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# A comment to Fratini's "Rent as a share of product and Sraffa's price equations"

Yoann Verger

Contact: yoann.verger@grignon.inra.fr

## Abstract

This comment from Fratini's article "Rent as a share of product and Sraffa's price equations" corrects Fratini's demonstration that rent as a share of the product and rate of profits could both increase at the same time. By doing so, this comment underlines the specificity of Sraffa's theory of value, and especially the need to only analyze systems of production where exchanges are needed to reproduce the system.

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In a recent paper, Fratini (2016) investigates rent as a share of the product. He brilliantly shows how, in the Sraffian framework, this view of rent can help to better understand the institutions governing the distribution of the national product (in the line of Adam Smith, Ricardo and Marx), instead of only considering rent per unit of land. However, while trying to show that, in some cases of joint production, the rent and the rate of profits could increase at the same time, he uses an example that is outside the scope of Sraffa's analysis. After showing that the point he was trying to make can be demonstrated inside the scope of Sraffa's analysis, I will try to underline the relevance of staying inside the boundaries of Sraffa's analysis to defend the political project of Sraffa.

First, Sraffa already showed that, in some cases of joint production, the wage and the rate of profit could both increase:

"The possibility that the price of a product may fall faster than the wage has some notable consequences. The first is that the rule that the fall of the wage in *any* standard involves a rise in the rate of profits must now admit of an exception. " (Sraffa, 1960, § 72).

The same reasoning can apply to the relation between the rate of profits and rent *as a share of product*. In a Sraffian standard system of joint production, goods that are produced in surplus are in the same proportions they are used

as inputs. When the value of the surplus is set to unity (i.e. the numeraire is the standard commodity) and when the wage is kept constant to zero, it is possible to know the evolution of the rate of profits when the rent increases, even without knowing the prices. In this case, the maximum total rent is 1, i.e. the entirety of the surplus. In such system, at a certain level of the rent, the price of a product could increase more quickly than the rent, if this product is jointly produced with a good whose price increases more slowly than the rent. If this good is chosen as the new numeraire, it is possible that, while the rent increases in terms of the old numeraire (the standard commodity), it decreases in terms of the new numeraire. As the rate of profits has a physical meaning, it will decrease, whatever the numeraire, if the rent increases in terms of the standard commodity.

Then, with an appropriate numeraire and at a certain level of the rent, a decrease of the rent can be followed by a decrease of the rate of profits, and the reverse will be true. If the numeraire is the standard commodity, however, this can never happen, and Sraffa is right to claim:

“A tax on a basic product then will affect all prices and cause a fall in the rate of profits that corresponds to a given wage, while if imposed on a non-basic it will have no effect beyond the price of the taxed commodity and those of such other non-basics as may be linked with it.” (Sraffa, 1960, § 65).

This sentence almost conclude a chapter where the numeraire was always the standard commodity, and this may explain why Sraffa does not specify that this sentence is only true if the standard commodity is the numeraire.

In order to show that the rent and the rate of profits can sometimes increase together, Fratini uses as an example a system where only two industries jointly produce the same two goods, commodity  $a$  and commodity  $b$  (Fratini, 2016, section 4, example 3, represented below):

$$\begin{aligned} q_{i1}(a) + l_1 &\rightarrow q_{o1}(a) + q_1(b) \\ q_{i2}(a) + l_2 &\rightarrow q_{o2}(a) + q_2(b) \end{aligned} \tag{1}$$

While commodity  $a$  is used in both processes as input, commodity  $b$  is never used as input. Fratini calculates the rate of profits depending on the wage and the rent as a share of the product on commodity  $a$ , and he shows that, depending on the input coefficients of commodity  $a$  and labor, it may happen that the rate of profits increases at all levels of the rent if the rent on commodity  $a$  increases, given a constant wage.

The problem with this example is that at least one of the two processes must be self-reproducing. A process may be said self-reproducing if it produces enough commodities to reproduce itself without the need to exchange some commodities with other processes (Dupertuis and Sinha, 2009). In Fratini’s example, commodity  $a$  is the only commodity input of the system, it must then be produced as output in a greater quantity than it is used as inputs, *once the system is brought to a self-replacing state*. Therefore, either one of the two

processes produces more of the commodity  $a$  than it uses it as input, so that after an appropriate multiplication it can cover the needs of both processes, which means that this process is self-reproducing. Or each process produces more of commodity  $a$  than it uses it as input, which means that each process is self-reproducing. The numerical example developed by Fratini in his Appendix belongs to this second case:

$$\begin{aligned} 2a + 20l &\rightarrow 10a + b \\ 5a + 10l &\rightarrow 9a + b \end{aligned} \tag{2}$$

Sraffa only deals with systems that can possibly be transformed, through appropriate multiplications, into self-reproducing systems:

“This formulation presupposes the system to be in a self-replacing state; but every system of the type under consideration is capable of being brought to such a state merely by changing the proportion in which the individual equations enter it”, (Sraffa, 1960, § 3, footnote).

However, inside a self-reproducing system, all processes must be non-self-reproducing. This is because prices are defined in the theory of Sraffa as the exchange ratios that will allow the system to reproduce itself (and to distribute the surplus according to a given social rule):

“There is a unique set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated” (Sraffa, 1960, § 1).

If a process is self-reproducing, there is no need for this process to make exchanges with other processes in order to reproduce itself. The price of the commodity produced by this process and its rate of profits are then determined by the conditions of production of this process only.

Furthermore, if a non-self-reproducing process produces two outputs, the right method to find the Sraffian prices of each single output is to add an other non-self-reproducing process producing the same two outputs, as Sraffa itself explains it (Sraffa, 1960, § 50, footnote 2). But, if a self-reproducing process jointly produces two outputs, it is not possible to define the Sraffian price of each single output. It is of course possible to calculate prices, if we add to this self-reproducing process an other process of production (either self-reproducing or not). But these prices cannot be the *Sraffian* prices of the two joint products, as no exchanges are needed between both processes. These prices are not the “exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated”.

At this point, two questions arise. First what are Fratini’s prices? These prices are the values that satisfy the equations, providing that a unique price is found for each commodity. In his example, Fratini finds a price for each single commodity, because he has two equations with two unknowns: but these prices

are not Sraffian prices, because at least one process is self-reproducing. Second can we construct Sraffian prices from Fratini's example?

Sraffa (1960, § 6) states: "we shall assume throughout that any system contains at least one basic product". Hence, Sraffa's theory may well concern the determination of relative prices associated with a uniform rate of profits across sectors, but only for systems of single production that contains at least one basic product. I add to the previous restriction that, in case of joint production, the system of production must only contain one self-replacing subsystem, as the notion of "mutual dependence" associated with basic commodity in single production (Sraffa, 1960, § 7)<sup>1</sup> is lost if there is more than one self-replacing subsystem in joint production (see Dupertuis and Sinha, 2009). In Fratini's example, there is a basic commodity,<sup>2</sup> but either one or both processes are self-reproducing, so there is no "mutual dependence" between them. In his numerical example, where both processes are self-reproducing, he does not have one circular system of production, but two side by side.

The implication of the lack of such conditions is the apparent impossibility to build a standard system (e.g. Manara, 1980 and Samuelson, 2008, in order to prove that a standard system can not be constructed in some cases of joint production, use systems of joint production with "hidden" self-reproducing processes). However this difficulty is only apparent: following the method displayed in Dupertuis and Sinha (2009), in order to construct the standard system, we will have to limit the system to the self-reproducing sub-system. If there is several self-reproducing sub-systems, several standard systems can be constructed. But, as we already said, it is true that, if in the self-reproducing sub-system the number of commodities produced is higher than the number of processes, some prices will stay indeterminate.

The standard system is important for Sraffa, as he can show that the maximum rate of profits is independent from the prices (it is a "physical" property of the system) and that, by using the standard commodity as the numeraire, the distribution also appears to be independent from the prices (i.e. it can be known prior to prices). Only in one case, it is not possible to calculate a standard system: it is the case where all industries produce non-basic commodities with the help of labour only (e.g. see Pasinetti, 1993 for a deep investigation of labor economy without capital). Interesting results can be found by analyzing such systems, but it must be underlined that such analyses move away from the

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<sup>1</sup>It is desirable at this stage to explain why the ratios which satisfy the conditions of production have been called 'values' or 'prices' rather than, as might be thought more appropriate, 'costs of production'.

The latter description would be adequate so far as *non*-basic products were concerned, since, as it follows from what we have seen in the preceding section, their exchange ratio is merely a reflection of what must be paid for means of production, labour and profits in order to produce them—there is no mutual dependence.

But for a basic product there is another aspect to be considered. Its exchange-ratio depends as much on the *use* that is made of it in the production of other basic commodities as on the extent to which those commodities enter its own production" (Sraffa, 1960, § 7).

<sup>2</sup>Commodity *a* is a basic commodity, even following the definition given by Sraffa after the introduction of joint production (Sraffa, 1960, § 60).

kind of analysis developed by Marx and pursued by Sraffa. They are interested in the description of a real capitalist system, and they try to highlight the possibility to decouple the problem of the distribution of wealth from the problem of value. This is why the possibility that an “ultimate” commodity is produced only by labour is explicitly rejected by Sraffa (Sraffa, 1960, Appendix D, § 3).

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