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Toward variational data assimilation for coupled models: first experiments on a diffusion problem

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Context

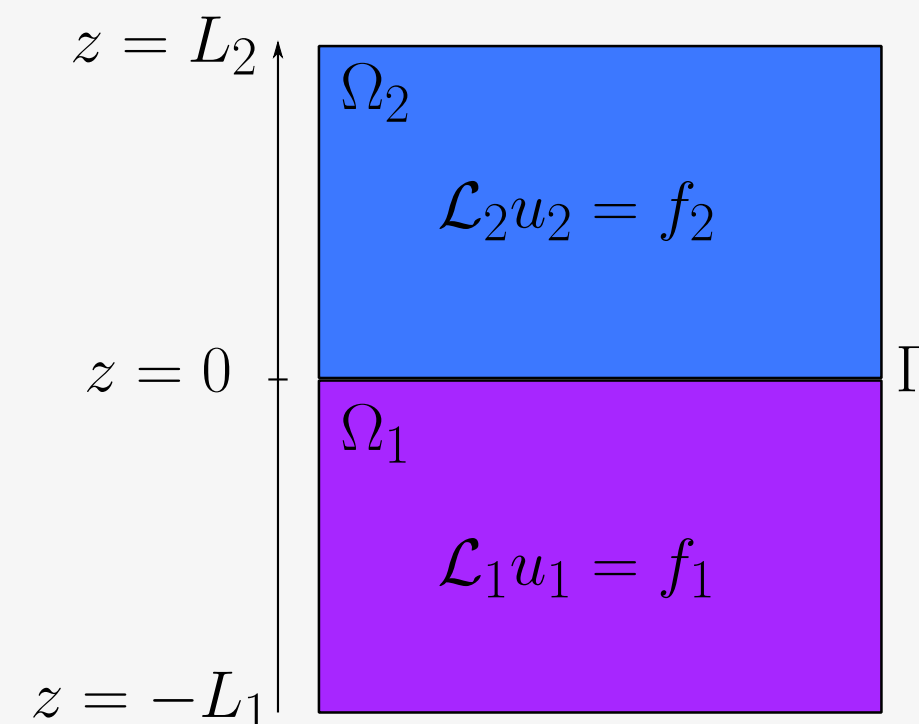
- **Ocean-atmosphere coupled models** have a key role in weather forecast nowadays
- The coupling methods may severely impact the model solution. An exact solution of the coupling problem can be obtained using a **Global-in-Time Schwarz method** (Lemarié et al. [2014])
- The **initialisation of coupled models** also has a major impact on the forecast solution (Mulholland et al. [2015])
- Few **coupled DA methods** started to be developed (Smith et al. [2015], Laloyaux et al. [2015]...) for coupled systems, and showed promising results

Our approach

- The dynamical equations of our system are coupled using an **iterative Schwarz domain decomposition method** (Gander [2008])
- We are using **variational DA techniques**, which require minimization iterations and we are looking to **take benefit of the minimization iterations to converge toward the exact solution** of the coupling problem: the minimisations iterations substitute the Schwarz iterations
- Three general variational DA algorithms, are presented here and applied to a simple coupled system (Pellerej et al. [2016])

1. Model problem and coupling strategy

Let us define two models on each space-time domain $\Omega_d \times [0, T]$ ($d = 1, 2$), with a common interface $\Gamma = \{z = 0\}$.



Problem: How to **strongly couple** the two models at their interface Γ ?

⇒ We propose to use a global-in-time Schwarz algorithm (Gander [2008])

For a given initial condition $u_0 \in H^1(\Omega_1 \cup \Omega_2)$ and **first-guess** $u_1^0(0, t)$, the coupling algorithm reads

$$\begin{cases} \mathcal{L}_2 u_2^k = f_2 & \text{on } \Omega_2 \times T_W \\ u_2^k(z, 0) = u_0(z) & z \in \Omega_2 \\ \mathcal{G}_2 u_2^k = \mathcal{G}_1 u_1^{k-1} & \text{on } \Gamma \times T_W \end{cases} \quad \begin{cases} \mathcal{L}_1 u_1^k = f_1 & \text{on } \Omega_1 \times T_W \\ u_1^k(z, 0) = u_0(z) & z \in \Omega_1 \\ \mathcal{F}_1 u_1^k = \mathcal{F}_2 u_2^k & \text{on } \Gamma \times T_W \end{cases} \quad (1)$$

\mathcal{F}_d and \mathcal{G}_d are the interface operators, k is the iteration number, $T_W = [0, T]$, and $f_d \in L^2(0, T; L^2(\Omega_d))$ is a given right-hand side

- **At convergence, this algorithm provides a mathematically strongly coupled solution which satisfies $\mathcal{F}_1 u_1 = \mathcal{F}_2 u_2$ and $\mathcal{G}_2 u_2 = \mathcal{G}_1 u_1$ on $\Gamma \times T_W$**
- **The convergence speed of the method greatly depends on the choice for \mathcal{F}_d and \mathcal{G}_d operators, and the choice of the first-guess**

2. Classic data assimilation

Let us introduce the classic cost function for variational data assimilation in the **uncoupled case**, for a domain Ω_d

- $\mathbf{x}_{0,d} = u_{0,d}(z) = u_0(z)$, $z \in \Omega_d$ ($d = 1, 2$) is the controlled state vector

$$J_{Upl}(\mathbf{x}_{0,d}) = \underbrace{\langle \mathbf{x}_{0,d} - \mathbf{x}_d^b, \mathbf{B}^{-1}(\mathbf{x}_{0,d} - \mathbf{x}_d^b) \rangle_{\Omega_d}}_{J^b(\mathbf{x}_{0,d})} + \underbrace{\int_0^T \langle \mathbf{y} - H(\mathbf{x}_d), \mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x}_d)) \rangle_{\Omega_d} dt}_{J^o(\mathbf{x}_{0,d})} \quad (2)$$

where $\langle \cdot \rangle_{\Sigma}$ is the usual Euclidian inner product on a spatial domain Σ .

3. Toward a coupled variational data assimilation

If the DA process is done separately on each subdomain, the initial condition $u_0 = (\mathbf{x}_{0,1}^o, \mathbf{x}_{0,2}^o)^T$ obtained on Ω does not satisfy the interface conditions. The interface imbalance in the initial condition can severely damage the forecast skills of coupled models (Mulholland et al. [2015])

Objective: properly take into account the coupling in the assimilation process

Full Iterative Method (FIM)

- $\mathbf{x}_0 = u_0(z)$, $z \in \Omega$
- We iterate the models till convergence of the Schwarz algorithm (k_{cvg} iterations)
- The first-guess u_1^0 in (1) is updated after each minimization iteration

$$J_{FIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_0^T \langle \mathbf{y} - H(\mathbf{x}^{cvg}), \mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x}^{cvg})) \rangle_{\Omega} dt \quad (3)$$

where $\mathbf{x}^{cvg} = (u_1^{k_{cvg}}, u_2^{k_{cvg}})^T$

Truncated Iterative Method (TIM)

- $\mathbf{x}_0 = (u_0(z), u_1^0(0, t))^T$
- The Schwarz iterations are truncated at k_{max} iterations
- Extended cost function (misfit in the interface conditions) (Gejadze and Monnier [2007])

$$J_{TIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_0^T \langle \mathbf{y} - H(\mathbf{x}^{trunc}), \mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x}^{trunc})) \rangle_{\Omega} dt + J^s \quad (4)$$

where $J^s = \alpha_{\mathcal{F}} \|\mathcal{F}_1 u_1(0, t) - \mathcal{F}_2 u_2(0, t)\|_{[0, T]}^2 + \alpha_{\mathcal{G}} \|\mathcal{G}_1 u_1(0, t) - \mathcal{G}_2 u_2(0, t)\|_{[0, T]}^2$ with $\|a\|_{\Sigma}^2 = \langle a, a \rangle_{\Sigma}$ and $\mathbf{x}^{trunc} = (u_1^{k_{max}}, u_2^{k_{max}})^T$

Coupled Assimilation Method with Uncoupled models (CAMU)

- $\mathbf{x}_0 = (\mathbf{x}_{0,1}, \mathbf{x}_{0,2})^T$ with $\mathbf{x}_{0,d} = (u_0|_{z \in \Omega_d}, u_d^0(0, t))$
- We suppress the coupling between both models

The cost function for the CAMU is

$$J_{CAMU}(\mathbf{x}_0) = \left\{ \sum_{d=1}^2 (J^b(\mathbf{x}_{0,d}) + J^o(\mathbf{x}_{0,d})) \right\} + J^s \quad (5)$$

Algo	Control vector	# of coupling iterations	extended cost function	Adjoint of the coupling	Coupling
FIM	$(u_0(z))$	k_{cvg}	no	yes	strong
TIM	$(u_0(z), u_1^0)^T$	k_{max}	yes	yes	~strong
CAMU	$(u_0(z), u_1^0, u_2^0)^T$	0	yes	no	weak

Table 1 : Overview of the properties of the coupled variational DA methods described

The originality of these algorithms is the use of a **Schwarz algorithm** to couple our models jointly to the DA process with an **extended cost function**.

4. Application to a 1D diffusion problem

Previous algorithms are applied on a **1D linear diffusion problem**. We consider:

- $\mathcal{L}_d = \partial_t + \nu_d \partial_z^2$
- $\nu_1 \neq \nu_2$ the diffusion coefficients in each subdomain
- $\mathcal{F}_d = \nu_d \partial_z$ and $\mathcal{G}_d = \text{Id}$ the interface operators on Γ (Dirichlet-Neumann)
- $u_d^*(z, t) = \frac{U_0}{4} e^{-\frac{|z|}{\alpha d}} \left\{ 3 + \cos^2 \left(\frac{3\pi t}{\tau} \right) \right\}$ on $\Omega_d \times T_W$ the analytical solution

Single column observation experiment:

- Observations are available in $\Omega \setminus \{\Gamma\}$ at the end of the time-window (i.e. at $t = T$)
- We define the **interface imbalance indicator**, equal to J^s with $\alpha_{\mathcal{F}} = 0.01$ and $\alpha_{\mathcal{G}} = 40$

5. Single column observation experiment results

Algo	$\alpha_{\mathcal{G}}$	$\alpha_{\mathcal{F}}$	k_{max}	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in $^{\circ}\text{C}$
FIM	-	-	k_{cvg}	58	1169	$3.69 \cdot 10^{-12}$	0.220
TIM	0	-	k_{cvg}	48	2016	$5.63 \cdot 10^{-12}$	0.220
TIM	0	-	5	245	1225	$2.91 \cdot 10^{-2}$	0.216
TIM	0	-	2	1518	3036	3.77	0.272
TIM	0.01	-	2	425	850	$9.89 \cdot 10^{-7}$	0.217
TIM	0.01	-	1	344	344	$8.38 \cdot 10^{-7}$	0.215
CAMU	0.01	40	0	2957	2957	$1.40 \cdot 10^{-4}$	0.231
CAMU	0.001	4	0	268	268	$9.38 \cdot 10^{-3}$	0.240
CAMU	0.0001	0.4	0	742	742	$3.29 \cdot 10^{-1}$	0.327
Uncoupled	0	0	0	101	101	29.0	1.717

Table 2 : Results obtained for the three coupled variational DA methods

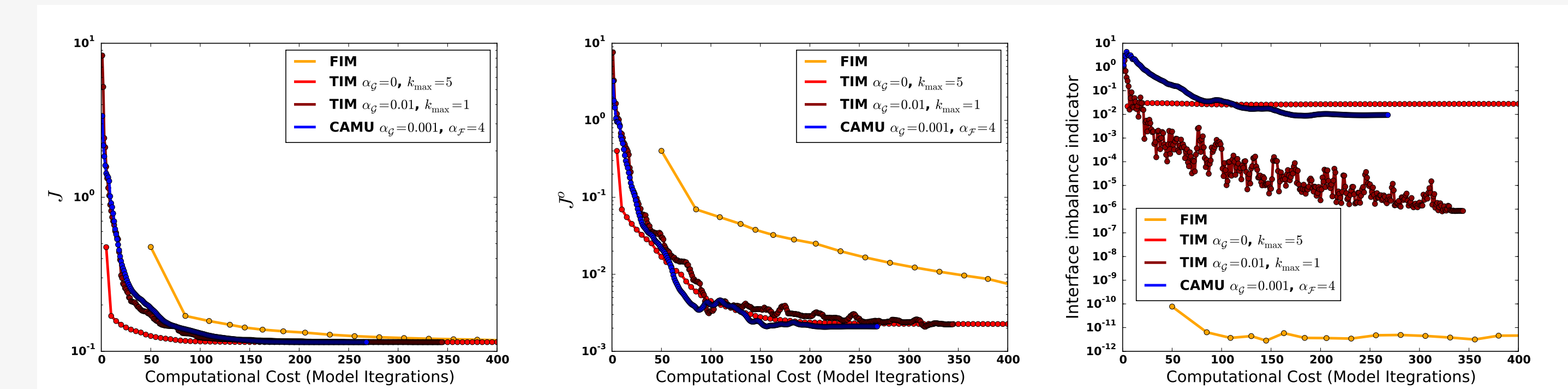


Figure 1 : Evolution of different terms with respect to number of model iterations for few configurations

6. Conclusions and perspectives

In the framework of an **iterative coupling**, we set up few data assimilation algorithms.

- **Adding a physical constraint on the interface conditions in the cost function can have a beneficial effect on the performance of the method and allow to save coupling iterations**
- **An approach which only requires the adjoint of each individual model but not the adjoint of the coupling showed promising results**
- **The methods are very sensitive to the parameters choices**
- **We only test the algorithms on a simple linear problem**

Perspectives

- Algorithm **convergence** and **conditioning** problem when J_S is part of the cost function will be studied
- Since the objective is to apply such methods to ocean-atmosphere coupled models, increasingly complex models including **physical parameterisations** for subgrid scales, and **non-linearities** will be considered

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