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Mathematical thought in the light of Matte Blanco's work

Giuseppe Iurato

E-mail: giuseppe.iurato@community.unipa.it

Abstract. Taking into account some basic epistemological considerations on psychoanalysis by Ignacio Matte Blanco, it is possible to deduce some first simple remarks on certain logical aspects of schizophrenic reasoning. Further remarks on mathematical thought are also made in the light of what has been established, taking into account the comparison with schizophrenia pattern.

Ignacio Matte Blanco (1908-1995) has been a notable psychiatrist and psychoanalyst who has devoted many years of his work in studying the mathematical thought, starting from his many-years experience with schizophrenic patients. His definitive results were published in the celebrated work entitled *Unconscious as Infinite Sets: An Essay on Bilogic* (1975), which has seen various editions.

In this paper, we mainly follow the synthetic work [1] which explain the main psychoanalytic considerations, also from an epistemological viewpoint, on the logic of unconscious deduced by Matte Blanco from his already mentioned fundamental psychiatric experience on schizophrenic patients.

First, the preface by Professor Adriano Ossicini to [1] argues on the general epistemological aspects of the psychological sciences, since the work of Matte Blanco is just directed toward these last, that is to say, he tries to establish fundamental relationships between the psychoanalysis and the exact sciences, in particular the mathematics. The work of Matte Blanco¹ is an original, interpretative afterthought of the Freudian theory through the methods of elementary logic. He starts from certain Freudian postulates which characterize the dynamical structure of unconscious, namely 1) displacement, 2) condensation, 3) absence of time, 4) substitution of the external reality with the psychic one (literal interpretation of the metaphor) and 5) absence of mutual contradiction among the presentation of the various instincts (or drives).

In particular, according to Freud², the usual ordinary logic rules of conscious thought are no valid for the unconscious since it operates according to another logic system. The former is ruled by a classical, assertoric (i.e., not modal) logic founded on the material implication and having, as fundamental laws, the identity principle, the non-contradiction principle, the bivalent principle, the principle of sufficient reason and the principle of the excluded third (*tertium non datur*). Instead, according to the studies on schizophrenia made by Matte Blanco, the fundamental principles of the unconscious are the *generalization principle*³ and the *symmetric principle*⁴ (see [1, Chap. I, Section 2]), through which it carries out the primary process (whereas the secondary process concerns the modus operandi of conscious thought). Subsequently, through them, Matte Blanco tries to explain the previous Freudian characteristic principles of unconscious (see [1, Chap. II]). In particular, he re-examines (see [1, Chap. II, Section 2]) the classical Freudian agencies in the light of his

¹ Of such a theoretical system, herein we give only a few hints, possibly reserving us to study in-depth the thought of this Author, in its relationships with the mathematics, in further papers.

² We recall that the above mentioned primary unconscious logic mechanisms were deduced by Freud mainly by two his clinical cases, precisely the case of the *Rat Man* (of 1909) and that of the *Wolf Man* (of 1914-1915), thanks to which Freud reached the discovery of the neurotic-obsessive disorders (see [2]). Besides, from the Rat Man paper, Freud established some analogies between the neurosis mechanisms and the primitive mind (see [2]).

³ According to this principle, the unconscious treats a single thing (individual, object, concept, etc) as it were a member, or an element, of a class which contains other members or elements; in turn, this class is considered as a subclass of another more general class, and so forth.

⁴ According to which, in the realm of unconscious, every relation is symmetric (just in the mathematical sense of this term).

principles. According to Matte Blanco, the conscious and unconscious are two different *modes of being* respect to the psychophysics unity of the human individual, *asymmetric* and in becoming the first one, *symmetric* and static the second one: this terminology is due to the fact that the latter is regulated by the above mentioned symmetric principle, contrarily to the first one.

Following [1, Chap. III], the symmetry and staticity, characterizing the unconscious, do not allow any finite-dimensional space-time idea as well as any sequential logic reasoning (which relies on asymmetry, as we will see later), so that the asymmetric conscious thought seems to be the result of a sort of ‘symmetry breaking’ of the infinite symmetric unconscious world (recalling, besides, that the symmetry breaking mechanisms, according to the modern physics, are at the basis of any fundamental physical phenomenology from the dynamical viewpoint). Nevertheless, according to Matte Blanco, the conscious becoming cannot do without the being unconscious, so that it may seem to be, in a certain sense, solved the secular *vexata quaestio* concerning the known *Parmenides-Heraclitean dialectic* between the logic of being and the logic of becoming (see also [3, Chap. 6, Section 6.2]). Indeed, according to this Author, the pair unconscious-conscious is inseparable. The symmetric thought is unthinkable without the asymmetric one, and the limit between normality and abnormality is given by the degree of reciprocal compenetration of these two modes of being.

In [1, Chap. IV], it is discussed the Matte Blanco’s notion of unconscious as infinite set, resuming the distinction between *set* and *class*, as typical of formal set theory. The unconscious does not distinguish between *partial* and *total* object and, moreover, each element of any set is conceived as having only human qualities (anthropomorphization attempts). This last property is a fundamental epistemological assumption common to many theory of the history of human thought, even if Matte Blanco deduced it from psychoanalytic considerations.

In [1, Chap. IV], it is also discussed the notion of infinite set in Mathematics, analogically compared with the symmetric mode of being of the unconscious, precisely with its property of indistinguishability between the part and the whole, in the sense that they both have the same cardinality, this just being the first notion of infinite set according to R. Dedekind (that, inter alia, has considered the notion of infinite set as a tool to explain the world of the human thought – see [1, Chap. IV, pp. 47-48, footnote 3]). Again according to Matte Blanco, many other mathematical concepts (like that of limit process) have their origins by the attempts to asymmetrically explain the properties of the symmetric one.

In [1, Chap. V], it is explained some useful concepts about consciousness according to Matte Blanco. Exactly, it cannot do without the asymmetric thought, in the sense that a conscious act consists in a continuous setting-up of asymmetric relations around the *cathexis object* (that is to say, the not-well defined *thing* invested by the human desire). The main consciousness’ activity is essentially analytic because it fundamentally subdivides every analyzed object into its constitutive components or parts (partial objects), unlike by an emotion, or an affection, which is a globally conceived symmetric sentiment⁵.

Nevertheless, the symmetric and asymmetric modes are inseparable amongst them, because an entirely symmetric mode is typical of any state of loss of consciousness whereas a complete asymmetric mode is also impossible since it would imply a total absence of any cathexis object, which is impossible for each human being. Every normal psychic state varies within an interval (or range) including a right mixing of both these modes, but whose ratio is continuously changing. Moreover, if we consider, for instance, a mathematical argument – hence a full asymmetric thought, at least in the involved theoretical principles and at the end of his formulation – then there is always an unavoidable emotional involvement which may be described as an involvement of asymmetrical type⁶. Therefore, albeit a certain human result – like a mathematical proof – may seem to be the

⁵ It will also be interesting to discuss the essential difference among emotions, sentiments and affects – as, for instance, made by the notable recent work of the neuroscientist Antonio R. Damasio – and their possible role in Mathematics (see also next footnote), and this also taking into account other remarkable works of Matte-Blanco himself on the same subject.

⁶ This phenomenon can be identified in many historical cases of fundamental creative innovation: for instance, the celebrated Einstein’s (as well as the main Riemannian) ideas on space-time, have been mainly based, at least in their initial theoretical

result of a completely asymmetrical work, indeed its production is never separated from an emotive-affective component of symmetric nature. This last remark is a fundamental one for understanding the nature of a creative thought. Here, we simply observe that this fact gives a line of overall consistency to the whole present paper: indeed, a *creative thought* is just of this last type, that is to say, it is the result of a dialectical (inseparable) interaction of the two modes of being, symmetric and asymmetric⁷. Further, according to Matte Blanco, the consciousness may think only in a three-dimensionally way, plus eventually a fourth temporal dimension, so that the three-dimensional space seems to be the dimension of consciousness and imagination. The human thought thinks mainly by three-dimensional images, also in abstraction (confirming a suggestion by J. Hadamard – see [4]).

On the other hand, some consciousness contents are available only by means of introspection, which is an asymmetric phenomenon. According to Matte Blanco, it has a precise characteristic: namely, it never concerns the instant in which takes place the introspection, but it concerns the immediately previous moments (hence, the past). The human thought exists only if it is reflected on itself, or else, the most peculiar character of the human thought is just this *reflectivity*. The elusive character of the conscious thought is due to the fact that the real nature of the consciousness is temporally located between these two modes of being, the symmetric and the asymmetric one, so that each time we try to think a conscious content, then we diachronically restrict ourselves to the asymmetric mode, so completely excluding the (synchronically inevitable) symmetric components. Only historically thinking, it is possible to avoid (or minimize) the latter. In [1, Chap. VI], it is discussed the concept of emotion (see previous footnotes ⁵ and ⁶) which plays a fundamental role for all the psychic life, above all in the formation of the thought. It is also describable by means of introspection. Nevertheless, it is mainly (but not completely) a symmetric phenomenon (see [1, Chap. VI, p. 68]).

Finally, in [1, Chap. VII], it is delineated one of most important Matte Blanco's notions, precisely that of *Bilogic*. According to this Author, the unconscious logic (or *symmetric logic*) is, as already said, mainly based on the principle of symmetry and on the principle of generalization, which regulate the so-called mode of being *symmetric*. The latter is inseparable from the mode of being *asymmetric*, regulated by the bivalent logic (as already said, the usual 'definitions' are possible only with the asymmetric thought), and vice versa, that is to say, any human psychic manifestations is the result of the interaction and/or the cooperation between these two modes of being. And this implies that any human reasoning is the result of the combination of the rules of two main logics, that *symmetric* and the bivalent (or *asymmetric*) one, which, in turn, are interpreted as components of a unique *bilogic*. Therefore, every human psychic phenomenon turns out to be a bilogic process which is a mixed chain of symmetric and asymmetric subprocesses whose combination modes are, a priori, various and infinities, giving rise to the rich variety of the human thoughts.

The emergence, at the threshold of consciousness, of a bilogic process is related with the concept of *triad* by Matte Blanco. This last concept should be meant as a fundamental structure of the Mathematical Logic, according to which it is the entity formed by two theoretical objects related with each other by a third object called *relation*. Matte Blanco thinks that the logic-mathematical structures are the results of the application of his theory of human psyche structure based on the notion of bilogic process. The bilogic process has been analyzed in many therapeutic cases treated by Matte Blanco, in both normal and pathological (schizophrenic) cases. At the end, he has

formulation, on given philosophical ideas, hence with a certain sentimental (hence emotive, according to Damasio) participation (see also next footnote ⁹) if one considers the arising process of philosophical systems.

⁷ This point of view is also supported by the theses of one of the foremost scholars of schizophrenia, Silvano Arieti (1914-1981). In fact, in his remarkable studies on this disorder, exposed in [23], he subsequently distinguishes (in [24]) three main processes with which the human being thinks: a *primary* process which is prelogic with a paleosymbolic function, primitive, not abstract and with individual referents; a *secondary* process which regulates the logic-formal models of the vigil thought; and a *tertiary* process which is the result of the interaction of the first two, and culminating into the creative intuition. The first two of these three Arieti's mental processes are substantially those corresponding to the Freudian ones.

concluded that the normal thought takes place in a context of (logic) causality, whereas the schizophrenic one (which, as an emblematic paradigm, permits us to shed a look within the unconscious realm) seems to follow an acausal context.

Finally, in [1, Chap. VII, Section 2], it is summarized other very interesting analyses of the bilogic structure in schizophrenic patients according to the studies conducted by Matte Blanco. For our purposes, it is simply enough to observe that the chronic schizophrenic thought continuously uses the symmetric and generalized principles in her or his reasoning. On the other hand, these last, from a formal (or mathematical) viewpoint, imply an impossibility to establish the so-called *axiom of specification* (or of *separation*) of the Formal Set Theory, according to which (see [5, Chap. 1, Section 1]), if A is a set and $p(x)$ is a statement for each x of A , then there exists a set B such that $y \in B$ if and only if $y \in A$ and $p(y)$ is true⁸; in such a case, we will write $B \doteq \{y; y \in A, p(y)\}$.

The chronic schizophrenic patient, according to Matte Blanco, is unable to use such a fundamental axiom, hence he or she is also unable to (mentally) construct the Boolean algebra $P(A)$ given by all the subsets of an arbitrary set A (to which every complete and completely distributive Boolean algebra is isomorphic, by means of the theorems of A. Tarski – see [6, Chap. V, Section 2] – and other representation theorems like those of M.H. Stone). This is due to the almost total use of symmetry and generalization principles in her or his reasoning. On the other hand, according to the just mentioned Stone's representation theorems (see [6, Chap. V, Section 2], [7, Chap. I] and [17, Proposition 1.4.4]), the two-element Boolean algebra $\mathbf{2}$ (following Halmos' notations) with support set $\{0,1\}$, is isomorphic to a Boolean algebra of the type $P(A)$ for some set A (like, for instance, the set of all maximal ideals of $\mathbf{2}$, that is to say, its maximal spectrum). On the other hand, since $\mathbf{2}$ is the mathematical structure which formalizes the *propositional calculus* (that is to say, it is its *propositional algebra* – see [5, Chap. 6] and [3, Chap. 4]), it follows that a chronic schizophrenic patient is basically unable to construct such an algebra, that is to say, he or she is unable to perform a rigorous *sylogistic inference* (which is the general element of $\mathbf{2}$). This might explain, amongst other things, the great difficulties to perform a makes sense reasoning by a chronic schizophrenic patient. Furthermore, and this is a crucial point of our discussion, the two-element Boolean Algebra $\mathbf{2}$ is also the truth's value algebra of the semantics (meant as the study of the various, possible interpretations of the propositional calculus – see [14, Chap. 5]), whence it follows too that a chronic schizophrenic patient is unable to integrate syntax-semantics and semantics-pragmatics, as also witnessed by recent neuropsycholinguistic researches (see [15]). Moreover, from an historical viewpoint, we remember that the original motivations to the same George Boole's work entitled *An investigation of the laws of thought* (1854), in which he introduced this algebra structure (see [8, Chap. 2, Section 2.1.2]), were just due to the attempts to formally analyze the laws of Logic considered as a result of human thought.

Moreover, in [8, Chap. 7, Section 7.3], it is confirmed that many people reason according to an incorrect inference that, in the light of what has been said above, may be explained (following Matte Blanco) through the unavoidable presence of the symmetric thought together the asymmetric one, that is to say, by means of a bilogic process. In [8, Chap. 7, Section 7.3.1], it is discussed the structure of a syllogistic inference of the type $A \Rightarrow C$ considered as the main elementary and basic logic tool of reasoning, the primary epitome of consciousness: it is, according to the Mental Models Theory (see [9]), the result of the action of three inseparable phases, the construction, the integration and the verification. The *construction* consists in the interpretation of the premises (relatives to A), namely, in the individual construction, for each premise, of a mental model representing the state of things that every premises describes. The *integration* consists in the coherent integration of these various models into a unique integrated model (for instance, individuating one or more so-called *intermediate terms* B between the extreme terms A and C) for the syllogistic conclusion (relatives to C). Finally, the *verification* consists in the epistemological analysis of the validity of the reached conclusion (for instance, by means of the K.R. Popper

⁸ $p(x)$ true [false] means that x has [does not have] the property expressed by the statement p , supposing that every element of A has, or does not have, such a property p .

falsification method, constructing suitable counterexamples). Often, the integration phase includes the institution of possible effect-cause links between the two extreme terms. Moreover, the establishing of these last relations is mainly a creative activity, since it is not based on pre-constituted laws.

On the other hand, following [10], a *conditional enunciate* is a (rational) enunciate of the type «if A then C » where A are the initial assumptions (like, for example, a single enunciate or a conjunction or a set of enunciates) considered as axioms of a certain theory, whereas C is the conclusion. Usually, in the conditional enunciates of the type $A \Rightarrow C$, A are the *hypotheses* while C is the *thesis*. Not every conditional enunciate is a theorem, since as *theorem* we mean a conditional enunciate in which C is a logical consequence of A , that is to say, if, in every interpretation of the formal language in which are formulated A and C , C is true whenever A is true, and, in such a case, we write more specifically $\models A \Rightarrow C$. Hence, in each theorem, the conditional enunciate $A \Rightarrow C$ means that this syllogistic inference is valid, namely it is true for every interpretation. This leads us towards the *syntax*, independently by the *semantics* (which studies all the possible interpretations). From here, we are naturally led towards the general notion of *proof* of a theorem of the type $\models A \Rightarrow C$ which is the search for the chain (called *derivation*) of the formal passages each constituted by an elementary conditional enunciation through which it is overall possible to deduce C from A . Hence, it is formally explainable as the search for a series of intermediate terms B_1, \dots, B_n such that the proof is formed by the chain $A \Rightarrow B_1 \Rightarrow \dots \Rightarrow B_{n-1} \Rightarrow B_n \Rightarrow C$ explicable by means of the correct application of the rational logic rules. The quest for a proof is therefore a fundamental creative process considered as a transcendental mental function in searching for the structure of being. The existence of almost one derivation of C from A for a given theorem $\models A \Rightarrow C$, is guaranteed by the well-known *Gödel's completeness theorem* (1930): this theorem has mainly a psychological function because it does not suggest any operative or methodological indication about the search or individuation of the proof strategy as well as one of its possible derivations, this confirming the nature prevalently creative of it⁹.

On the other hand, considering, for example, an arbitrary insight process, it is therefore plausible to think that the long unconscious work in finding a proof (mentioned, amongst others, by J. Hadamard and H.J. Poincaré¹⁰) is owned to the (indivisible and homogeneous unity or) *syncretic* character of unconscious which has mainly an immediate unifying and multiple logic character¹¹,

⁹ In this regards, one of the most paradigmatic examples is given by the recent proof of *Poincaré conjecture* by Grigorij J. Perel'man, who has outlined it in three papers only without many formal details, between 2002 and 2003, but the experts, called to explicate all the formal passages of the proof, later found that these papers already contained all the *ideas* need for constructing a complete proof of this geometrisation conjecture. For the relative validation, it has been necessary to explicitly deduce any elementary passage $B_i \Rightarrow B_{i+1}$ involved in it, reaching to a complete proof of several hundred pages (more than six hundred – see [20]), whilst the Author achieved it with few ones in an utter and inexplicable (if not at the intuitive level) creative manner. This is an epitome of the mathematical creative work.

¹⁰ On the other hand, Poincaré himself, in [19], stated that “in Mathematics, it is the intuition that discovers, whilst it is the logic that proves”, coherently with what we have here established. The philosophical thought of this Author is indeed very close to the ideas here exposed and based on Matte Blanco's work. Thereafter, I. Lakatos said (in [26, Vol. II]) that the mathematics grows up and develops just thanks to a main dialectic relationship between rigour (*formal* proofs) and intuition (*preformal* and *postformal* proofs). Furthermore, following [21, pp. 285-286], in an emblematic and explicit way, Federigo Enriques affirms as «*such an extralogic dimension, called “unconscious”, is powerfully present within the same mental processes. The knowledge acquisition does not wholly fall into the lucid consciousness and volition domain, but extends itself through unconscious and instinctive associations and links. At the same sources of the rational processes, it is present an “original impulse” which strongly acts in the infancy of the individual and in that of the humankind; in short, the development of pure science embeds its roots into an affective disposition*». Finally, Giulio Vivanti, in [27, Preface, p. VII], in regards to the creative Weierstrass' approach to analytic functions, states that «[with this method] *one replaces to the calculus – which proves, but doesn't illuminate – the reasoning*». Finally, it would also be interesting to see what Krzysztof Maurin states in [29, Foreword] about the role played by unconscious (see above all [29, Foreword, pp. xxi-xxii]) in the creative thought, taking, as main model, the work achieved by Riemann, so confirming most of what has been said in this paper.

¹¹ In [16], we have taken into account some similar structural aspects of the yet distinct notions of unconscious according to C. Lévi-Strauss and C.G. Jung. The first Author considers a notion of unconscious meant as the place from which originates every form of possible thought, that is to say, it is considered as the ‘place of every science’, whereas the second Author considers a collective

impossible to the asymmetric (or conscious) thought. Indeed, following what is mentioned in [28, Chap. 1, Section 10], according to Poincaré the most insights spring out of an unexpected inner decisive inspiration often taking place in a moment in which the mind is very far from the solution to the problem under examination which has yet been, for a long time, inconclusively discussed and unconsciously incubated. It is as if, all the elements of this searched solution, put in movement from the previous conscious study (of the problem under examination), continue to mechanically (and unconsciously) roam within a sort of ‘cerebral maze’ until up when, suddenly, they finally find a road along which link themselves, in a continuous chain (that is, a derivation), from the hypotheses towards the thesis. Afterwards, Hadamard says as, amongst the infinite possible choices (namely, the above road), that is to say, amongst the infinite possible (unconscious) association of ideas (which pursue the solution), just our own unconscious seems to choose the one satisfying a kind of ‘beauty criterion’ (*d’après* P.A.M. Dirac) which is ruled by a certain instinctive sense of scientific-artistic elegance. In turn, the latter is influenced by our scientific education (as said in [28, Chap. 1]), that is to say, it is just the *method* that will become an instinct, in a manner that it is impossible to explain in words. Maybe, this might be related with the continuous content exchange between explicit and implicit memories. On the other hand, following [8], a syllogism is nothing but an *explicitation* process of the valid or interesting conclusions of what *implicitly* contained into the premises. Furthermore, taking into account what has been just said by Poincaré and Hadamard, Mario Ageno (in [28, Chap. 1]) adds that however only the (scientific) *method* cannot open the way to the finding of the solution to a problem if one does not learn to discovery as well new problems and to correctly formulate them.

Then, when one conducts a logic derivation of a mathematical proof, we essentially follow a bilogic process made by symmetric steps (which have a unifying and creative character in finding the various, not a priori given, intermediate terms B_i $i=1, \dots, n$) and asymmetric steps (in applying the necessary logic tools and rules for the various partial elementary syllogistic inferences $B_i \Rightarrow B_{i+1}$ for each i); in these two inseparable types of thought, it is then important to consider their degree of more or less contemporaneity. Hence, the Matte Blanco bilogic process may be considered as one of the most suitable candidate in trying to explain the primary bases of a creative process, whence it follows the unescapable role (or influence) played by unconscious (or symmetric) thought in finding a new mathematical proof (which is not otherwise rationally deducible – for instance, by means of generalization, analogy, extension, reductio ad absurdum, etc¹²), specially as concerns the affective-emotive aspects involved in it. The unavoidable presence of symmetric thought aspects also explains why, in certain cases, the new proof of a theorem is almost never perfect in its first initial form¹³, but it shall reach its perfect and correct (in a certain sense, definitive) form after the subsequent correction made upon the possible initial imperfections, that is, by elimination of any symmetric thought interference.

unconscious that assures, amongst other, the existence of opposite pairs of *dialectic* type and not of *polar* type, inasmuch the former reach a (Hegelian dialectic) synthesis’ conclusion from the initial dual elements of the opposite pair, whereas the latter keep, into a dynamical tensional state, such opposite elements without reaching any their synthesis (see also [18]). Thus, it would also be possible trying to explain the above syncretic character by means of the well-known *Pseudo Scotto Law* (see [3]) according to which, if a proposition and its negation (like in an opposite pair) contemporaneously subsist as true, then it is deductible every other possible proposition, so reaching to touch clear syncretic characters.

¹² In this regards, it is fundamental taking into account the distinction between *logic* and *mathematical processes* according to Hermann Weyl (see [22, Chap. XII, Section 1]), the second ones corresponding to the creative processes here considered. From the various, reciprocal interaction between these two basic *generative principles* of an arbitrary mathematical construction, it follows a remarkable thought’s line of the modern Mathematical Philosophy.

¹³ See the further, recent example given by the creative proof of the *last Fermat’s theorem* by Andrew J. Wiles, with the collaboration of Richard Taylor, whose first form (given in the celebrated 1993 Cambridge lectures) had needed for some corrections (worked out upon the previous Cambridge version, just with the aid of his former PhD student Richard Taylor) in order that it were correct (last 1995 proof). Nevertheless, this detracts nothing to the exceptional initial creative work of Wiles and to the subsequent as many creative error corrections by Wiles and Taylor. In this regards, see the interesting survey paper [25] and references therein.

In [4], Hadamard has hence pointed out that the major part of the mathematical insights take place through images¹⁴ and not through verbal processes. His conjecture is now also confirmed by the *double codex* theory due to W. Bucci (see [11], [12] and [13, Vol. 1, Chap. 5, Section 5.4]) and according to which, starting from the ideas of A. Paivio (see [13, Vol. 2, Chap. 5, Section 5.1]), there exist two equal rank codices, that *verbal* and the *non-verbal* one. The mature thought may be placed in both these codices. In particular, the non-verbal codex is that appointed to the emotional functions and to other holistic type of thoughts. It works by synchronous information through parallel multiple channels (hence following a *synchronous* logic). Therefore, we would be tempted to suggest that the asymmetric thought is ruled by an *asynchronous* logic, whereas the symmetric thought is mainly ruled by a synchronous logic. The brain perceives and works out the reality simultaneously by means of both codices, and they are continuously in reciprocal communication through bidirectional connections and continuous referential links. The thought by images is the primary (but not the unique one) and specific expression of complex emotions like desires, beliefs, expectations and other holistic sensory-motor experienced, representational and mathematical contents and others, which cannot be verbally expressed. It is evident that the Bucci's double codex theory may be closely correlated with the Matte Blanco's bilogic process and vice versa, at least from a historiographical and epistemological perspective.

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¹⁴ These Hadamard's suppositions seem having been recently confirmed too by some last neuroscience researches (mainly, by the neurocognitive researches on consciousness due to Stanislas Dehaene and co-workers) according to which there are fundamental and typical functional intercorrelations (*neural circuits*) between the frontal and prefrontal areas (mainly deputed to the rational and abstract thought) and the occipital-parietal ones (mainly deputed to elaboration of visual stimuli and images), considered to be at the basis of consciousness acts.

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