Towards a general characterization of conceptual neighborhoods in temporal and spatial reasoning

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Abstract

We present a general method for describing the conceptual neighborhood structure of qualitative relations in temporal and spatial reasoning. This method is based on representing the topological structure of the configuration spaces in terms of incidence structures. We illustrate this approach on some of the qualitative calculi introduced in the literature, including Allen's temporal calculus and Freksa's directional calculus.

1 Introduction

The notion of conceptual neighborhoods has been explicitly introduced in temporal reasoning by C. Freksa ([4]). His motivations are of a cognitive nature: although knowledge about temporal or spatial situations tends to be imprecise and qualitative, the range of possibilities in terms of relations can often be expressed in terms of relations which in some sense are neighbors.

In the temporal reasoning framework proposed by Allen, and more generally in the context of generalized interval calculi, convex relations, which are one of the possible interpretations of conceptual neighborhoods, have interesting computational properties. One is the fact that composition of elements can be computed in constant time, because of the "min-max formula". Another is that although temporal reasoning in the full framework of Allen's algebra is NP-hard, limitation to the subclass of convex relations leads to polynomial problems.

Conceptual neighborhoods in spatial reasoning have been introduced and studied along similar lines by Freksa ([5]), Cohn and Randell ([10,11,1]), Egenhofer ([2]). In particular, it is an important notion in modeling physical processes which vary continuously. As a consequence, the intervening spatial relations vary along paths of vicinity, imposing constraints on the set of evolutions or envisionments which can be reached from a given state.

The purpose of this paper is to propose a general principle and tools for investigating the concept of conceptual neighborhoods. The general principle consists in considering the *configuration spaces* of the qualitative relations, which gives a particular metric model of the set of relations, and abstracting to get its topological structure in terms of incidence structures. The tools are mainly derived from the theory of regular cell complexes, which have been used in a similar context by M. Fleck ([3]), and are used as modeling tools in geographic information systems.

In the first part of the paper, we put this program to work on the familiar case of temporal reasoning in the case of Allen's calculus, and in the less familiar extensions to generalized interval calculi. We then correlate the results we get to the point of view related to the order structures of the qualitative relations. In the second part, we attack spatial reasoning, covering in particular some of the cases where spatial reasoning is basically recoverable from temporal reasoning. We then discuss some calculi in the family of qualitative triangulation, including the flip-flop calculus ([8]) and Freksa's calculus ([5]). We conclude this exploration by discussing the nature of the notion of conceptual neighborhood we get in this way.

2 The qualitative structure of configuration spaces

2.1 The topological structure of Allen's relations

Let us start with the well known example of intervals on the real line, which we consider here as couples (X_1, X_2) of real integers, with $x_1 < x_2$. Hence, the set of intervals can be identified with the subset of the real plane defined by $X_1 < X_2$, which is the upper half-plane **H** limited by the first bisector (Fig. 1). Now fix a given interval (y_1, y_2). The set of primitive Allen's relations then corresponds to a decomposition of H into disjoint regions. For instance, relation m (meets) corresponds to the subregion of H defined by $X_2 = y_1$. The decomposition is shown in Fig. 1.

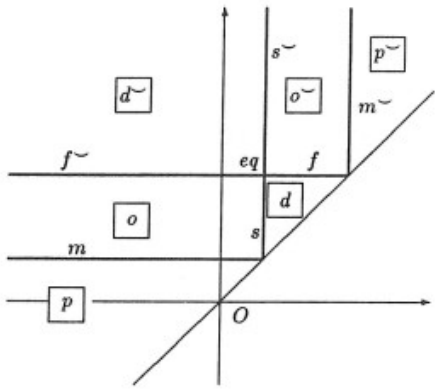

Figure 1: Regions associated to atomic relations in Allen's algebra

This decomposition contains metric information, which has been used e.g. by Rit [12]. For example, the set of intervals with a given duration is represented by a line parallel to the first bisector. However, we are interested in qualitative, specifically topological information. To express this type of information, we introduce the *incidence structure* associated to the decomposition.

First observe that each region of the decomposition, as a topological subspace of the plane, is homeomorphic (isomorphic as a topological space) to a p-cell (p is the dimension of the cell, which here is 0, 1, or 2). A 0-cell is a point, a 1-cell an open segment, a 2-cell the interior of a disk. Hence the decomposition is in fact a *cell-decomposition* of $\mathbf{H}$. Also observe that the closure of each region α in $\mathbf{H}$ is the disjoint union of α and a finite number of regions of lower dimension. For instance, the closure of o (overlaps) is the union of o itself (a 2-cell), m, s, $f^\smile$ (which are 1-cells), and eq (a 0-cell).

Definition 1 *The edge relation ∂ holds between α and β if the topological closure of α contains β and the dimension of β is one less than the dimension of α. This is denoted by $\partial(\alpha) \ni \beta$.*

Definition 2 *The incidence structure of a cell decomposition is the graph with one vertex for each cell, and an arc from α to β if and only if $\partial(\alpha) \ni \beta$.*

The incidence structure of the decomposition of H corresponding to the relations of Allen is represented in Fig. 2.

Clearly, this incidence graph contains the topologically invariant part of the cell-decomposition. However, for reasons which will become apparent later, we consider the *opposite* graph, that is, the graph with

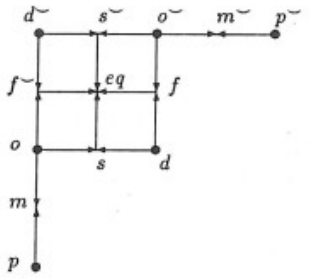

Figure 2: The incidence structure of Allen's relations

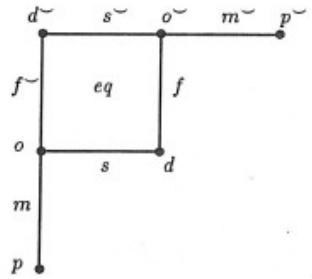

Figure 3: A compact model of the opposite incidence structure

the same vertices and the arcs reversed. The fact is that this graph is also an incidence graph. It is even more well-behaved, because it is the incidence graph of a compact cell decomposition, which belongs to the class of *regular cell-complexes*. For this class, in fact, the incidence structure uniquely determines the cell-complex (this is shown in [3]). In the case of Allen's relations, the corresponding cell-complex corresponds to the decomposition of the polygon shown in Fig. 3.

Because the incidence structure of the original cell-decomposition of the half plane can be recovered from the polygon, this contains in essence the whole topological information about Allen's relations. Before proceeding further, however, we look at an alternative way of getting essentially the same structure.

2.2 Topology and order

Here the idea is to use the order structure on intervals. A convenient way of getting it for free is introduced in [7]. A fixed interval (y_1, y_2) being given, call 0, 1, 2, 3, 4 the five zones defined by $X < y_1$, $X = y_1$, $y_1 < X < y_2$. $X = y_2$, and $X > y_2$ respectively. Then each

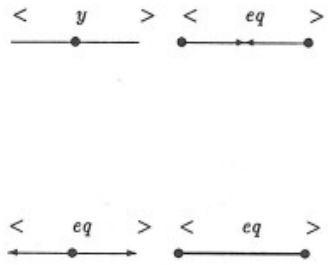

Figure 4: Relations in the $\mathbf{A}(1)$-calculus

one of the primitive relations of Allen is characterized by a pair of integers specifying in which of the five zones the start and finish points of a given interval lie with respect to (y_1, y_2). We call the representation of Allen's relations as pairs of integers the *canonical representation*.

In this way, the set of primitive relations is identified with a subset of $\mathbf{N} \times \mathbf{N}$. The order structure on the reals is reflected by the natural order on the integers, hence it is a reasonable choice to consider the product order on the set of relations. In this way, the set of relations becomes a distributive lattice, i.e. a partially ordered set in which each pair of elements has a lower upper bound and an upper lower bound, and such that the distributivity axiom is satisfied.

The important remark here is that the polygon is recoverable from the lattice, or the lattice from the polygon (by 2-subdivision of each cell).

2.3 Conceptual neighborhoods and generalized intervals

The preceding considerations about Allen's relations basically extend to the more general framework of generalized intervals ([9,7]).

The simplest case is the case of instant calculus: given a fixed instant y, it defines a decomposition of the real line into three cells, corresponding to the three relations $<, eq, >$. The decomposition, its incidence graph, the opposite graph, and the decomposition corresponding to the opposite graph, are represented in Fig. 4. (We adopt the same presentation in the next four Figures).

More generally, let a q-interval $y = (y_1, \ldots, y_q)$ be a sequence $y_1 < \ldots < y_q$. The set of relations between p-intervals and q-intervals has a canonical embedding as a distributive lattice in $\mathbf{R}^p$. This lattice corresponds to the 2-subdivision of a polytope of dimension $inf(p, q)$ embedded in $\mathbf{R}^p$, which is in a natural way a compact model of the opposite incidence structure defined by the configuration space.

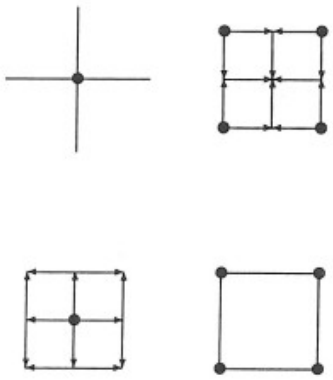

Figure 5: Relations in the $\mathbf{A}(1) \times \mathbf{A}(1)$-calculus

3 Conceptual neighborhoods in spatial reasoning

3.1 The $\mathbf{A}(1) \times \mathbf{A}(1)$ and $\mathbf{A}(2) \times \mathbf{A}(2)$-calculus

We remarked in [6] that the North-East- South-West calculus of geographers is exactly the kind of calculus we get in 2D-space if we reason about points with fixed directions of reference, i.e. the $\mathbf{A}(1) \times \mathbf{A}(1)$ calculus, where $\mathbf{A}(1)$ is the relation algebra which describes the instant calculus. In terms of cell decompositions, a fixed point in 2D-space defines a decomposition into 9 regions as represented in Fig. 5. The corresponding incidence structure, opposite incidence structure and compact incidence structure are also shown in the same Figure. As expected, the latter is just the product of two copies of the structure described for the instant calculus.

In the same way, the compact incidence structure associated to the $\mathbf{A}(2) \times \mathbf{A}(2)$-calculus is the 4-dimensional cell-complex which is the product of two copies of the cell-complex associated to Allen's calculus.

3.2 Qualitative triangulation calculi

First consider the flip-flop calculus ([8]). A pair of points in 2-D space defines seven regions. The corresponding cell-decomposition, incidence structures, and compact incidence structure are represented in Fig. 6.

Next consider Freksa's calculus [5]. We now get a decomposition into 15 regions. The incidence information is summarized in Fig. 7.

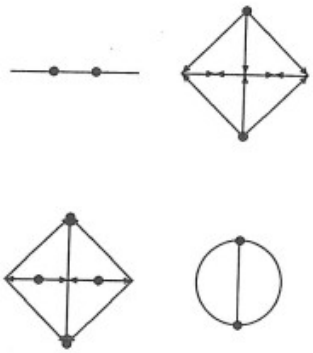

Figure 6: The incidence structure of the flip-flop calculus

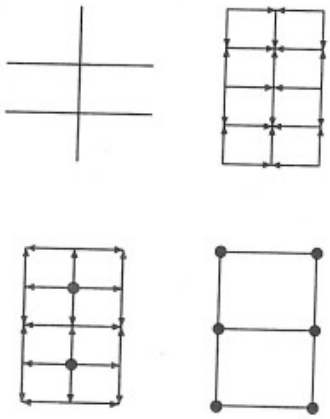

Figure 7: The incidence structure of Freksa's calculus

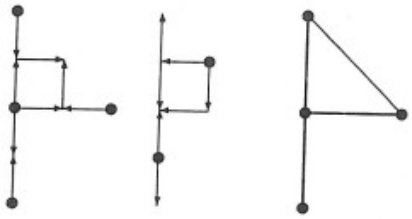

Figure 8: The incidence structure of the 8-relation calculus

3.3 The 8-relation calculus

We also observed in [6] that, at least as far as its relations are concerned, the qualitative spatial calculus about regions considered by Cohn, Randell ([10,11,1]), Egenhofer ([2]) can be obtained from the interval calculus by quotienting relatively to time reversal. The set of relations is represented in Fig. 8 shows the corresponding structures and cell-complex. which can be obtained from Allen's situation as quotients.

4 Discussion

We observed in a number of cases that abstracting the topological structure of qualitative relations, both in the temporal and spatial cases, can be conveniently explained in terms of cell decompositions and their incidence structures.

What light does this point of view shed on the problem of defining conceptual neighborhood structures? First, it seems that this is a good way of considering continuously varying structures. Intuitively, transition between regions represented by the edge relation correspond to minimal changes in the qualitative structure. This fact is best illustrated by considering for instance relations between two 3-intervals. A transition corresponding to the edge relation or its inverse occurs when two distinct points are merged. If 3-intervals represent deformable, elastic objects, such a merge can occur with all other relations being fixed. This is no longer true if we reason about rigid objects. Here, the relative lengths of (x_1, x_2), (x_1, x_3), and (x_2, x_3) with respect to their y counterparts determine the degree of "collapsing" or "blowing-up" induced by continuous change. This is best observed on the lattice representing the set of relations between 3-intervals. Imagine a fixed 3-interval y and a 3-interval x moving from left to right. Initially, x is way to the left of y, corresponding to the relation whose canonical representation is (0, 0, 0). If all six lengths are distinct, then the point in the lattice representing the position of x with respect to y will follow a path along the arcs of the lattice, since each transition will only

involve two points and result in a well-behaved (in the sense that it leads from a relation to another relation whose dimension differs by one). Hence one and only one among the increasing paths between (0, 0, 0) and (6, 6, 6) is followed. In the particular case where two or more lengths are the same, the representing point will take short-cuts, corresponding to jumps between dimension of 2 or more.

This suggests that the neighborhood structures we get are in some sense the most general ones, and that specific neighborhood structures can be obtained from them by considering further properties of the objects considered.

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