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# On the application of a novel interface element for the simulation of delamination and fracture

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**Abstract** — Purpose of this communication is to discuss the performaces of a novel interface element, recently proposed by the author, that possesses drilling degrees of freedom. The capabilities of the proposed element are demonstrated with reference to a typical delamination problem where fracture propagation is simulated using the cohesive-zone approach.

**Keywords** — Interface elements, drilling rotations, cohesive zone.

#### 1 Introduction

Interface elements are a valuable tool for the analysis of problems where discontinuous behaviour occurs, and in particular for situations where interface positions are a priori known. Successful applications in Solid Mechanics include soil-structure interaction, simulation of joints between dissimilar materials and, more generally, delamination and fracture problems.

Probably a rigorous definition of *optimal* interface elements is not available; however, a reasonable characterization of *good* interface elements is that they should satisfy some essential requisites, namely: (a) be sufficiently stiff within the elastic range, (b) have the correct rank, i.e. no zero-energy modes, (c) be fully compatible with the surrounding continuum elements.

In this study the performances of a novel interface element with rotational (drilling) degrees of freedom, recently proposed by the author in [7], are evaluated. The interest of such element is motivated by the fact that it can be coupled with membrane elements with in-plane rotations, which are well-known to exhibit excellent accuracy and convergence characteristics both for regular and distorted meshes, see e.g. [2, 3]. More generally, refined elements with low number of nodes are highly attractive for identification purposes since they result in low connectivity of the structural stiffness matrix and, hence, reduced computational effort.

The outline of the paper is as follows. In section 2 the element formulation based on the mixed functional of Hughes and Brezzi [1] is briefly recalled. Element performances are then evaluated in section 3 via eigenvalue analysis in order to assess the impact of quadrature schemes and to find out potential sources of weakness. Robustness, convergence for successive mesh refinements and accuracy in the solution of the process zone are not secondary aspects. To this end in section 4 a typical delamination problem is considered, where interface elements are implemented with a traction-relative displacement relationship taken from [6].

#### 2 Element formulation

The general variational framework for problems including in-plane rotational degrees of freedom has been presented by Hughes and Brezzi in [1]. In particular, in their formulation the stress tensor is not a priori assumed to be symmetric, whereby its role it plays in the theory is complementary to that of the (infinitesimal) rotation field.

The stiffness matrix of the drilling interface element can be obtained starting from the following degenerated Hughes-Brezzi functional [7]:

$$\Pi_{\gamma}(\mathbf{u}, \boldsymbol{\phi}, \boldsymbol{\tau}) = \frac{1}{2} \int_{\Omega} \mathbf{D}[\![\mathbf{u}]\!] \cdot [\![\mathbf{u}]\!] \, d\Omega + \int_{\Omega} \boldsymbol{\tau} \cdot (\mathbf{u}' - \boldsymbol{\phi}) \, d\Omega - \frac{1}{2} \gamma^{-1} \int_{\Omega} \boldsymbol{\tau}^2 \, d\Omega - \int_{\Omega} \mathbf{f} \cdot \mathbf{u} \, d\Omega \qquad (1)$$

where  $\mathbf{u}$  is the displacement vector,  $\boldsymbol{\phi}$  is the drilling rotation,  $\boldsymbol{\tau}$  is a Lagrange multiplier that plays the same role of the skew-symmetric part of the stress in the membrane element [3] and  $\boldsymbol{\gamma}$  is a regularizing parameter to be chosen in accordance with the ellipticity condition.

Introducing the standard linear continuous finite element interpolations for displacements and rotations and adding the Allman-type quadratic modes for linking displacements to rotations, the discrete equations obtained by taking the variations of the above functional can be written in matrix form as [7]:

$$\begin{bmatrix} \mathbf{K}_{u}^{e} & \mathbf{g}^{e} \\ \mathbf{g}^{e,T} & -\frac{\Omega^{e}}{\gamma} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{I} \\ \tau^{I} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}$$
 (2)

The variable  $\tau$  is in general assumed element-wise discontinuous, so that it can be eliminated using static condensation; hence, the stiffness matrix to be assembled at the global level is:

$$\mathbf{K}^e = \mathbf{K}_u^e + \frac{\gamma}{\Omega^e} (\mathbf{g}^e \otimes \mathbf{g}^e) \tag{3}$$

The individual terms of the stiffness matrix are evaluated using a three-point quadrature scheme; as shown in next section, either Newton-Cotes or Gauss quadrature yield the same identical result.

## 3 Eigenvalue analysis

The question of influence of the quadrature rule adopted for the numerical computation of the tangent stiffness matrix and the internal force vector of interface elements has been analyzed by many authors and with different arguments.

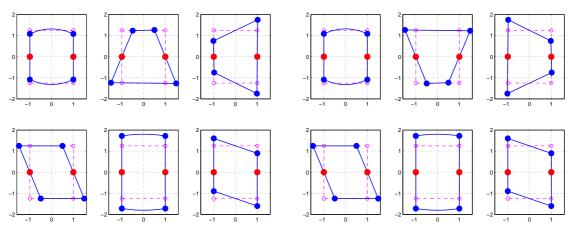


Figure 1 – Eigenmodes of the drilling interface element. Cotes (left) VS Gauss quadrature (right).

In particular, one of the most cited articles on the subject [4] reports an eigenvalue analysis of the element stiffness matrix and a detailed discussion on the conditions under which spurious oscillations in the traction profiles are observed.

In the author's opinion the conclusions drawn in [4] have been probably misunterstood in some cases, since no superiority of Newton-Cotes or Simpson rules over Gaussian quadrature can be a priori established. For line interface elements there is however a general preference for Newton-Cotes quadrature probably originating from the fact that in this last case the eigenmodes of the element stiffness matrix exhibit no coupling between node displacements.

Whether the absence of such a coupling is really benefic or not is not our concern here because for the proposed interface element with drilling degrees of freedom the coupling of nodal displacements occurs both with Newton-Cotes and Gauss scheme. Actually, provided that a number of integration points sufficient to ensure a stiffness matrix with the correct rank is used, the eigenvalues and eigenmodes that are obtained are exactly the same, see Figure 1. This eliminates any ambiguity in the choice of the quadrature rule.

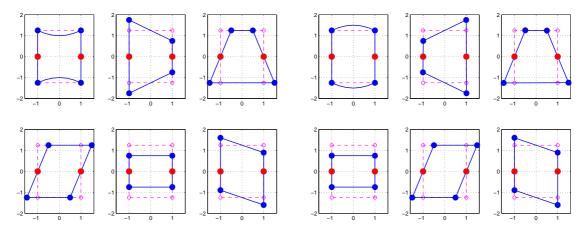


Figure 2 – Eigenmodes of the drilling interface element after mode filtering. Cotes (left) VS Gauss quadrature (right).

Figure 1 also reveals that the eigenmodes of the drilling interface element exhibit a coupling between the rotational degrees of freedom and the displacements in the direction normal to the interface. An immediate consequence of this fact is that the consistent nodal loads of the drilling interface element for pure mode-I deformation are not identical to those of the standard interface element possessing only two translational dofs per node. Clearly, this is an undesired effect but it can be easily removed using a standard mode-filtering technique [8]. The result is shown in Figure 2, where the new eigenmodes are depicted.

### 4 Numerical example

The interface element with drilling rotations has been implemented in a customized version of the finite element code FEAP [5]. For the numerical simulations described in this section classical 4-noded interface elements have been used in conjunction with Enhanced Assumed Strain (EAS) quadrilaterals whilst the proposed interface element has been coupled with an enhanced version of the quadrilateral membrane element with drilling degrees of freedom described in [3].

In order to show the capabilities of the new element we consider as an example a Double Cantilever Beam (DCB) test, for which the material data set is given in Table 1, see also [6].

Loading is simulated via displacement control and the dimensions of the specimen are similar to those adopted for typycal experimental tests used for measuring the mode-I fracture energy, see also Figure 3.

Bulk material	E=70 (GPa)	v = 0.3	
Interface	$k_n (N/mm^3)$	$G_{o}$ (N/mm)	$G_{c}$ (N/mm)
	1.00E+04	4.50E-02	5.00E-01

Table 1 – DCB test. Material data

Figure 3 also shows a schematic representation of the FE mesh adopted in computations; in particular,  $x_1$  and  $x_2$  respectively denote the number of elements at the ends and in the middle region of the FE mesh.

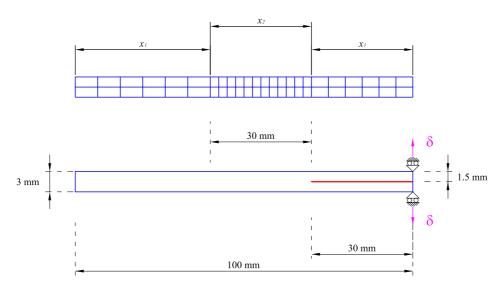


Figure 3 – DCB test. Model problem and scheme of FE mesh

In figure 4 are plotted the global responses of the structure obtained using EAS quadrilaterals for the bulk material along with standard interface elements, that have been integrated with the Newton-Cotes and Gauss rules, respectively. A rough comparison shows that a load-deflection curve that does not exhibit instabilities can be obtained using a greater length of cohesive elements if Gauss quadrature is adopted. One can then conclude that accuracy in the integration of the constitutive relationship is not a secondary aspect and that, at least for this class of problems, Gauss quadrature should be preferred.

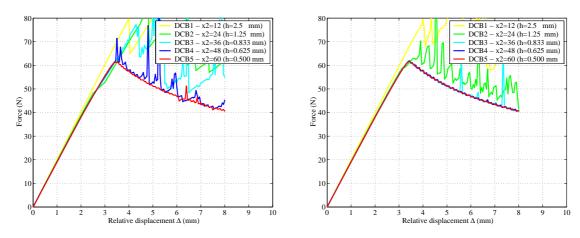


Figure 4 – Comparison of computed load-deflection curves for a DCB delamination test. Standard interface elements. Cotes (left) VS Gauss quadrature (right).

In figure 5 the load-deflection response obtained using standard interface elements and Gauss quadrature is now compared with the one computed using the new interface elements coupled with membrane elements with in-plane rotations. At first sight the inclusion of drilling degrees of freedom seems to produce a very limited effect and no significant performance improvement is apparent since the length of cohesive elements in the middle region of the structure that allows to obtain a converged solution equals 0.833 mm for the drilling element and 0.625 mm for the standard one.

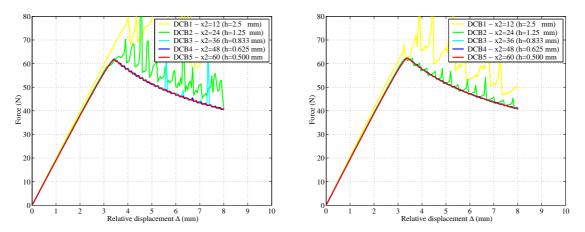


Figure 5 – Comparison of computed load-deflection curves for a DCB delamination test. Gauss quadrature. Standard interface elements (left) VS drilling interface elements (right).

The analysis of the DCB problem is now repeated for an irregular mesh obtained by introducing a small distorsion  $\delta$  of the same order of magnitude of the arm thickness h of the DCB, see also figure 6, where a schematic representation of the supernodes of the two meshes is reported.

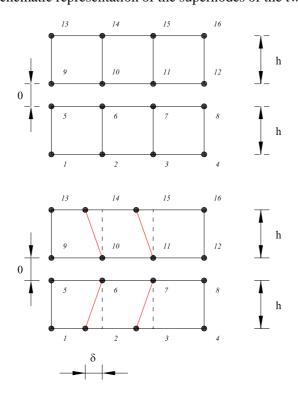


Figure 6 – Supernodes of the regular and distorted meshes.

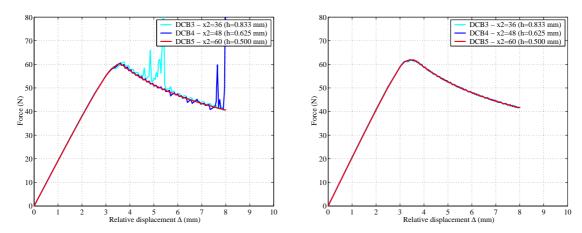


Figure 7 – Comparison of computed load-deflection curves for a DCB delamination test. Distorted mesh. Standard interface elements (left) VS drilling interface elements (right).

As shown in figure 7 the differences in the computed load-deflection responses are now dramatically apparent. In particular, the standard interface element now is likely to converge for an element size of  $0.5 \, mm$  while the drilling element still gives the correct answer for  $h = 0.833 \, mm$ .

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