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Propagation of local alterations using Level-Sets within the Arlequin framework

O. Jamond^{1,2}, H. Ben Dhia¹

¹ MSSMAT, UMR 8579 (CNRS, Ecole Centrale Paris)
92295 Chatenay Malabry Cedex France
{olivier.jamond,hachmi.ben-dhia}@ecp.fr

² Centre de Technologie de Ladoux (Michelin)
63000 Clermont-Ferrand

Résumé — A Heaviside function is used within the Arlequin method to derive an enriched Arlequin methodology labeled the *Shooting Star Method*. The potential of the latter to propagate cracks with great flexibility and effectiveness is exemplified by a numerical test.

Mots clés — Arlequin method, X-FEM, crack propagation.

1 Introduction

Generating repetitively meshes that match material discontinuities, propagating cracks or evolving zones with strains or stresses localizations in order to achieve accurate finite element solutions, is a rather tedious and time consuming task. Many flexible numerical methodologies have been developed during the last two decades to tackle efficiently this issue. Among other approaches, let us mention the Partition of Unity Method [8] and the Extended Finite Element Method (X-FEM) (e.g. [9]). These methods, quite reminiscent to Trefftz approach, enrich basically the finite element space, associated to a given coarse mesh by a local analytical or semi-analytical knowledge about the globally unknown solution. Typically, Level-Sets functions and asymptotic local behaviours of the solution of the considered problem are used locally in the X-FEM. These methods are particularly relevant for the approximation of problems where the local behaviour of the solution is known. The s-method [6] and the finite element method with patches [7] are full finite element-based approach that enriches the coarse finite element space by a local one.

The Arlequin modeling framework [2, 3, 4, 5]) has a wider range of application and presents as a real flexible multimodel and multiscale paradigm : by superposing a local model (say a patch) to a global model and using a partition of energies, it allows for almost any different representation for the super-imposed models. For the simulation of crack propagation or the evolution of fatigue or damage into a solid that may transform into a macroscopic crack, the project of using the Arlequin framework with possible blending with other partition of unity methods has been initiated few years ago. First steps of this project, using only the Arlequin framework, were achieved and significant numerical results have been reported and/or communicated [10, 11]. Two typical results, extracted from [10, 11], are given in the numerical section. In this work, a step forward is achieved in our project : a new methodology that keep the coarse mesh unchanged during the whole propagation process is developed. For this, the fine Arlequin patch is always relocalized in the near critical crack tip zone and the coarse space is enriched by a Heaviside function, which is

adapted during the propagation process. This approach which could be labeled the *shooting star* method is developed and numerically tested in this work.

2 Continuous Arlequin Formulation of a cracked elastic body problem

We consider a bidimensional linear elastic cracked solid, occupying the closure of a bounded domain Ω_0 . The boundary of the solid is partitioned into $\Gamma_u, \Gamma_g, \Gamma$. Γ_u is the part where the solid is clamped, and Γ defines the crack whose tip is denoted by o . Free boundary conditions are applied on the faces of the crack and on Γ_g . The solid is submitted to a volume density of loads f .

Let Ω_1 be a subdomain of Ω_0 , located in the near crack tip zone, which is partitioned into two regular non overlapping domains Ω_c and Ω_f such that (see figure 1) :

$$o \in \Omega_f \quad (1)$$

$$\partial\Omega_1 \subset \partial\Omega_c \quad (2)$$

$$meas(\Omega_i) > 0, i = c, f \quad (3)$$

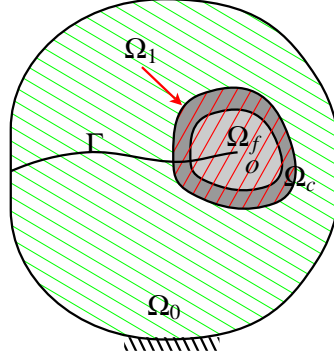


Figure 1: Arlequin domains

A Lagrangian Arlequin formulation of this problem reads as following : (see e.g. [3, 4, 5])

$$\text{Find } (u_0, u_1, \Phi) \in V_0 \times V_1 \times M ; \quad (4)$$

$$\forall v_0 \in V_0, \quad a_0(u_0, v_0) + c(\Phi, v_0) = l_0(v_0) \quad (5)$$

$$\forall v_1 \in V_1, \quad a_1(u_1, v_1) - c(\Phi, v_1) = l_1(v_1) \quad (6)$$

$$\forall \Psi \in M, \quad c(\Psi, u_0 - u_1) = 0 \quad (7)$$

where $V_0 = \{ \mathbf{v} \in H^1(\Omega_0); v = 0 \text{ on } \Gamma_u \}$ and where $V_1 = H^1(\Omega_1)$. The mediator space $M = H^1(\Omega_c)$. Furthermore, for $i = 0, 1$, the weighted internal and external virtual works are defined by :

$$\forall u_i, v_i \in V_i, \quad a_i(u_i, v_i) = \int_{\Omega_i} \alpha_i \sigma(u_i) : \varepsilon(v_i) dx \quad (8)$$

$$\forall v_i \in V_i, \quad l_i(v_i) = \int_{\Omega_i} \beta_i f \cdot v_i dx \quad (9)$$

The weight parameter functions $(\alpha_i, \beta_i), i = 0, 1$, are defined in Ω_0 and satisfy : ([5])

$$\alpha_i, \beta_i \in [0, 1] \text{ in } \Omega_0 \quad (10)$$

$$\alpha_0 + \alpha_1 = 1, \beta_0 + \beta_1 = 1 \text{ in } \Omega_0 \quad (11)$$

$$\alpha_0, \beta_0 = 1 \text{ in } \Omega_0 \setminus \Omega_1 \quad (12)$$

$$\exists \alpha_0 > 0; \alpha_i \geq \alpha_0 \text{ in } \Omega_f \quad (13)$$

The coupling operator c is defined by :

$$\forall(\Psi, v) \in M \times V_1, c(\Psi, v) = \int_{\Omega_c} \kappa_0 \{\Psi \cdot v\} + \kappa_1 \{\varepsilon(\Psi) : \varepsilon(v)\} dx \quad (14)$$

This problem has been analyzed mathematically in [5] and proved to be well-posed.

3 Discrete formulation : the *shooting star* method

To solve efficiently the Arlequin problem defined in the previous section, we suggest the following scheme : let T_h^0 be a mesh of the unfractured domain Ω_0^Γ . Let T_h^1 be a mesh of Ω_1 , significantly finer than T_h^0 , especially in the very near crack tip zone. Let Ω_o^s and Γ_0 denote the set of elements of T_h^0 containing the crack tip o and the part of the crack Γ exterior to Ω_o^s , respectively. Let $B_0 = (\varphi_{0i}, i = 1, n_0)$ be a classical vector-valued finite element basis associated to the mesh T_h^0 . We denote by V_{0h}^r the finite element space spanned by B_0 . This coarse space is enriched as follows. Let K_0^d be the set of indices k such that $\overset{\circ}{\omega}_k \cap \Gamma_0 \neq \emptyset$, $\overset{\circ}{\omega}_k$ standing for the interior of the support of the basis function φ_{0k} . Let Ω_d denote the union of these supports. Ω_d is divided by the crack Γ into two domains, denoted by Ω_d^+ and Ω_d^- . A precise definition of the domain Ω_d^+ (and thus of Ω_d^-) relies on the signed distance function : defining at each point $p \in \Gamma$ a classical positively oriented frame (τ_p, ν_p) , one can associate to each point $x \in \Omega_d$ (one of) its nearest point(s) x^* on Γ . The domain Ω_d^+ is then defined by :

$$\Omega_d^+ = \{x \in \Omega_d ; (x - x^*) \cdot \nu_{x^*} \geq 0\} \quad (15)$$

The following finite dimensional space :

$$V_{0h} = V_{0h}^r + V_{0h}^d = span\{(\varphi_{0i})_{i=1, n_0}, (H\varphi_{0j})_{j \in K_0^d}\} \quad (16)$$

is then taken as an approximation of V_0 , where H refers to the sign-like function, defined by :

$$H(x) = \begin{cases} +1 & \text{in } \Omega_d^+ \\ -1 & \text{elsewhere} \end{cases} \quad (17)$$

Now, based on the fine mesh T_h^1 of the cracked subdomain Ω_1 , a classical finite element subspace V_{1h} is constructed for the approximation of the local space V_1 . The discrete mediator space M_h is taken equal to the space of restriction of V_{1h} to the coupling zone Ω_c (though other choices are possible, as reported in [4, 5]). Finally, we assume that :

$$\Omega_o^s \subset \Omega_f \quad (18)$$

The condition (18) is a relaxation of the condition (1) that takes into account the finite scale introduced by the discretization.

With these finite dimensional spaces, our discrete enriched Arlequin approximation of the problem defined by (5)-(14) reads as follows.

$$\begin{aligned} \text{Find}(u_{0h}^r, u_{0h}^d, u_{1h}, \phi_h) &\in V_{0h}^r \times V_{0h}^d \times V_{1h} \times M_h; \\ \forall(v_{0h}^r, v_{0h}^d, v_{1h}, \Psi_h) &\in V_{0h}^r \times V_{0h}^d \times V_{1h} \times M_h \\ a_0(u_{0h}^r, v_{0h}^r) + a_0^d(u_{0h}^d, v_{0h}^d) + c(\phi_h, v_{0h}^r) &= l_0(v_{0h}^r) \end{aligned} \quad (19)$$

$$a_0^d(u_{0h}^r, v_{0h}^d) + a_0(u_{0h}^d, v_{0h}^d) + c^d(\phi_h, v_{0h}^d) = l_0(v_{0h}^d) \quad (20)$$

$$a_1(u_{1h}, v_{1h}) - c(\phi_h, v_{1h}) = l_1(v_{1h}) \quad (21)$$

$$c(\Psi_h, u_{0h}^r - u_{1h}) + c^d(\Psi_h, u_{0h}^d) = 0 \quad (22)$$

where :

$$a_0^d(u_{0h}^r, v_{0h}^d) = \int_{\Omega_0 \cap \Omega_d^+} \alpha_0 \sigma(u_{0h}^r) : \varepsilon(v_{0h}^d) dx + \int_{\Omega_0 \cap \Omega_d^-} \alpha_0 \sigma(u_{0h}^r) : \varepsilon(v_{0h}^d) dx \quad (23)$$

$$c^d(\Psi_h, u_{0h}^d) = \int_{\Omega_c \cap \Omega_d^+} \left\{ \kappa_0 \Psi_h \cdot u_{0h}^d + \kappa_1 \varepsilon(\Psi_h) : \varepsilon(u_{0h}^d) \right\} dx \quad (24)$$

$$+ \int_{\Omega_c \cap \Omega_d^-} \left\{ \kappa_0 \Psi_h \cdot u_{0h}^d + \kappa_1 \varepsilon(\Psi_h) : \varepsilon(u_{0h}^d) \right\} dx$$

This system involves terms related to the coupling between the enrichment degrees of freedom of the coarse model and the ones of the fine model. These terms are new when referring to specific computational developments for the standard Arlequin formulation, given in [4] and for the X-FEM, given in [9]. They require additional computational efforts that will be detailed during the conference.

4 Numerical results

4.1 Previous results

Figure 2, extracted from [10], shows the propagation of damage into a bimaterial solid submitted to a cyclic loading acting on its upper edge. From a numerical point of view, the critical mechanical phenomena in the near crack tip zone are taken into account by a local Arlequin patch, evolving and moving with the evolution of the critical zone during the cyclic loading. Observe that the coarse global mesh is adapted when needed to take into account the part of the crack which is not contained in the Arlequin patch. Observe also that, for the considered test, the direction of propagation of the crack is the (known) straight material interface ; which simplifies significantly the simulation and particularly the adaptation of the coarse mesh to match the part of the crack outside of the patch.

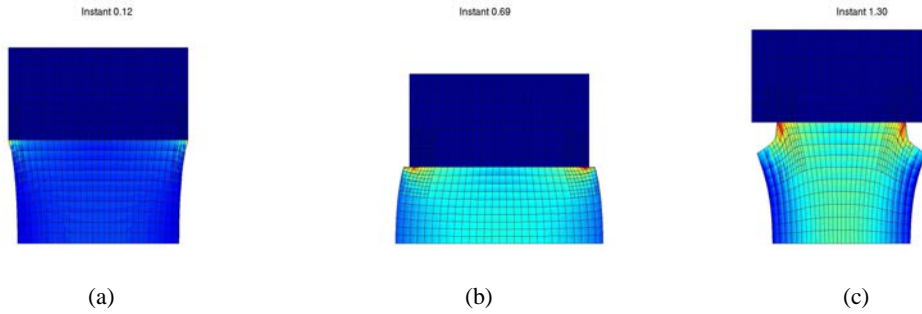


Figure 2: Propagation of damage and creation of a crack within the Arlequin framework (three selected snapshots)

Figure 3, extracted from [11], shows the propagation of a crack in an elastic structure by means of a refined finite element Arlequin patch, adapted to the evolution of the cracked zone, while the coarse mesh is unchanged during the whole process. Data for this problem (called the double cantilever beam test) were taken from the paper by Belytschko and Black [1].

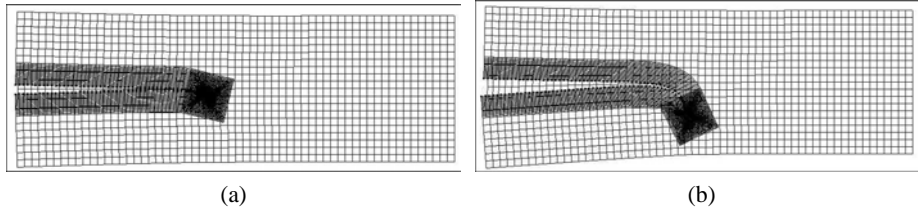


Figure 3: Propagation of a crack for the double-cantilever elastic beam

4.2 New results

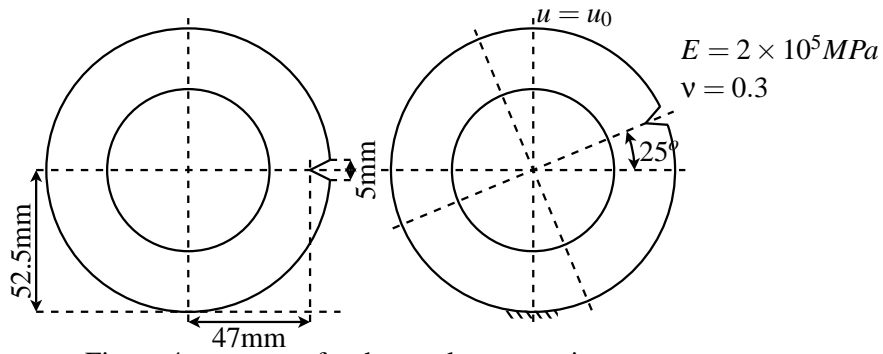


Figure 4: geometry for the crack propagation test

We simulate here a quasi-static propagation of a crack in a linear homogeneous and isotropic bidimensional elastic crown (see figure 4) in plane strain conditions. The top of the ring is assigned a displacement u_0 in the y direction. A small crack is initiated in the crown. The procedure used for the simulation of the propagation is the following simplified one : for a given size of the crack, the solution is calculated. The polar angle giving the maximum orthonormal stress component on the first crown of elements of the patch surrounding the crack tip is taken as the direction of propagation of the crack. Then a constant increment of $0.5mm$ is assigned to the crack.

Some steps of the propagation are reported in figures 5-a-b-c. A zoom is given in figure 5-d. These qualitative numerical results aim only at illustrating the effectiveness of our Arlequin/level-set methodology to simulate with enhanced flexibility the propagation of a crack. The main point is that this methodology prevents the remeshing of the whole structure, while capturing the main mechanical features in the critical zones. Only the patch surrounding the crack tip is locally remeshed at each step of the crack propagation.

5 Conclusion

A new methodology using the Heaviside enrichment within the multiscale Arlequin framework is developed here. The effectiveness of this methodology to introduce and propagate cracks in sound meshes is exemplified. The extension of this methodology to the simulation of damage propagation is an ongoing work that will be discussed during the conference.

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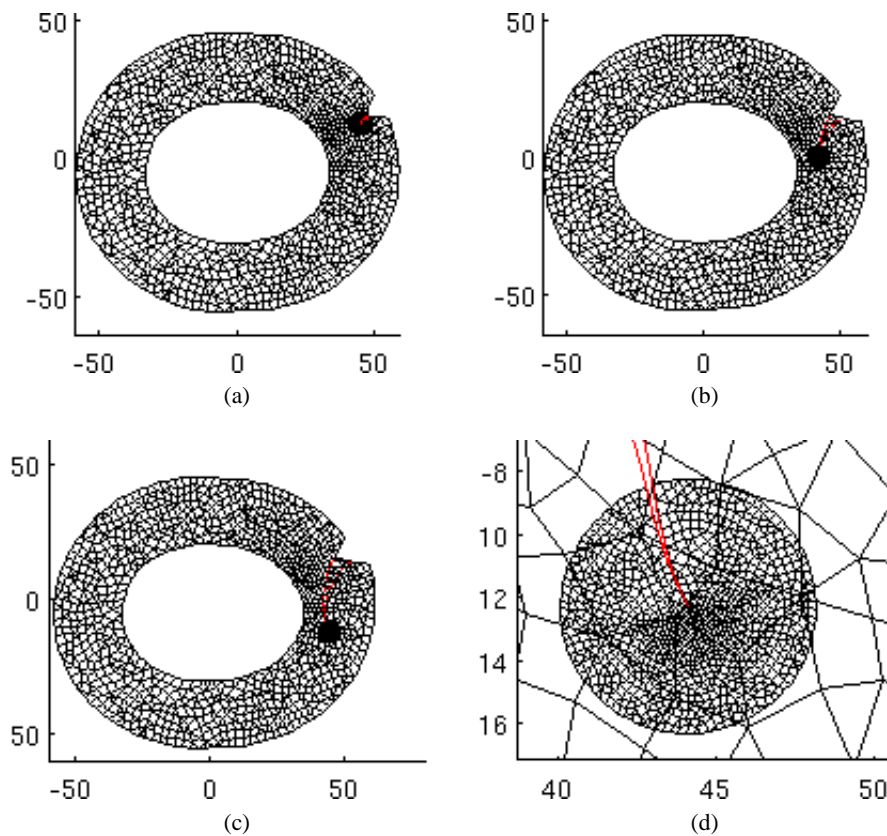


Figure 5: propagation of a crack

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