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Light rings and light points of boson stars

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This work is devoted to the study of photon trajectories around rotating boson stars. The basic properties of boson star models are given with a particular emphasis on the high compactness those objects can have. Using an effective potential method, circular orbits of photon around rotating boson stars are then obtained, at least for relativistic enough configurations. A particular class of light rings is exhibited, where the photons are at rest. It is proposed to call those orbits *light points*. Their existence is only possible for very specific boson stars and the link between light points and ergoregions is investigated.

I. INTRODUCTION

Boson stars were first introduced in the late sixties in [1–3]. They consist of a complex scalar field coupled to gravity. Various families of boson stars have been constructed, with various potentials for the scalar field, with or without rotation, in various dimensions (see [4–7] and references therein)...

One of the main motivations for the study of boson stars is the fact that they can act as black mimickers [8]. What it means is that they can have a large mass with a small size without developing any horizon nor singularity. Moreover boson stars have no hard surface, the scalar field interacting with ordinary matter only through gravitation. Possible scenarios for the formation of boson stars have been studied. There has also been a number of studies that proposed observational means of discriminating boson stars from black holes. This could for instance be done by direct imaging of the galactic center [9]. Observations of gravitational waves could also be a fruitful way of investigating the nature of the compact objects observed in the universe [10, 11].

Apart from being an alternative to black holes, boson stars are very interesting testbeds for a lot of situations where intense gravitational fields play a role. For instance, in a previous paper, we investigated a class of peculiar orbits for massive particles [12]. Other specific effects concerning the imaging of accretion disks around boson stars were studied in [9, 13].

The main goal of the current paper is to investigate the existence of light rings around boson stars. Those are closed (circular) orbits of photons. They can only appear for very compact objects (dubbed ultracompact in [14]). It is known that black holes can admit light rings and that they can lie outside the horizon. The link between light rings and various instabilities is explored in a recent study by V. Cardoso *al.* [14]. Light rings can also be important in the context of gravitational waves. It has been shown in [15] that the first gravitational wave signals observed by the LIGO observatory [16] could not discriminate between black holes and alternative compact objects. Indeed, the observed ringdown signal is shown to be an indication of the existence of a light ring and not of an horizon. Horizonless objects with light rings would have the same ringdown signal as the one observed. The existence of boson stars (even if unlikely), could not be excluded, provided some of them admit light rings.

This paper is organized as follows. In Sec. II the reader is reminded about models of boson stars. In particular, the high compactness of such objects is exhibited. Section III describes the study of light rings by means of an effective potential method. In Sec. IV a particular class of light rings, where the photons are *at rest*, is investigated. The existence of those *light points* is a new effect that is typical of boson stars. Indeed, their appearance requires an intense gravitational field but they cannot exist in the case of black holes due to the presence of the horizon. Conclusions are given in Sec. V.

II. BOSON STAR MODELS

In this section we recall the setting used in [12] to compute boson star spacetimes. Boson stars consist of a complex scalar field Φ coupled to gravity. This is achieved by considering the following action

$$S = \int (\mathcal{L}_g + \mathcal{L}_\Phi) \sqrt{-g} dx^4, \quad (1)$$

where \mathcal{L}_g is the Hilbert-Einstein Lagrangian and \mathcal{L}_Φ the Lagrangian of the complex scalar field. They are given by the standard expressions

$$\mathcal{L}_g = \frac{1}{16\pi} R \quad (2)$$

$$\mathcal{L}_\Phi = -\frac{1}{2} \left[\nabla_\mu \Phi \nabla^\mu \bar{\Phi} + V(|\Phi|^2) \right]. \quad (3)$$

∇ denotes the covariant derivative associated to \mathbf{g} and R the Ricci scalar. $V(|\Phi|^2)$ is a potential that can be modified to construct various models of boson stars. In this paper one considers the simplest possible choice where the scalar field is a free-field and which implies that

$$V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2. \quad (4)$$

m is the mass of the individual boson and the factor m/\hbar then appears as a scale factor for the various quantities (distances, masses...). Throughout this paper geometric units are used, such that $G = 1$ and $c = 1$.

Rotating boson stars are computed by demanding that the scalar field takes the form

$$\Phi = \phi(r, \theta) \exp[i(\omega t - k\varphi)], \quad (5)$$

where ϕ is the amplitude of the field (and so a real), ω is a real constant (smaller than m/\hbar) and k an integer known as the rotational quantum number. Due to the $U(1)$ symmetry of the action (1), it does not depend on t and φ , leading to axisymmetric solutions. This is why the amplitude ϕ depends on (r, θ) only. The numbers ω and k appears as parameters of the various boson stars. Let us point out that the case $k = 0$ corresponds to spherically symmetric boson stars.

For the metric, quasi-isotropic coordinates are well adapted to the symmetry of the problem. The metric reads

$$g_{\mu\nu} dx^\mu dx^\nu = -N^2 dr^2 + A^2 (dr^2 + r^2 d\theta^2) + B^2 r^2 \sin^2 \theta (d\varphi^2 + \beta^\phi d\varphi^2). \quad (6)$$

In the 3+1 language, N is the lapse, the shift vector is $\beta^i = (0, 0, \beta^\varphi)$ and the spatial metric $\gamma_{ij} = \text{diag}(A^2, B^2 r^2, B^2 r^2 \sin^2 \theta)$. The unknowns are the functions N , A , B and β^φ , all of them depending only on (r, θ) . The unknown fields obey the Einstein-Klein-Gordon system which results from the variation of the action (1) with respect to both the metric \mathbf{g} and the scalar field Φ .

Those equations are solved by means of highly accurate spectral methods, implemented by the KADATH library [17]. This enabled us to compute sequences of rotating boson stars for k ranging from 0 to 4. Numerical methods and tests are given in details in [12]. We present here the sequences in a different way: by plotting the compactness of the various configurations. In order to do so, one needs to define the radius of the boson stars. This cannot be done unambiguously because the scalar field extends up to infinity and those objects have no true surface. In the following, the radius R of the boson star is defined as the radius where the field is 0.1 times its maximal value, in the equatorial plane. The precise value of the thus defined radius obviously depends on the value of the threshold (here 0.1). However, due to the fast decay of the field far from the origin, it is a small effect. Figure 1 shows the radii of the various boson star models. One can clearly see the known facts that the size of a boson star increases with ω and k .

With this definition of the radius, it is possible to define the compactness of the objects, by just taking the adimensional ratio M/R , where M is the standard ADM mass of the boson stars. It is shown in Fig. 2. First one can notice that the result depends very moderately on the value of k . The variation of the mass and radius with k almost cancel so that the compactness depends almost only on ω . One can also notice that it can reach very high values of the order unity (remember that typical neutron stars have a compactness of about 0.2), illustrating the fact that boson stars are indeed good black hole mimickers. Let us also mention that spherically symmetric boson stars (i.e. for which $k = 0$) can only reach compactness of about 0.25. This is mainly due to the fact that they cannot attain small enough values of ω , due to the turning point in the sequences (see Fig. 1 for instance).

III. LIGHT RINGS

Given the high compactness of boson stars, there is a good hope that those objects could have some light rings. Those are closed orbits of photons. Mathematically speaking one needs to find closed null geodesics of the spacetime. Given the fact that boson stars are axisymmetric objects, one looks for circular orbits in the equatorial plane, as in the case of a Kerr black hole. In this work such orbits are localized using an effective potential method which proceeds as follows.

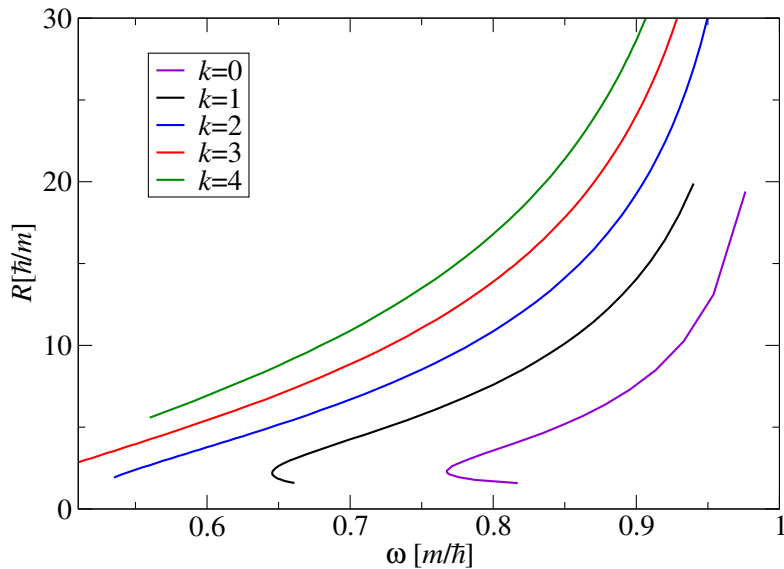


FIG. 1: Radii of the various boson stars. It is defined as the radius where the field is 0.1 times its maximal value, in the equatorial plane.

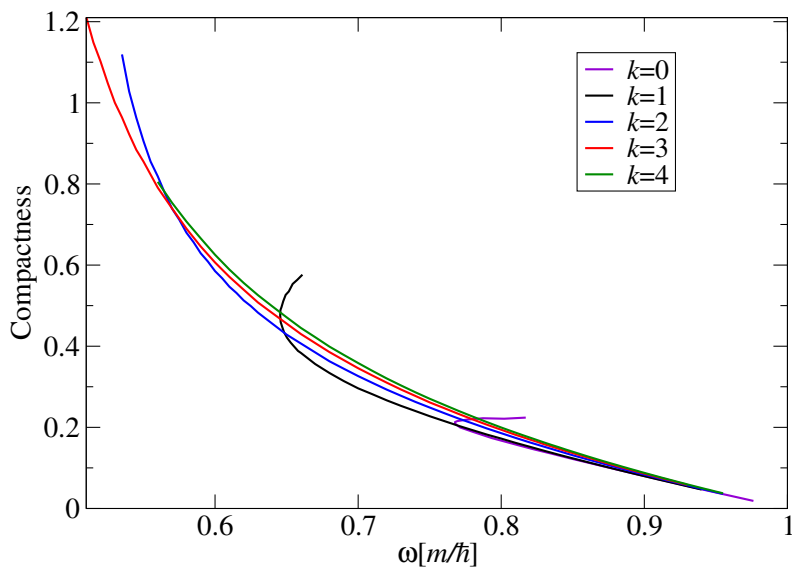


FIG. 2: Compactness of the various boson stars.

Let us call U^μ the tangent vector of a photon trajectory. As we consider only orbits in the equatorial plane, it reduces to $(U^t, U^r, 0, U^\varphi)$. Boson star spacetimes admit two independent Killing vectors : $(\partial_t)^\mu$ and $(\partial_\varphi)^\mu$. Their existence leads to two conserved quantities along the geodesics, being the scalar products of U^μ with the two Killing vectors:

$$U_\mu (\partial_t)^\mu = (-N^2 + B^2 r^2 \beta^\varphi{}^2) U^t + B^2 r^2 \beta^\varphi U^\varphi = E \quad (7)$$

$$U_\mu (\partial_\varphi)^\mu = B^2 r^2 \beta^\varphi U^t + B^2 r^2 U^\varphi = L, \quad (8)$$

where E and L are two constants along the geodesic. From those equations, one can express the components U^t and U^φ , as functions of E and L :

$$U^t = \frac{\beta^\varphi L - E}{N^2} \quad (9)$$

$$U^\varphi = \frac{L}{B^2 r^2} - \frac{\beta^\varphi}{N^2} (\beta^\varphi L - E). \quad (10)$$

Injecting those expressions into the fact that the geodesic is null (i.e. $U_\mu U^\mu = 0$) leads to an equation on U^r . It can be put into the form $(U^r)^2 + V_{\text{eff}} = 0$ where V_{eff} is an effective potential, given by

$$V_{\text{eff}} = \frac{1}{A^2} \left[-\frac{(\beta^\varphi L - E)^2}{N^2} + \frac{L^2}{B^2 r^2} \right]. \quad (11)$$

Circular orbits are such that $V_{\text{eff}} = 0$ and $\partial_r V_{\text{eff}} = 0$. The second condition is necessary to prevent being only at the periastron or the apoastron of an elliptic orbit. The first condition leads to

$$\frac{E}{L} = \beta^\varphi - \epsilon \frac{N}{Br}, \quad (12)$$

where $\epsilon = \pm 1$. Contrary to the massive particle case, E and L are not constrained independently but only via their ratio. This is basically due to the fact that there is no scale of mass in the case of photons.

The condition $\partial_r V_{\text{eff}} = 0$, along with $V_{\text{eff}} = 0$ leads to an equation involving the radius of the orbit and the various metric fields :

$$I(r) = \left(\epsilon \frac{\partial_r \beta^\varphi}{NB} \right) r^2 + \left(\frac{\partial_r B}{B^3} - \frac{\partial_r N}{NB^2} \right) r + \frac{1}{B^2} = 0. \quad (13)$$

This is a purely radial equation (remember one works in the equatorial plane) but it is not a second order equation on r , the various fields depending on r . Figure 3 shows the functions $I(r)$ for two values of ω , in the case $k = 1$. Let us first mention that choosing $\epsilon = +1$ leads to functions $I(r)$ that always remain strictly positive. This is the situation illustrated by the dashed curves of Fig. 3. When $\epsilon = -1$ the function $I(r)$ can sometimes vanish, depending on the boson star considered. Basically, only the most relativistic boson stars (i.e. boson stars with small value of ω) are in that case and thus admit light rings. In Fig. 3, the boson star $k = 1; \omega = 0.8$ has no light ring (the solid blue curve is always positive), whereas the boson star $k = 1; \omega = 0.7$ has two light rings, corresponding to the two values of r where $I(r)$ vanishes (see solid black curve).

Figure 4 shows the value of the minimum of $I(r)$, in the case $\epsilon = -1$. When this minimum is below zero the boson star admits light rings. This correspond to the configurations with low values of ω and it can happen for all values of $k > 0$. The case $k = 0$ is not shown in Fig. 4 because the minimum is much above zero. Spherically symmetric boson stars do not have light rings. The light ring of a Schwarzschild black hole is located at $3M$. If the radius of the $k = 0$ boson stars was much smaller than this value, light rings would have been expected to exist. However, from Fig. 2 one can see that the compactness of the spherically symmetric boson stars cannot exceed 0.3. This moderate compactness is an argument against the existence of light rings in that particular case. This is confirmed from our study of the function $I(r)$.

The shape of the function $I(r)$ depicted in Fig. 3 is very generic : when the minimum of $I(r)$ is negative there are two light rings, corresponding to two radii R_{minus} and R_{plus} . Moreover, it is easy to see that $\partial_r V_{\text{eff}}$ and $I(r)$ have opposite signs. It follows that R_{minus} corresponds to a minimum of V_{eff} and is stable whereas R_{plus} corresponds to a maximum of V_{eff} and so to an unstable orbit. Figure 4 shows the minimum values of $I(r)$. All the configurations for which it is below zero admit light rings. The radii of the light rings are show in Fig. 5.

By inserting Eq. (12) into Eqs. (9-10) one can find the value of $\frac{d\varphi}{dt}$ for the light rings. It leads to

$$\frac{d\varphi}{dt} = -\frac{N}{Br} - \beta^\varphi. \quad (14)$$

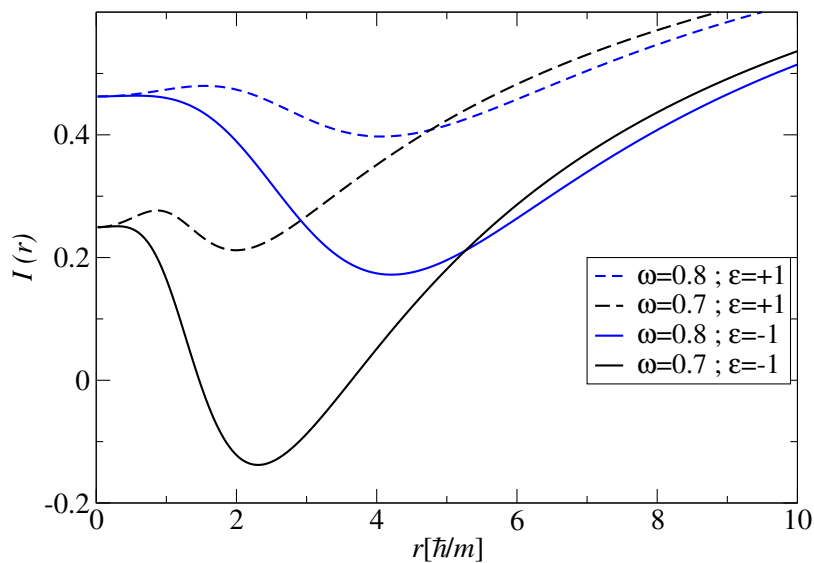


FIG. 3: Function $I(r)$ for $k = 1$. The blue curves correspond to $\omega = 0.8$ and the black ones to $\omega = 0.7$. The dashed lines denote the cases where $\epsilon = +1$ and the solid lines the cases $\epsilon = -1$.

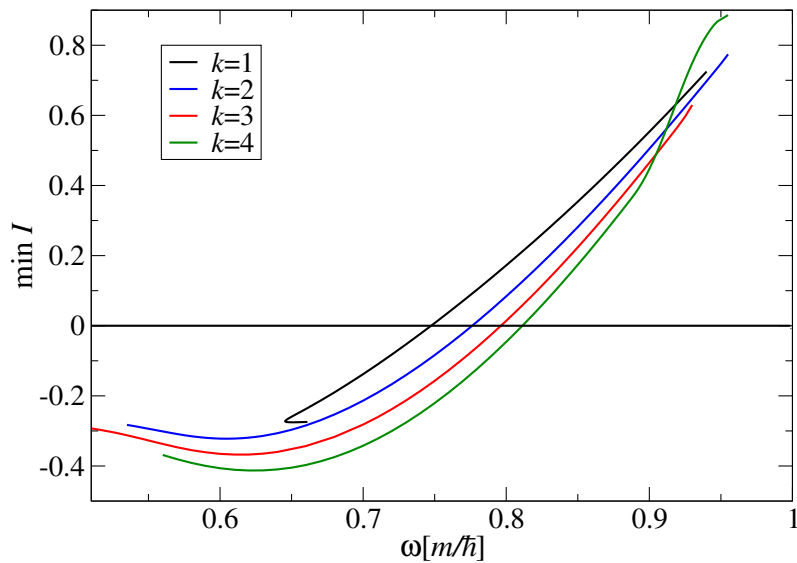


FIG. 4: Minimum of the function $I(r)$, when $\epsilon = -1$. The configurations for which the minimum is below zero admit light rings.

The right hand side of Eq. (14) must be evaluated at R_{minus} or R_{plus} depending on the light ring considered. The tangent vector of the trajectories is then given by $U^\mu = \left(1, 0, 0, \frac{d\varphi}{dt}\right)$ where the parameter along the trajectory has

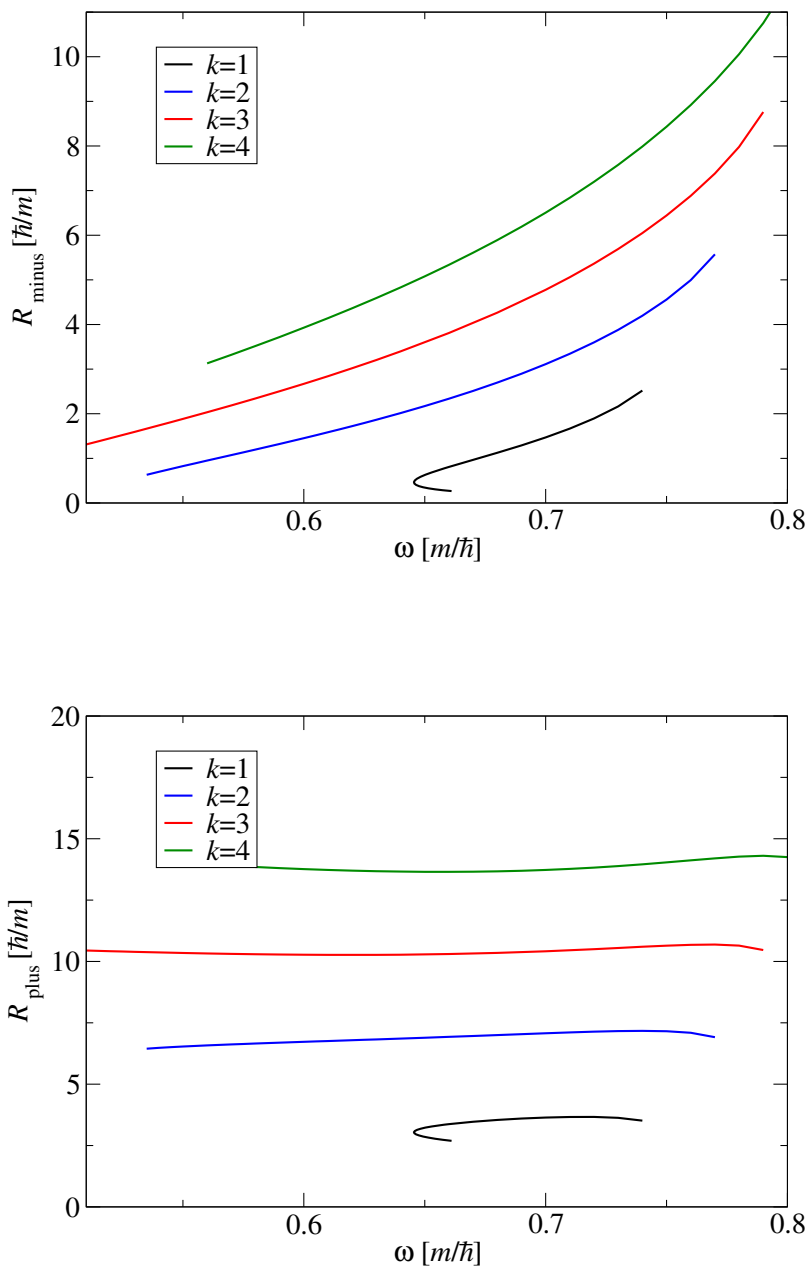


FIG. 5: Radii of the inner light rings (first panel) and the outer ones (second panel).

been chosen to be the coordinate t . Orbital frequencies of the inner (resp. outer) light rings are shown in Fig. 6. They will be discussed furthermore in Sec. IV.

IV. LIGHT POINTS

The most striking result from Fig. 6 is the fact that the orbital velocity of the inner light ring can change sign and so that there are boson stars for which it is zero. The associated trajectories are simply given by $U^\mu = (1, 0, 0, 0)$. In

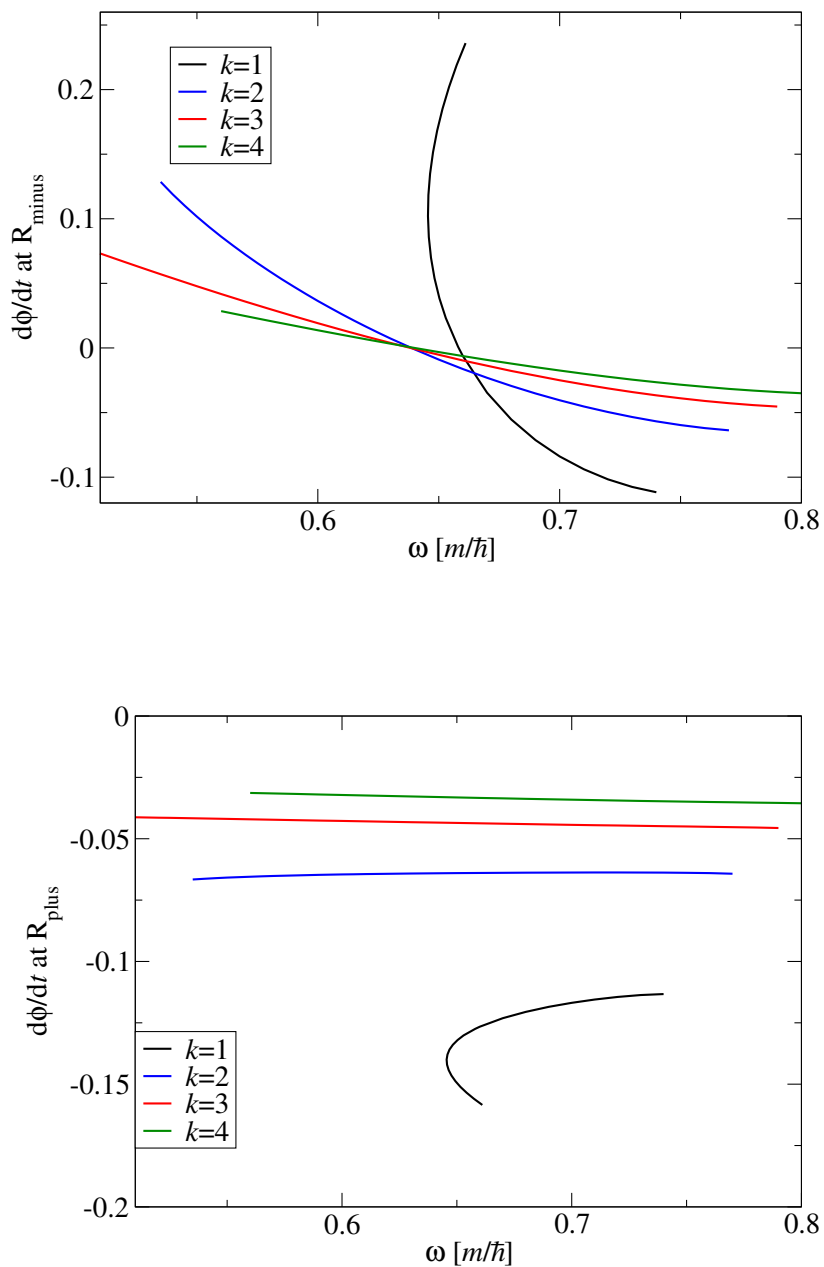


FIG. 6: Orbital frequency $\frac{d\varphi}{dt}$ of the inner light rings (first panel) and the outer ones (second panel).

term of spatial coordinates (r, θ, φ) , those photons are not moving. They correspond more to light points than light rings.

Even if those light points seem to be peculiar, a detailed study can confirm their existence and location. One first condition is that the vector $U^\mu = (1, 0, 0, 0)$ must be a null vector. Given the fact that the quasi-isotropic metric is diagonal it is equivalent to $g_{tt} = 0$. This condition is the same as the one defining the boundary of an ergoregion (see Sec. IV-E of [12] for a detailed study of ergoregions of boson stars). So light points necessarily lies on the boundary of an ergoregion.

However not all the null curves are geodesics. The geodesic equation is $\frac{dU^\mu}{dt} + \Gamma_{\alpha\beta}^\mu U^\alpha U^\beta = C(t) U^\mu$ where $C(t)$ is a function along the curve. Note that, a priori, one cannot make $C = 0$ because there is no guarantee that t is an affine parameter. Γ denotes the four-dimensional Christoffel symbols. Inserting $U^\mu = (1, 0, 0, 0)$ in the geodesic equation leads to the condition

$$\Gamma_{tt}^\mu = \frac{1}{2} g^{\mu\alpha} [-\partial_\alpha g_{tt}] = C(t) U^\mu, \quad (15)$$

where all the time derivatives have been set to zero. Moreover, $\partial_t g_{tt}$ and $\partial_\varphi g_{tt}$ vanish due to the stationarity and axisymmetry of the problem. $\partial_\theta g_{tt}$ is also zero because the considered orbits are in the equatorial plane which is a surface of symmetry. Equation (15) then reduces to $g^{\mu r} [-\partial_r g_{tt}] = 2C(t) U^\mu$. Given the form of the metric (i.e. the use of quasi-isotropic coordinates), one can show that g^{tr} , $g^{\theta r}$ and $g^{\varphi r}$ all vanish. It first implies that one must necessarily set $C(t) = 0$ which means that t is indeed an affine parameter. Only the radial component of the geodesic equation is not trivially satisfied and it reduces to

$$\partial_r g_{tt} = 0. \quad (16)$$

This condition on the derivative, along with the fact that g_{tt} must vanish at the light points implies that they can only be situated exactly when an ergoregion starts to form. This is illustrated in Fig. 7 where the quantity $-g_{tt}$ is plotted, in the plane $z = 0$, for three different boson stars. The curve $\omega = 0.7$ never goes to zero hence the associated boson star has no ergoregion. The curve $\omega = 0.6$ gets below zero and vanishes at two different radii. They are the inner and outer radii of the ergoregion (for boson stars ergoregions have the shape of a torus ; see Sec. IV-E of [12]). The curve $\omega \approx 0.638$ corresponds to the case where an ergoregion appears. g_{tt} vanishes at just one point, which is also the minimum of the curve. This point is such that $g_{tt} = 0$ and $\partial_r g_{tt} = 0$ and so is a light point.

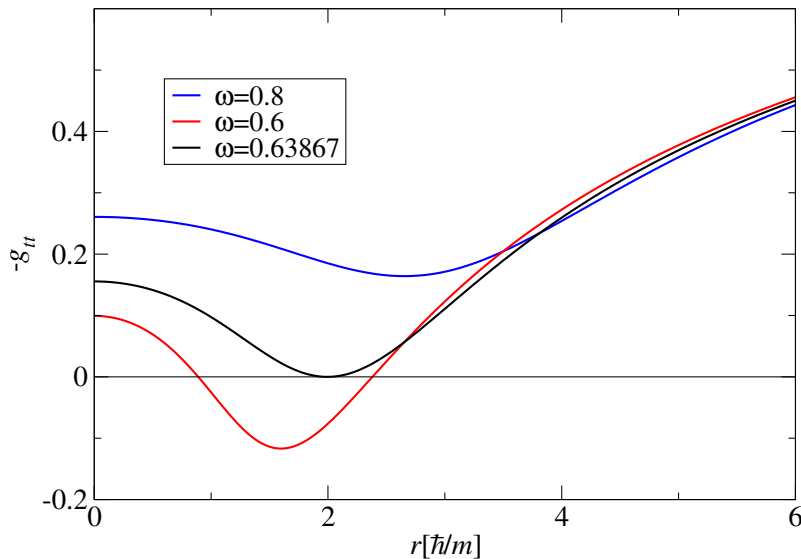


FIG. 7: $-g_{tt}$ for three boson stars with $k = 2$. The upper curve has no ergoregion. The lower one has a standard ergoregion in the shape of a torus. The middle curve corresponds to the critic case where an ergoregion just starts to appear.

Figure 8 gives another illustration of the location of the light points (for the sequence $k = 2$). The solid black curve denotes the inner radius of the ergoregions and the dashed black one the one of the outer radius. Those two curves join at $\omega \approx 0.638$ which is the frequency at which an ergoregion starts to develop. The blue curve shows the radius of the inner light ring for which $d\varphi/dt < 0$ whereas the red curve denotes the same radius but for configurations with $d\varphi/dt > 0$. It is clear from Fig. 8 that the light point lies at the intersection of the various curves. Moreover, it is located exactly at the radius where the ergoregion appears. This is not so surprising. Ergoregions are defined as

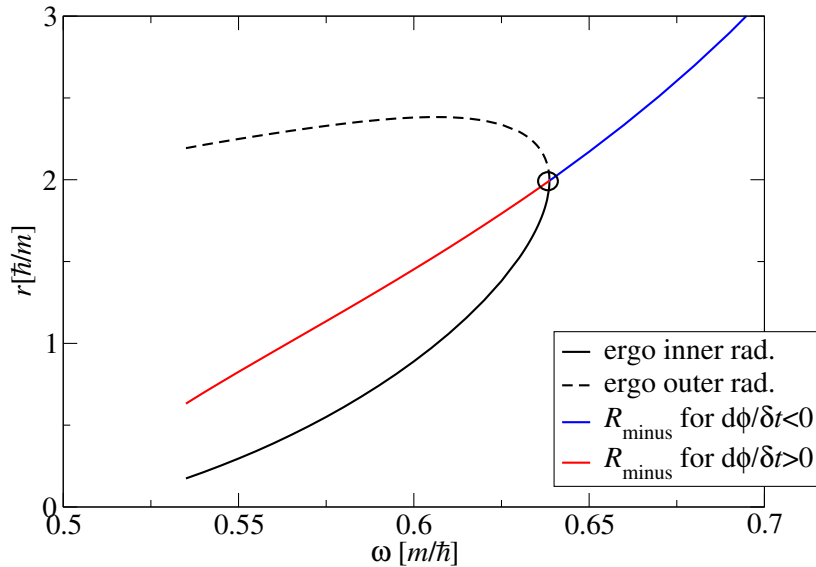


FIG. 8: For boson stars with $k = 2$, inner and outer radii of the ergoregion (black curves). The red and blue curves denote the radius of the inner light ring, with different signs in the orbital frequency. All the curves intersect at the location of the circle which is the light point.

TABLE I: Parameters of the light points for $k = 1$ to $k = 4$.

k	ω	R_{minus}
1	0.6582	0.779
2	0.6387	1.99
3	0.6383	3.36
4	0.6401	4.83

being locations where nothing can remain at rest, due to the frame-dragging effect generated by rotation. When it just appears only massless particles moving at the speed of light can overcome this effect and remain at rest. In other words, at the onset of the ergoregion, the frame-dragging effect is just compensated by a velocity of c . Light points are not seen in the case of a Kerr black hole because the ergoregion starts to appear exactly at the location of the horizon. Each sequence of boson stars with $k > 0$ admits a single light point. The corresponding values of ω and R_{minus} are summarized in Tab. I.

The obtained results can be checked by integrating numerically the geodesic equation. This is done using the tool Gyoto [18]. One can compute the radius of a light ring, put a photon there, with the right angular velocity (i.e. given by Eq. (14)) and check whether the computed orbit is indeed a circle. Figure 9 shows such a computation, in the case of a boson star $k = 2$ and $\omega = 0.7$. The first panel corresponds to the inner light ring ($R_{\text{minus}} = 3.11139$ and $d\varphi/dt = -0.0404863$). The second panel corresponds to the outer light ring ($R_{\text{plus}} = 7.07489$ and $d\varphi/dt = -0.0637682$). The photon on the inner light ring remains nicely on a circular orbit. The one on the outer light ring starts by describing a circular orbit but is eventually kicked out. This is consistent with the fact that only the inner light ring is stable, corresponding to a minimum of the effective potential. The last panel corresponds to the light point of the $k = 2$ sequence (i.e. $\omega = 0.6387$ and $R = 1.99$). As expected, the photon stays at the same spatial location and so appears as a single point in Fig. 9.

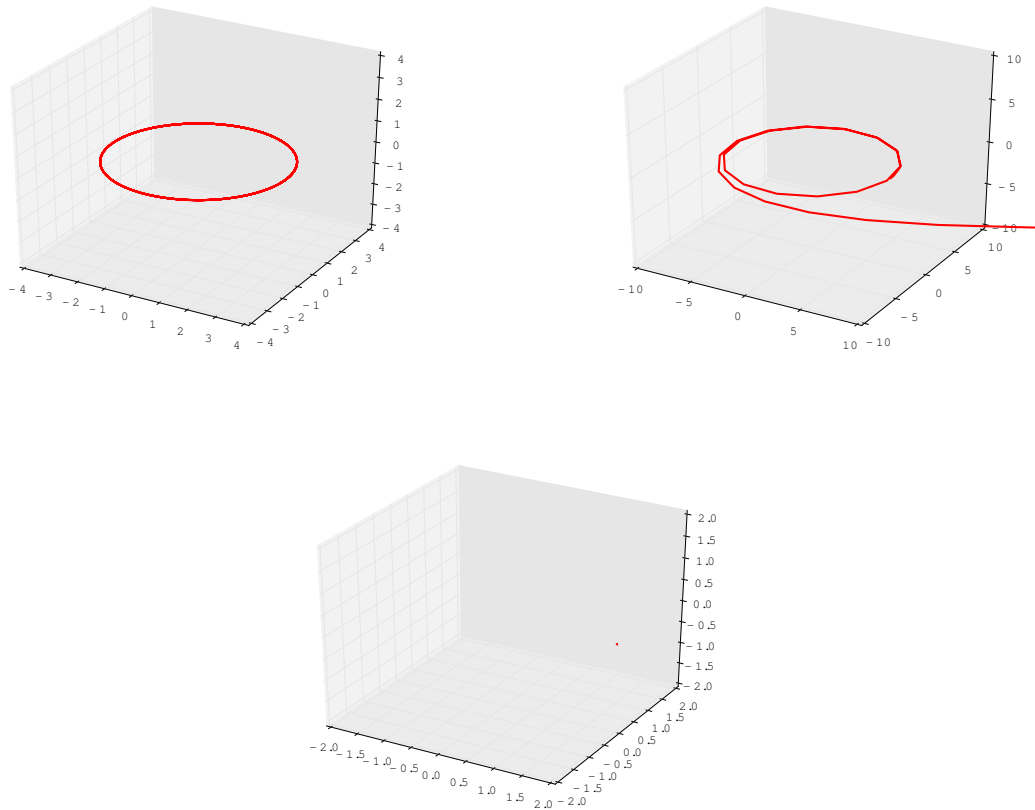


FIG. 9: Orbits integrated numerically. The first panel corresponds to a stable inner light ring whereas the second one depicts the orbit of an unstable outer one (for $k = 2$ and $\omega = 0.7$). The last panel corresponds to the light point of the $k = 2$ sequence.

V. CONCLUSION

In this paper, models of rotating boson stars are studied. They were obtained numerically in a previous work [12]. It is confirmed that rotating boson stars are objects that can reach very high compactness. It implies that boson stars can exhibit effects where strong gravitational fields are required. The existence of closed orbits of photons, such as light rings, is confirmed for a large class of boson stars (i.e. the one with high enough compactness). Existence of those light rings could have strong implications on the stability of the boson stars themselves [14].

A new class of light rings has been exhibited, which corresponds to trajectories where the photons are at rest, with respect to an observer at infinity. It is proposed to call those orbits light points. Their existence is closely linked to the appearance of an ergoregion. They are very specific of boson stars in the sense that they cannot exist in the black hole case, due to the presence of the horizon. It is believed it is the first time that those light points are exhibited. Their existence and stability has been confirmed by a direct integration of the geodesic equation.

Acknowledgments

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