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# Shift Design Personnel Task Scheduling Problem with Equity: Improving a constraint-based approach with a Large Neighborhood Search

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## Abstract

This paper introduces a new approach to solve the Shift-Design Personnel Task Scheduling Problem with an Equity criterion. This problem has been introduced in the context of drug evaluation, which requires to assign a set of medical tasks, fixed in time, to a set of skilled employees, whose shifts are to design. Employees' schedules must consider both legal and organizational constraints. The optimality criterion concerns the equity of the workload sharing. Two dedicated methods, one based on Constraint Programming and the other on a two-phase method, have been proposed in the literature to solve this problem. In this paper, we propose a new approach by improving the Constraint Programming model with a Large Neighborhood Search and we show that this new method clearly outperforms existing literature.

*Keywords:* Personnel Task Scheduling Problem, Fixed Job Scheduling Problem, Task Assignment, Shift Design, Equity, Large Neighborhood Search

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## 1. Introduction

The Shift-Design Personnel Task Scheduling Problem with an Equity criterion (SDPTSP-E) has been introduced by [10]. This has been designed to model a particular problem that arises in a drug evaluation company and that cannot be addressed with classical methods.

As many staff scheduling problems in that field, this problem addresses in fact two different questions: how to organize a given workforce in order to reduce the costs to cover a workload and how to build employee rosters in order to respect employees constraints and wishes. According to all the possible constraints, objective functions and context, such problems may be very different. This justifies the whole research field dealing with staff scheduling as described in [5]. Nevertheless, none of those problems deals with such structured workload as the one here.

Indeed, the SDPTSP-E has been designed to address some particularities of a case-study problem. To evaluate the impact of new drugs on the human body, the company hires some healthy volunteers to consume given drugs and go through a set of tests, such as blood analysis for example. Those analysis have to be done at a fixed time to respect the protocol of the drug evaluation. By considering all the ongoing protocol, it leads to a large set of fixed tasks that have to be assigned to employees according to their availability and skills. This assignment problem can be considered as a complete problem, close to the Personnel Task Scheduling Problem ([7]) or the Fixed Jobs Scheduling Problem defined by [6]. However, these problems hardly consider legal constraints. On the contrary, the Nurse Rostering problem [3] or the Tour Scheduling Problem ([12, 1]), which deal with such constraints, do not consider fixed tasks.

Hence, the combination of fixed tasks assignment and rosters design leads to consider a new problem, named Shift-Design Personnel Task Scheduling Problem (SDPTSP). Moreover, to the best of our knowledge,

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the equity criterion has not been investigated in such a way in any other problem. Within SDPTSP-E, the objective is to minimize the biggest gap between a targeted workload and the assigned workload. The targeted workload is different for each employee and it is fixed by the decision-maker depending on many criteria such as employees' availabilities and skills.

The SDPTSP-E has been studied in [10] with a constraint-based approach, that integrates every part of the problem and in [14] using a two-phase method, that iteratively investigates the shifts design and the tasks assignment. In this paper, we keep the constraint-based approach introduced in [10] and use it within a Large Neighborhood Search. This kind of approach has proven very efficient on some problems, such as explained in [13].

The remaining of the paper is organized as follows. In Section 2, we recall the formal definition of the problem. Then, for the sake of completeness, we briefly recall in Section 3 the main features of the constraint-based model (CP). In Section 4 we explain in details the design of the Large Neighborhood Search (LNS) that we propose. Finally, we present in Section 5 a wide range of computing benchmarks, which highlight the interest of using the LNS on top of the CP model.

## 2. Problem description

For the sake of completeness, this section describes briefly the SDPTSP-E. This model has been introduced to tackle a particular problem that arises in a drug evaluation company. Note that an extensive description of this challenging problem is given in [10].

Let us consider a set of tasks with fixed starting and finishing times. Tasks have to be assigned to a set of workers on a one-week horizon, while trying to minimize the inequity among workers. In this particular context, a task corresponds to a fixed interval of work that cannot be preempted and must be assigned to exactly one worker, respecting skills and availabilities requirements. We call *shift* of a worker the time he/she spends at work for a given working day, i.e. a shift corresponds to a period when the worker is available to perform tasks. No task can be assigned to a worker out of a shift. While designing shifts, note that only meetings and medical tasks are considered. Other work activities, such as writing medical reports, are not scheduled, so as to let each worker organize her/himself freely. Indeed, administrative work is the only work activity that is not fixed in time and that can be preempted, which explains why they are not taken into account in this study. As a consequence, designed shifts may be shorter than the real ones, because employees may choose to do their administrative work at the start or at the end of their designed shifts, according to personal and practical issues. While designing shifts, one has to respect a large set of complex constraints, some of them being related to work regulation, while others focus on the internal organization of the company. These constraints are detailed below.

### 2.1. Constraints

Within the SDPTSP-E, we consider two different types of constraints, the organizational and the legal ones. All those constraints have to be respected to ensure the feasibility of the overall schedule.

#### 2.1.1. Organizational constraints

The organizational constraints are the following ones:

- Every task must be assigned to one employee.
- Employees cannot perform tasks which require unmastered skills.
- Employees cannot perform tasks while unavailable.
- The assignment of meetings must be respected.
- Employees can do only one task at a given time
- Tasks starting after 6 a.m. on a given day must not belong to the same shift as tasks starting before 6 a.m.

The purpose of this last constraint is to avoid shifts starting in the middle of the night and ending in the middle of the morning, because they are not appreciated by employees. Indeed, tasks starting before 6 a.m. should be considered as night tasks, meaning that they should be performed during a night shift, whereas tasks starting after 6 a.m. should be considered as morning tasks, meaning that they should be performed during a morning shift.

### *2.1.2. Legal constraints*

In the following, we introduce the notion of daily working time and duration of a working day. Note that there is a difference between these two terms: If the shift starts before noon and ends after 2:30 p.m., then a one-hour lunch break has to be removed from the duration of the shift to obtain the working time. For example, if a shift starts at 11 a.m. and ends at 5 p.m., the corresponding working time is equal to 5 hours, but the duration is 6 hours. We also use the notion of working day for an employee: A day is considered as worked if the employee is working between 6 a.m. and 6 a.m. in the next morning.

Legal constraints are the following ones:

- The daily working time must not exceed 10 hours.
- The weekly working time must not exceed 48 hours.
- The duration of a working day must not exceed 11 hours.
- The duration of a rest period must not be less than 11 hours.
- The duration of the weekly rest must not be less than 35 hours.
- Series of consecutive working days must not exceed 6 days.
- Employees do not work more than one shift per working day.
- Depending on the starting and ending times of their shifts, workers must have a lunch break or not.

This last constraint needs further explanations. In [10], it has been proposed to deal with lunch break, by ensuring that an employee who should have a lunch break does not work during at least one hour in his/her shift. This corresponds to the industrial way of dealing with lunch break. Note that the break is not necessarily placed in the time window [12a.m.; 2 : 30p.m.], since it is possible that an employee has to do a long task that completely overlaps lunch hours. In that case, we cannot enforce the lunch break to lie in this interval, because there would be no solution. In practice, when such a case occurs, the employee is flexible and does a break before or after such a task. In order to deal correctly with the lunch break, we hence decide to bound the sum of the processing times of tasks assigned to a worker in a given shift by the maximum between the daily working time and five hours (this corresponds to the length of the longest tasks in the instances). It means that, if a shift is starting at 11 a.m. and ending at 7 p.m., then the sum of tasks' processing times cannot exceed seven hours (we have to keep one hour for the lunch break). If the shift is starting at 11 a.m. and ending at 3 p.m., then the sum of the processing times is bounded by four hours (no need to keep time for the lunch break, if the length of the shift is less than five hours, break can be done before or after time of the assigned shift).

### *2.2. Equity*

In order to build good schedules, decision-maker has to share the workload among workers in a fair way. The underlying idea is to find a solution where employees have the same global amount of work. However, as explained previously, administrative activities are not taken into account while designing individual schedules. As a consequence, one cannot hope to share the workload in a fair way based on the designed shifts only. Since the amount of administrative work may be different for each employee, they should not perform the same amount of fixed tasks so as to obtain at the end an equivalent global workload. Consequently, it is required to distinguish workers according to the amount of administrative work they

have to perform. For that purpose, the decision-maker associates with each employee a weekly *targeted workload*, corresponding to the amount of time each worker should dedicate to the fixed tasks we want to schedule.

The idea of the objective function is to assign to each worker a set of fixed tasks that corresponds to an amount of work as close as possible to her/his targeted workload. Indeed, if a worker is above his/her targeted workload, then he/she will have great difficulties to perform all his/her administrative work. On the contrary, a worker that is under his/her targeted workload, will have free time. Of course, workers prefer to be under their targeted workload, but we assume that the sum of the processing times of the fixed tasks is equal to the sum of the targeted workloads. Therefore, if some workers are under their targeted workload, it means that other workers are above, which is not fair.

In a more formal way, the objective function we consider in this paper is the *equity* among workers, that we want to maximize. It is equivalent to minimize the inequity, defined as the difference between the highest and the lowest worker's gap value, itself being the difference  $\Delta_i$  between the targeted workload and the real workload of the worker  $w_i$ . We hence want to minimize  $\Delta = \max_i \Delta_i - \min_i \Delta_i$ . A small difference between the highest gap and the smallest gap means that the workload is well balanced among workers. This objective criterion is illustrated on Figure 1. In Figure 1a, the difference between the targeted and the real workload is null for the worker  $w_2$ , but not for  $w_1$  and  $w_3$ , leading to a strictly positive value of the objective function. On the contrary, on Figure 1b, the real workload perfectly matches with the targeted workload for each worker, and we hence have an optimal solution.

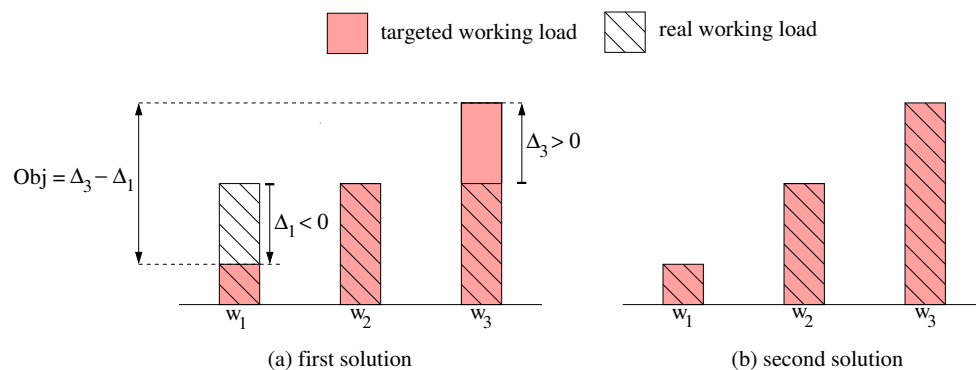


Figure 1: Objective function: equity

### 2.3. Feasibility

According to the global workload and the way it is spread along time-horizon, it may be impossible to find a solution that respects all the constraints. In particular, it may be difficult to cover every fixed tasks. If the decision-maker is not able to find an agreement with some employees in order to relax a few constraints (i.e. longer days or shorter breaks), then externals have to be hired to strengthen the regular workforce. In this case, the decision-maker tries to hire as few as possible externals, which requires a deep insight on the problem at hand to evaluate the different time-windows where additional workforce is needed on an acceptable timetable for regular workers, according to the structure of activity peaks, or unassigned tasks distribution. Therefore, in this paper (as in [10] and [14]), we relax the constraint which specifies that each task has to be assigned to exactly one employee. Hence, we can reach solutions with a partial assignment (i.e. some tasks are left unassigned). These solutions will be used by the decision-maker to evaluate externals requirements. However, a complete assignment is always better than a partial one. So, in this paper, we consider the minimization of the number of unassigned tasks and the minimization of the inequity among workers in a lexicographic order.

### 3. Constraints Modeling

This section is dedicated to the description of the Constraint Programming (CP) model introduced in [10] to handle the SDPTSP-E. The main idea of the model is to assign tasks to workers using a set variable model. More precisely, set variables  $Ta[n]$  give the set of tasks assigned to worker  $n$  on the whole horizon, while set variables  $Da[d][n]$  give the set of tasks assigned to worker  $n$  on day  $d$ . The domain of these variables is initially defined according to employee skills and availabilities. While solving the problem, these sets are reduced by constraints filtering. Constraints over shift sequences are posted on boolean variables  $Sh[d][n]$ , which define whether shift  $d$  is a working day or a day-off. Most of the legal constraints are posted on integer variables  $Fi[d][n]$  and  $La[d][n]$ , which stand for the starting and ending time of each day of each worker. These variables are linked with variables  $Da[d][n]$  so as to give the start (respectively the end) of the earliest (respectively latest) task assigned to a worker a day. The model also consider integer variables  $At[d][n]$ , which define for each day and worker the minimal attendance time (i.e. the attendance time resulting from fixed tasks) and integer variables  $Ld[d][n]$  to tackle the duration of the lunch break. A matrix  $Wd$  gives for each day and each worker the daily working time corresponding to the shift performed by the employee. In most cases,  $Wd$  is equal to the length of the shift performed by the employee, however, if the employee has a lunch break, then  $Wd$  is equal to the length of the shift minus the duration of the lunch break (one hour). Finally, the vector  $Bs$  gives for each worker the starting time of the weekly break.

Using those variables, it is possible to consider all the organizational and legal constraints within a CP approach as explained in [10].

The basic branching scheme considers the original variables  $Ta[n]$ , which corresponds pick an employee  $n$  and assign him/her a task among the possible ones. The different variables ordering strategies either consider first employee with a small amount of work (Less Working strategy) or employee with the highest amount of work (Most Working strategy). In the first case, the objective is to reduce unfairness of the partial solution, while in the second one the objective is to assign an amount of work to employees already involved. The principle here is to respect as much as possible the structure of the shifts currently in building and keep other employees free if possible. A parameter is fixed in order to have a limit on the amount of work that can be assigned to an employee. The interest here is to give priority to complete solutions.

Once a variable has been chosen, the Value Ordering strategy fixes the way to branch on the different possible values for this variable. Authors proposed different strategies based on biggest tasks or biggest overlap among tasks for example. the most interesting one here is the one named Lack of Nurse. This strategy leads to consider tasks that are difficult to cover, which means the number of eligible employees is the smallest.

The CP method leads in 5 minutes of computation to acceptable results regarding the number of unassigned tasks and inequity, nevertheless in [14], it has been proven that it is possible to improve these results on several instances with a two-phase method. After investigation, it appears that the CP model is quite efficient to reach a solution, but is sometimes unable to improve it for a while, even if the solution is far away from the best one. On the opposite, the two-phase method is able to explore much more solutions, that may be quite far away from each other. For that reason, we decided to mix both advantages in a Large Neighborhood Search, based on the CP model.

### 4. Large Neighborhood Search

This section describes the Large Neighborhood Search (LNS) we propose to solve the SDPTSP-E. Note that LNS approaches have proven very efficient on many problems, such as Vehicule Routing Problems for example ([15]). Generally speaking, such an approach is a good way to improve the performance of exact approaches such as CP models, especially on large scale instances. Indeed, CP approaches are usually very good at finding a first solution, but they often encounter difficulties to improve them due to a lack of diversification during the search. On the contrary, LNS approaches enable to explore very different and interesting part of the search space. Therefore, using a CP model within an LNS framework allows to benefit from the filtering and propagation processes of CP, while exploring very different parts of the search space. Since the CP model is used only on a subpart of the whole problem, its ability to filter is much higher than

on the whole problem, leading to an efficient exploration. In the following, we first introduce the global structure of the LNS, before describing more precisely how we destroy and repair solutions.

#### 4.1. Principles and global structure

Basically, LNS are based on an iterative process which successively destroy and repair a given solution so as to improve it step by step. The main idea is to focus on the part of the solution which looks suboptimal according to some heuristic indicators, so as to destroy it. This destruction leads to a partial solution, which maybe used as a starting point to reach a whole neighborhood of new solutions. The repair step consists in exploring a part of this neighborhood so as to find an improving solution. The improvement we are able to reach mainly depend on the subpart of the solution which is chosen to be destroyed. The most important thing is to destroy decision variables that are somehow related to each others, so as to be able to reach new solutions. To do so, a very effective way is to take into account the structure of the problem, so as to work on a sub-problem that is quite independent from the rest of the solution. Otherwise, one may have no choice but to rebuild the exact same solution, which is not desired. As a consequence, the destroyed part has to be large enough to allow the method to reach new solutions, and if possible to escape from a local optima, but not too large, in order to keep the reparation step efficient. Using a CP approach to repair the current solution enables to benefit from strong filtering, but this comes at computing price (compared to a simpler heuristic repair process). As a consequence, we choose to use the best solution at each iteration so as to avoid spending too much time in diversification (LNS already enables enough diversification). Indeed, we use two different and randomized operators to make the neighborhood varying at each step. The approach ends up when reaching a given time-limit.

In the context of a CP based LNS, the destroy step amounts to choosing a subset of the decision variables to unstantiate, while other decision variables stay instantiated to their value in the best solution. As mentionned, a good way to design the destruction operators is to take into account the structure of the problem. In our context, we deal with both shifts design and tasks assignment, which naturally leads to consider two operators, one for each aspect. To handle the tasks assignment aspect, we use a time operator which unassigned a set of tasks that are fixed within the same time interval, so as to allow tasks exchanges between employees. To deal with the shifts design problem, we use an employee operator that unassigned all the tasks assigned to a subset of workers, so as to allow changes on their working shifts. One of those two different operators is randomly chosen at each iteration in order to destroy a part of the current solution. Then, this subpart of the solution is rebuilt with the CP approach, leading to a given neighbor.

#### 4.2. Destruction operators

As mentionned previously, destroying a part of the current solution amounts to uninstantiating some decision variables. While doing this, it is important to keep the number of uninstantiated variables under control, so as to have an efficient repair step. We propose to use a parameter *Size* to fix the number of uninstantiated variables.

##### 4.2.1. Time-oriented operator

This time-oriented destruction operator  $T_{op}$  focuses on the task assignment part of the SDPTSP-E. The idea is to focus on a specific time-period so as to destroy the assignment of tasks that are overlapping in time (or close to each other). The time-interval is built randomly but in such a way that it covers a task that was left unassigned in the best solution. If every task is already assigned, one is picked randomly.

Once the task  $t_i$  is chosen, a given number of tasks which are as close as possible to  $t_i$  regarding their starting and ending times is picked. This number is given by the parameter *Size*, and discussed further. Note that the corresponding unassign time interval may not be centered around  $t_i$  in order to allow this operator reaching several neighborhoods even when picking the same initial task. All these considered tasks are unassigned, before using the CP model to rebuild a solution.

This operator enables a very high number of exchanges between employees who work on the selected time interval. Since tasks may last till 5 hours, it is important to consider a large time interval so as to give enough flexibility to the CP model in order to reach a better assignment between involved employees.



The use of this operator is illustrated on Figure 2. On Figure (2a), we can see a part of a solution where task  $t$  is currently not assigned. Destroying the assignments of the three workers on that particular time interval allows to rebuild this part of the solutions in a better way, as we can see on Figure (2b), where every task is assigned. Note that, this improving solution may not be found by the CP model in case of a partial exploration of the neighborhood.

On the whole, the time-oriented operator is able to deeply impact a specific working day, but it may encounter big difficulties to change the sequence of working days. For instance, changing a day-off into a working day may be very difficult. Indeed, while trying to assign new tasks on an employee who was previously in a day-off, her/his other working days remain fixed, which would probably lead to violate some legal constraints. To that purpose, we propose next another operator that better fits the need of deep changes in the structure of an individual schedule.

#### 4.2.2. Employee-oriented operator

In order to be able to perform deep changes in the structure of an individual schedule, we have to consider legal constraints that may block the shift assignment of an employee regarding the previous and followings shifts. To that purpose, we propose a second operator  $P_{op}$  that focuses on shifts design, and allows to unassign the tasks of a subset of workers on the whole week. To limit the number of decision variables that are uninstantiated, we now use the parameter *Size* to control the number of employees whose schedule is destroyed.

This operator randomly picks a subset of workers and destroy their whole schedule so as to be able to design new shifts sequences. This operator may be useful to build new shifts that cover some tasks. Figure 3 illustrates the interest of this operator. Figure (3a) shows a partial assignment, with an unassigned task lying in the middle of the resting time of all employees. As a consequence, this task cannot be assigned to any employees without changing at least two days (and probably the whole week). Using the employee-oriented operator enables to reach an interesting neighbor presented in Figure (3b), where new shifts have been created to cover task  $t$ .

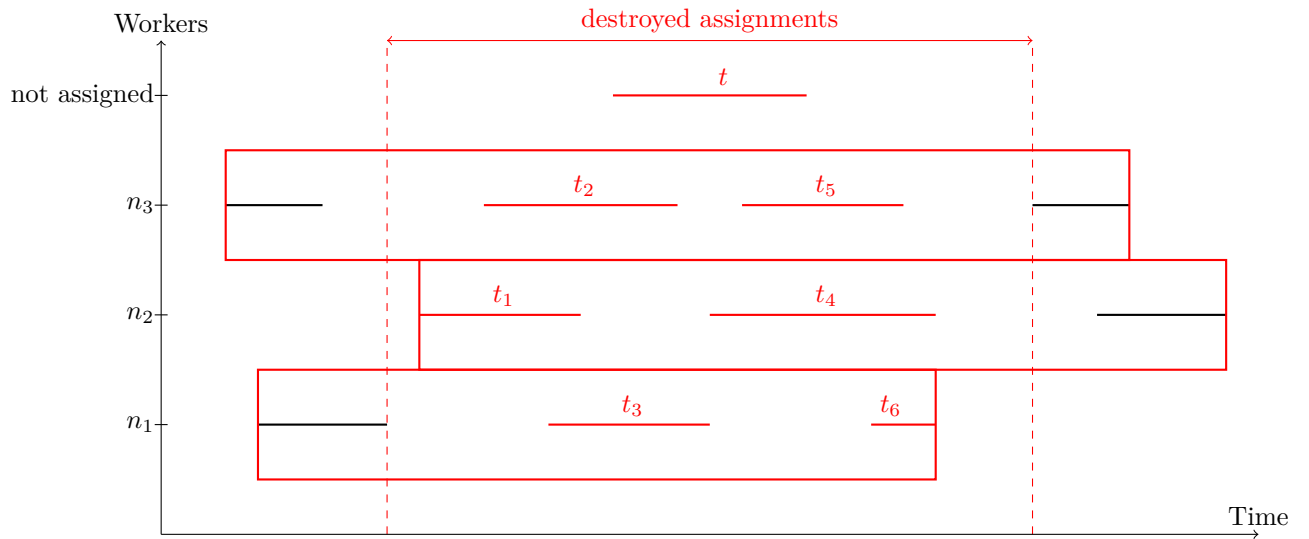
#### 4.3. Reparation step

After using one of the previously described operators, we use the CP model to explore the corresponding neighborhood by repairing the current partial solution. However, a complete exploration would presumably be too time-consuming, because of the size of the neighborhood which may be quite large. For that reason, and according to the fact that the CP model is better reaching a solution than improving it, we limit the search to the first solution reached at each iteration. Nevertheless, we add a new constraint to filter solutions that are strictly worse than the previous best one regarding the number of unassigned tasks. This configuration is quite unusual, because repairing the solution with the CP model takes more time than with a simple heuristic. Thus, one usually does not want to loose time exploring solutions that are not strictly better than the current one. However, it turns out this way of exploring the neighborhood is much more efficient in our context, as explained in the analysis of experimental results.

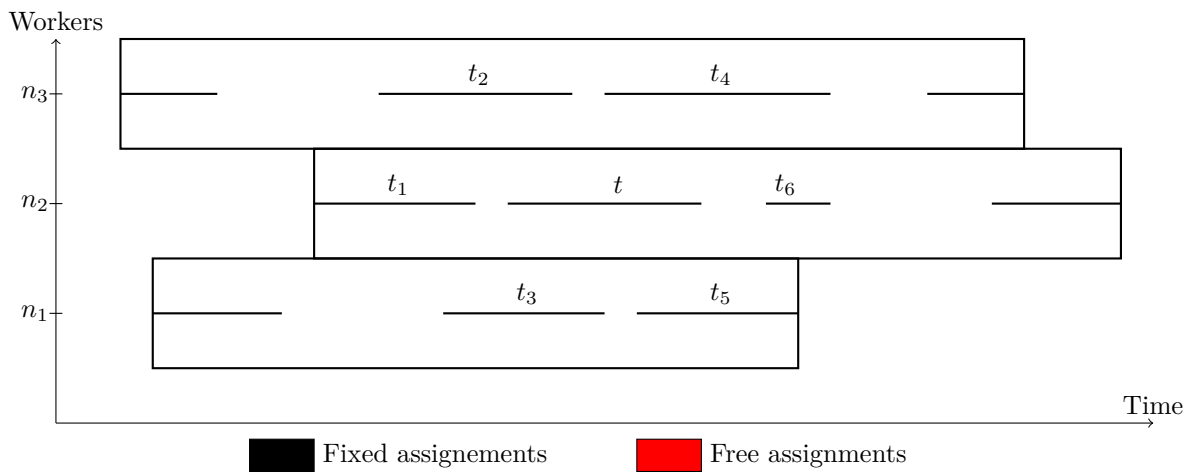
In order to repair the solution in an efficient way, one has to pay attention to the search strategy. In [10], the authors use several deterministic strategies, some of them being good at finding fair solutions, and others at assigning a maximum number of tasks. However, within a LNS framework, we assume that the same neighborhood (or nearly the same) may be explored several times, at different iterations. Therefore, using a deterministic strategy is presumably not a good idea, because it would lead to build the same solution many times. To deal with this issue, we propose to introduce some randomness in the search strategy while keeping the general idea of the best search strategies described in [10]. To that purpose, we introduced a new vector of integer variables  $emp[t]$ , which gives the employee assigned to the task  $t$ . Obviously, we also post channelling constraints between  $emp[t]$  and  $Ta[n]$ . Then, it is possible to branch directly on  $emp[t]$ , which makes it possible to benefit from some interesting search strategies of the literature.

To stay close to the strategy Lack of Nurse (LN), we could branch on the variable  $emp[t]$  with the smallest domain, but this strategy would also be deterministic. Therefore, we propose to branch on variables  $emp[t]$  according to a dom/wdeg strategy [2]. It means that we branch on the variable  $emp[t]$  with the smallest



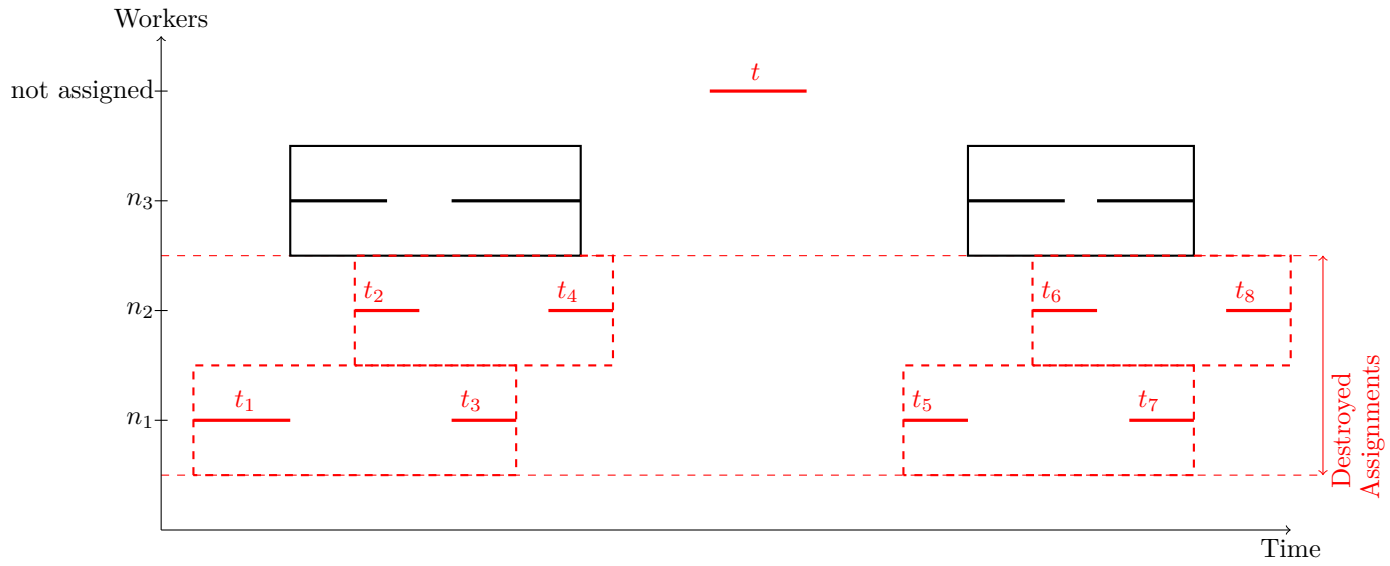


(a) Neighborhood of the current solution (3 workers and 11 tasks, with 1 task unassigned)

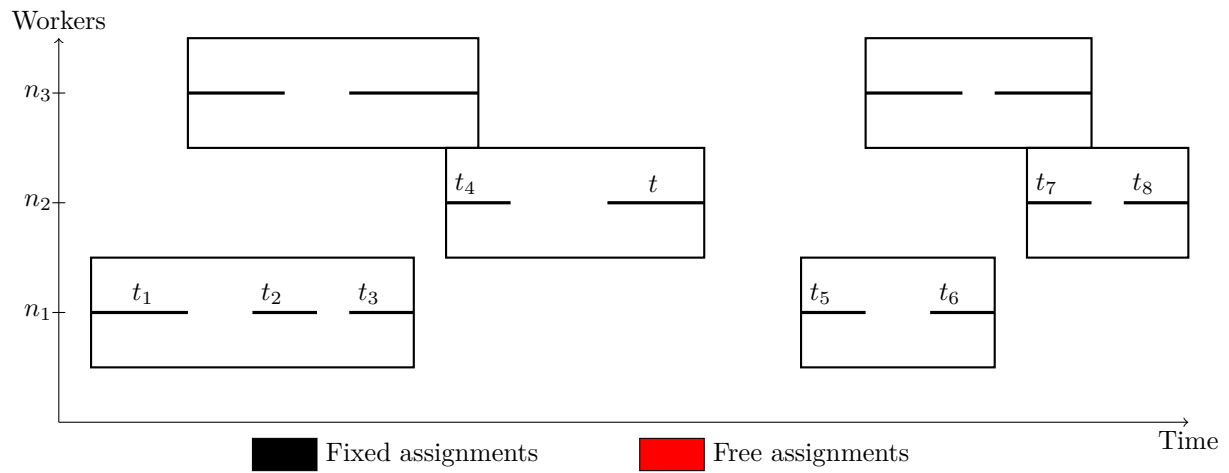


(b) Improved solution, all tasks are assigned

Figure 2: Using the "Time-oriented" destruction operator  $T_{op}$



(a) Neighborhood of the current solution (3 workers and 13 tasks, with 1 task unassigned)



(b) Improved solution, all tasks are assigned

Figure 3: Using the "Employee-oriented" destruction operator  $P_{op}$

ratio between its domain size and the number of backtracks due to a related constraint. Moreover, we use the last-conflict heuristic [11] on top of dom/wdeg, in order to give priority to the variable that has led to the last backtrack. Once the variable is chosen, we branch on the smallest possible value in its domain. This could be seen as a variant of the strategy Most Working (MW), because it leads to assign the workload in an unbalanced way (first workers are chosen in priority). On the whole, we obtain an adaptative search strategy that is close to the MW-LN strategy, which was one of the best strategy to build complete solutions.

#### 4.4. Improvement step

Because of its structure, the proposed search strategy is presumably not very good at finding fair solutions, which is consistent with our goal to deal with the equity only in a second step, after finding a solution as complete as possible. Therefore, we need to deal with the equity in a post processing step. To do that, we use the local descent  $H_{\Delta}$  introduced in [14]. Basically, it amounts to moving and swapping tasks between workers so as to continuously improve the equity. Surprisingly, this descent is very efficient, even on solutions with a poor equity. Since the descent is quite fast, we decided to use it on each complete solution. In our case, it turned out to be very effective because, the LNS is able to generate a lot of different solutions, even for the same number of unassigned tasks.

## 5. Experimental results

The CP model has been implemented with Choco Solver, a Java Constraint Satisfaction Problem Solver [4]. However, while keeping the CP model proposed in [10], we updated its results because they were reached with the second version of Choco Solver, whereas the proposed LNS uses the third one. Instances have been run on an Intel Core i3-540 (3.06 GHz & 8G RAM) with a time limit of 5 minutes, which is the same configuration as in the CP approach of [10] and the two-phases method (M2P) of [14]. Note that instances have been introduced in [10], and can be found in [9]. We first present the results obtained with the LNS, then establish a comparison on all existing methods.

### 5.1. Large Neighborhood Search results

The results of the LNS are given in details in this section. We recall that a solution with a complete assignment of tasks to workers is referred to as a *complete solution*, by opposition to a *partial solution*. Within this section, we use the indicator «C» to indicate the number of complete solutions reached, «I» is the mean inequity value of the best solution in minutes (over complete solutions only), «A» is the average percentage of assigned tasks (over partial solutions only) and «T» is the mean computation time of best solutions, in seconds. Note that it is not possible to always find a complete solution in the instances of the benchmark. It has been proven that (at least) 102 instances do not admit a complete solution by an Integer Programming approach [8]. Hence, even if the benchmark is composed of 720 instances, we compare the number of complete solutions that we can reach with the 618 instances for which it may be possible to find a complete solution.

First of all, we focus on the parameter *Size*. Based on some preliminary computational tests, we decided to bound it depending on the size of the instance. Thus, when dealing with the employee-oriented destruction operator  $P_{op}$ , it ranges from 5 to 30 percents of the workers. Likewise, when dealing with the time-oriented destruction operator  $T_{op}$ , it ranges from 5 to 30 percents of the tasks. The parameter is initially set at its lowest value, and increased after 50 iterations without improvements. To be able to escape local optima quickly, we do not simply increment the parameter *Size*, but we increase it by one quarter of its varying range (leading to increase *Size* by about 6 percents). To balance this quick and strong increase, the parameter *Size* is decreased when an improving solution is found.

Before evaluating the interest of the LNS, which focus on improving an existing solution, we need to evaluate the performance of the new branching strategy on variables  $emp[t]$ . To do that, we compare the initial solution of the CP model with the initial solution of the LNS (Tables 1 and 2). Interestingly, the LNS branching strategy turned out to be more efficient than the best one of the CP model, over all possible branching strategies introduced in [10], which highlights the interest of the variables  $emp[t]$ . More precisely,

we can see that the LNS branching strategy leads to 513 complete solutions against 481 with the CP branching strategy, on a total of 618 instances where it is possible to have complete solutions. However, as expected, the inequity of those initial solutions is still very high.

I <sub>t</sub>	Crit.	I <sub>v</sub>			Total
		600	800	1 000	
100	C	53/54	31/47	4/21	88/122
	I	129	154	108	137
200	C	59/60	44/55	8/35	111/150
	I	133	144	164	140
300	C	58/58	51/58	30/56	139/172
	I	132	132	139	134
400	C	59/59	52/59	32/56	143/174
	I	141	129	162	141
Total	C	229/231	178/219	74/168	<b>481/618</b>
	I	134	138	150	<b>138</b>

Table 1: Best initial solutions obtained with CP model and  $H_{\Delta}$

I <sub>t</sub>	Crit.	I <sub>v</sub>			Total
		600	800	1 000	
100	C	52/54	37/47	5/21	94/122
	I	280	487	684	383
200	C	59/60	48/55	12/35	119/150
	I	260	368	416	319
300	C	58/58	51/58	36/56	145/172
	I	271	308	471	334
400	C	59/59	56/59	40/56	155/174
	I	225	325	497	331
Total	C	228/231	192/219	93/168	<b>513/618</b>
	I	258	362	486	<b>339</b>

Table 2: Iniales solutions with LNS (`dom/wdeg + last conflict-min value-HΔ`)

After those tests, we would like to introduce the results of the LNS with only one of the two destruction operators we proposed. We wanted to evaluate the impact of the two destruction operators.

First, we ran the LNS on all instances with only one destruction operator. Our goal was to determine wether or not these two operators are indeed complementary or not. As illustrated by Tables 3 and 4 it is clear that each operator leads to an improvement both on the number of complete solutions and the inequity. Therefore, we may conclude that these two operators allow to escape from local optima by reaching neighborhoods that are more difficult (if not impossible) to reach by the other one alone.

All those results include the improvement step  $H_{\Delta}$ , used on all complete solutions. The local descent is very effective, even when starting from poor solutions, and may lead to huge improvement on the inequity. Nevetheless, the resulting gap with the original equity of the considered solution is not possible to predict. Therefore, it is more effective to look for many different solutions, so as to call  $H_{\Delta}$  as many times as possible. Improvements that this step showed for the two-phase method [14] is there confirmed on all preliminary tests.

I <sub>t</sub>	Crit.	I <sub>v</sub>			Total
		600	800	1 000	
100	C	53/54	44/47	14/21	111/122
	I	14	16	106	27
	A	97,86	98,13	96,76	97,19
	T	265	220	71	185
200	C	59/60	50/55	19/35	128/150
	I	20	17	25	20
	A	99,00	98,75	98,11	98,25
	T	295	251	99	215
300	C	58/58	54/58	44/56	156/172
	I	24	19	44	28
	A	99,67	99,22	98,92	99,06
	T	290	270	221	260
400	C	59/59	57/59	43/56	159/174
	I	31	24	48	33
	A	99,75	99,75	99,18	99,29
	T	295	285	221	267
Total	C	229/231	205/219	120/168	<b>554/618</b>
	I	23	19	50	<b>27</b>
	A	98,46	98,63	97,85	<b>98,06</b>
	T	286	257	153	<b>232</b>

Table 3: LNS Results while using only T<sub>op</sub> (5 min)

I <sub>t</sub>	Crit.	I <sub>v</sub>			Total
		600	800	1 000	
100	C	53/54	45/47	15/21	113/122
	I	10	10	16	11
	A	97,86	98,07	97,04	97,36
	T	265	225	77	189
200	C	59/60	51/55	25/35	135/150
	I	18	13	19	16
	A	99,00	98,83	98,06	98,23
	T	295	255	128	226
300	C	58/58	54/58	46/56	158/172
	I	27	22	20	23
	A	99,67	99,33	98,93	99,11
	T	290	272	232	265
400	C	59/59	57/59	44/56	160/174
	I	35	28	29	31
	A	99,75	99,75	99,19	99,30
	T	294	285	229	269
Total	C	229/231	207/219	130/168	<b>566/618</b>
	I	23	19	22	<b>21</b>
	A	98,46	98,66	97,92	<b>98,12</b>
	T	286	259	167	<b>237</b>

Table 4: LNS Results while using only P<sub>op</sub> (5 min)

## 5.2. Overall comparison

To evaluate the performance of the proposed LNS, we compare its results with those of the M2P, introduced in [14], because it is the best known method in the litterature. Tables 5 and 6 show that the LNS is able to find much more complete solutions than M2P and with a better equity. Moreover, partial solutions have on average a smaller number of unassigned tasks than with M2P. Note that the LNS is also slightly faster than the M2P to reach its best solution.

Even if the LNS gives very good results on average, we should also pay attention to its robustness regarding to the quality of its results. In this context, we would like to know if it gives very bad results on a few instances, so as to catch the configuration it fails to solve. We hence counted the number of best solutions computed by each method. On Table 7 the quality of a solution is evaluated in a classical way, whereas we just look at the number of unassigned tasks in Table 8. Based on these results, it is clear that the LNS dominates the other approaches, since it reaches 531 best solutions on a total of 720 instances, compared to around one hundred for the CP model and M2P. When considering only the number of unassigned tasks, the LNS performs even better, since it finds the best solution 719 times out of 720.

I <sub>t</sub>	Crit.	I <sub>v</sub>			
		600	800	1 000	Total
100	C	54/54	46/47	20/21	120/122
	I	17	14	17	16
	A	97,67	98,07	97,10	97,38
	T	144	117	128	127
200	C	59/60	53/55	29/35	141/150
	I	27	24	26	26
	A	99,00	98,71	98,90	98,13
	T	73	175	162	162
300	C	58/58	54/58	47/56	159/172
	I	33	26	29	29
	A	99,67	99,33	98,90	99,10
	T	180	170	171	172
400	C	59/59	58/59	44/56	161/174
	I	34	31	38	34
	A	99,75	99,75	99,27	99,34
	T	278	289	195	209
Total	C	230/231	211/219	139/168	<b>581/618</b>
	I	28	24	30	<b>27</b>
	A	98,41	98,60	97,95	<b>98,12</b>
	T	158	154	155	<b>155</b>

Table 5: LNS Results in 5 minutes

I <sub>t</sub>	Crit.	I <sub>v</sub>			
		600	800	1 000	Total
100	C	53/54	42/47	11/21	106/122
	I	28	35	72	35
	A	97,50	97,78	96,71	97,05
	T	145	155	167	156
200	C	59/60	50/55	22/35	131/150
	I	34	35	42	36
	A	99,00	98,60	97,72	97,93
	T	166	138	156	154
300	C	58/58	53/58	42/56	153/172
	I	40	38	46	41
	A	99,67	99,19	98,76	98,94
	T	191	186	174	184
400	C	59/59	55/59	40/56	154/174
	I	47	44	51	47
	A	99,75	99,70	99,01	99,17
	T	236	202	196	211
Total	C	229/231	200/219	115/168	<b>544/618</b>
	I	38	38	49	<b>40</b>
	A	98,28	98,47	97,68	<b>97,90</b>
	T	184	170	173	<b>176</b>

Table 6: M2P results in 5 min

I <sub>t</sub>	Method	I <sub>v</sub>			
		600	800	1 000	Total
100	CP		2	3	5
	LNS	56	54	41	151
	M2P	4	4	16	24
200	CP	7		4	11
	LNS	47	52	50	149
	M2P	9	9	7	25
300	CP	28	6	3	37
	LNS	26	50	46	122
	M2P	8	5	11	24
400	CP	40	9	6	55
	LNS	20	42	47	109
	M2P	1	13	9	23
Total	CP	75	17	16	108
	LNS	149	198	184	531
	M2P	22	31	43	96

Table 7: Number of best overall results

I <sub>t</sub>	Method	I <sub>v</sub>			
		600	800	1 000	Total
100	CP	58	50	17	125
	LNS	60	60	59	179
	M2P	57	48	29	134
200	CP	60	49	27	136
	LNS	60	60	60	180
	M2P	60	54	32	146
300	CP	60	58	41	159
	LNS	60	60	60	180
	M2P	60	57	49	166
400	CP	60	59	40	159
	LNS	60	60	60	180
	M2P	60	57	43	160
Total	CP	238	216	126	580
	LNS	240	240	239	719
	M2P	237	216	153	606

Table 8: Number of best results on the number of unassigned tasks

## 6. Concluding remarks

This paper deals with a new method for the Shift-Design Personnel Task Scheduling Problem with Equity. This real-life problem deals simultaneously with the assignment of fixed tasks and the design of personnel shifts, so as to minimize the number of unassigned tasks while sharing the workload between employees in a fair way.

We presented here a Large Neighborhood Search, based on a CP model and a local descent. This method clearly improves the state-of-the-art results and it gives new best solutions on several instances. The research

avenues that still stand for this problem now deal with two main points. On one hand, it would be useful to improve the quality of lower bounds so as to evaluate more precisely the quality our approach. On the other hand, it would be interesting to try an approach based on a column generation model, because it has proven very efficient on personnel scheduling, and the structure of the problem is suitable for this kind of method.

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