

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<sup>1</sup>In this paper the terms "source localization" and "source mapping" are equivalent. These refer to the goal of the paper, which is to map noise sources inside a global vehicle (here a boat) during a pass-by experiment.

1 scaling and shift ambiguities due to the nonlinear  
2 relation between the blurring kernel and the signal,  
3 as also discussed in [15, 16]. Felix *et al.* extended  
4 this result for the case of  $\ell_p$ , ( $p < 1$ )-norm in [17].  
5 In particular, both of these articles showed that us-  
6 ing the  $\ell_1/\ell_2$  function can overcome this difficulty.  
7 However, the  $\ell_1/\ell_2$  function creates some difficulties  
8 when solving the nonconvex and nonsmooth mini-  
9 mization problems that prevent the use of such a  
10 penalty term in current restoration methods. In the  
11 present paper, we propose to use the smoothed  $\ell_1/\ell_2$   
12 ratio mentioned in [18] as a penalty. This penalty  
13 also overcomes the scaling and shift-ambiguity is-  
14 sues. Moreover, this ratio can be used to force the  
15 sparse representation of the signal in a blind de-  
16 convolution of moving-source mapping. Note that  
17 this is a different problem from classical blind de-  
18 convolution in underwater acoustics, as we consider  
19 several sources in a nonmultipath environment. The  
20 problem is presented in the power spectrum domain  
21 where the noise variance is a parameter that has  
22 to be estimated jointly with the autospectra and  
23 the blur. This fundamental problem was not taken  
24 into account with the original SOOT algorithm in  
25 [18], while it is explicitly dealt with in our proposed  
26 method.

27 This paper is organized as follows. Following this  
28 Introduction, Section 2 is devoted to a review of the  
29 related framework for moving-source mapping us-  
30 ing deconvolution. Section 3 presents the proposed  
31 forward model, and Section 4 describes the mini-  
32 mization problem, the proposed algorithm, and some  
33 mathematical tools that are essential to this method-  
34 ology. The performance of the proposed method  
35 is assessed in Section 5, where we detail the cho-  
36 sen optimization criteria and provide comparisons  
37 with two methods: the deconvolution approach for  
38 the mapping of acoustic sources (DAMAS-MS) and  
39 the sound density modeling (SDM) methods. The  
40 proposed methodology is first evaluated on realistic  
41 synthetic data, and then it is applied to real data  
42 recorded in Lake Castillon (Verdon Gorges, France).  
43 The conclusions and perspectives are drawn up in  
44 Section 6.

## 45 2. Related work

46 In this section, we briefly present the classical  
47 methods that have been developed for acoustic-  
48 source localization.

49 Many methods have been developed to solve this  
50 problem based on array processing. The most clas-

51 sical one is beamforming [19], which has been exten-  
52 sively used due to its robustness against noise and  
53 environmental mismatch. However, classical beam-  
54 forming cannot be used for pass-by experiments,  
55 where the 'vehicle' is moving and the goal is to map  
56 the different acoustic noise sources in the vehicle.  
57 Here instead, source mapping is achieved by the ex-  
58 tension to beamforming for moving sources (BF-MS)  
59 [20].

60 Nevertheless, the spatial resolution of BF-MS is  
61 limited, as the image of a point source is the array  
62 transfer function, which is comprised of a main lobe  
63 and secondary lobes. Consequently, many improve-  
64 ments have been proposed to overcome this problem,  
65 including a hardware strategy to reduce the side-  
66 lobe levels, where the resolution of the main lobes is  
67 through optimization of the antenna geometry. In  
68 particular, several optimizations of the sensor posi-  
69 tions of linear antennas have been proposed through  
70 the use of pseudo-random distributions [21, 22, 23].  
71 Furthermore, a numerical strategy classically uses  
72 the weighting coefficients, which shade the array  
73 aperture and thus taper the side lobes, and as a con-  
74 sequence, also enlarge the main lobe [24]. Another  
75 common approach is to use deconvolution methods.

76 Recently, Sijtsma proposed an extended version of  
77 the deconvolution method CLEAN [25, 26] for mov-  
78 ing sources, which is known as CLEAN-SC (*i.e.*,  
79 CLEAN based on spatial-source coherence) [27].  
80 This method follows an approach similar to the  
81 matching pursuit method [28]. CLEAN-SC pro-  
82 vides satisfactory results for high signal-to-noise  
83 ratios (SNRs), but requires *a-priori* knowledge of  
84 the number of sources which is not always known  
85 in practical cases. Brooks and Humphreys devel-  
86 oped another approach, known as DAMAS [29], and  
87 its extensions [20, 30]. These algorithms use the  
88 iterative Gauss-Seidel method for solving the linear  
89 inverse problem under the nonnegative constraint  
90 on source powers. A particular extension was dedi-  
91 cated to moving sources, as DAMAS-MS [20], which  
92 improves moving-source mapping in the context of  
93 a high SNR. Another popular method that was also  
94 developed for moving sources is the SDM method  
95 of [31], which is based on a gradient-descent opti-  
96 mization technique. This represented the first use  
97 of optimization techniques with a noise prior and  
98 constraints on the signal. These two methods (*i.e.*,  
99 DAMAS-MS and SDM) will be used as the refer-  
100 ences for comparison with our proposed method.

101 In the case of low SNRs, the problem is difficult  
102 to solve, and thus some other approaches need to

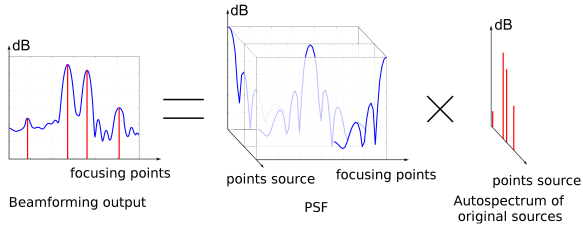


Figure 1: Modeling of the forward problem

be developed. In array processing, Swindlehurst and Kailath [32] proposed a first-order perturbation analysis of the multiple signal classification (MUSIC) and root-MUSIC algorithms for various model errors. Another possibility is to include a regularization term to stabilize the solution. The Tikhonov regularization is applied to jet noise-source localization [33]. The sparse distribution of sources is also commonly used, as in [12, 34, 35, 13, 36, 37, 38]. These methods were developed for fixed-source localization and have not currently been extended to moving sources.

The goal of the present paper is to propose a new blind deconvolution method that is applied to BF-MS results to improve moving-source mapping.

The strategy is to formulate the forward problem as an optimization problem, with constraints that are derived from the physical context. The proposed cost function contains several parts: (i) a data-fidelity term that accounts for the noise characteristics; (ii) the smoothed  $\ell_1/\ell_2$  ratio [18] that promotes sparsity in the moving-source locations; and (iii) the knowledge of some physical properties of the sources and the system, and of the variance noise, introduced through indicator functions.

### 3. Observation model

#### 3.1. Beamforming for moving sources

Beamforming for moving sources compensates for the Doppler effect and back-propagates the pressures measured by the  $M$  sensor array to a calculation grid of  $N$  points, which correspond to the possible source locations. We consider the classical case of pass-by experiments, in the far-field, with sources that share the same global movement and have low Mach numbers,  $\|\overline{Ma}\| \ll 1$ . For these conditions, some assumptions can be made over short time intervals of duration  $T$ , which are referred to as *snapshots* [20], whereby:

- 1) The sources are in fixed positions;
- 2) The Doppler effect is negligible at the frequencies and speeds of interest (*i.e.*, it does not exceed the frequency resolution defined for the localization results).

Under these assumptions, BF-MS can be implemented in a simple way in the frequency domain. The measured acoustic pressures are temporally sliced into  $K$  snapshots that are indexed by  $k$ . The calculation grid of  $N$  points is defined for the snapshot  $k$  using the *a-priori* known global trajectory of the vehicle. Note that this grid moves according to this trajectory.

For the snapshot  $k$ , the pressures measured by these  $M$  sensors at time  $t \in [1, T]$  are denoted as  $\check{\mathbf{p}}_t^k \in \mathbb{R}^M$ , which can be divided into two parts:

$$\check{\mathbf{p}}_t^k = \check{\overline{\mathbf{p}}}_t^k + \check{\mathbf{r}}_t^k \quad (1)$$

in which  $\check{\overline{\mathbf{p}}}_t^k$  are the pressures measured by the  $M$  sensors at time  $t$  for the ideal case without noise, and  $\check{\mathbf{r}}_t^k$  is an additive noise in the recording domain. This defines the vectors:

$$\begin{aligned} \check{\mathbf{P}}^k &= [(\check{\mathbf{p}}_1^k)^\top, (\check{\mathbf{p}}_2^k)^\top, \dots, (\check{\mathbf{p}}_T^k)^\top]^\top \in \mathbb{R}^{MT}, \\ \check{\overline{\mathbf{P}}}^k &= [(\check{\overline{\mathbf{p}}}_1^k)^\top, (\check{\overline{\mathbf{p}}}_2^k)^\top, \dots, (\check{\overline{\mathbf{p}}}_T^k)^\top]^\top \in \mathbb{R}^{MT}, \\ \check{\mathbf{R}}^k &= [(\check{\mathbf{r}}_1^k)^\top, (\check{\mathbf{r}}_2^k)^\top, \dots, (\check{\mathbf{r}}_T^k)^\top]^\top \in \mathbb{R}^{MT}, \end{aligned}$$

where  $\cdot^\top$  denotes the transpose.

We now consider the pressures measured in the frequency domain between  $\zeta_1$  and  $\zeta_F$  Hz, which is related to a vector  $\mathcal{F} = [\zeta_1, \dots, \zeta_F] \in \mathbb{R}^F$ . We define the Fourier transforms of  $\check{\mathbf{P}}^k$ ,  $\check{\overline{\mathbf{P}}}^k$  and  $\check{\mathbf{R}}^k$  for each snapshot  $k$ , as following:

$$\begin{aligned} \mathbf{P}^k &= [(\mathbf{p}_1^k)^\top, (\mathbf{p}_2^k)^\top, \dots, (\mathbf{p}_F^k)^\top]^\top \in \mathbb{C}^{MF} \\ \overline{\mathbf{P}}^k &= [(\overline{\mathbf{p}}_1^k)^\top, (\overline{\mathbf{p}}_2^k)^\top, \dots, (\overline{\mathbf{p}}_F^k)^\top]^\top \in \mathbb{C}^{MF}, \\ \mathbf{R}^k &= [(\mathbf{r}_1^k)^\top, (\mathbf{r}_2^k)^\top, \dots, (\mathbf{r}_F^k)^\top]^\top \in \mathbb{C}^{MF} \end{aligned}$$

where for every  $f \in \{1, \dots, F\}$ ,  $\mathbf{p}_f^k$ ,  $\overline{\mathbf{p}}_f^k$ , and  $\mathbf{r}_f^k$  are the vectors that contain the Fourier transform coefficients  $p_f^k(m)$ ,  $\overline{p}_f^k(m)$ , and  $r_f^k(m)$  of the vectors  $[\check{\mathbf{p}}_1^k(m), \check{\mathbf{p}}_2^k(m), \dots, \check{\mathbf{p}}_T^k(m)]^\top$ ,  $[\check{\overline{\mathbf{p}}}_1^k(m), \check{\overline{\mathbf{p}}}_2^k(m), \dots, \check{\overline{\mathbf{p}}}_T^k(m)]^\top$ , and  $[\check{\mathbf{r}}_1^k(m), \check{\mathbf{r}}_2^k(m), \dots, \check{\mathbf{r}}_T^k(m)]^\top$  at the frequency  $\zeta_f \in \mathcal{F}$  ( $\zeta_f$  is the  $f^{\text{th}}$  element of vector  $\mathcal{F}$ ), for

1 every  $m \in \{1, \dots, M\}$ , respectively.

2 The BF-MS computed for the  $n^{\text{th}}$  calculation  
3 point at the frequency  $\zeta_f \in \mathcal{F}$ , and for the snapshot  
4  $k$ ,  $b_f^k(n)$ , is given by:

$$b_f^k(n) = |(\mathbf{w}_{f,n}^k)^H \mathbf{p}_f^k|^2 \quad (2)$$

5 where  $\cdot^H$  is the conjugate transpose, and  $\mathbf{w}_{f,n}^k$  is the  
6 steering vector of length  $M$  between the  $M$  sensors  
7 and the  $n^{\text{th}}$  calculation point. The  $m^{\text{th}}$  element  
8  $w_{f,n}^k(m)$  of  $\mathbf{w}_{f,n}^k$  is:

$$w_{f,n}^k(m) = \left( \sum_{m'=1}^M \left( \frac{1}{d_{n,m'}^k} \right)^2 \right)^{-1} \frac{\exp(-j\zeta_f d_{n,m}^k)}{d_{n,m}^k} \quad (3)$$

9 where  $j$  is the square root of -1, and  $d_{n,m}^k$  is the  
10 distance between the  $m^{\text{th}}$  sensor and the  $n^{\text{th}}$  calculation  
11 point during the snapshot  $k$ . We then define  
12 the vector  $\mathbf{b}_f \in \mathbb{R}^N$  with its  $n^{\text{th}}$  element  $b_f(n)$  as  
13 the estimate of the BF-MS output for the  $n^{\text{th}}$  calculation  
14 point, through averaging over all of the  $K$   
15 snapshots; *i.e.*,

$$b_f(n) = \frac{1}{K} \sum_{k=1}^K b_f^k(n). \quad (4)$$

16 Note that in the case considered, the receiver array  
17 is a linear array along the  $x$ -axis. Consequently,  
18 BF-MS is performed along the  $x$  dimension, and the  
19 calculation grid is a one-dimension vector of length  
20  $N$  along  $x$ . For more details on these computations,  
21 we refer the reader to [39, 40].

### 22 3.2. Inverse problem formulation

23 We set the following assumptions:

24 (H1) : The sources are random variables that are  
25 mutually independent and stationary;

26 (H2) : The number  $M$  of the sensors is greater than  
27 the number  $N_s$  of the sources (*i.e.*,  $M > N_s$ ),  
28 and these  $N_s$  sources are sparsely distributed  
29 on the calculation grid;

30 (H3) : The noise components are mutually independent,  
31 and independent of the sources.

32 Using the expression of the BF-MS, and assuming  
33 that the sources are located at the  $N$  points of the  
34 calculation grid, it is possible to express the BF-MS  
35 output at a given frequency  $\zeta_f$ ,  $\mathbf{b}_f \in \mathbb{R}^N$ , by:

$$\mathbf{b}_f = \mathbf{A}_f \bar{\mathbf{q}}_f + \mathbf{z}_f \quad (5)$$

36 where  $\mathbf{A}_f \in \mathbb{R}^{N \times N}$  (Fig. 1, middle) is the array  
37 transfer function matrix that contains the beam-  
38 forming point-spread functions. The  $(n, n') \in$   
39  $\{1, \dots, N\}^2$  element  $a_f(n, n')$  of  $\mathbf{A}_f$  is:

$$a_f(n, n') = \frac{1}{K} \sum_{k=1}^K \left| \sum_{m=1}^M (w_{f,n}^k(m))^H \frac{\exp(-j\zeta_f d_{n',m}^k)}{d_{n',m}^k} \right|^2 \quad (6)$$

40  $\mathbf{z}_f \in \mathbb{R}^N$  is the measurement noise, and  $\bar{\mathbf{q}}_f \in \mathbb{R}^N$  is  
41 the autospectra of the possible sources located at  
42 the  $N$  calculation points (Fig. 1, right), which are  
43 the unknowns to be estimated. This expression is  
44 frequently used in deconvolution [20, 27, 31, 41], although  
45 it needs the knowledge of matrix  $\mathbf{A}_f$  related to  
46 the environment and to the array to perform the  
47 deconvolution. Nowadays, some research projects  
48 are focused on uncertain cases with partially known  
49 or unknown ocean environments and experimental  
50 configurations [42]. Consequently, when  $\mathbf{A}_f$  is  
51 unknown, we propose in this paper to formulate  
52 the BF-MS output at the frequency  $\zeta_f$  as a blind  
53 deconvolution problem:

$$\mathbf{b}_f = \bar{\mathbf{h}}_f * \bar{\mathbf{q}}_f + \mathbf{z}_f \quad (7)$$

54 where  $\bar{\mathbf{h}}_f \in \mathbb{R}^N$  is an unknown blur kernel, which  
55 needs to be estimated, as well as the autospectra  
56 of the sources.  $*$  is a discrete-time convolution  
57 operator (with appropriate boundary processing).  
58 We now turn our attention to the term  $\mathbf{z}_f$ , which  
59 corresponds to the additional noise. In the literature,  
60 several methods have been proposed with  $\mathbf{z}_f$  as a  
61 Gaussian noise with zero mean (which is not adapted  
62 to the BF-MS signal). We propose to introduce the  
63 noise in the time recording domain and to model  
64 its transformation through BF-MS. In acoustics,  
65 the noise components  $\check{\mathbf{r}}_{m,t}^k$  in the time domain can  
66 commonly be considered to be Gaussian, with zero  
67 mean and variance  $\bar{\sigma}^2$ . From Equations (1) and (2),  
68 and assumptions (H1) - (H3), we have:

$$b_f(n) = \frac{1}{K} \sum_{k=1}^K \left( |(\mathbf{w}_{f,n}^k)^H \bar{\mathbf{p}}_f^k|^2 + |(\mathbf{w}_{f,n}^k)^H \mathbf{r}_f^k|^2 \right) \quad (8)$$

69 Using Equation (8), we assume in this paper that  
70 the observation noise  $\mathbf{z}_f$  can be divided into two  
71 terms:

$$\mathbf{z}_f = \frac{1}{K} \left( \sum_{k=1}^K \|\mathbf{w}_{f,n}^k\|^2 \right) \bar{\sigma}^2 \mathbf{1}_N + \mathbf{e}_f \quad (9)$$

1 where  $\|\cdot\|$  is the  $\ell_2$ -norm (which is also known as  
2 the Euclidean norm),  $\mathbf{1}_N$  is a vector of ones of length  
3  $N$ , and  $\mathbf{e}_f \in \mathbb{R}^N$  represents the remaining unknown  
4 effects, where the amplitude of these remaining ef-  
5 fects is much lower than that of the variance  $\bar{\sigma}^2$  of  
6 the Gaussian noise. Note that:

$$\|\mathbf{w}_{f,n}^k\|^2 = \left( \sum_{m=1}^M \left( \frac{1}{d_{n,m}^k} \right)^2 \right)^{-1}$$

7 Consequently, Equation (7) can be expressed as the  
8 following nonlinear problem in the standard form:

$$\mathbf{B} = \bar{\mathbf{H}} \circledast \bar{\mathbf{Q}} + \bar{\sigma}^2 \delta \mathbf{1}_{NF} + \mathbf{E} \quad (10)$$

9 where

$$\begin{aligned} \mathbf{B} &= [\mathbf{b}_1^\top, \mathbf{b}_2^\top, \dots, \mathbf{b}_F^\top]^\top \in \mathbb{R}^{NF}, \\ \bar{\mathbf{H}} &= [\bar{\mathbf{h}}_1^\top, \bar{\mathbf{h}}_2^\top, \dots, \bar{\mathbf{h}}_F^\top]^\top \in \mathbb{R}^{NF}, \\ \bar{\mathbf{Q}} &= [\bar{\mathbf{q}}_1^\top, \bar{\mathbf{q}}_2^\top, \dots, \bar{\mathbf{q}}_F^\top]^\top \in \mathbb{R}^{NF}, \\ \delta &= \frac{1}{K} \sum_{k=1}^K \left( \sum_{m=1}^M \left( \frac{1}{d_{n,m}^k} \right)^2 \right)^{-1}, \\ \mathbf{E} &= [\mathbf{e}_1^\top, \mathbf{e}_2^\top, \dots, \mathbf{e}_F^\top]^\top \in \mathbb{R}^{NF}, \end{aligned}$$

10 and the discrete-time convolution operator  $\circledast$  be-  
11 tween  $\bar{\mathbf{H}}$  and  $\bar{\mathbf{Q}}$  is defined as follows:

$$\bar{\mathbf{H}} \circledast \bar{\mathbf{Q}} = [(\bar{\mathbf{h}}_1 * \bar{\mathbf{q}}_1)^\top, (\bar{\mathbf{h}}_2 * \bar{\mathbf{q}}_2)^\top, \dots, (\bar{\mathbf{h}}_F * \bar{\mathbf{q}}_F)^\top]^\top.$$

## 12 4. Proposed method

### 13 4.1. Criterion to be minimized

14 The purpose of this study is to identify  $(\bar{\mathbf{H}}, \bar{\mathbf{Q}}, \bar{\sigma}^2)$   
15 from  $\mathbf{B}$  through Equation (10), which leads to an  
16 inverse problem. To solve this, we propose the  
17 following optimization problem:

$$\text{Find } (\hat{\mathbf{H}}, \hat{\mathbf{Q}}, \hat{\sigma}^2) \in \underset{\mathbf{H} \in \mathbb{R}^{2NF}, \mathbf{Q} \in \mathbb{R}^{2NF}, \sigma^2 \in \mathbb{R}_+}{\text{argmin}} \theta(\mathbf{H}, \mathbf{Q}, \sigma^2) \quad (11)$$

18 where:

$$\theta(\mathbf{H}, \mathbf{Q}, \sigma^2) = \psi(\mathbf{H}, \mathbf{Q}, \sigma^2) + \rho(\mathbf{H}, \mathbf{Q}, \sigma^2). \quad (12)$$

19 The first term of Equation (12) can be split into two  
20 new terms

$$\psi(\mathbf{H}, \mathbf{Q}, \sigma^2) = \phi(\mathbf{H}, \mathbf{Q}, \sigma^2) + \varphi(\mathbf{Q}), \quad (13)$$

21 where  $\phi : \mathbb{R}^{NF} \times \mathbb{R}^{NF} \times \mathbb{R}_+ \rightarrow \mathbb{R}$  is a data fidelity  
22 term that is related to the observation model. In this  
23 case, we choose the least-squares objective function,  
24 *i.e.*,

$$\phi(\mathbf{H}, \mathbf{Q}, \sigma^2) = \frac{1}{2} \|\mathbf{H} \circledast \mathbf{Q} + \sigma^2 \delta \mathbf{1}_{NF} - \mathbf{B}\|^2. \quad (14)$$

25  $\varphi$  models a regularization function that accounts for  
26 the sparsity of the solution. In the present paper,  
27 we propose to use a new regularization function, the  
28 smoothed  $\ell_1/\ell_2$  ratio, as proposed by [18]; *i.e.*, for  
29 every  $\mathbf{Q} \in \mathbb{R}^{NF}$ ,  $(\lambda, \alpha, \beta, \eta) \in ]0, +\infty[^4$ :

$$\varphi(\mathbf{Q}) = \lambda \log \left( \frac{\ell_{1,\alpha}(\mathbf{Q}) + \beta}{\ell_{2,\eta}(\mathbf{Q})} \right) \quad (15)$$

30 with,

$$\begin{aligned} \ell_{1,\alpha}(\mathbf{Q}) &= \sum_{f=1}^F \sum_{n=1}^N \left( \sqrt{q_f(n)^2 + \alpha^2} - \alpha \right), \\ \ell_{2,\eta}(\mathbf{Q}) &= \sqrt{\sum_{f=1}^F \sum_{n=1}^N q_f(n)^2 + \eta^2}. \end{aligned}$$

31 Note that empirically, the SOOT algorithm provides  
32 better results if the condition  $\beta < \eta^2/\alpha$  is satisfied.  
33 The second term of Equation (12),  $\rho : \mathbb{R}^{NF} \times \mathbb{R}^{NF} \times$   
34  $\mathbb{R}_+ \rightarrow \mathbb{R}$  is a regularization term that is related to  
35 some *a-priori* constraints on the solution. In the  
36 following, we assume that  $\rho$  can be split into three  
37 new terms that concern the three quantities to be  
38 estimated:

$$\rho(\mathbf{H}, \mathbf{Q}, \sigma^2) = \rho_1(\mathbf{H}) + \rho_2(\mathbf{Q}) + \rho_3(\sigma^2)$$

39 where  $\rho_1$ ,  $\rho_2$  and  $\rho_3$  are (not necessarily smooth)  
40 proper, lower semicontinuous, convex functions [49,  
41 Ch. 1], that are continuous on their domain, and  
42 which introduce the prior knowledge on the kernel  
43 blur (system),  $\mathbf{H}$ , the source autospectra,  $\mathbf{Q}$ , and  
44 the noise variance,  $\sigma^2$ . Due to these properties, the  
45 problem can be addressed with the block coordi-  
46 nate variable metric forward-backward algorithm  
47 [44]. Moreover, in practice,  $\mathbf{H}$ ,  $\mathbf{Q}$  and  $\sigma^2$  have dif-  
48 ferent properties, and this choice allows the *a-priori*  
49 information to be taken into account independently  
50 from the searched quantities.

### 51 4.2. Proposed algorithm

52 The objective here is to provide a numerical solu-  
53 tion to the optimization problem of Equation (12),

1 which is a nonlinear blind deconvolution with three  
2 unknowns  $(\mathbf{H}, \mathbf{Q}, \sigma^2)$ . One class of popular solutions  
3 to solve this problem is the alternating minimiza-  
4 tion algorithm, which iteratively performs the three  
5 steps: (i) updating  $\mathbf{H}$  given  $\mathbf{Q}$  and  $\sigma^2$ ; (ii) updating  
6  $\mathbf{Q}$  given  $\mathbf{H}$  and  $\sigma^2$ ; and (iii) updating  $\sigma^2$  given  $\mathbf{H}$   
7 and  $\mathbf{Q}$  [43]. Furthermore, the criterion to minimize,  
8 which is formed as the sum of the smooth and non-  
9 smooth functions, can be addressed with a block  
10 alternating forward-backward method [44, 45]. This  
11 method combines explicitly the (forward) gradient  
12 step with respect to the smooth (not necessarily  
13 convex) functions and the proximal (backward) step  
14 with respect to the nonsmooth functions. The con-  
15 vergence of the algorithm can be accelerated using a  
16 majorize-minimize approach [46, 44, 47]. In this pa-  
17 per, we extend the smoothed one-over-two (SOOT)  
18 algorithm proposed in [18] by including a step for  
19 the noise variance estimation. This algorithm of  
20 noise-robust SOOT (NR-SOOT) is proposed, as pre-  
21 sented in Algorithm 1. As previously mentioned, the  
22 block-variable metric forward-backward algorithm  
23 combines two steps of the process that requires two  
24 optimization principles. We now recall the defini-  
25 tion of these: The first is related to the choice of a  
26 variable metric that relies upon the majorization-  
27 minimization properties [47]; *i.e.*,

28 **Definition 1** Let  $\psi : \mathbb{R}^N \rightarrow \mathbb{R}$  be a differentiable  
29 function. Let  $x \in \mathbb{R}^N$ . Let us define, for every  
30  $x' \in \mathbb{R}^N$ :

$$\varrho(x', x) = \psi(x) + (x-x')^\top \nabla \psi(x) + \frac{1}{2}(x-x')^\top U(x)(x-x'),$$

31 where  $U(x) \in \mathbb{R}^{N \times N}$  is a semidefinite positive ma-  
32 trix. Then,  $U(x)$  satisfies the majoration condi-  
33 tion for  $\psi$  at  $x$  if  $\varrho(\cdot, x)$  is a quadratic majorant  
34 of the function  $\psi$  at  $x$ ; *i.e.*, for every  $x' \in \mathbb{R}^N$ ,  
35  $\psi(x') \leq \varrho(x', x)$ .

36 A function  $\psi$  has a  $\mu$ -Lipschitzian gradient on a  
37 convex subset  $C \in \mathbb{R}^N$ , with  $\mu > 0$ , if for every  
38  $(x, x') \in C^2$ ,  $\|\nabla \psi(x) - \nabla \psi(x')\| \leq \mu \|x - x'\|$ . Then,  
39 for every  $\mathbf{x} \in C$ , a quadratic majorant of  $\psi$  at  $\mathbf{x}$  is  
40 easily obtained taking  $U(\mathbf{x}) = \mu \mathbf{I}_N$ , where  $\mathbf{I}_N$  is the  
41 identity matrix of  $\mathbb{R}^{N \times N}$ .

42 The second optimization principle is the definition  
43 of the proximity operator of a proper, lower semi-  
44 continuous, convex function, relative to the metric  
45 induced by a symmetric positive definite matrix,  
46 which is defined in [48] as follows:

47 **Definition 2** Let  $\rho : \mathbb{R}^N \rightarrow ]-\infty, +\infty]$  be a proper,  
48 lower semicontinuous, and convex function, let  
49  $U \in \mathbb{R}^{N \times N}$  be a symmetric positive definite ma-  
50 trix, and let  $x \in \mathbb{R}^N$ . The proximity operator of  
51  $\rho$  at  $x$  relative to the metric induced by  $U$  is the  
52 unique minimizer of  $\rho + \frac{1}{2}(\cdot - x)^\top U(\cdot - x)$ , and it  
53 is denoted by  $\text{prox}_{U, \rho}(x)$ . If  $U$  is equal to  $\mathbf{I}_N$ , then  
54  $\text{prox}_\rho := \text{prox}_{\mathbf{I}_N, \rho}$  is the proximity operator origi-  
55 nally defined in [50].

The convergence property of the NR-SOOT al-  
56 gorithm can be derived from the general results  
57 established in [44]:

59 **Proposition 1** Let  $(\mathbf{Q}^l)_{l \in \mathbb{N}}$ ,  $(\mathbf{H}^l)_{l \in \mathbb{N}}$  and  
60  $(\sigma^{2,l})_{l \in \mathbb{N}}$  be sequences generated by Algorithm  
61 1. Assume that:

- 62 1. There exists  $(\underline{\nu}, \bar{\nu}) \in ]0, +\infty[^2$  such that, for all  
63  $l \in \mathbb{N}$ ,

$$(\forall i \in \{0, \dots, I_l - 1\})$$

$$\underline{\nu} \mathbf{I}_{NF} \preceq G_1(\mathbf{H}^{l,i}, \mathbf{Q}^l, \sigma^{2,l}) \preceq \bar{\nu} \mathbf{I}_{NF},$$

$$(\forall j \in \{0, \dots, J_l - 1\})$$

$$\underline{\nu} \mathbf{I}_{NF} \preceq G_2(\mathbf{H}^{l+1}, \mathbf{Q}^{l,j}, \sigma^{2,l}) \preceq \bar{\nu} \mathbf{I}_{NF},$$

$$\underline{\nu} \preceq G_3(\mathbf{H}^{l+1}, \mathbf{Q}^{l+1}, \sigma^{2,l}) \preceq \bar{\nu}.$$

- 64 2. Step-sizes  $(\gamma_1^{l,i})_{l \in \mathbb{N}, 0 \leq i \leq I_l - 1}$ ,  $(\gamma_2^{l,j})_{l \in \mathbb{N}, 0 \leq j \leq J_l - 1}$   
65 and  $(\gamma_3^l)_{l \in \mathbb{N}}$  are chosen in the interval  $[\underline{\gamma}, 2 -$   
66  $\bar{\gamma}]$  where  $\underline{\gamma}$  and  $\bar{\gamma}$  are some given positive real  
67 constants.

- 68 3.  $\rho$  is a semi-algebraic function.<sup>2</sup>

69 Then, the sequence  $(\mathbf{H}^l, \mathbf{Q}^l, \sigma^{2,l})_{l \in \mathbb{N}}$  converges to the  
70 critical point  $(\hat{\mathbf{H}}, \hat{\mathbf{Q}}, \hat{\sigma}^2)$  of Equation (11). Moreover,  
71  $(\theta(\mathbf{H}^l, \mathbf{Q}^l, \sigma^{2,l}))_{l \in \mathbb{N}}$  is a nonincreasing sequence that  
72 converges to  $\theta(\hat{\mathbf{H}}, \hat{\mathbf{Q}}, \hat{\sigma}^2)$ .

73 In NR-SOOT algorithm 1,  $\nabla_1$ ,  $\nabla_2$ , and  $\nabla_3$  are the  
74 partial gradients of  $\psi$  with respect to the variables  
75  $\mathbf{H}$ ,  $\mathbf{Q}$ , and  $\sigma^2$ .  $G_1$ ,  $G_2$ , and  $G_3$  are the semidefi-  
76 nite positive matrix used to build the majorizing  
77 approximations of  $\psi$  with respect to  $\mathbf{H}$ ,  $\mathbf{Q}$ , and  $\sigma^2$ ,  
78 and their expressions are given by the following  
79 proposition, as established in [18]:

<sup>2</sup>Semi-algebraicity is a property satisfied by a wide class  
of functions, which means that their graph is a finite union  
of sets defined by a finite number of polynomial inequalities.

---

**Algorithm 1** The NR-SOOT algorithm.

---

For every  $l \in \mathbb{N}$ , let  $I_l \in \mathbb{N}^*$ ,  $J_l \in \mathbb{N}^*$ . Let  $(\gamma_1^{l,i})_{0 \leq i \leq I_l - 1}$ ,  $(\gamma_2^{l,j})_{0 \leq j \leq J_l - 1}$ , and  $\gamma_3^l$  be positive sequences. Initialize with  $\mathbf{H}^0 \in \text{dom}(\rho_1)$ ,  $\mathbf{Q}^0 \in \text{dom}(\rho_2)$ , and  $\sigma^{2,0} \in \text{dom}(\rho_3)$ .

**Iterations:**

For  $l = 0, 1, \dots$

$$\begin{array}{l}
 \mathbf{Q}^{l,0} = \mathbf{Q}^l, \mathbf{H}^{l,0} = \mathbf{H}^l, \\
 \text{For } i = 0, \dots, I_l - 1 \\
 \quad \tilde{\mathbf{H}}^{l,i+1} = \mathbf{H}^{l,i} - \gamma_1^{l,i} G_1(\mathbf{H}^{l,i}, \mathbf{Q}^l, \sigma^{2,l})^{-1} \nabla_1 \psi(\mathbf{H}^{l,i}, \mathbf{Q}^l, \sigma^{2,l}) \\
 \quad \mathbf{H}^{l,i+1} = \text{prox}_{(\gamma_1^{l,i})^{-1} G_1(\mathbf{H}^{l,i}, \mathbf{Q}^l, \sigma^{2,l}), \rho_1}(\tilde{\mathbf{H}}^{l,i+1}) \\
 \mathbf{H}^{l+1} = \mathbf{H}^{l,I_l} \\
 \text{For } j = 0, \dots, J_l - 1 \\
 \quad \tilde{\mathbf{Q}}^{l,j+1} = \mathbf{Q}^{l,j} - \gamma_2^{l,j} G_2(\mathbf{H}^{l+1}, \mathbf{Q}^{l,j}, \sigma^{2,l})^{-1} \nabla_2 \psi(\mathbf{H}^{l+1}, \mathbf{Q}^{l,j}, \sigma^{2,l}) \\
 \quad \mathbf{Q}^{l,j+1} = \text{prox}_{(\gamma_2^{l,j})^{-1} G_2(\mathbf{H}^{l+1}, \mathbf{Q}^{l,j}, \sigma^{2,l}), \rho_2}(\tilde{\mathbf{Q}}^{l,j+1}) \\
 \mathbf{Q}^{l+1} = \mathbf{Q}^{l,J_l} \\
 \tilde{\sigma}^{2,l} = \sigma^{2,l} - \gamma_3^l G_3(\mathbf{H}^{l+1}, \mathbf{Q}^{l+1}, \sigma^{2,l})^{-1} \nabla_3 \psi(\mathbf{H}^{l+1}, \mathbf{Q}^{l+1}, \sigma^{2,l}) \\
 \sigma^{2,l+1} = \text{prox}_{(\gamma_3^l)^{-1} G_3(\mathbf{H}^{l+1}, \mathbf{Q}^{l+1}, \sigma^{2,l}), \rho_3}(\tilde{\sigma}^{2,l})
 \end{array}$$


---

1 **Proposition 2** For every  $(\mathbf{H}, \mathbf{Q}, \sigma^2) \in \mathbb{R}^{NF} \times$  18  
 2  $\mathbb{R}^{NF} \times \mathbb{R}_+$ , let:

$$G_1(\mathbf{H}, \mathbf{Q}, \sigma^2) = \mu_1(\mathbf{Q}, \sigma^2) \mathbf{I}_{NF},$$

$$G_2(\mathbf{H}, \mathbf{Q}, \sigma^2) = \left( \mu_2(\mathbf{H}, \sigma^2) + \frac{9\lambda}{8\eta^2} \right) \mathbf{I}_{NF} \\ + \frac{\lambda}{\ell_{1,\alpha}(\mathbf{Q}) + \beta} G_{\ell_{1,\alpha}}(\mathbf{Q})$$

$$G_3(\mathbf{H}, \mathbf{Q}, \sigma^2) = \mu_3(\mathbf{H}, \mathbf{Q}),$$

3 where:

$$G_{\ell_{1,\alpha}}(\mathbf{Q}) = \text{Diag} \left( \left( (q_f(n)^2 + \alpha^2)^{-1/2} \right)_{1 \leq f \leq F, 1 \leq n \leq N} \right), \quad (16)$$

4 and  $\mu_1(\mathbf{Q}, \sigma^2)$ ,  $\mu_2(\mathbf{H}, \sigma^2)$ , and  $\mu_3(\mathbf{H}, \mathbf{Q})$  are  
 5 the Lipschitz constants for  $\nabla_1 \phi(\cdot, \mathbf{Q}, \sigma^2)$ ,  
 6  $\nabla_2 \phi(\mathbf{H}, \cdot, \sigma^2)$ , and  $\nabla_3 \phi(\mathbf{H}, \mathbf{Q}, \cdot)$ , respectively.<sup>3</sup>  
 7 Then,  $G_1(\mathbf{H}, \mathbf{Q}, \sigma^2)$ ,  $G_2(\mathbf{H}, \mathbf{Q}, \sigma^2)$ , and  
 8  $G_3(\mathbf{H}, \mathbf{Q}, \sigma^2)$  satisfy the majoration condition  
 9 for  $\psi(\cdot, \mathbf{Q}, \sigma^2)$  at  $\mathbf{H}$ ,  $\psi(\mathbf{H}, \cdot, \sigma^2)$  at  $\mathbf{Q}$ , and  
 10  $\psi(\mathbf{H}, \mathbf{Q}, \cdot)$  at  $\sigma^2$ , respectively.

11 To conclude, we have proposed a blind deconvolution  
 12 method to apply to the BF-MS that imposes  
 13 sparsity on the noise acoustic-source locations. This  
 14 method is validated in the next section, and compared  
 15 to the classical methods of DAMAS-MS and  
 16 SDM, used in acoustics for moving-source deconvolution.  
 17

<sup>3</sup>These Lipschitz constants are straightforward to derive since  $\phi$  is a quadratic cost.

**5. Results**

19 We consider synthetic and real data for the  
 20 method validation. The synthetic data allow the  
 21 use of quantitative indicators, whereas real data  
 22 only provide subjective results. For both cases,  
 23 we perform comparative evaluation with the stan-  
 24 dard algorithms DAMAS-MS and SDM. In prac-  
 25 tice, the kernel blur related to the array trans-  
 26 fer function has finite energy, and thus  $\rho_1$  can  
 27 be chosen as an indicator function of set  $C =$   
 28  $\{\mathbf{H} \in [h_{\min}, h_{\max}]^{NF} \mid \|\mathbf{H}\| \leq \kappa\}$  (equal to 0 if  $\mathbf{H} \in$   
 29  $C$ , and  $+\infty$  otherwise), where  $\kappa > 0$ , and  $h_{\min}$   
 30 and  $h_{\max}$  are the minimum and maximum values  
 31 of  $\bar{\mathbf{H}}$ , respectively. In the real data case, we choose  
 32  $h_{\min} = 0$  and  $h_{\max} = 1$ . As mentioned before, the

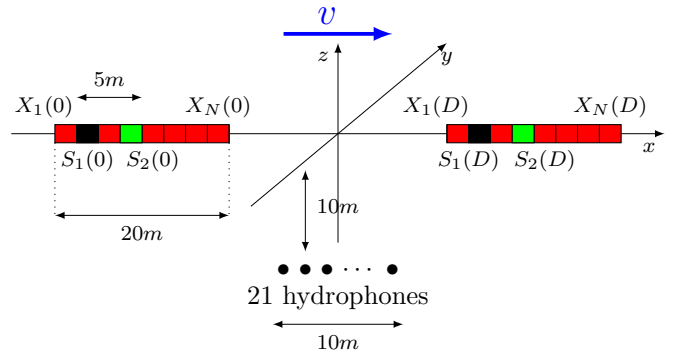


Figure 2: Simulated configuration of a pass-by experiment. Black, source  $S_1$ ; green, source  $S_2$ ; red, calculation grid; blue arrow, global movement of the sources.



1 autospectra of sources  $\mathbf{Q}$  is sparse; moreover, it  
 2 is limited in amplitude. Then, one natural choice  
 3 for  $\rho_2$  is the indicator function of the hypercube  
 4  $[q_{\min}, q_{\max}]^{NF}$ , where  $q_{\min}$  (resp.  $q_{\max}$ ) is the lower  
 5 (resp., upper) boundary of  $\mathbf{Q}$ . In practice, we choose  
 6  $q_{\min}$  as 0, which leads to a nonnegative constraint  
 7 on the source power variables, and  $q_{\max}$  is the max-  
 8 imum value of  $\mathbf{B}$ . Finally, the function  $\rho_3$  related  
 9 to the constraint on the noise variance is equal to  
 10 the indicator function of the interval  $[\sigma_{\min}^2, \sigma_{\max}^2]$ ,  
 11 where  $\sigma_{\min}^2 = 0$  and  $\sigma_{\max}^2 = 1$ . Note that the prox-  
 12 imity operators can be easily explicitly expressed  
 13 (see Appendix).

14 The NR-SOOT algorithm with the penalty  
 15 smoothed  $\ell_1/\ell_2$  function and the classical DAMAS-  
 16 MS and SDM algorithms are applied to the BF-MS  
 17 result. For every  $l \in \mathbb{N}$ , the number of inner-loops  
 18 are fixed as  $I_l = 1$  and  $J_l = 100$ . The NR-SOOT  
 19 algorithm is launched on 5000 iterations, and it  
 20 can stop earlier at iteration  $l$  if  $\|\mathbf{Q}^l - \mathbf{Q}^{l-1}\| \leq$   
 21  $\sqrt{NF} \times 10^{-6}$ .

### 22 5.1. Synthetic data

23 The simulated configuration is presented in Fig-  
 24 ure 2. Here, we consider two sources: a random  
 25 broadband source located at  $S_1 = (-4m, 0m, 0m)$   
 26 (Fig. 2, black) and a sum of 3 sine functions at fre-  
 27 quencies 1200 Hz, 1400 Hz, and 1800 Hz located at  
 28  $S_2 = (1m, 0m, 0m)$  (Fig. 2, green), in the coordi-  
 29 nate system where the origin is the center of the  
 30 moving calculation grid all the time. The sources are  
 31 moving jointly, and they follow a linear trajectory of  
 32 length 20 m at constant speed  $v = 2m/s$ . A linear  
 33 antenna of 21 hydrophones that are equally spaced  
 34 (with an inter-sensor distance of 0.5 m) records the  
 35 propagated acoustic signals over  $D = 10s$ . Zero-  
 36 mean white Gaussian noise is added to the recorded  
 37 signals. To perform BF-MS, the moving calculation  
 38 grid  $X_n(t), \forall n \in \{1, \dots, N\}$  has a length of 20 m  
 39 and contains  $N = 101$  points.

40 Concerning the initialization of the methods;  $\mathbf{Q}^0$   
 41 is the BF-MS output  $\mathbf{B}$ . The initialization of the  
 42 blur  $\mathbf{H}^0$  for the SOOT and NR-SOOT algorithms is  
 43 a centered Gaussian filter, such that  $\mathbf{H}^0 \in C$ . The  
 44 regularization parameters of SDM, and  $(\lambda, \alpha, \beta, \eta) \in$   
 45  $]0, +\infty[^4$  (depending on the SOOT or NR-SOOT  
 46 algorithms) are empirically adjusted, although it  
 47 can be noted that the method is not too sensitive  
 48 to their initialization.

49 Figure 3 summarizes the quantitative results in  
 50 terms of reconstruction error  $\overline{\mathbf{Q}}$ . The relative error  
 51 is defined with the  $\ell_2$ -norm (Fig. 3, top) and

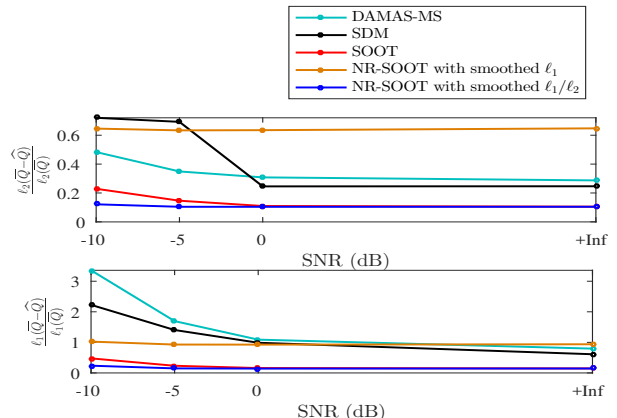


Figure 3: Comparison of the results for input data without noise and for three different signal-to-noise ratios (SNR)  $\in \{-10, -5, 0\}$  dB.

52  $\ell_1$ -norm (Fig. 3, bottom) between the real  $\overline{\mathbf{Q}}$  and  
 53 the estimated  $\widehat{\mathbf{Q}}$ . It demonstrates that the method  
 54 can reconstruct accurately in terms of amplitude  
 55 (observing  $\ell_2$ -norm) and in terms of sparse source  
 56 positions (observing  $\ell_1$ -norm). From Figure 3, we  
 57 observe that SDM performs better in terms of source  
 58 localization sparsity than DAMAS-MS for the case  
 59 considered (the  $\ell_1$ -norm values of the residual error  
 60 by SDM are always smaller than those by DAMAS-  
 61 MS). However, the performance of SDM decreases  
 62 significantly when the SNR decreases. The NR-  
 63 SOOT method with  $\ell_{1,\alpha}$  as the penalty function,  
 64 shown to compare  $\ell_{1,\alpha}$  with  $\ell_1/\ell_2$  approach, gives  
 65 satisfactory results in terms of sparsity of the source  
 66 localization, but not in terms of amplitude recon-  
 67 struction. This result confirms the conclusion of  
 68 [41], which shows that  $\ell_{1,\alpha}$  should not be used in  
 69 this case. The original SOOT provides very satisfy-  
 70 ing results compared to DAMAS-MS and SDM. Its  
 71 performances for cases of high SNR are similar to  
 72 the proposed method NR-SOOT with smoothed  
 73  $\ell_1/\ell_2$ . Nevertheless, for the cases of low SNR, the  
 74 NR-SOOT algorithm with smoothed  $\ell_1/\ell_2$  is the  
 75 only one that provides a satisfactory source localiza-  
 76 tion estimation. To summarize, in all of these cases,  
 77 the NR-SOOT algorithm with smoothed  $\ell_1/\ell_2$  as  
 78 the penalty function has the smallest error for the  
 79 source localization estimation in terms of sparsity  
 80 and amplitude. In the following, for the NR-SOOT  
 81 method, we only perform it with the smoothed  $\ell_1/\ell_2$   
 82 penalty function (and call it NR-SOOT).

83 After this quantitative study, it is necessary to  
 84 investigate the performance of these methods quali-  
 85 tatively, directly on the localization maps for input

1 data without noise and for a SNR of -10 dB. Figure 4  
2 and Figure 5 show the results for the DAMAS-MS,  
3 SDM, original SOOT, and NR-SOOT algorithms  
4 at frequencies of 1400 Hz and 770 Hz, respectively.  
5 In these Figures, the green lines ( $a_1, b_1$ ) represent  
6 the theoretical sources to estimate (in terms of position  
7 and amplitude), the magenta lines represent  
8 the BF-MS results, which are the starting points  
9 of the DAMAS-MS, SDM, original SOOT, and NR-  
10 SOOT algorithms. In Figure 4 and Figure 5, the  
11 results obtained by DAMAS-MS are in greenish-  
12 blue, those of SDM are in black ( $a_2, b_2$ ), those of  
13 original SOOT are in red, and those of NR-SOOT  
14 are in blue ( $a_3, b_3$ ).

15 At the frequency of 1400 Hz (Fig. 4), for which  
16 both sources exist, for the case without noise on  
17 the recorded data (Fig. 4a) both the original SOOT  
18 and the NR-SOOT algorithms detect the source  
19 positions accurately. DAMAS-MS gives some false  
20 alarms at  $x = -3 m$  and  $x = 2.5 m$ . These false  
21 sources have small amplitudes, but they are a real  
22 problem because the number of sources is gener-  
23 ally unknown. SDM locates two sources, but the  
24 amplitude estimation is not satisfactory and these  
25 sources are spread in the space. For the case of a  
26 SNR of -10 dB, DAMAS-MS does not succeed at  
27 all, and it shows several false alarms with significant  
28 amplitudes. With the SDM method, there is one  
29 false alarm around  $x = -1 m$ , and the amplitudes  
30 are not correct. The original SOOT algorithm gives  
31 good results, although there are two false alarms  
32 around  $x = -9 m$  and  $x = 9 m$ . In contrast, the  
33 NR-SOOT algorithm gives perfect results in terms  
34 of localization and source amplitude estimation.

35 We now turn our attention to the case at the  
36 low frequency 770 Hz (Fig. 5), for which only the  
37 source  $S_1$  exists. In this case, DAMAS-MS and SDM  
38 give unsatisfactory results, with a spatially extended  
39 source and false alarms even in the noise-free case for  
40 DAMAS-MS. The original SOOT gives satisfactory  
41 results without noise (Fig. 5a<sub>3</sub>), although when the  
42 SNR decreases, the original SOOT algorithm creates  
43 false alarms (Fig. 5b<sub>3</sub>), while the NR-SOOT algo-  
44 rithm shows excellent results in terms of position and  
45 amplitude (Fig. 5b<sub>3</sub>). The NR-SOOT algorithm is  
46 robust against noise. In the following, for the sake of  
47 simplicity, and as it always provides the best results,  
48 we only consider the NR-SOOT algorithm for blind  
49 deconvolution of the two-dimensional illustrations.

50 The two-dimensional localization maps are shown  
51 in Figure 7 (without noise) and Figure 8 (SNR of  
52 -10 dB), with each Figure showing the initial BF-MS

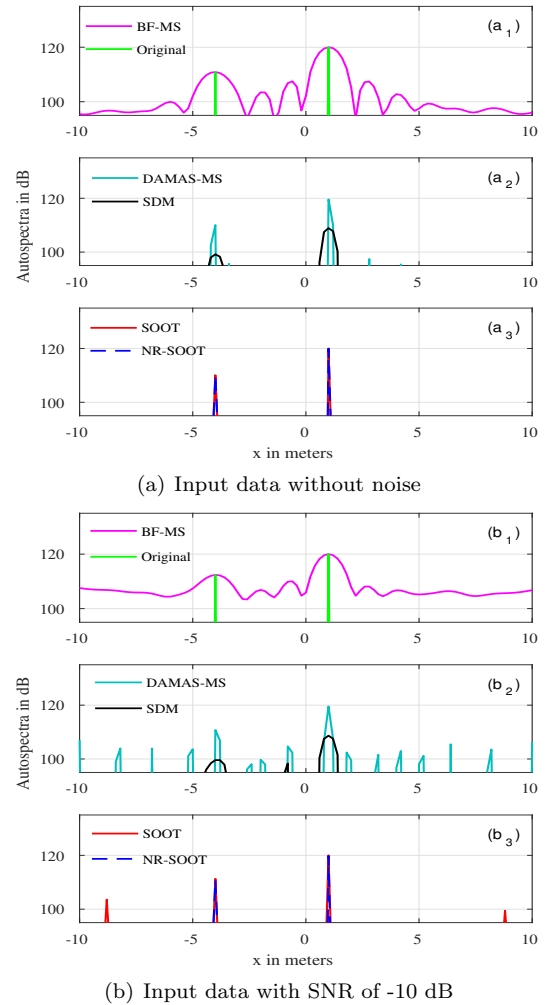


Figure 4: Comparison of the results at the frequency of 1400 Hz, without noise (a), and with SNR of -10 dB (b).

53 and the results obtained by DAMAS-MS, SDM, orig-  
54 inal SOOT, and NR-SOOT. For the case without  
55 noise of Figure 7, all of the methods improve the  
56 BF-MS output, localize the two sources, and allow  
57 identification as one broadband source and a sum-  
58 of-sine source. However, the results obtained using  
59 DAMAS-MS and SDM are not as good as those  
60 using the original SOOT or NR-SOOT algorithms,  
61 because the source localizations are spread over sev-  
62 eral  $x$  positions. Moreover, by studying the different  
63 zones indicated in the red ellipses in Figure 7 and  
64 Figure 8, which are related to the autospectrum of  
65 the sine source at the three frequencies of 1200 Hz,  
66 1400 Hz, and 1800 Hz, some other conclusions can  
67 be drawn. The results obtained using the DAMAS-  
68 MS method are not performing well, as some noise

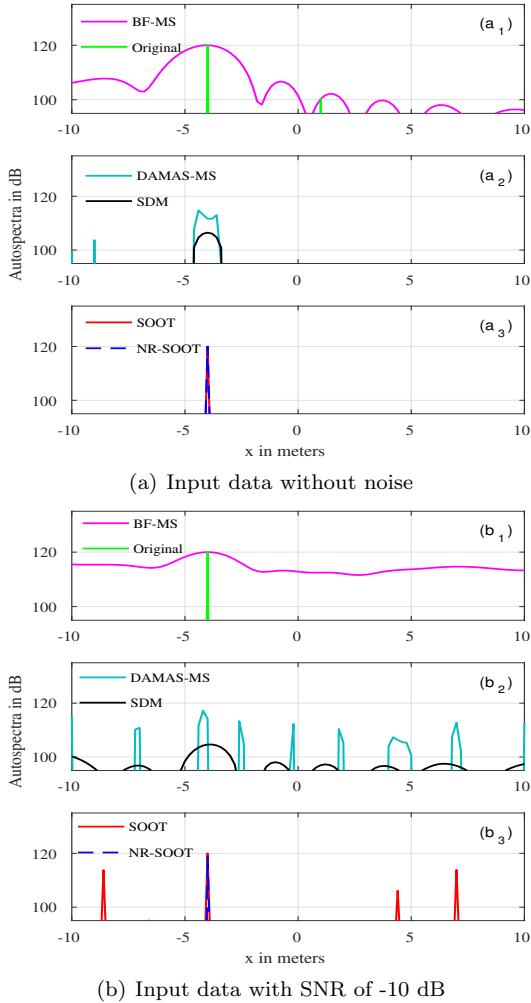


Figure 5: Comparison of the results at the frequency of  $770\text{ Hz}$ , without noise (a), and with SNR of  $-10\text{ dB}$  (b).

1 appears, as indicated by the red arrows in Figure 7b.  
 2 For the case of a SNR of  $-10\text{ dB}$  (Fig. 8), DAMAS-  
 3 MS, SDM do not identify the sources and give many  
 4 false alarms. The original SOOT manages to estimate  
 5 a point source at the true source positions  
 6 but also gives many false alarms. In contrast, the  
 7 NR-SOOT algorithm still gives good results and provides  
 8 the best performance compared to the three  
 9 other methods.

10 Figure 6 shows the computational time (in minutes)  
 11 for the different methods and for different noise  
 12 levels. The computational time corresponds to the time  
 13 required to satisfy the stopping criterion, i.e.  
 14  $\|\mathbf{Q}^l - \mathbf{Q}^{l-1}\| \leq \sqrt{NF} \times 10^{-6}$ , with the simulations  
 15 performed on the same CPU. The SDM computa-  
 16 tional cost is four-fold the SOOT and NR-SOOT

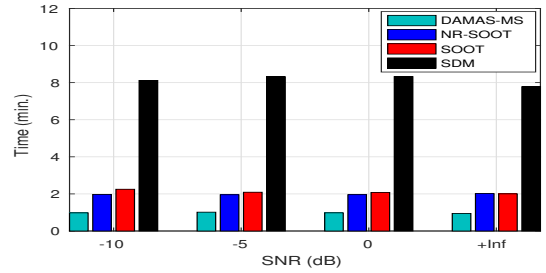


Figure 6: Computational times for input data without noise and for three different signal-to-noise ratios, SNR  $\in \{-10, -5, 0\}$  dB.

17 computational costs. As SDM is an approach that  
 18 is similar to the forward-backward method, these  
 19 results confirm the performance of the proposed  
 20 majorant. In all cases, DAMAS-MS is the fastest  
 21 algorithm, but the computational costs for DAMAS-  
 22 MS, SOOT and NR-SOOT are of the same order  
 23 (from 1-2 min).

24 To conclude, the NR-SOOT algorithms have better  
 25 performances than the DAMAS-MS, SDM meth-  
 26 ods in terms of localization, as the source  $S_2$  is  
 27 spread over several  $x$  positions by DAMAS-MS and  
 28 SDM, whereas the SOOT and the proposed NR-  
 29 SOOT algorithm manage to estimate a point source  
 30 at the true source position. Nevertheless, in term  
 31 of robustness against noise, the NR-SOOT method  
 32 is the only one that provides satisfactory results for  
 33 low SNRs.

## 34 5.2. Real data

35 We finally compare the proposed NR-SOOT al-  
 36 gorithm with the classical methods using real data.  
 37 The experiment was conducted in January 2015  
 38 by DGA naval systems at Lake Castillon, a moun-  
 39 tain lake in the French Alps with an average depth  
 40 of  $100\text{ m}$  and a maximum width of  $600\text{ m}$ . This  
 41 consisted of towing a 21-m-long scale model of a  
 42 surface ship. The ship hull included two shakers,  $S_1$   
 43 and  $S_2$ , that generated two point acoustic sources  
 44 outside the hull: a sum of 3 sine functions at frequen-  
 45 cies of  $1200\text{ Hz}$ ,  $1400\text{ Hz}$ , and  $1800\text{ Hz}$ , located at  
 46  $x = -5.9\text{ m}$ , and a random broadband source located  
 47 at  $x = 2.3\text{ m}$ . A linear antenna of nine hydrophones  
 48 that were equally spaced by  $0.5\text{ m}$  recorded the propa-  
 49 gated acoustic signals over  $D = 14.15\text{ s}$  for the  
 50 source speed of  $v = 2\text{ m/s}$  (Fig. 9). We also con-  
 51 sider the same configuration with the source speed  
 52 of  $v = 5\text{ m/s}$  over  $D = 5.3\text{ s}$ . (Fig. 10). The coordi-  
 53 nate system of the array was used to describe all of

the geometries, with the origin corresponding to the array center. The array was immersed at 10 m in depth and was positioned at 2.50 m from the closest point of approach in the  $y$  direction. The source trajectory was calculated using a tachymeter system on the idler pulley. The acquisition time considered for the array processing is sufficient, such that the ship model passed by entirely above the antenna. In these Figures, the zones indicated in the red ellipses correspond to the estimated autospectrum of the sine source at the three frequencies of 1200 Hz, 1400 Hz, and 1800 Hz, and the red arrows show the remaining noise or the false alarms.

First, we consider the results in the case of the source speed  $v = 2\text{ m/s}$  (Fig. 9). The three methods improve the BF-MS output and identify the sources. DAMAS-MS and NR-SOOT have better performances than SDM in terms of localization. However, for the result of the DAMAS-MS method, there are some false alarms that are indicated by the red arrows in Figure 9b.

Secondly, we consider the case with the source speed  $v = 5\text{ m/s}$ , for which the signal in the recording is more noisy. In this configuration, one new ‘natural’ source appears at the wake of the ship (Fig. 9d, bottom left). Three methods identify three sources, whereby the sine source is better localized by the NR-SOOT algorithm than the other methods. Both the DAMAS-MS and SDM methods show many false alarms, which are indicated by the red arrows in Figure 10b, c. In particular, the localization of the ‘natural’ source is only possible with the NR-SOOT algorithm. In conclusion, our results from this experiment remain true to our hypothesis, as well as our predictions. The results shown in Figure 9 and Figure 10 present the best results with perfect source location and improved robustness against noise for the NR-SOOT algorithm.

## 6. Conclusions

This paper proposes a new method, known as NR-SOOT, that is an extension of the SOOT algorithm [18], for moving-source localization based on blind deconvolution in underwater acoustic data. As the number of sources is small enough and they do not spread spatially, its autospectrum has a sparse representation, and it is possible to obtain more accurate results for blind deconvolution through a regularization function. The smooth approximation of  $\ell_1/\ell_2$  shows very good performances in terms of localization and suppression of false alarms, and

provides better results than DAMAS-MS and SDM, particularly for low SNRs.

## Appendix

In this appendix, we give the explicit expressions of the proximity operators involved in the NR-SOOT algorithm. For every  $(\mathbf{H}, \mathbf{Q}, \sigma^2) \in \mathbb{R}^{NF} \times \mathbb{R}^{NF} \times \mathbb{R}_+$  and  $\gamma \in ]0, +\infty[$ , let  $G_1$ ,  $G_2$ , and  $G_3$  be the majorant matrix of  $\psi$  at  $\mathbf{H}$ , at  $\mathbf{Q}$ , and at  $\sigma^2$ , respectively, that are given by Proposition 2. Then

1.  $\text{prox}_{(\gamma_1)^{-1}G_1, t_C} = \Pi_C$ ,
2.  $\text{prox}_{(\gamma_2)^{-1}G_2, t_{[q_{\min}, q_{\max}]^{NF}}} = \Pi_{[q_{\min}, q_{\max}]^{NF}}$ ,
3.  $\text{prox}_{(\gamma_3)^{-1}G_3, t_{[\sigma_{\min}^2, \sigma_{\max}^2]}} = \Pi_{[\sigma_{\min}^2, \sigma_{\max}^2]}$ ,

which can be easily computed.

## Acknowledgements

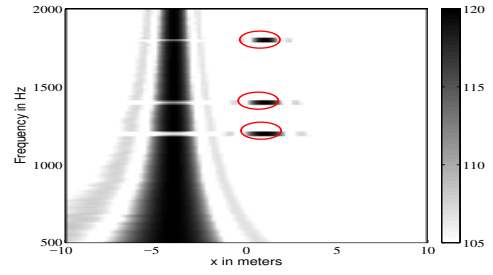
This work was financially supported by the Ministère du Redressement Productif (Direction Générale de la Compétitivité, de l’Industrie et des Services) and by the DGA-MRIS, grant RAPID ARMADA N°122906030.

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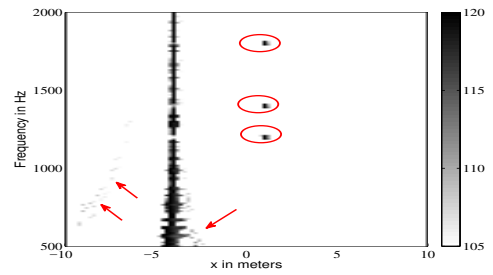
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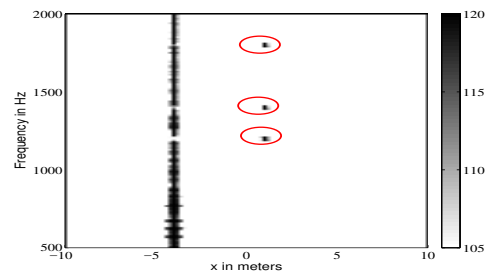
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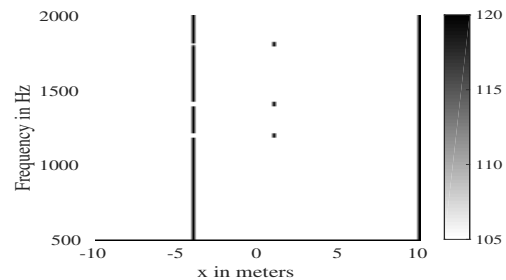
(a) Initial BF-MS



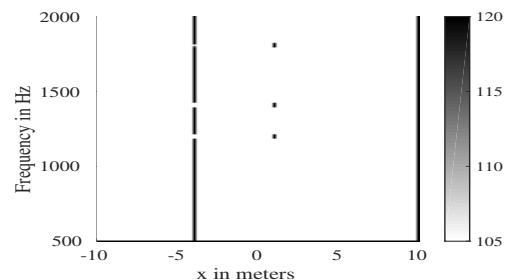
(b) Deconvolution with DAMAS-MS



(c) Deconvolution with SDM

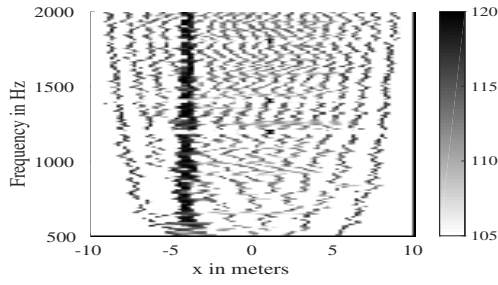


(d) Blind deconvolution with SOOT

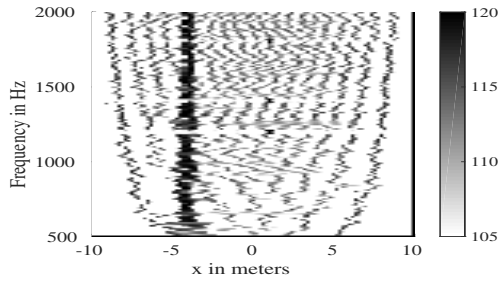


(e) Blind deconvolution with NR-SOOT

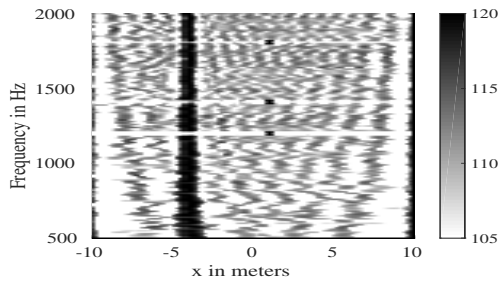
Figure 7: Localization in the frequency-distance domain obtained (in the case without noise). (a) Initial BF-MS. (b) DAMAS-MS. (c) SDM. (d) original SOOT. (e) NR-SOOT (the dynamic ranges shown are 15 dB).



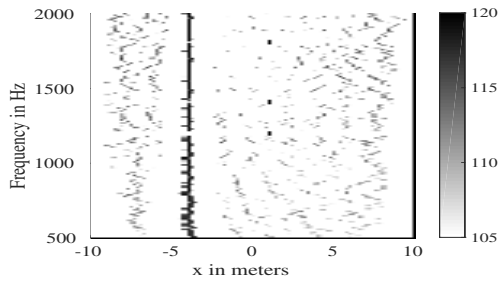
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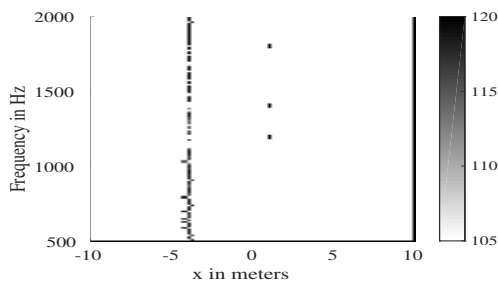
(b) Deconvolution with DAMAS-MS



(c) Deconvolution with SDM

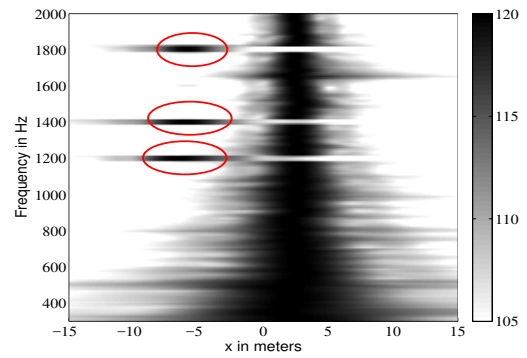


(d) Blind deconvolution with SOOT

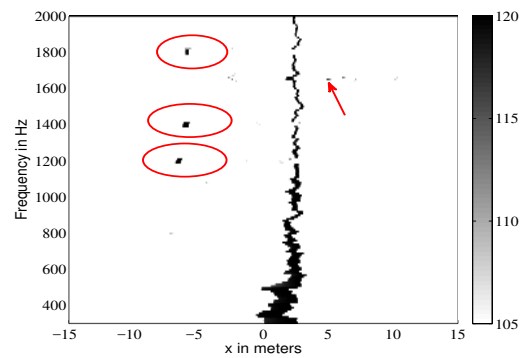


(e) Blind deconvolution with NR-SOOT

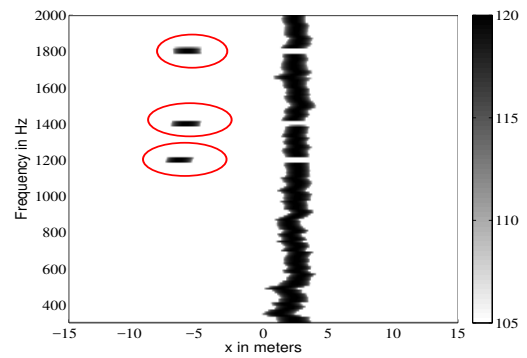
Figure 8: Localization in the frequency-distance domain obtained (with SNR of -10 dB). (a) Initial BF-MS. (b) DAMAS-MS. (c) SDM. (d) original SOOT. (e) NR-SOOT (the dynamic ranges shown are 15 dB).



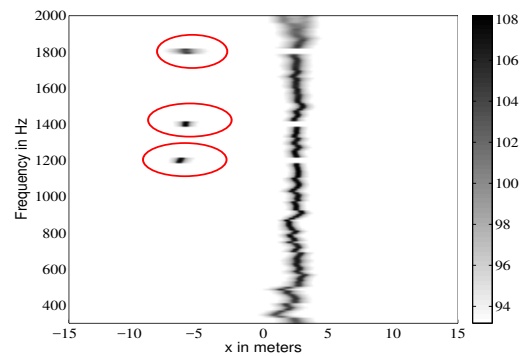
(a) Initial BF-MS



(b) Deconvolution with DAMAS-MS

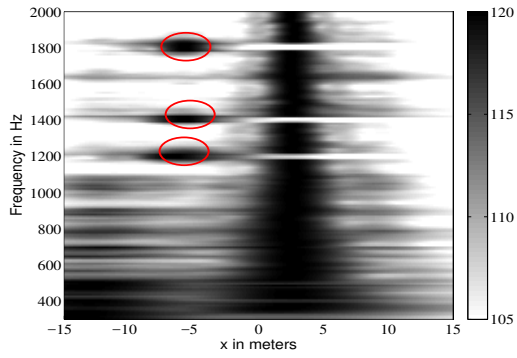


(c) Deconvolution with SDM

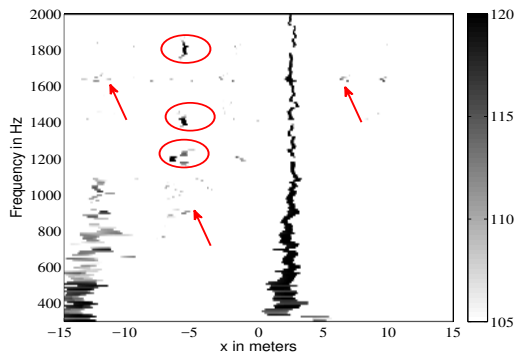


(d) Blind deconvolution with NR-SOOT

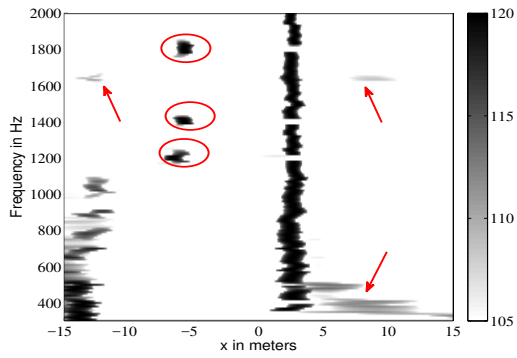
Figure 9: Localization obtained in the frequency-distance domain for the model ship with two artificial sources, traveling at 2 m/s. (a) Initial BF-MS. (b) DAMAS-MS. (c) SDM. (d) NR-SOOT (the dynamic ranges shown are 15 dB).



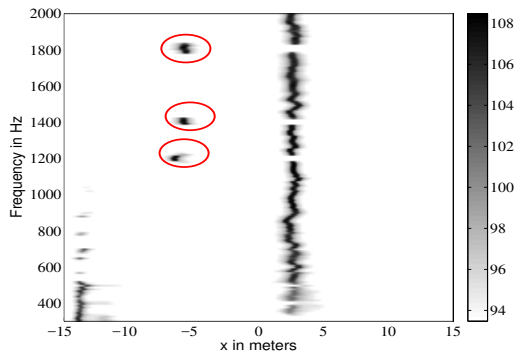
(a) Initial BF-MS



(b) Deconvolution with DAMAS-MS



(c) Deconvolution with SDM



(d) Blind deconvolution with NR-SOOT

Figure 10: Localization obtained in the frequency-distance domain for the model ship with two artificial sources, traveling at 5 m/s. (a) Initial BF-MS. (b) DAMAS-MS. (c) SDM. (d) NR-SOOT (the dynamic ranges shown are 15 dB).