



Widely linear detection in MIMO communications from non-circularity to circularity

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Outline

- ◆ **Widely linear processing concepts**
- ◆ **MIMO detection model**
- ◆ **Widely linear detection I: non-circular signals**
- ◆ **Widely linear detection II: circular signals**
- ◆ **Performance evaluation**
- ◆ **Conclusions**

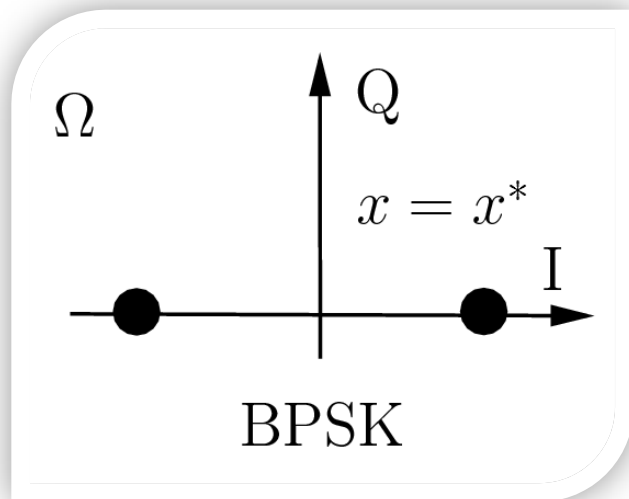
Widely linear processing (WLP) concepts

WLP: exploiting the information in received signal and its conjugate version, such as $\mathbf{x} = \mathbf{C}_1 \mathbf{y} + \mathbf{C}_2 \mathbf{y}^*$ if $\mathbf{y} = f(\mathbf{x})$.

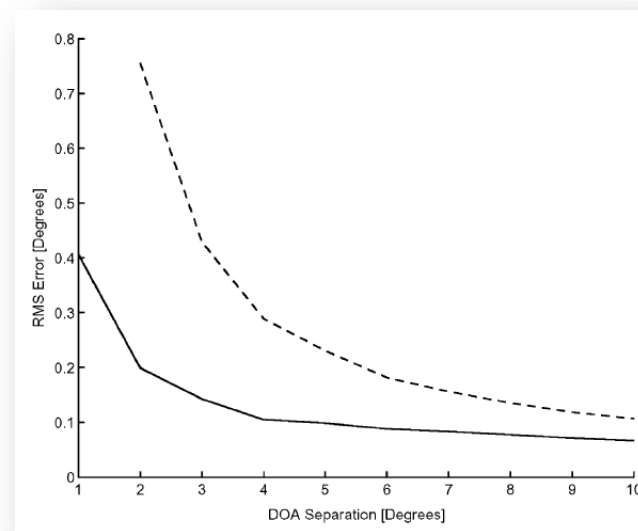
Potential case: non-circular signals

Applications: array signal processing, beamforming

◆ Example of BPSK



DoA finding [2]





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MIMO detection model

MIMO model:

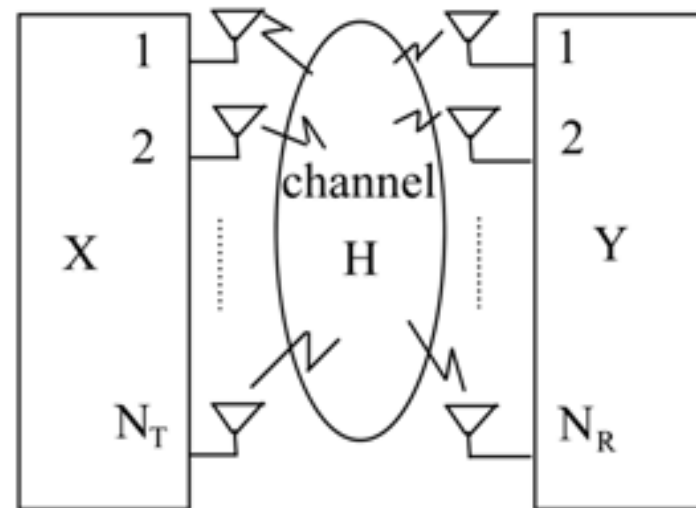
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

\mathbf{x} : transmitted signal vector

\mathbf{y} : received vector

\mathbf{w} : white Gaussian noise

\mathbf{H} : frequency flat fading





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Widely linear detection I: non-circular signals

Circular signals

A random signal \mathbf{x} is said to be second-order circular if it satisfies the following conditions

$$E[\mathbf{x}(t)\mathbf{x}^T(t+\tau)] = \mathbf{0}$$

$$E[\mathbf{x}(t)] = \mathbf{0}$$

Examples: QPSK, MPSK, MQAM in MIMO communications

Non-circular signals

A signal \mathbf{x} is second-order non-circular if $E[\mathbf{x}(t)\mathbf{x}^T(t+\tau)] \neq \mathbf{0}$ for at least one couple (t, τ)

Examples: BPSK, ASK, real signals in MIMO communications

Widely linear detection I: non-circular signals

Exploitation of real signals in MIMO

$$\mathbf{x}(t) = \mathbf{x}^*(t)$$

MIMO system model:

Signal property: $\mathbf{x}^* = \mathbf{x}$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \Rightarrow \mathbf{y}^* = \mathbf{H}^*\mathbf{x}^* + \mathbf{w}^*$$

$$\left. \begin{array}{l} \mathbf{x}^* = \mathbf{x} \\ \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \Rightarrow \mathbf{y}^* = \mathbf{H}^*\mathbf{x}^* + \mathbf{w}^* \end{array} \right\} \Rightarrow \mathbf{y}^* = \mathbf{H}^*\mathbf{x} + \mathbf{w}^*$$

Widely linear model:

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{w} \\ \mathbf{w}^* \end{bmatrix}$$

Widely linear detection I: non-circular signals

◆ **Detection:**
$$\hat{\mathbf{x}} = \mathbf{C} \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} = [\mathbf{C}_1 \quad \mathbf{C}_2] \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} = \mathbf{C}_1 \mathbf{y} + \mathbf{C}_2 \mathbf{y}^*$$

◆ **Extended zero-forcing (EZF)**
$$\mathbf{C} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \end{bmatrix}^\dagger$$

$$\mathbf{C}_1 = \mathbf{C}_2^*$$

$$\mathbf{C}_1 = (2 \operatorname{Re}[\mathbf{H}^H \mathbf{H}])^{-1} \mathbf{H}^H$$

◆ **Extended MMSE (EMMSE)**

$$\mathbf{C}_1 = \mathbf{C}_2^*$$

$$\mathbf{C}_1 = (2P \operatorname{Re}[\mathbf{H}^H \mathbf{H}] + \sigma^2 \mathbf{I})^{-1} P \mathbf{H}^H$$

◆ **EZF/EMMSE-OSIC**



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Widely linear detection II: circular signals

General case: MPSK signal

- ◆ In the complex plane of a MPSK signal constellation Ω , $|\Omega|=M$, scalar MPSK signal $x \in \Omega$ can be generally represented as:

$$x = \sqrt{P} \exp(j \frac{2\pi}{M} m) \quad m = 0, 1, \dots, M-1$$

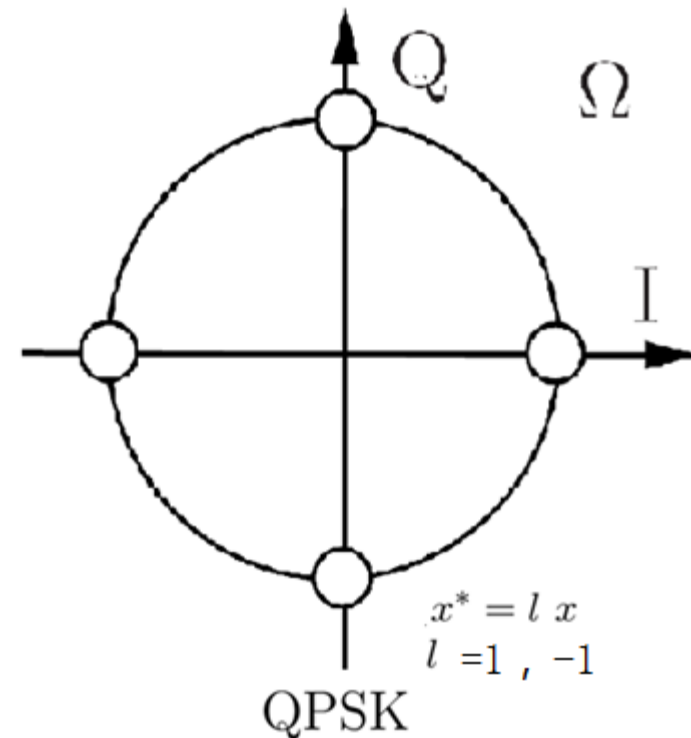
- ◆ Property 1 : Scalar signal x is drawn from MPSK constellation Ω , the following equation holds:

$$x^* = lx$$

where l is called rotation factor, $l \in \Phi$.

$$\Phi = \{ \exp(\pm j \frac{4\pi}{M} k) \mid k = 0, 1, \dots, \frac{M}{4} - 1 \}$$

Example





Widely linear detection II: circular signals

MPSK vector

Property 2 : For a n-dimensional signal vector $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ with elements drawn from an MPSK constellation, the following equation holds:

$$\mathbf{x}^* = \mathbf{M}_n \mathbf{x}$$

where \mathbf{M}_n is called rotation matrix,

$$\mathbf{M}_n = \begin{bmatrix} l_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & l_n \end{bmatrix}$$

with $l_i \in \Phi$, $|\Phi| = \frac{M}{2} = K$

Widely linear detection II: circular signals

Exploitation model in MIMO detection

$$\left. \begin{array}{l} \text{signal property: } \mathbf{x}^* = \mathbf{M}_{N_T} \mathbf{x} \\ \mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \Rightarrow \mathbf{y}^* = \mathbf{H}^* \mathbf{x}^* + \mathbf{w}^* \end{array} \right\} \Rightarrow \mathbf{y}^* = \mathbf{H}^* \mathbf{M}_{N_T} \mathbf{x} + \mathbf{w}^*$$

widely linear model:

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \mathbf{M}_{N_T} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{w} \\ \mathbf{w}^* \end{bmatrix}$$

Widely linear detection II: circular signals

◆ Detection:

$$\hat{\mathbf{x}} = \mathbf{C} \begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix}$$

◆ Extended zero-forcing (EZF)

$$\mathbf{C} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \mathbf{M}_{N_T} \end{bmatrix}^\dagger = (\mathbf{H}^H \mathbf{H} + \mathbf{M}_{N_T}^* \mathbf{H}^T \mathbf{H}^* \mathbf{M}_{N_T})^{-1} [\mathbf{H}^H \quad \mathbf{M}_{N_T}^* \mathbf{H}^T]$$

◆ Extended MMSE (EMMSE)

$$\mathbf{C} = (P \mathbf{H}^H \mathbf{H} + P \mathbf{M}_{N_T}^* \mathbf{H}^T \mathbf{H}^* \mathbf{M}_{N_T} + \sigma^2 \mathbf{I})^{-1} [\mathbf{H}^H \quad \mathbf{M}_{N_T}^* \mathbf{H}^T]$$

◆ EZF/EMMSE-OSIC

Widely linear detection II: circular signals

◆ **Difficulty: unknown** $\mathbf{M}_{N_T} \in \Phi^{N_T}$, $|\Phi^{N_T}| = (M/2)^{N_T}$

◆ **Complexity:**

$(M/2)^{N_T}$	matrix inverse
$(M/2)^{N_T}$	candidate vector \mathbf{x}
$(M/2)^{N_T}$	likelihood test

◆ **Proposed algorithm: WLSD**


Widely linear detection II: circular signals

Widely linear sphere decoding

widely linear model:

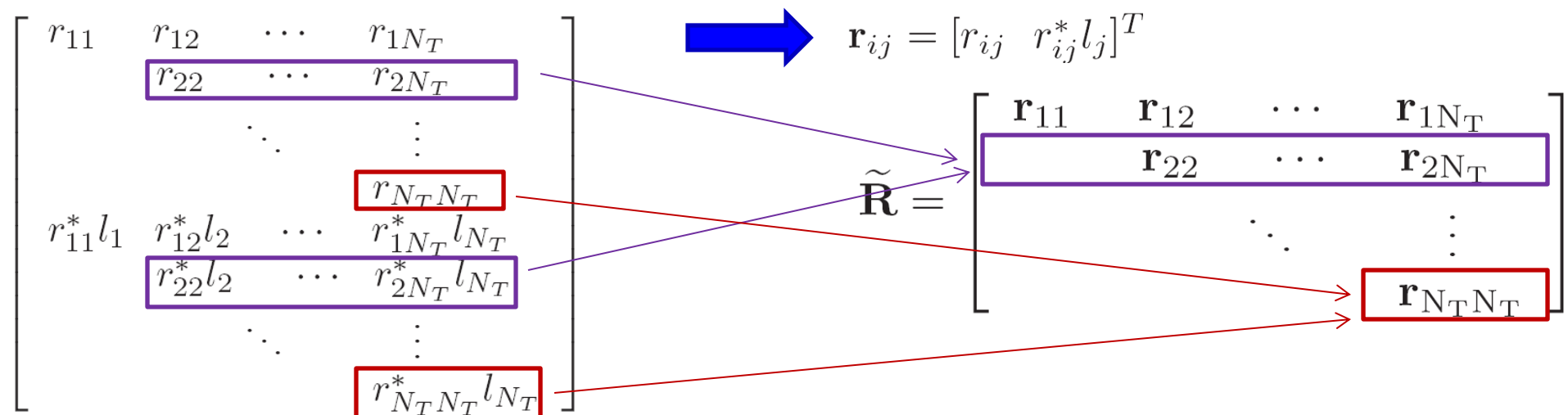
$$\begin{bmatrix} \mathbf{y} \\ \mathbf{y}^* \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}^* \mathbf{M}_{N_T} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{w} \\ \mathbf{w}^* \end{bmatrix}$$

Transformation by exploiting QR decomposition $\mathbf{H} = \mathbf{QR}$:

$$\begin{bmatrix} \mathbf{Q}^H \mathbf{y} \\ \mathbf{Q}^T \mathbf{y}^* \end{bmatrix} = \begin{bmatrix} \mathbf{R} \\ \mathbf{R}^* \mathbf{M}_{N_T} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{Q}^H \mathbf{w} \\ \mathbf{Q}^T \mathbf{w}^* \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & \cdots & r_{1N_T} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{N_T N_T} \\ r_{11}^* l_1 & \cdots & r_{1N_T}^* l_{N_T} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{N_T N_T}^* l_{N_T} \end{bmatrix}$$

Widely linear detection II: circular signals

By gathering the inter-conjugate rows:



New model:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{R}}\mathbf{x} + \tilde{\mathbf{w}}$$

$$\tilde{\mathbf{y}} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_{N_T}^T]^T \quad \mathbf{y}_i = [y_i, y_i^*]^T$$

Widely linear detection II: circular signals

New model:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{R}}\mathbf{x} + \tilde{\mathbf{w}}$$

$$\tilde{\mathbf{y}} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_{N_T}^T]^T \quad \mathbf{y}_i = [y_i, y_i^*]^T$$

Sphere searching based on rotation factor

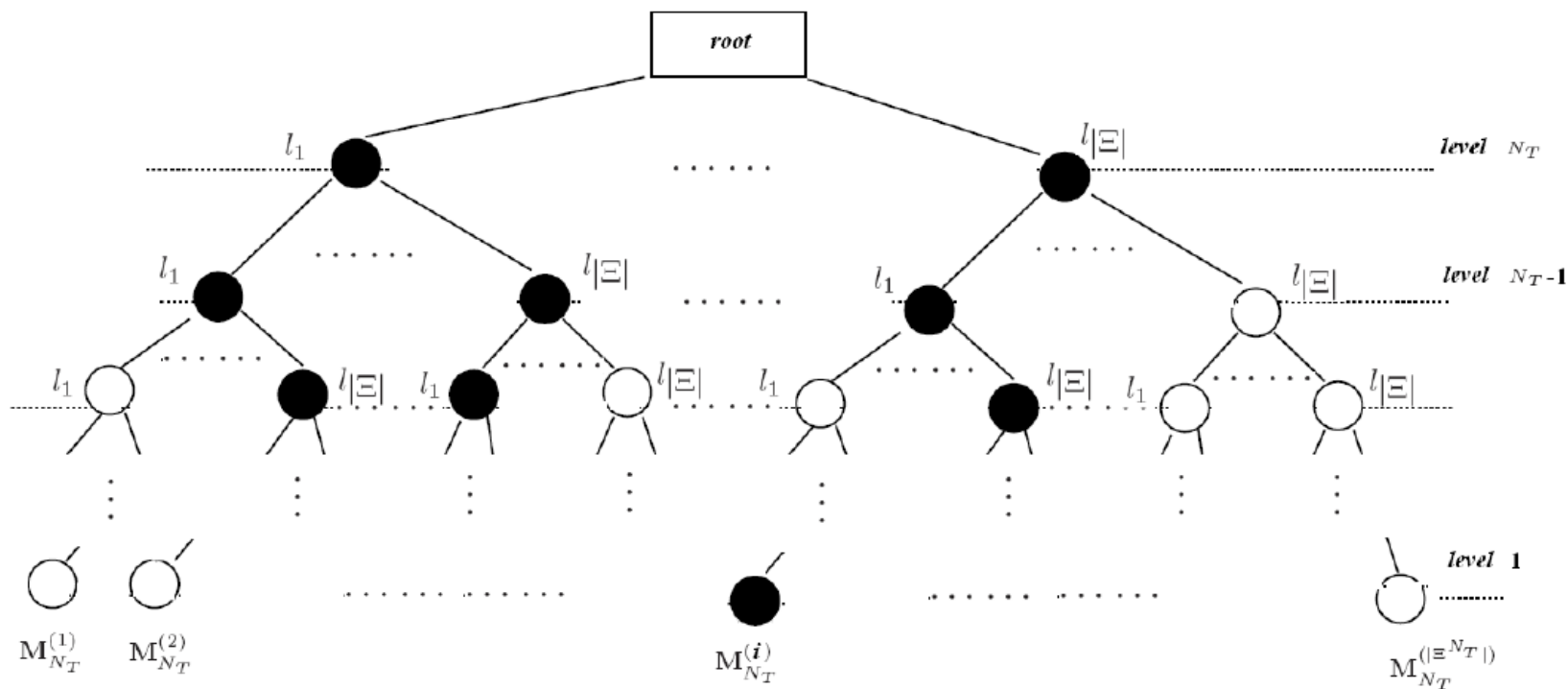
$$\mathbf{M}_{N_T} = \arg \min_{\mathbf{1} \in \Xi^{N_T}} \|\tilde{\mathbf{y}} - \tilde{\mathbf{R}}\mathbf{x}\|^2 \leq d^2$$

For each element x_i , we have:

$$\|\mathbf{y}_{N_T} - \mathbf{r}_{N_T N_T} x_{N_T}\|^2 \leq d^2$$
$$\|\mathbf{y}_j - \sum_{i=j}^{N_T} \mathbf{r}_{ji} x_i\|^2 \leq d^2 - \sum_{k=j+1}^{N_T} \|\mathbf{y}_k - \sum_{i=k}^{N_T} \mathbf{r}_{ki} x_i\|^2$$

Widely linear detection II: circular signals

Widely linear sphere decoding





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Performance evaluation

Detection for non-circular signal

Detector:	EZF/EMMSE	EZF-OSIC/EMMSE-OSIC
Diversity:	$N_R - \frac{N_T - 1}{2}$	$[N_R - \frac{N_T - 1}{2}, N_R]$
Complexity:	polynomial	

Detector:	ZF/MMSE	ZF-OSIC/MMSE-OSIC
Diversity:	$N_R - N_T + 1$	$[N_R - N_T + 1, N_R]$
Complexity:	polynomial	

Performance evaluation

Detection for circular signal

Detector: WLSD

Diversity: $[N_R - \frac{N_T - 1}{2}, N_R]$

Candidate set:

$$\mathbf{M}_{N_T} \in \Phi^{N_T}, \quad |\Phi^{N_T}| = (M/2)^{N_T}$$

Complexity: exponential

Detector: SD

Diversity: N_R

Candidate set:

$$\mathbf{x} \in \Omega^{N_T} \quad |\Omega^{N_T}| = M^{N_T}$$

Complexity: exponential

Comparison

$$|\Phi^{N_T}| = \frac{|\Omega^{N_T}|}{2^{N_T}}$$

Performance evaluation

Variations from non-circularity to circularity

Non-circular signal

Candidates set

$$\mathbf{M}_{N_T} = \mathbf{I}_{N_T}, \quad \Phi^{N_T} = \{ \mathbf{I}_{N_T} \}, \quad |\Phi^{N_T}| = 1$$

Detector: EZF/EMMSE+OSIC

Complexity: polynomial

Diversity:

$$\left[N_R - \frac{N_T - 1}{2}, N_R \right]$$

Circular signal:

Candidates set

$$\mathbf{M}_{N_T} \in \Phi^{N_T}, \quad |\Phi^{N_T}| = (M / 2)^{N_T}$$

Detector: WLSD

Complexity: exponential

Diversity :

$$\left[N_R - \frac{N_T - 1}{2}, N_R \right]$$

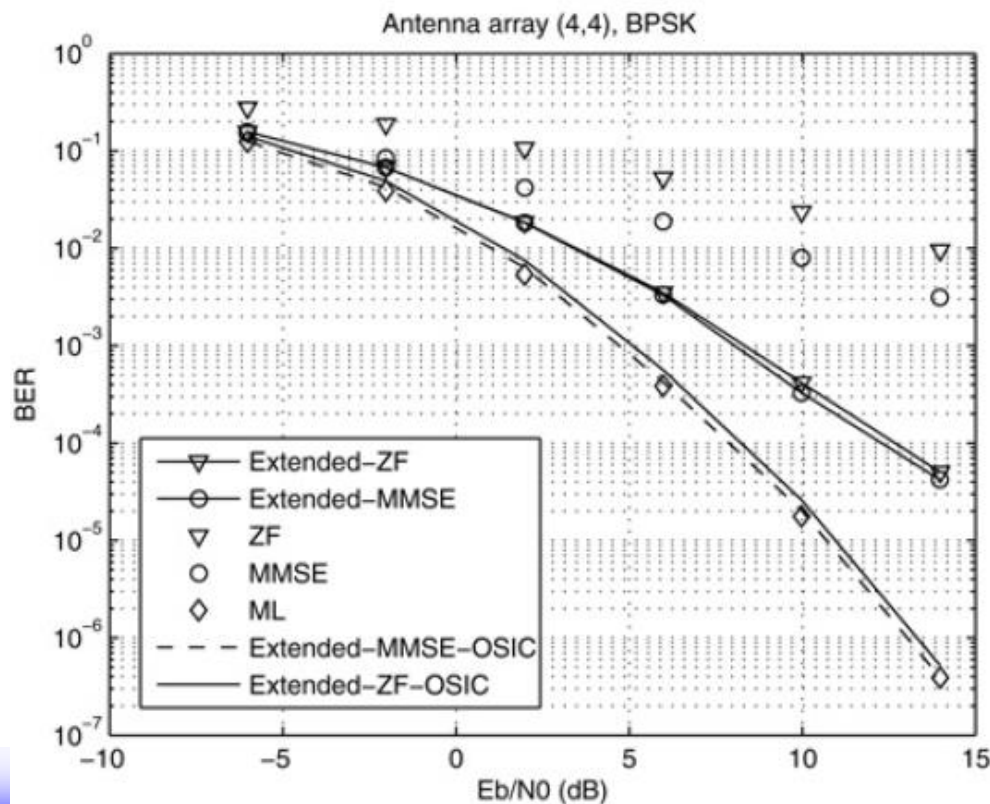
Performance evaluation

BER performance 1:

Simulation setting:

◆ (4,4) array

◆ BPSK modulation signal



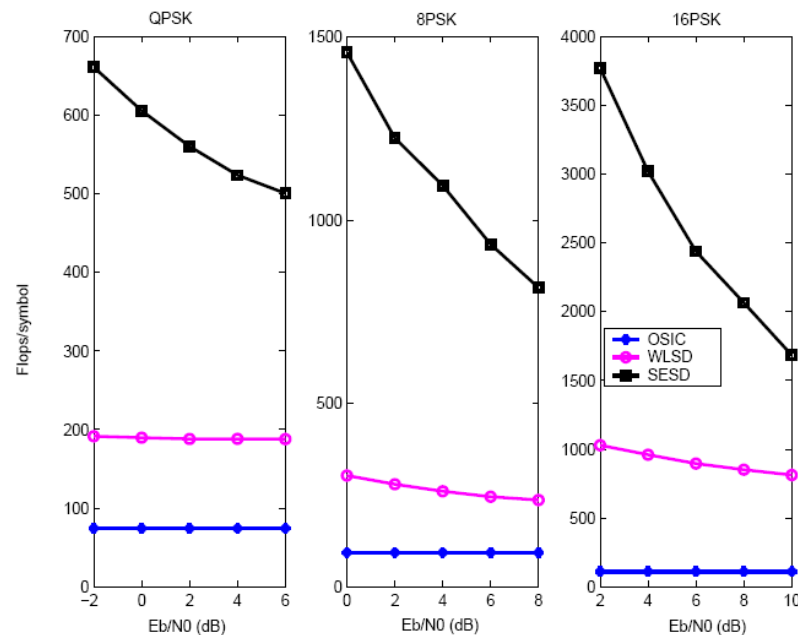
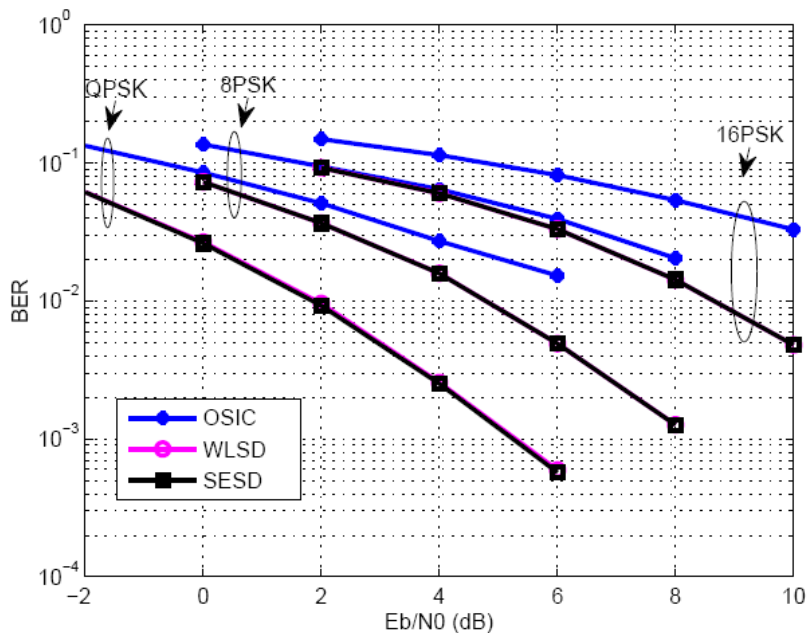
Performance evaluation

BER performance 1:

Simulation setting:

◆ (4,4) array

◆ QPSK, 8PSK, 16PSK modulation signal



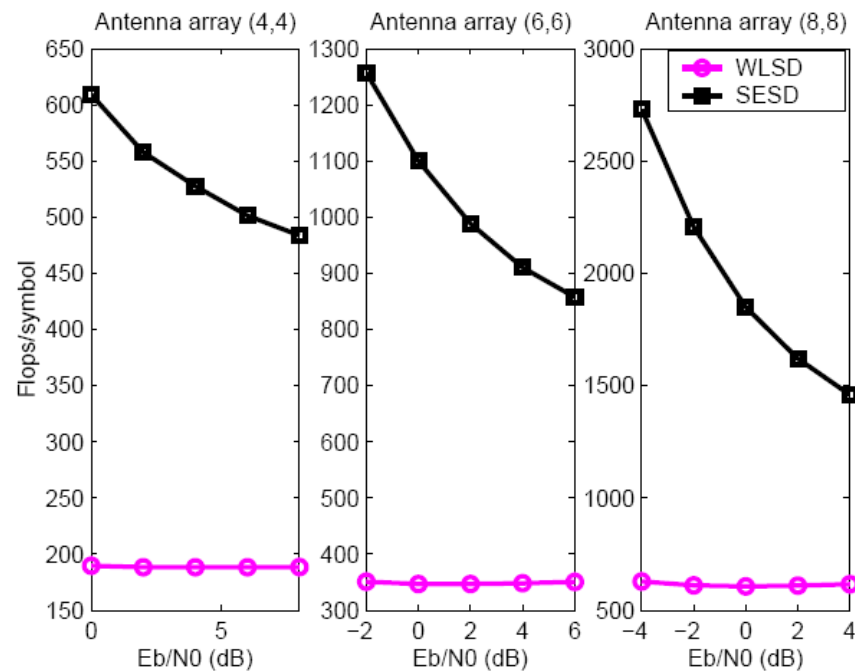
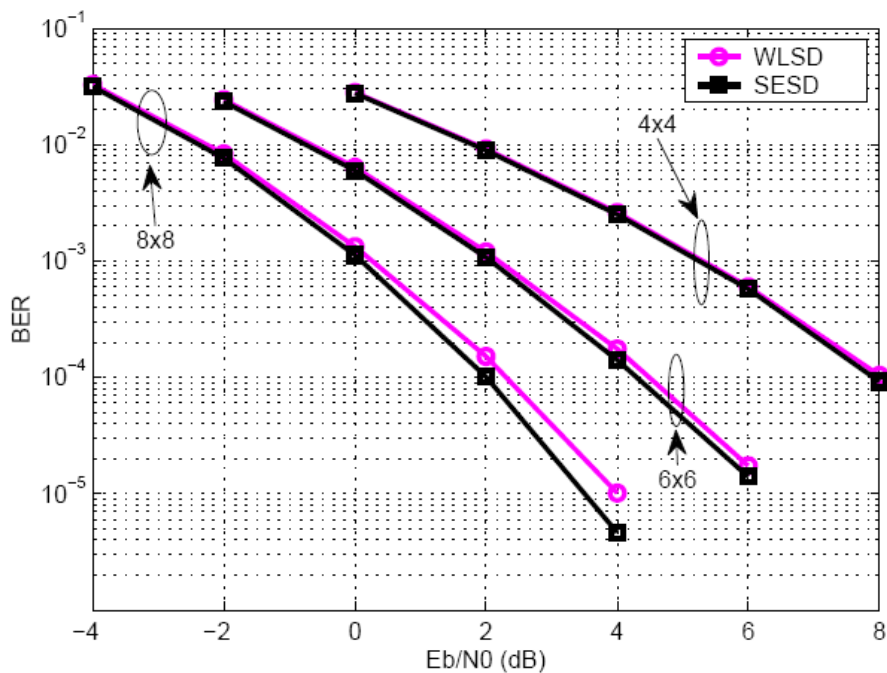
Performance evaluation

BER performance 2:

Simulation setting:

◆ QPSK modulation signal

◆ (4,4) (6,6) (8,8) array





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Conclusions

Contributions:

- ◆ Properties of modulation signals are modeled by a unified mathematical model;
- ◆ Widely linear MIMO receiver is proposed for non-circular signals
- ◆ Widely linear sphere decoding is proposed for MIMO detection by exploiting the properties of circular signals.



Thanks!!