A unified approach to Nevanlinna-Pick interpolation problems

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Abstract— **This work deals with Complex-valued interpolation by a Schur rational function of given degree at a set of nodes located in the closed lower half-plane, with prescribed maximum points for the modulus (i.e. points where it is equal to 1) on the real axis. The motivation comes from broadband matching, for which the technique we develop offers a new tool.**

I. INTRODUCTION

From the seventies on, Nevanlinna-Pick interpolation has been widely studied in connection with various problems in signal processing and control [2]. The theoretical approach developed in [16], in which interpolants are viewed as operator dilations, is at the origin of a wide literature including constructive results [19], [10]. Among the most relevant (and also historical) application domains, we mention sensitivity minimization ([18], [19]), filter design and broadband matching [17].

Pick's theorem yields necessary and sufficient conditions on a set of interpolation data in order that there exists a complex analytic function bounded by 1 in modulus (a so-called Schur function) to meet these conditions on the disc or the half-plane. Composing with the Cayley transform $z \mapsto (1-z)/(1+z)$, the problem may be equivalently be stated in terms of Carathéodory functions rather than Schur functions (a Carathéodory function is an analytic function with non-negative real part).

Nevanlinna's theorem gives a description of all solutions to this interpolation problem [9, Ch. I, sec. 2, Ch. IV, sec. 6]. However, in applications, it is essential to control the degree of the interpolant. Interpolation with a degree constraint was considered in connection with the covariance extension problem in [12], in which the spectral density plays a key role in controlling the degree. Subsequently in [10], the following interpolation problem was studed: describe all Caratheodory functions $Y(z)$ of **degree at most** l solving $l+1$ interpolation conditions

$$Y(z_j) = w_j, \quad j = 0, \dots, l,$$

where $z_0, z_1, z_2 \dots z_l$ lie interior to the analyticity domain and the interpolation values $w_0, w_1, w_2 \dots, w_l$ are such that the associated Pick matrix P is positive definite; the last condition corresponds to the indeterminate case (i.e. the problem has more than one solution [8], [1]). An existence result was proved [10, Theorem 5.2], showing that to each polynomial $d(z)$ of degree at most $2l$ there is a pair of polynomials (π, χ) such that $Y = \frac{\pi}{\chi}$ is a Carathodory function of degree at most l satisfying the above interpolations conditions, and having $d(z)$ as "dissipation polynomial":

$$d(z) = \pi(z)\xi^*(z) + \pi^*(z)\xi(z) \tag{1}$$

where the superscript * indicates the paraconjugate polynomial (see definitions below). In fact, after normalization, the correspondence between dissipation polynomials and rational solutions π/χ is one-to-one. This result was proved in [6] for the covariance extension problem when $d(z)$ is strictly positive on the boundary of the analyticity domain, and in [11] in the general case where $d(z) \geq 0$ there. Later, the theory was further adapted to issues in control and filter design, where freedom in the choice of $d(z)$ can been used to shape the interpolants in order to meet additional design specifications (see e.g. [14]).

In the present work, we analyze a still more general version of this problem where both the interpolation conditions and the zeros of the dissipation polynomial may lie on the boundary. Our motivation comes from broadband matching, and it is more natural to formulate the problem in terms of the scattering matrix representation of a two-port, which is a 2×2 matrix of complex-valued functions of the frequency. Both for technical and practical reasons, we choose to work on the lower half-plane $\mathbb{C}^-$ rather than the disk or the more classical right-half-plane. In this framework, the paraconjugate of a polynomial p is $p^*(s) = p(-s)$. The scattering matrix of a lossless filter is thus a 2×2 function, analytic in $\mathbb{C}^-$, and assuming unitary values on the real line (real points represent frequencies). Such a matrix is called inner. The filter is finite-dimensional if its scattering matrix is rational, and finite-dimensional filters are those realizable in practice.

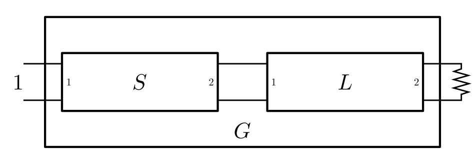

Fig. 1. Filter plugged to a load L with reflection coefficient L_{11}

Hereafter, we identify two-ports with their scattering matrix. The problem of synthesizing the filter S in Figure 1 so that the reflexion coefficient G_{11} of the scattering matrix of the global system is smallest possible on some bandwidth is a very old one. It corresponds to the need of conveying energy to the system L in the bandwidth, rather than having the energy bouncing back. When the filter is finite-dimensional, the so-called matching theory of Fano and Youla [7] provides

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one with a parametrization of all G that can be realized as in Figure 1 for fixed L and varying S of given degree. However, it is still unknown at present how to set up matching filtering characteristics from this parametrization, as soon as the load has degree greater that one. Another approach was proposed by J. Helton [13] in an infinite dimensional setting, where the matching problem gets reformulated as a H^∞ approximation problem of Nehari type, the solution to which is elegantly produced in terms of the norm and maximizing vectors of a Hankel operator. However, the "infinite degree" of this optimal filter makes it hardly realizable or even computable in practice.

We propose here an intermediate approach [3] where a finite-dimensional filter response of prescribed degree is being synthesized by imposing matching and stopping frequencies with respect to some general frequency varying load. A classical computation show that the reflection coefficient G_{11} of the overall system at frequency ω is

$$G_{11}(\omega) = \det(S(\omega)) \frac{\overline{S_{22}(\omega)} - L_{11}(\omega)}{1 - S_{22}(\omega) L_{11}(\omega)}.$$

A matching frequency, that is, a frequency ω for which $G_{11}(\omega) = 0$, satisfies (whenever $|L_{11}(\omega)| < 1$) that

$$S_{22}(\omega) = \overline{L_{11}(\omega)}. \tag{2}$$

In contrast, a stopping frequency ω, defined by the property that $|G_{11}(\omega)| = 1$, satisfies (whenever $|L_{11}(\omega)| < 1$) that $|S_{22}(\omega)| = 1$ or equivalently that

$$S_{12}(\omega) = S_{21}(\omega) = 0. \tag{3}$$

Writing S (of McMillan degree l) in Belevitch form [5], normalized so that $\lim_{s\to\infty} S(s) = I_2$ (see remark 2.1), we have that

$$S = \frac{1}{q} \begin{bmatrix} p^* & -r \\ r^* & p \end{bmatrix}, \tag{4}$$

where p, q are monic complex polynomials of degree l and r is a complex polynomial of degree at most $l-1$, satisfying the Feldtkeller equation:

$$qq^* = pp^* + rr^*. \tag{5}$$

If $(x_1, x_2, \ldots, x_l)$ denotes a set of matching frequencies and if we set $\gamma_j = \overline{L_{11}(x_j)}$ for $j = 1, \ldots, l$, the matching problem amounts to solve (7) whith fixed transmission polynomial r by imposing $l-1$ stopping frequencies. This problem looks quite similar to the one considered in [10], however there is a major difference: interpolation takes place on the boundary. In [3], existence and uniqueness of the solutions were proved under the restrictive assumption that r has no root on the real axis, which is not suited for the application to broadand matching just described. In this paper, we remove this requirement. Moreover, we consider two different normalizations and deal with mixed type of interpolation conditions (in the analyticity domain and on the boundary). We propose a topological approach that provides an algorithmic way to compute the solutions. The tricky part is to address both interpolation points on the real line and non strictly positive dissipation polynomials. This is possible upon moving back and forth from Schur to Carathéodory functions and using representation formulas in the Hardy space $H^2(\mathbb{C}^-)$ for Carathéodory functions.

II. MAIN RESULTS.

Let $X = (x_1, x_2 \ldots x_m)$ be m distinct real points associated to m interpolation values $\gamma = (\gamma_1, \gamma_2 \ldots \gamma_m)$ in $\mathbb{D}^m$, with $\mathbb{D}$ the unit disk. Let further $Z = (z_1, z_2 \ldots z_l) \in (\mathbb{C}^-)^l$ be l distinct complex numbers associated to l complex interpolation values $\beta = (\beta_1, \beta_2 \ldots, \beta_l)$. We define $P(Z, \beta)$ to be the so-called Pick matrix associated with the interpolation data (z_k, β_k), namely the Hermitian $l \times l$ matrix defined by:

$$P_{k,j}(Z, \beta) = \frac{1 - \beta_k \overline{\beta_j}}{i(z_k - \overline{z_j})}. \tag{6}$$

We denote by $\mathbb{P}^+_Z$ (or simply $\mathbb{P}^+$), the set of those $\beta \in \mathbb{C}^l$ such that (Z, β) is positive definite. The total number of interpolation conditions is thus equal to $N = m + l$.

We consider the following two problems, each of which corresponds to a specific normalization :
Given $X \in \mathbb{R}^m$, $\gamma \in \mathbb{D}^m$, $Z \in (\mathbb{C}^-)^l$, $\beta \in \mathbb{P}^+$ and $r \neq 0$ a complex polynomial of degree at most $N-1$, such that $r(x_k) \neq 0$, $k = 1, \ldots, m$,
to find (p, q) a pair of complex polynomials such that,

$$\begin{aligned} \frac{p}{q}(x_k) &= \gamma_k, \quad \text{for } k = 1, .., m && (7) \\ \frac{p}{q}(z_k) &= \beta_k, \quad \text{for } k = 1, .., l && (8) \\ qq^* - pp^* &= rr^* && (9) \end{aligned}$$

where
Problem $\mathscr{P}$: p and q are monic of degree N and q is stable in the broad sense (no root in the open lower half-plane $\mathbb{C}^-$).
Problem $\hat{\mathscr{P}}$: p and q have degree at most $N-1$, q is stable in the broad sense, and verifies the normalization $q(-i) > 0$.

Remark 2.1: Problem $\mathscr{P}$ imposes a normalization at infinity on the interpolant p/q, since p, q are monic of degree N hence $(p/q)(\infty) = 1$. In connection with the matching problem discussed in Section I, where p/q is thought of as the entry S_{22} of the scattering matrix S of a filter, this is justified in that the low-pass equivalent model of LC-resonant filters behaves like an open circuit at infinite frequency, which translates into the normalization $S(\infty) = Id$. However, if for example we add a transmission line in front of the filter, this line can be modeled in the narrow band approximation by a reflection coefficient which is unimodular with free phase at infinity. This extra design parameter can be used to meet an additional interpolation condition or, in a dual way, to reduce the degree of p and q while keeping the interpolation properties of p/q. This leads us naturally to problem $\hat{\mathscr{P}}$.

Let $\mathbf{P}_N$ designate the set of complex polynomials of degree at most N and $\mathbf{PM}_N$ the subset of monic polynomials. To each $Q \in \mathbf{P}_{2N}$ which is non-negative on $\mathbb{R}$, we associate by spectral factorization a unique polynomial $\hat{\varphi}(Q) \in \mathbf{P}_N$ of degree half the degree of Q such that $\hat{\varphi}(Q)(\hat{\varphi}(Q))^* = Q$, which is stable in the broad sense and satisfies $\hat{\varphi}(Q)(-i) > 0$. If in addition Q is monic of degree $2N$, we put $\varphi(Q) \in \mathbf{PM}_N$ for the corresponding polynomial, normalized this time so as to be monic. We also put $\hat{N} = N-1$. Now, for fixed r, Equation (5) associates

- to each $p \in \mathbf{PM}_N$ a unique monic polynomial $q = \varphi(pp^* + rr^*) \in \mathbf{PM}_N$,
- to each $p \in \mathbf{P}_{\hat{N}}$ a unique polynomial $q = \hat{\varphi}(pp^* + rr^*) \in \mathbf{P}_{\hat{N}}$.

The two spectral factorizations φ and $\hat{\varphi}$ allow us to define two evaluation maps which play an essential role in the study of problem $\mathscr{P}$ and problem $\hat{\mathscr{P}}$:

$$\psi: \quad p \in \mathbf{PM}_N \to \begin{pmatrix} p(x_1)/q(x_1) \\ \vdots \\ p(x_m)/q(x_m) \\ p(z_1)/q(z_1) \\ \vdots \\ p(z_l)/q(z_l) \end{pmatrix}, \tag{10}$$

where $q \in \mathbf{PM}_N$ is computed from p using the map φ, and

$$\hat{\psi}: \quad p \in \mathbf{P}_{\hat{N}} \to \begin{pmatrix} p(x_1)/q(x_1) \\ \vdots \\ p(x_m)/q(x_m) \\ p(z_1)/q(z_1) \\ \vdots \\ p(z_l)/q(z_l) \end{pmatrix}, \tag{11}$$

where $q \in \mathbf{P}_{\hat{N}}$ is computed from p using the map $\hat{\varphi}$.

Since $|p|^2 \le |p|^2 + |r|^2 = |q|^2$ on $\mathbb{R}$ in both cases, the rational function p/q has no real pole and no pole in $\mathbb{C}^-$ since q is stable in the broad sense. Thus, by the maximum principle, we conclude that $|p/q| \le 1$ on $\overline{\mathbb{C}^-}$. In addition, since no x_k is a root of r by assumption, we have that $|p(x_k)/q(x_k)| < 1$ hence p/q is not a Blaschke product. Therefore the Pick matrix associated with the interpolation data $(z_k, p(z_k)/q(z_k))$ must be positive definite whence ψ and $\hat{\psi}$ take their values in $\mathbb{D}^m \times \mathbb{P}_Z^+$.

Below, we say that a polynomial $p \in \mathbf{P}_{\hat{N}}$ has n zeros at infinity if p has degree $\hat{N} - n$. Zeros at infinity are considered to lie on the real line. Hereafter the degree of p is abbreviated as $\deg p$. The main result of the paper may now be stated as follows.

Theorem 2.1: 1) ψ is a homeomorphism from $\mathbf{PM}_N$ onto the product space $\mathbb{D}^m \times \mathbb{P}^+$. If $p \in \mathbf{PM}_N$ has no real root in common with r, then ψ is continuously differentiable in a neighborhood of p with invertible derivative, hence it is a local diffeomorphism at p.
2) $\hat{\psi}$ is a homeomorphism from $\mathbf{P}_{\hat{N}}$ onto $\mathbb{D}^m \times \mathbb{P}^+$. If $p \in \mathbf{P}_{\hat{N}}$ has no common real root with r (including at infinity), then $\hat{\psi}$ is differentiable at p and its differential is invertible, hence it is a local diffeomorphism at p.

Remark 2.2: Theorem 2.1 shows that, if no root of r coincides with an interpolation point on the real line, Problems $\mathscr{P}$ and $\hat{\mathscr{P}}$ have one and only one solution. If p has a common real root with r, then the associated scattering matrix (4) drops in degree.

We now come to a Proposition that enables us to use continuation techniques in order to practically solve for problem $\mathscr{P}$. We define $\mathbf{PM}_N(r)$ to be the open subset of $\mathbf{PM}_N$ comprised of those polynomials that have no common real root with r.

Proposition 2.1: $\psi(\mathbf{PM}_N(r))$ is an open, dense and connected subset of $\mathbb{D}^m \times \mathbb{P}^+$. Suppose that ν_0, ν_1 both lie in $\psi(\mathbf{PM}_N(r))$, and that γ is a continuous path from ν_0 to ν_1 in $\mathbb{D}^m \times \mathbb{P}^+$. Then, for every $\varepsilon > 0$ there exists a continuous path $\hat{\gamma}$ from ν_0 to ν_1 in $\psi(\mathbf{PM}_N(r))$ such that

$$\sup_{t \in [0,1]} \|\hat{\gamma}(t) - \gamma(t)\| \le \varepsilon,$$

where $\|.\|$ designates an arbitrary but fixed norm on $\mathbb{R}^{2N} \sim \mathbb{C}^N \supset \mathbb{D}^m \times \mathbb{P}^+$.

We also point out an interesting relation between problem $\mathscr{P}$ and $\hat{\mathscr{P}}$. In the statement we write ψ_r and $\hat{\psi}_r$ to stress the dependency of ψ, $\hat{\psi}$ with respect to the polynomial r.

Proposition 2.2: Suppose $\nu \in \mathbb{D}^m \times \mathbb{P}^+$, and (α_k) is a sequence of real numbers tending to $+\infty$. Let $p_k = \psi_{\alpha_k r}^{-1}(\nu)$ and write the leading term of $\hat{\varphi}(\alpha_k^2 rr^* + p_k p_k^*)$ in the form $e^{i\beta_k}$, noting that it is unimodular. Then, we have that

$$\lim_{k \to \infty} e^{i\beta_k} \frac{p_k}{\alpha_k} = \hat{\psi}_r^{-1}(\nu).$$

III. A TASTE OF PROOFS.

We give an idea of the proof of Theorem 2.1, part 1). A complete proof may be found in [4]. The proof of part 2) must be adapted, but follow the same lines.

Step 1. The map ψ is continuous at every p and if p has no real common root with r, then ψ is differentiable at p. This mainly relies on analogous properties for the spectral factorization φ.

Step 2. ψ is injective.

Step 3. ψ is proper: the pre-image of each compact set in $\mathbb{D}^m \times \mathbb{P}^+$ is compact.

Step 4. ψ is a homeomorphism from $\mathbf{PM}_N$ onto $\mathbb{D}^m \times \mathbb{P}^+$. This result is established using a famous result by Brouwer, known as *invariance of the domain* [15, chap. 10, sect. 62]. If $\Omega \subset \mathbb{R}^n$ is open and $f : \Omega \to \mathbb{R}^n$ is continuous and injective, it says that f is an open map, meaning that it maps open sets to open sets. Hence $f(\Omega)$ is open and the inverse map $f^{-1} : f(\Omega) \to \Omega$ is continuous, that is: f is a homeomorphism onto its image. Finally, the fact that the image of $\mathbf{PM}_N$ is all of $\mathbb{D}^m \times \mathbb{P}^+$ rests on the properness of ψ which implies that $\psi(\mathbf{PM}_N)$ is closed (and open according to the above) in the connected space $\mathbb{D}^m \times \mathbb{P}^+$.

Step 5. The map ψ is a diffeomorphism from $\mathbf{PM}_N(r)$ onto its image.

Our proof of the injectivity of ψ as well as the fact that the differential of ψ is locally invertible in a neighborhood of $p \in \psi(\mathbf{PM}_N(r))$ relies on representation formulas in the Hardy space $H^2(\mathbb{C}^-)$. Every $f \in H^2(\mathbb{C}^-)$ is the Cauchy integral of the non-tangential limit of its real part:

$$f(z) = -\frac{1}{i\pi} \int_{\mathbb{R}} \frac{\Re f(t)}{t-z}\, dt, \qquad z \in \mathbb{C}^-, \tag{12}$$

and the non-tangential limit of its imaginary part is the Hilbert transform of the nontangential limit of its real part [9, Ch. III, sec. 2]:

$$\Im f(x) = \frac{1}{\pi} \lim_{\varepsilon \to 0^+} \int_{|x-t|>\varepsilon} \frac{\Re f(t)}{t-x}\, dt, \qquad \text{a.e.} \quad x \in \mathbb{R}. \quad (13)$$

Consequently, the nontangential limit of f can be recovered from its real part as

$$f(x) = \Re f(x) + \frac{i}{\pi} \lim_{\varepsilon \to 0^+} \int_{|x-t|>\varepsilon} \frac{\Re f(t)}{t-x}\, dt, \qquad \text{a.e. } x \in \mathbb{R}. \quad (14)$$

To use these formulas, we move from the Schur class $\mathscr{S}$ to the Carathéodory class $\mathscr{C}$. The map $f \mapsto (1-f)/(1+f)$ maps $\mathscr{C}$ to $\mathscr{S}$ and back. However, the image of $f \in \mathscr{S}$ under this map may not be in $H^2(\mathbb{C}^-)$. This has to do with the fact that real parts of Carathéodory functions, unlike those of Schur functions, may be Poisson integrals of measures rather than functions, and therefore cannot in general be recovered from their pointwise trace on $\mathbb{R}$.

We now briefly explain how injectivity is proved. Assume that there exist distinct polynomials $p_1(z)$ and $p_2(z)$ in $\mathbf{PM}_N$ such that $\psi(p_1) = \psi(p_2)$. For $j = \{1,2\}$, let $q_j = \varphi(p_j)$, we have

$$\frac{p_1}{q_1}(x_k) \quad = \quad \frac{p_2}{q_2}(x_k), \ 1 \le k \le m, \quad (15)$$

$$\frac{p_1}{q_1}(z_\ell) \quad = \quad \frac{p_2}{q_2}(z_\ell), \ 1 \le l \le l. \quad (16)$$

Since, there are at most $\deg r$ real numbers t for which $|p_j(t)/q_j(t)| = 1$, we can find a complex number ξ of modulus 1, distinct from -1, such that $1 + \xi p_j/q_j$ is never zero on $\mathbb{R}$ for $j = \{1,2\}$. Consider the Cayley transforms of the Schur functions $\xi p_j/q_j$, that is

$$\frac{1 - \xi \frac{p_j(z)}{q_j(z)}}{1 + \xi \frac{p_j(z)}{q_j(z)}} = \frac{1-\xi}{1+\xi} + Y_j(z), \quad (17)$$

where

$$Y_j := \left(\frac{2\xi}{1+\xi} \right) \frac{q_j(z) - p_j(z)}{q_j(z) + \xi p_j(z)} \quad (18)$$

is a Carathéodory function which belongs to $H^2(\mathbb{C}^-)$. By a straightforward computation we obtain

$$\Re(Y_j) = Y_j + Y_j^* = \frac{d}{g_j}, \quad (19)$$

where

$$d = \frac{2rr^*}{|1+\xi|^2}, \quad g_j = \frac{(q_j + \xi p_j)(q_j + \xi p_j)^*}{|1+\xi|^2}, \quad (20)$$

are non-negative polynomials, g_j being monic. At each real interpolation point x_k, we rewrite (15) using (13):

$$J(x_k) = \int_{-\infty}^{\infty} d(t) \frac{g_2(t) - g_1(t)}{g_1(t) g_2(t)} \frac{dt}{t - x_k} = 0, \quad (21)$$

where we omitted the principal value in the integral (21) because the integrand is in fact nonsingular. At each complex interpolation point z_k, we rewrite (16) using (12)

$$I(z_k) = \int_{-\infty}^{\infty} d(t) \frac{g_2(t) - g_1 t)}{g_1(t) g_2(t)} \frac{dt}{t - z_k} = 0, \quad (22)$$

and taking conjugates

$$\overline{I(z_k)} = \int_{-\infty}^{\infty} d(t) \frac{g_2(t) - g_1(t)}{g_1(t) g_2(t)} \frac{dt}{t - \overline{z_k}} = 0. \quad (23)$$

We now combine linearly equations (22), (23) and (21) using $2l + m$ arbitrary complex coefficients. Putting everything over a common denominator yields

$$\int_{-\infty}^{\infty} d(t) \frac{g_2(t) - g_1(t)}{g_1(t) g_2(t) \prod_{k=1}^{l} |t - z_k|^2} \frac{Q(t)dt}{\prod_{k=1}^{m} (t - x_k)} = 0, \quad (24)$$

for any polynomial Q in $\mathbf{P}_{2l+m-1}$. Observing now that $g_2 - g_1$ vanishes at the x_k, and choosing $Q(t) = (g_2(t) - g_1(t))/\prod_1^m (t - x_k)$ (note that g_2 and g_1 being monic $g_2 - g_1$ has degree at most $2l + m - 1$), we conclude since the integrand is nonnegative that $g_1 = g_2$. But, given d, g_j non negative polynomials such that d/g_j belongs to $L^2(\mathbb{R})$, there exist a unique $Y_j \in H^2(\mathbb{C}^-)$ satisfying (19). Thus $Y_1 = Y_2$, which in turn implies $p_1 = p_2$ and $q_1 = q_2$.

REFERENCES

[1] L.V. Ahlfors. *Conformal invariants*. AMS Chelsea, 1973.

[2] A. Ball, Joseph and ter Horst, Sanna. *Robust Control, Multidimensional Systems and Multivariable Nevanlinna-Pick Interpolation*, volume 203 of *Operator Theory: Advances and Applications*, chapter 2, pages 13–88. Birkhaüser, 2010.

[3] L. Baratchart, M. Olivi, and F. Seyfert. Generalized Nevanlinna-Pick interpolation on the boundary. Application to impedance matching. In *Proceedings of the MTNS (Groninguen, Netherlands)*, 2014.

[4] L. Baratchart, M. Olivi, and F. Seyfert. Generalized Nevanlinna-Pick interpolation on the boundary. Application to impedance matching. Working paper or preprint: https://hal.inria.fr/hal-01249330, December 2015.

[5] V. Belevitch. Elementary applications of scattering formalism to network design. *IRE Trans. Circuit Theory*, 1956.

[6] S. V. Gusev C. I. Byrnes, A. Lindquist and A. S. Matveev. A complete parameterization of all positive rational extensions of a covariance sequence. *IEEE Trans. Autom. Control*, 40(11):18411857, 1995.

[7] H.J. Carlin and P.P. Civalleri. *Wideband Circuit Design*. CRC Press, 1997.

[8] H. Dym. *J-contractive matrix functions, reproducing kernel spaces and interpolation*, volume 71 of *CBMS lecture notes*. American Mathematical Society, Rhode Island, 1989.

[9] J. Garnett. *Bounded Analytic Functions*. Academic Press, 1981.

[10] T. Georgiou. A topological approach to Nevanlinna-Pick interpolation. *SIAM J. MATH ANAL.*, 18(5):1248–1260, 1987.

[11] T. Georgiou. The interpolation problem with a degree constraint. *IEEE Trans. Automat. Contro*, 1999.

[12] T. T. Georgiou. A topological approach to nevanlinna-pick interpolation. *SIAM J. Math. Anal.*, 18(5):12481260, 1987.

[13] J.W. Helton. Broadbanding:gain equalization directly from data. *Circuits and Systems, IEEE Transactions on*, 28(12):1125–1137, 1981.

[14] T. T. Georgiou J. Karlsson and A. G. Lindquist. The inverse problem of analytic interpolation with degree constraint and weight selection for control synthesis. *IEEE Trans on automatic control*, 55(2):405–418, 2010.

[15] James Munkres. *Elements of algebraic topology*. Addison-Wesley, 1930.

[16] D. Sarason. Generalized interpolation in H^∞. *Trans. American Math. Society*, 127:179–203, 1967.

[17] D. C. Youla and M. Saito. Interpolation with positive real functions. *J. Franklin Institute*, 1967.

[18] O. Zames and B. A. Francis. Feedback, minimax sensitivity, and optimal robustness. *IEEE Trans. Automatic Control*, 28(5):585–600, 1983.

[19] O. Zames and B. A. Francis. H^∞-optimal sensitivity theory for siso feedback systems. *IEEE Trans. Automatic Control*, 29(1):9–16, 1984.