

Toward variational data assimilation for coupled models: first experiments on a diffusion problem

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informatiques mathématiques

Context

- Ocean-atmosphere coupled models have a key role in weather forecast nowadays
- ► The coupling methods may severely impact the model solution. An exact solution of the coupling problem can be obtained using a Global-in-Time Schwarz method (Lemarié et al. [2014]
- The initialisation of coupled models also has a major impact on the forecast solution (Mulholland et al. [2015])
- Few coupled DA methods started to be developed (Smith et al. [2015], Laloyaux et al. [2015]...) for coupled systems, and showed promising results

Our approach

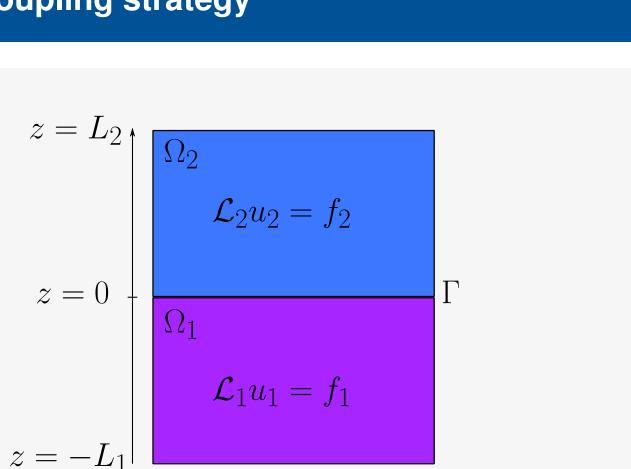
- The dynamical equations of our system are coupled using an iterative Schwarz domain decomposition method (Gander [2008])
- ► We are using variational DA techniques, which require minimization iterations and we are looking to take benefit of the minimization iterations to converge toward the exact **solution** of the coupling problem: the minimisations iterations substitute the Schwarz iterations
- Three general variational DA algorithms, are presented here and applied to a simple coupled system (Pellerej et al. [2016])

1. Model problem and coupling strategy

Let us define two models on each space-time domain $\Omega_d \times [0,T]$ (d = 1,2), with a common interface $\Gamma = \{z = 0\}$.

Problem: How to strongly couple the two models at their interface Γ ?

 \Rightarrow We propose to use a global-in-time Schwarz algorithm (Gander [2008])



on $\Omega_1 \times T_W$

on $\Gamma \times T_W$

 $z \in \Omega_1$

For a given initial condition $u_0 \in H^1(\Omega_1 \cup \Omega_2)$ and *first-guess* $u_1^0(0, t)$, the coupling algorithm reads

$$\begin{cases} \mathcal{L}_2 u_2^k = f_2 & \text{on } \Omega_2 \times T_W \\ u_2^k(z,0) = u_0(z) & z \in \Omega_2 \\ \mathcal{G}_2 u_2^k = \mathcal{G}_1 u_1^{k-1} & \text{on } \Gamma \times T_W \end{cases} \begin{cases} \mathcal{L}_1 u_1^k = f_1 \\ u_1^k(z,0) = u_0 \\ \mathcal{F}_1 u_1^k = \mathcal{F}_2 \end{cases}$$

 \mathcal{F}_d and \mathcal{G}_d are the interface operators, k is the iteration number, $T_W = [0, T]$, and $f_d \in L^2(0,T;L^2(\Omega_d))$ is a given right-hand side

- ► At convergence, this algorithm provides a mathematically strongly coupled solution which satisfies $\mathcal{F}_1 u_1 = \mathcal{F}_2 u_2$ and $\mathcal{G}_2 u_2 = \mathcal{G}_1 u_1$ on $\Gamma \times T_W$
- The convergence speed of the method greatly depends on the choice for \mathcal{F}_d and \mathcal{G}_d operators, and the choice of the *first-guess*

2. Classic data assimilation

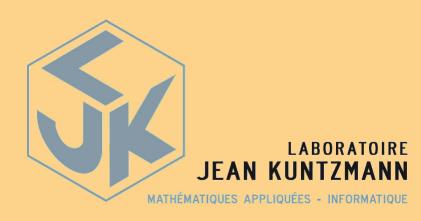
Let us introduce the classic cost function for variational data assimilation in the **uncoupled** case, for a domain Ω_d

► $\mathbf{x}_{0,d} = u_{0,d}(z) = u_0(z), \ z \in \Omega_d$ (d = 1, 2) is the controlled state vector $J^{o}(\mathbf{x}_{0,d})$

$$J_{Ucpl}(\mathbf{x}_{0,d}) = \overline{\left\langle \mathbf{x}_{0,d} - \mathbf{x}_{d}^{b}, \mathbf{B}^{-1}(\mathbf{x}_{0,d} - \mathbf{x}_{d}^{b}) \right\rangle_{\Omega_{d}}} + \overline{\int_{0}^{T} \left\langle \mathbf{y} - H\left(\mathbf{x}_{d}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}_{d}\right)) \right\rangle_{\Omega_{d}}} \,\mathrm{dt} \qquad (2)$$

where $\langle \cdot \rangle_{\Sigma}$ is the usual Euclidian inner product on a spatial domain Σ .





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(1)

3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition $u_0 = (\mathbf{x}_{0,1}^a, \mathbf{x}_{0,2}^a)^T$ obtained on Ω does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Mulholland et al. [2015])

Objective: properly take into account the coupling in the assimilation process

Full Iterative Method (FIM)

- $\mathbf{F} \mathbf{x_0} = u_0(z), \ z \in \Omega$
- We iterate the models till convergence of the Schwarz algorithm (k_{CVg} iterations)
- ▶ The first-guess u_1^0 in (1) is updated after each minimization iteration

$$J_{FIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_0 \left\langle \mathbf{y} - H\left(\mathbf{x}^{\text{evg}}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}^{\text{evg}}\right)) \right\rangle_\Omega dt$$
(3)

where
$$\mathbf{x}^{ ext{cvg}} = (u_1^{k_{ ext{cvg}}}, u_2^{k_{ ext{cvg}}})^{k_{ ext{cvg}}}$$

Truncated Iterative Method (TIM)

- ► $\mathbf{x}_0 = (u_0(z), u_1^0(0, t))^T$
- The Schwarz iterations are truncated at k_{max} iterations
- Extended cost function (misfit in the interface conditions) (Gejadze and Monnier [2007])

$$J_{TIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_0^T \left\langle \mathbf{y} - H\left(\mathbf{x}^{\text{trunc}}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}^{\text{trunc}}\right)) \right\rangle_\Omega dt + J^s$$

$$\text{here } J^s = \alpha_{\mathcal{F}} \|\mathcal{F}_1 u_1(0,t) - \mathcal{F}_2 u_2(0,t)\|_{[0,T]}^2 + \alpha_{\mathcal{G}} \|\mathcal{G}_1 u_1(0,t) - \mathcal{G}_2 u_2(0,t)\|_{[0,T]}^2 \text{ with}$$

$$u\|_{\Sigma}^2 = \langle a, a \rangle_{\Sigma} \text{ and } \mathbf{x}^{\text{trunc}} = (u_1^{k_{\text{max}}}, u_2^{k_{\text{max}}})^T$$

Coupled Assimilation Method with Uncoupled models (CAMU)

- ► $\mathbf{x}_0 = (\mathbf{x}_{0,1}, \mathbf{x}_{0,2})^T$ with $\mathbf{x}_{0,d} = (u_0|_{z \in \Omega_d}, u_d^0(0,t))$
- We suppress the coupling between both models
- The cost function for the CAMU is

$$J_{CAMU}(\mathbf{x}_0) = \left\{ \sum_{d=1}^{2} (J^b(\mathbf{x}_{0,d}) + J^o(\mathbf{x}_{0,d})) \right\} + J^s$$
(5)

Algo	Control vector	# of coupling iterations	extended cost function	Adjoint of the coupling	Coupling
FIM	$(u_0(z))$	$k_{ m cvg}$	no	yes	strong
TIM	$(u_0(z), u_1^0)^T$	k_{\max}	yes	yes	\sim strong
CAMU	$(u_0(z), u_1^0, u_2^0)^T$	0	yes	no	weak

openies of the coupled variational DA methods described

The originality of these algorithms is the use of a **Schwarz algorithm** to couple our models jointly to the DA process with an **extended cost function**.

4. Application to a 1D diffusion problem

- Previous algorithms are applied on a **1D linear diffusion problem**. We consider: $\blacktriangleright \mathcal{L}_d = \partial_t + \nu_d \partial_z^2$
- $\nu_1 \neq \nu_2$ the diffusion coefficients in each subdomain
- $\mathcal{F}_d = \nu_d \partial_z$ and $\mathcal{G}_d = \mathrm{Id}$ the interface operators on Γ (Dirichlet-Neumann)
- $u_d^{\star}(z,t) = \frac{U_0}{4} e^{-\frac{|z|}{\alpha_d}} \left\{ 3 + \cos^2\left(\frac{3\pi t}{\tau}\right) \right\}$ on $\Omega_d \times T_W$ the analytical solution

Single column observation experiment:

• Observations are available in $\Omega \setminus \{\Gamma\}$ at the end of the time-window (i.e. at t = T) • We define the interface imbalance indicator, equal to J^s with $\alpha_{\mathcal{G}} = 0.01$ and $\alpha_{\mathcal{F}} = 40$

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Algo	$\alpha_{\mathcal{G}}$	$\alpha_{\mathcal{F}}$	$k_{ m max}$	# of minin iterati
FIM	-	-	$k_{ m cvg}$	58
TIM	0	-	$rac{k_{ m cvg}}{k_{ m cvg}}$	48
TIM	0	-	5	245
TIM	0	-	2	151
TIM	0.01	-	2	425
TIM	0.01	-	1	34
CAMU	0.01	40	0	295
CAMU	0.001	4	0	26
CAMU	0.0001	0.4	0	742
Uncoupled	0	0	0	101
	Table	2:	Results	obtained for
101	FIM	1 1		10 ¹

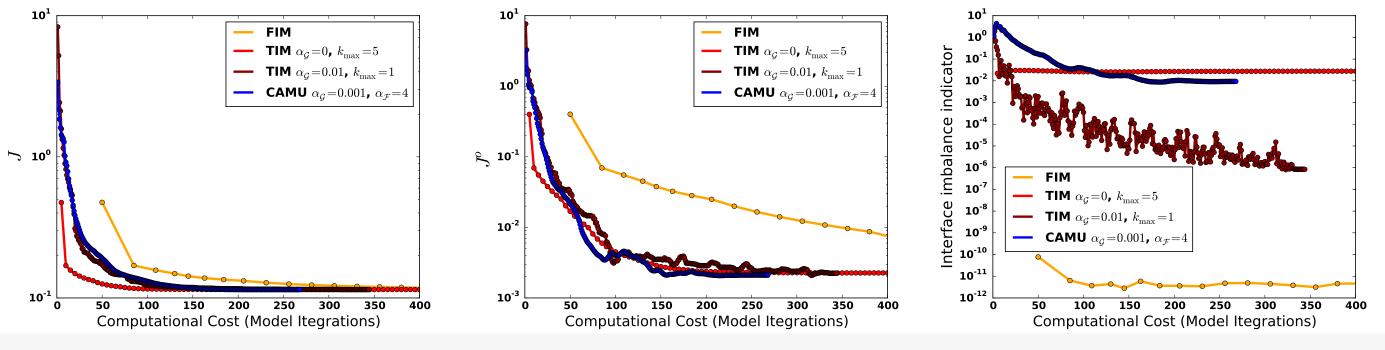


Figure 1 : Evolution of different terms with respect to number of model iterations for few configurations

6. Conclusions and perspectives

In the framework of an **iterative coupling**, we set up few data assimilation algorithms. Adding a physical constraint on the interface conditions in the cost function can have a beneficial effect on the performance of the method and allow to save coupling iterations ► An approach which only requires the adjoint of each individual model but not the adjoint of the

- coupling showed promising results
- The methods are very sensitive to the parameters choices
- ► We only test the algorithms on a simple linear problem

Perspectives

- studied
- will be considered

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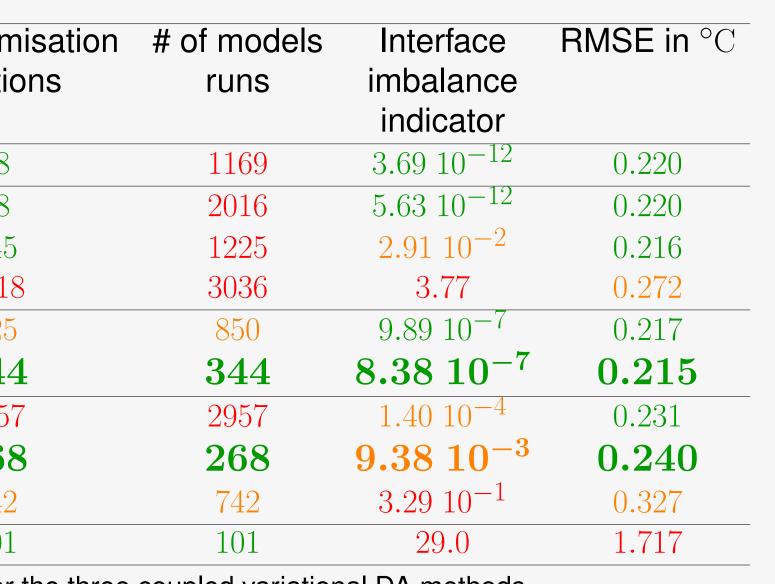
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bservation experiment results



or the three coupled variational DA methods

• Algorithm convergence and conditioning problem when J_S is part of the cost function will be

Since the objective is to apply such methods to ocean-atmosphere coupled models, increasingly complex models including physical parameterisations for subgrid scales, and non-linearities

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