



HAL
open science

Modelling of a Simulated Moving Bed in case of non-ideal hydrodynamics

Leonel Fangueiro Gomes, Frédéric Augier, Damien Leinekugel-Le-Cocq, Ivana Vinkovic, S Simoëns

► **To cite this version:**

Leonel Fangueiro Gomes, Frédéric Augier, Damien Leinekugel-Le-Cocq, Ivana Vinkovic, S Simoëns. Modelling of a Simulated Moving Bed in case of non-ideal hydrodynamics. Chemical Engineering Science, 2016, 153, pp.188 - 198. 10.1016/j.ces.2016.07.027 . hal-01408721

HAL Id: hal-01408721

<https://hal.science/hal-01408721>

Submitted on 5 Dec 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 Modelling of a Simulated Moving Bed in case of non-ideal hydrodynamics

2
3 L. Fangueiro Gomes^{1,2}, F. Augier^{1*}, D. Leinekugel-le-Cocq¹, I. Vinkovic², S. Simoëns²

4
5 ¹IFP Energies nouvelles, Rond-point de l'échangeur de Solaize, BP3, 69360 Solaize, France

6 ²LMFA, UMR CNRS 5509, Ecole Centrale de Lyon, Université de Lyon 1, INSA Lyon, 69131, Ecully
7 Cedex, France

8 *Corresponding author (frederic.augier@ifpen.fr)

9 10 Abstract

11 The one-dimensional hydrodynamic model proposed by Gomes et al. (2015) is coupled with
12 adsorption and validated by comparing the concentration profiles of this one-dimensional model with
13 those given by the CFD model of one adsorption column including obstacles as distribution network
14 and beams. This one-dimensional model is capable of predicting the CFD results for different mass
15 transfer rates, while the traditional dispersed plug flow (DPF) model is accurate for slow mass transfer
16 rates only. The model proposed by Gomes et al. (2015) is capable of reproducing the adsorber
17 Residence Time Distribution (RTD) while dissociating the selective zones from the non-selective ones.
18 It is based on the CFD techniques developed by Liu and Tilton (2010) and Liu (2012) that transport the
19 moments of the fluid age distribution and consequently calculate the degree of mixing (Danckwerts,
20 1958 and Zwietering, 1959). Then, this new model is integrated in a cyclic solver in order to perform
21 Simulated Moving Bed (SMB) studies. The new model provides a detailed hydrodynamic description,
22 which appears to be mandatory especially when mass transfer exchanges are fast, without undergoing
23 the prohibitive simulation times of CFD models.

24 Keywords:

25 Adsorption, CFD, Simulated Moving Bed, 1D modelling

26 27 Research Highlights

- 28
- 29 • A 1D “multi-entrance multi-exit” model is developed to simulate adsorption processes
 - 30 • The model is more relevant than a Dispersive Plug Flow for fast mass transfer rates
 - 31 • It is successfully implemented in a cyclic Simulated Moving Bed Solver

32 1 INTRODUCTION

33 In adsorption separation processes, the difference of the affinities of an adsorbent for the adsorbates
34 (components present in the feed mixture), generates different retention times, enabling the separation
35 of molecules. However, in the presence of dispersive phenomena, the concentration fronts may be
36 spread, limiting the separation (Ruthven, 1984). In some cases, such phenomena can lead to peak
37 tailing: the concentration fronts advance asymmetrically through the separation unit, leaning to the
38 right when plotted in function of time.

39 The dispersive phenomena mentioned above can essentially be of intra-particle nature and/or
40 hydrodynamic nature. The intra-particle dispersive phenomena comprise the mass transfer
41 resistances between the bulk and adsorbed phases and the non-linearity of the sorption equilibrium.
42 The hydrodynamic dispersive phenomena are related to the adsorption vessel design: packing
43 heterogeneities, injection/withdrawal device and bed geometry.

44 In order to design and optimize adsorption processes, models are often used to study and quantify the
45 influence of different parameters. One simple approach is to consider that the motion of the eluent can
46 be described by a dispersed plug flow (DPF) model. In this case, the flow is assumed essentially one-
47 dimensional and all the hydrodynamic heterogeneities are taken into account by adjusting an axial
48 dispersion coefficient. This approach is sufficiently accurate when the intra-particle dispersive

49 phenomena are dominant relatively to the hydrodynamic ones. However, if intra-particle and
50 hydrodynamic dispersive phenomena are of the same order of magnitude the validity of this model is
51 at stake. In this case a better description of flow may be required. When in the need of a more detailed
52 perception of hydrodynamics of industrial apparatus, it is usual to make use of CFD simulations.

53 Several studies have already been published regarding the coupling between detailed hydrodynamics
54 and adsorption since the paper of Bart et al. (1996). These authors developed a two-dimensional
55 model to study the effect of maldistribution on the sorption of toluene in activated carbon. They found
56 a good agreement between simulation and experimental results. Wu et al. (2004) and Kwapinski et al.
57 (2010) performed CFD simulations in order to evaluate the impact of dispersion on the separation
58 efficiency of a binary separation in a low aspect ratio chromatographic column. In their case, they
59 observed a considerable effect of peak tailing due to mass transfer resistances, and showed the
60 inability of a 1D model to reproduce their results. Augier et al. (2008) showed that hydrodynamics
61 inside industrial adsorbers can strongly impair the efficiency of adsorption through the use of CFD.
62 Zheng et al. (2010) and Liu et al. (2014) developed two-dimensional models that can operate through
63 cyclic simulations to study a Pressure Swing Adsorption (PSA) process. These models allowed to
64 perceive the process hydrodynamics and to propose better adsorber geometries. They observed
65 concentration maldistributions that would not be visible using a one-dimensional model.

66 This work focuses on a SMB process for the *p*-xylene (PX) purification. The PX is used in the
67 production of monomers of the polyethylene terephthalate (PET), and is obtained in a mixture with its
68 isomers, *o*-xylene (OX), *m*-xylene (MX) and ethylbenzene (EB). Its purification is performed using a
69 set of superposed fixed adsorbers, ranging from 15 to 24 for example, and the simulated
70 countercurrent between the adsorbent and the eluent is achieved by the periodic commutation of the
71 inlet and outlet ports throughout the column that comprises the adsorbers. The SMB contains several
72 hydrodynamic defects: the pipes of the distribution and withdrawal system are placed inside the
73 packing, beams are used to support the weight of the beds, there are empty volumes between
74 consecutive beds and the injection is not ideal, as shown in figure 1(see Pavone and Hotier (2010)
75 and Minceva and Rodrigues (2002) for a more detailed description).

76 The plates of the distribution system are divided into panels that withdraw the flow from the bottom of
77 the adsorbers and mix it with an injected feed from the distribution network to send it to the packed
78 bed below (see for example the patent US20110303602 A1, Augier and Hotier, 2011).

79 Since its conception, numerous major improvements have been made in the field of the SMB
80 modelling, regarding the mass transfer modelling (Mendes et al., 1996), CSS prediction (Pais et al.,
81 1997a,b), SMB optimal design (Storti et al., 1995 and Mazzotti et al., 1997a,b) and column dead
82 volume modelling (Migliorini et al., 1999, Minceva and Rodrigues, 2003 and Silva et al., 2016). Yet,
83 plug flow models (axially dispersion or not) have been generally used to model the flow adsorption
84 columns and their suitability has not been questioned.

85 Augier et al. (2008) studied the impact of a round obstacle within a packed bed of adsorbent on the
86 breakthrough curves. By measuring the resulting variance Augier et al. (2008) were able to adjust a
87 DPF model to their case. When using most of the adsorbents (as it will be shown in this work) and
88 after adjusting the axial dispersion coefficient, the DPF model is sufficiently accurate for predicting the
89 hydrodynamic heterogeneities mentioned above. However, if other adsorbents with better mass
90 transfer performances are considered, the accuracy of the DPF model is unknown and has to be
91 investigated.

92 In order to study the SMB with a proper hydrodynamic description, ideally one should perform CFD
93 simulations coupled with adsorption. However, CFD simulations of multicomponent adsorption in a
94 single adsorber bed can take up to 72 hours to converge (8 cores 2.7 GHz, 16GB RAM) and the SMB
95 comprises a high number of beds. This makes CFD an inappropriate tool for engineers to study multi-
96 bed cyclic processes. Hence, it is interesting to develop a 1D hydrodynamic model simple enough to

97 converge in seconds but capable of giving a fine description of the adsorbents hydrodynamics. In a
98 previous paper Gomes et al., (2015) developed a 1D model using the CFD methodology developed by
99 Liu and Tilton (2010) and Liu (2012) based on the work of Danckwerts (1958) and Zwietering (1959).
100 This model has shown a good agreement with the CFD when comparing the inert tracer concentration
101 profiles.

102 In this work, the accuracy of this 1D model (here called 2MENCSTR model, *see abbreviations*)
103 coupled with adsorption is evaluated relatively to the DPF model. To do this a chromatographic
104 separation is simulated in a single adsorbent. The results obtained with the DPF and the 2MENCSTR
105 model are compared to those given by CFD. Then the DPF and 2MENCSTR models are integrated in
106 a cyclic solver in order to evaluate the importance of a detailed hydrodynamic description when
107 modelling SMB processes.

108

109 **2 MODELLING**

110 **2.1 Adsorption column geometry**

111 In both CFD and 1D models, the following hydrodynamic heterogeneities are taken into account: the
112 packing of adsorbent, a pipe of the distribution/withdrawal system, a beam, and two free flow volumes,
113 one above the packed bed that distributes the flow and one below that withdraws the flow. The
114 dimensions of the geometry used in the CFD model are presented in figure 2.

115 In this work the flow is considered to be essentially two-dimensional which is a reasonable
116 approximation as shown by Augier et al. (2008), allowing much shorter simulation times than if three
117 dimensions were considered. Nevertheless, the methodology proposed can also be used in a three-
118 dimensional geometry.

119 The studied geometry comprises an adsorbent packed bed with 1.2 m height and 0.5 m width, where
120 the half of a pipe (0.151 m radius) and half of a beam (0.268 m edge) are placed. The pipe and beam
121 dimensions were chosen so that each one would occupy around 6% of the packed bed volume. Two
122 free flow volumes of 0.015 m height are placed above and below the packed bed. The inlet and outlet
123 have a length of 0.025 m. The dashed lines in figure 2 represent the symmetry boundary conditions
124 used for both transport and momentum balance.

125 It is known that the porosity profiles near the walls are variable, however this variation occurs in a
126 range of few particles that constitute the bed. A preliminary study showed that, since the domain in
127 study contains a massive amount of particles, the results obtained were insensible to whether the
128 porosity is assumed to be constant or variable using the profiles proposed by Giese et al. (1998).

129

130 **2.2 Intra-particle phenomena**

131 In the adsorbent packed bed, three scales of porosity are used to describe the mass balance: the
132 interstitial porosity ε , the macro-porosity ε_p and the micro-porosity (or adsorbed phase). The transition
133 of the molecules between the bulk phase and the adsorbed phase is described by two linear driving
134 forces (LDF) (Ruthven, 1984). A given component i present in the bulk phase (the concentration of
135 this molecule is expressed as C_i) faces a fluid film mass transfer resistance (k_1) when migrating into
136 the macro-porous liquid phase ($C_{m,i}$). Then, while migrating from the macro-porous liquid phase to the
137 adsorbed phase (q_i) a second mass transfer resistance is felt (k_2), function of the radius of the
138 adsorbent crystal radius.

139 Hence, the mass flux of a component i between the bulk phase and the macro-porous phase is given
140 by:

$$N_i^\varepsilon = -(1 - \varepsilon)\varepsilon_p k_1 (C_i - C_{m,i}) \quad (1)$$

141

142 The mass balance of a component i in the macro-porous liquid phase is given by:

$$\varepsilon_p \frac{\partial C_{m,i}}{\partial t} = \varepsilon_p k_1 (C_i - C_{m,i}) - (1 - \varepsilon_p) k_2 (q_i^* - q_i) \rho_p \quad (2)$$

143

144 for which ρ_p is the apparent adsorbent mass density, q_i^* is the adsorbed concentration of i at
 145 equilibrium with the macro-porous liquid phase and q_i is the average adsorbed concentration. The
 146 instant average adsorbed concentration of i can be obtained using the adsorbed phase mass balance:

$$\frac{\partial q_i}{\partial t} = k_2 (q_i^* - q_i) \quad (3)$$

147

148 The adsorbed concentration at equilibrium with the macro-porous liquid phase can be calculated by
 149 the generalized Langmuir isotherm:

$$\frac{q_i^*}{q_{m,i}} = \frac{b_i C_{m,i}}{1 + \sum_{j=1}^{NC} b_j C_{m,j}} \quad (4)$$

150

151 for which $q_{m,i}$ is the adsorbent mass capacity for the component i , b_i is the adsorbent coefficient of
 152 affinity for the component i and NC is the number of components in equilibrium.

153 2.3 CFD Model

154 Even though adsorption is often an exothermic phenomenon, in this study the adsorbent is
 155 permanently saturated and the molecules show similar values of adsorption capacity and enthalpy.
 156 Thus, it is reasonable to consider it as isothermal. With this, the mass and momentum balance are
 157 calculated but the energy balance is not.

158 In a previous paper (Gomes et al., 2015) found a good agreement between the concentration profiles
 159 of an inert tracer obtained experimentally and numerically, giving a fair degree of trust in the current
 160 CFD model (COMSOL 4.3b software).

161 Since the geometry in study comprises a packed bed between two free flow volumes, the momentum
 162 and mass balance must be solved differently leading to two distinct sets of equations.

163 Given the fact that at the SMB operating conditions (180 °C and 9 bar, Minceva and Rodrigues, 2007)
 164 the xylenes are in liquid phase thus incompressible and that the inlet and outlet volumetric flows are
 165 constant the flow is assumed to be steady in both porous and free flow.

166 2.3.1 Free flow

167 In the free flow volumes placed above and below the packed bed the flow is described using the
 168 Reynolds averaged Navier-Stokes equations (RANS) for an incompressible Newtonian fluid and
 169 steady flow. The turbulent dynamic viscosity μ_T is modelled by the standard k- ε turbulence model
 170 (Jones and Launder, 1972).

171 With such macroscopic description of the flow, the mixing of the molecules due to eddy motion is not
 172 simulated. This phenomenon is then modelled by imposing a constant turbulent Schmidt number:

$$Sc_T = \frac{\mu_T}{\rho D_T} \quad (5)$$

173 μ_T is the turbulent dynamic viscosity calculated by the k- ε turbulence model, ρ is the fluid mass density
 174 and D_T is the turbulent diffusivity coefficient. This diffusivity coefficient must be added to the molecular
 175 diffusion D_M on the diffusive term of the mass balance equation, in the free flow regions. Based on the
 176 average values found in the bibliography (Koeltzsch, 2000; Flesch et al., 2002 and Combest et al.,
 177 2011) the Sc_T is assumed to be 0.7.

178 In the free flow volumes a component i is transported by diffusion and convection:

$$\frac{\partial C_i}{\partial t} = \nabla \cdot ((D_M + D_T) \cdot \nabla C_i - \vec{u} C_i) \quad (6)$$

179

180 The global dispersion coefficient results from the sum of the molecular diffusion and the turbulent
 181 diffusivity, both isotropic.

182 2.3.2 Porous media

183 The CFD model describes hydrodynamics at a scale much larger than the bed particles which is
 184 reasonable given the high ratio between the bed dimensions and the particles diameter. The
 185 interaction between the fluid and the particles is not simulated, but modelled by the addition of a
 186 diffusive term in the momentum and mass balance. Then, in order to simulate the flow in the porous
 187 media a modification of the laminar Navier-Stokes equations is used, called the Brinkman-
 188 Forchheimer equations (Vafai and Tien, 1982):

$$\frac{\rho}{\varepsilon} \left((\vec{u} \cdot \nabla) \frac{\vec{u}}{\varepsilon} \right) = \nabla \cdot \left[-p + \frac{\mu_e}{\varepsilon} (\nabla \vec{u} + (\nabla \vec{u})^T) - \frac{2\mu}{3\varepsilon} (\nabla \cdot \vec{u}) \right] - \left(\rho \cdot \beta \cdot |\vec{u}| + \frac{\mu}{K} \right) \vec{u} \quad (7)$$

$$\rho \cdot \nabla \vec{u} = 0 \quad (8)$$

189 \vec{u} is the superficial fluid velocity, ε the bed porosity, p the pressure, μ_e the effective dynamic viscosity,
 190 μ the dynamic viscosity, β is the non-Darcy coefficient and K the bed permeability. The validity of
 191 these equations has been discussed by several authors (Nield, 1991; Vafai and Kim, 1995; Chan et
 192 al., 2000; Benyahia, 2004; Augier et al., 2008 and Gomes et al., 2015). The value of μ_e has been a
 193 subject of debate, but similarly to the previous paper (Gomes et al., 2015), where the validity of the
 194 current CFD model is evaluated, its value is assumed to be equal to μ .

195 β and K can be calculated, for a bed of spheres by the Ergun's law (1952):

$$\beta = 1.75 \frac{1 - \varepsilon}{\varepsilon^3 d_p} \quad (9)$$

$$K = \frac{\varepsilon^3 d_p^2}{150(1 - \varepsilon)^2} \quad (10)$$

196

197 As aforementioned, a diffusive term must be added to the mass balance equation in order to model
 198 the dispersion caused by the packing. This is done by adding the mechanical dispersion coefficient D_p
 199 to the molecular diffusion coefficient, in the diffusive term of the mass balance equation of the bulk
 200 phase:

$$\varepsilon \frac{\partial C_i}{\partial t} + \nabla \cdot (\vec{u} C_i - \varepsilon (D_M + D_p) \cdot \nabla C_i) = N_i^\varepsilon \quad (11)$$

201

202 The mass flux of a component i between the bulk phase and the macro-porous phase, N_i^ε , is given by
 203 Eq. (1).

204 Contrarily to the molecular diffusion (D_M) and turbulent diffusivity (D_T) coefficients, the mechanical
205 dispersion coefficient (D_p) is anisotropic and described by a diagonal tensor containing the transversal
206 and axial dispersion coefficients. These are calculated by imposing two constant Peclet numbers
207 based on the particle diameter, one in the transversal direction (Pe_{tr}) and the other in the axial
208 direction (Pe_{ax}), relatively to the flow direction. The values of Pe_{tr} and Pe_{ax} are assumed to be 11 and
209 2 respectively, the asymptotic values for liquid flows with a particle Reynolds number higher than 20
210 (Delgado, 2006).

211 To couple the two free flow volumes with the porous media, the boundary conditions in use force the
212 pressure, the horizontal and the vertical components of the fluid velocity and the bulk phase
213 concentration of every component to be the same on both sides of the interface, for each mesh
214 element.
215

216 **2.4 Degree of mixing and transport of the fluid age distribution moments**

217 In 1952, Danckwerts showed that two reactors that share the same Residence Time Distribution
218 (RTD) can produce different conversions, if the chemical reaction follows a second order kinetics. To
219 explain this, Danckwerts (1958) and Zwietering (1959) formulated a criterion, the degree of mixing (J),
220 that quantifies the mixing earliness of a vessel. However, for about 50 years it was not possible to
221 calculate its value, except for the asymptotic values: 0 for an ideal Continuous Stirred-tank Reactor
222 (CSTR) and 1 for a Plug Flow Reactor (PFR). Recently, Liu (2012) showed that the degree of mixing
223 could be computed using the volume averages of the first and second raw moments of the fluid age
224 distribution (m_1 and m_2) whose spatial distribution can be obtained by coupling a steady moment
225 balance equation with the flow field simulated by CFD (Liu and Tilton, 2010). Such methodology can
226 only be applied to steady incompressible flows.

227 Adsorption is a non-linear exchange phenomena in which q_i^* varies non-linearly with $C_{m,i}$. Therefore,
228 when proposing a 1D model capable of reproducing the coupling between hydrodynamics and
229 adsorption, it is of the highest interest to preserve the CFD model degree of mixing. The degree of
230 mixing obtained for the adsorber geometry represented in figure 2 is close to 1 ($J=0.987$), indicating a
231 behaviour close to a completely segregated system, despite the hydrodynamic heterogeneities
232 (Gomes et al., 2015).

233 **2.5 1D Model**

234 As long as the degree of mixing is close to the one of a PFR it is reasonable to assume that no
235 transversal mixing occurs in the adsorber. For this reason, each trajectory of the fluid is modelled by
236 three consecutive cascades of N CSTR models (hereby called NCSTR models), one for each zone
237 (above the packed bed, below the bed and the bed itself), as shown in figure 3 a). Finally, when all the
238 fluid trajectories are modelled in this way, the CFD model can be condensed to a set of parallel
239 systems comprising three consecutive NCSTR models, as shown in figure 3 b) (Gomes et al., 2015).

240 Gomes et al. (2015) observed that the dispersive effect of the zone above the bed (zone A) is induced
241 by the heterogeneity of m_1 in the inlet of the porous media (the fluid elements take different times to
242 flow from the inlet to the entrance of the packed bed) rather than by local dispersion. This fact allows
243 to model both free flow volumes downstream of the bed, simplifying the model. One major assumption
244 that is made in order to develop this model is that the horizontal velocity profiles are quite flat, i.e. no
245 major velocity variations are found in the x-axis.

246 A given streamline, modeled by a cascade of N CSTR has a first portion identical to the cascade of M
247 CSTR corresponding to a smaller streamline. Therefore both streamlines can be modeled with the aid
248 of a single CSTR cascade comprising two different parts. A first part consists in a cascade of M CSTR
249 in which the flow passing through is equal to the sum of the flows of both streamlines. At the end of
250 this first part, the flow corresponding to the shortest streamline is withdrawn, and the flow
251 corresponding to the longest streamline flows through a cascade of $(N-M)$ CSTR. By doing this

252 individually for each streamline, it is possible to represent the porous zone with a Multiple Exit NCSTR
 253 model and the free flow zones with a Multiple Entrance NCSTR model, henceforth denominated
 254 2MENCSTR model (as for Multiple Exits NCSTR model connected to a Multiple Entrances NCSTR
 255 model), as shown in figure 3 c.

256 2.6 SMB modelling

257 In order to model a SMB process the switching of the inlet and outlet ports is accounted for, as shown
 258 in figure 4 a, using a time-dependent solver. The solid arrows represent the streams at an instant t
 259 while the dashed arrows represent the streams after the stream switching, at $t+\Delta t_{sw}$, where Δt_{sw} is the
 260 switching time.

261 Then, two different models are used to model the SMB adsorption columns: the 2MENCSTR model
 262 and the DPF model.

263 The 2MENCSTR model (figure 4 b) comprises a cascade of CSTR models with multiple side exits that
 264 are connected to a second CSTR cascade with multiple side entrances. The first cascade is used to
 265 model the porous bed, thus the CSTR are filled with adsorbent, while the second cascade is used to
 266 model the free flow regions above and below the bed, thus the CSTR are “empty”. The parameters
 267 regarding the CSTR sizes, the flow of each connection and connection arrangement are function of
 268 the spatial distributions of the moments of the fluid age distribution transported by CFD. The single
 269 arbitrary parameter is the number of interconnections.

270 DPF models are generally used for the modelling of adsorption columns in SMB studies. In this work
 271 the Peclet numbers of the DPF models are adjusted to match the variance of the associated
 272 2MENCSTR model. It is assured that for each SMB configuration and geometry in study the
 273 2MENCSTR and the DPF models have the same residence time, variance and mass of adsorbent.

274 Whether in the use of the 2MENCSTR or DPF model the following equations are used to perform the
 275 mass balance of a component i in the inlet and outlet nodes between two consecutive adsorption
 276 columns j and $j+1$:

277 Desorbent node

$$Q_{Z1} = Q_{Z4} + Q_D \quad (12)$$

$$Q_{Z1} C_{i,j+1}^{in} = Q_{Z4} C_{i,j}^{out} + Q_D C_{i,D} \quad (13)$$

278

279 Extract node

$$Q_{Z2} = Q_{Z1} - Q_X \quad (14)$$

$$C_{i,X} = C_{i,j+1}^{in} = C_{i,j}^{out} \quad (15)$$

280

281 Feed node

$$Q_{Z3} = Q_{Z2} + Q_F \quad (16)$$

$$Q_{Z3} C_{i,j+1}^{in} = Q_{Z2} C_{i,j}^{out} + Q_F C_{i,F} \quad (17)$$

282

283 Raffinate node

$$Q_{Z4} = Q_{Z3} - Q_R \quad (18)$$

$$C_{i,R} = C_{i,j+1}^{in} = C_{i,j}^{out} \quad (19)$$

284

285 where Q_{zi} is the volumetric flow rate of zone i , the index D stands for desorbent, X for extract, F for
286 feed and R for raffinate. By employing these equations and periodically commuting the inlet and outlet
287 nodes through the SMB column during several cycles it is possible to achieve a cyclic steady state
288 (CSS).

289 The equation systems reported above are implemented in a time-dependent solver in order to model
290 the separation of a binary mixture of p -xylene (PX) and m -xylene (MX), using p -diethylbenzene
291 (PDEB) as desorbent. Then, after the CSS is reached, the concentration profiles and performances
292 obtained by the DPF-based SMB model and the 2MENCSTR-based SMB model are compared, for the
293 same geometry and operating conditions.

294

295 **2.7 Computation**

296 The CFD simulations were performed with COMSOL 4.3b software. The flow field was solved using a
297 quadratic discretization for the pressure and linear discretization for the velocity. The mass balance
298 equations were solved using a quadratic discretization and a second order BDF for the time stepping.

299 The 1D models (DPF and 2MENCSTR) computing code has been developed in Fortran (including the
300 SMB solver). The ordinary differential equations system obtained by spatial discretization by cascades
301 of CSTRs is solved using the DDASPK subroutine based on the Petzold-Gear BDF method.

302

303 **2.8 Simulations parameters**

304 The parameters used in this study are shown in Table 1. The equilibrium values q_i and b_i (Minceva
305 and Rodrigues, 2007) and the LDF constants k_1 and k_2 (Bergeot et al. 2010) were found in the
306 bibliography. For the chromatographic simulations, the inlet velocity was adjusted with the aim of
307 setting the residence time inside the porous bed to 27.4 s, which leads to an average superficial
308 velocity of 1.4 cm/s, close to what is used in industrial units. It must be reminded that the
309 dimensionless numbers Schmidt and Peclet were not fitted but obtained from the bibliography. Two
310 different SMB configurations were considered in this work, comprising 6 and 12 adsorption columns.
311 The SMB operating conditions used are shown in Table 2.

312

313 **3 RESULTS AND DISCUSSION**

314 **3.1 DTS and model parameterization**

315 The characteristic parameters of the resulting 2MENCSTR model, that represents the geometry shown
316 in Fig. 2, are shown in figure 5.

317 In figure 5, the first column shows then cumulative number of CSTR in the first cascade of the
318 2MENCSTR model, i.e. the porous one. The number of CSTR is counted until a side exit, i.e. the first
319 side exit comes out from the 1846th CSTR, while the second side exit comes out from the 1852th
320 CSTR, and so on. The second column shows the total volume of each CSTR from a given excerpt of
321 the cascade, including the volume of the packing and the void fraction. All CSTR between 1 and 1845
322 have the same volume that the 1846th CSTR.

323 Similarly, the first and second rows show the number and volumes of the CSTR in the second
324 cascade. Within the matrix, the flows exchanged by the two cascades are represented as fractions of
325 the flow rate at the inlet of the first CSTR of the bed.

326 As mentioned above, the single arbitrary parameter of this model is the number of interconnections
327 between the two cascades. In this study, the number of interconnections was set to 20, since it is

328 enough to give a good temporal discretization of the adsorption column RTD while keeping the model
329 simple.

330 Concerning the DPF model, the axial dispersion coefficient is calculated through the RTD moments
331 ($Pe/2 = \mu_1^2/\sigma^2$ with a Peclet number involving the bed length and not the particle diameter). For
332 simplification, the equivalent DPF model is assimilated to a cascade of 73 CSTR (corresponding to a
333 Peclet number of 146), with a higher interstitial porosity (33.878%) in order to take into account of the
334 free flow zones above and below the bed..

335 The inlet flow considered for RTD simulation is 3600 m³/h. At t=0, a Dirac injection modelled by a
336 Gaussian shape ($e^{-50(t-1)^2}$ where t is time) is introduced. RTD calculated with CFD, DPF and
337 2MENCSTR models are compared in figure 6. The asymmetry of the CFD curve is well reproduced by
338 the 2MENCSTR. The DPF model generates a symmetric RTD.

339 3.2 Chromatographic separation

340 In order to compare the 2MENCSTR model with the CFD and simultaneously evaluate the dispersive
341 effect produced by each one of the hydrodynamic heterogeneities, in an initial state the adsorber is
342 filled with *p*-diethylbenzene (PDEB), the desorbent regularly used in the PX purification. Then, a Dirac
343 injection modelled by a Gaussian shape ($e^{-50(t-1)^2}$ where t is time) of a mixture comprising 95% of PX
344 and 5% of MX is introduced.

345 In figure 7, the concentration profiles of PX and MX obtained using the parameters shown in Table 1 in
346 the CFD model, a DPF model and the model proposed, the 2MENCSTR model are shown.

347 The deviations between the CFD and the DPF models are very slight but the 2MENCSTR fits better
348 the CFD results. In this case the use of the 2MENCSTR model, instead of the DPF model, does not
349 present a significant gain regarding the coupling between hydrodynamics and adsorption. Since the
350 RTD of a DPF model is symmetric, and that the concentration profiles obtained with this model are
351 visibly leaning to the right, it is possible to conclude that in this case most of the peak tailing noticed is
352 due to the slow mass transfer rates, rather than the adsorber design and the consequent flow patterns.

353 In order to investigate if this SMB-enhancing strategy is worth and also to further test the models in
354 study, in a scenario where the adsorbent in use has a better mass transfer capacity, the LDF
355 constants presented in Table 1 are multiplied by a factor of 10. The resulting concentration profiles are
356 shown in figure 8.

357 While the DPF model is unable to reproduce the asymmetric concentration profiles of the CFD model,
358 the 2MENCSTR shows a good agreement with the CFD, yet showing some deviations when the
359 concentration profiles are compared in a logarithmic scale.

360 In order to investigate the importance of a detailed hydrodynamic description in function of the mass
361 transfer scenario, the chromatographic separation is simulated with the three models in study, CFD,
362 DPF and 2MENCSTR for five different sets of LDF constants k_1 and k_2 . The LDF constants used by
363 Bergeot et al. (2010) ($k_1=0.5 \text{ s}^{-1}$ and $k_2=0.153 \text{ s}^{-1}$) were multiplied by 0.5, 1, 5, 10 and 50 and the
364 deviation between the 1D model (either DPF or 2MENCSTR) and the CFD model was measured by
365 equation 20 and reported on figure 9.

$$366 \text{ deviation} = \sqrt{\frac{\int_0^\infty (C_{PX}^{CFD} - C_{PX}^{1D})^2 dt}{\int_0^\infty C_{PX}^{CFD}{}^2 dt}} \quad (20)$$

367 Figure 9 shows that when an adsorbent with mass transfer performances equal or lower than the one
368 used by Bergeot et al. (2010) is used, the DPF model is capable of predicting the concentration

369 profiles with a reasonable error. The DPF model shows a deviation from the CFD concentration
 370 profiles of 7%, slightly higher than the 2MENCSTR model with 5% for k_{LDF} multipliers of 0.5 and 1. A
 371 fine description of hydrodynamics is not mandatory. However, if the adsorbent used is capable of
 372 inducing mass transfer rates faster than those indicated before (k_{LDF} multiplier of 5, 10 and 50), the
 373 deviations between the DPF and the CFD model are more important than the deviations between the
 374 2MENCSTR and the CFD (that also increase with the k_{LDF} multiplier). For instance, for a k_{LDF}
 375 multiplier of 10, the DPF model shows a deviation of 27% from the CFD concentration profiles while
 376 the 2MENCSTR deviation is 3 times lower. In the case of fast mass transfer rates, it is interesting to
 377 use a more detailed 1D hydrodynamic model in order to minimize the discrepancies between a CFD
 378 model (2D in this case but also applicable to 3D geometries) and a simpler and faster model.

379 It is then acceptable that the 2MENCSTR model has been validated for SMB studies by comparing its
 380 results with those obtained with a CFD model of a single adsorption column. The 2MENCSTR model
 381 is then integrated in a cyclic solver and compared to the DPF model to evaluate the usefulness of a
 382 detailed hydrodynamic description in SMB modelling.

383 3.3 SMB results

384 The PX and MX profiles obtained for a SMB unit comprising 6 adsorption columns are shown in the
 385 form of mass fraction for k_1 and k_2 equal to 5 and 1.53 s, respectively, at linear and logarithmic scales
 386 in figure 10.

387 In order to quantify the SMB performance, the purity and the recovery of PX in the extract stream are
 388 measured:

$$PUR_X = \frac{\langle C_{PX,X} \rangle}{\langle C_{PX,X} \rangle + \langle C_{MX,X} \rangle} \quad (21)$$

$$REC_X = \frac{Q_X \langle C_{PX,X} \rangle}{Q_F C_{PX,F}} \quad (22)$$

389

390 $\langle C_{PX,X} \rangle$ and $\langle C_{MX,X} \rangle$ denote that the concentration of PX and MX are averaged on the extract stream
 391 over a switching time (after the CSS is reached), due to the SMB time-variant concentration profiles.

392 When comparing the concentration profiles obtained with the 2MENCSTR and DPF models, slight
 393 deviations are visible, both at linear and logarithmic scales. In order to quantify these deviations the
 394 2MENCSTR model is taken as a reference and the deviations between the extract purity and recovery
 395 provided by both models is quantified. These deviations are more penalizing at high purities and
 396 recoveries, i.e. an absolute deviation of 1% is more prejudicial for a purity of 99% than for a purity of
 397 90%. Thus, the deviations between both models are calculated in terms of relative deviations from
 398 100%, and reported in table 3:

399 - considering the deviation of impurities fraction in the extract:

$$\epsilon_{PUR_X} = \frac{(1 - PUR_X^{DPF}) - (1 - PUR_X^{2MENCSTR})}{(1 - PUR_X^{2MENCSTR})} \quad (23)$$

- considering the deviation of PX fraction in the raffinate:

$$\epsilon_{REC_X} = \frac{(1 - REC_X^{DPF}) - (1 - REC_X^{2MENCSTR})}{(1 - REC_X^{2MENCSTR})} \quad (24)$$

400

401 By looking at Table 3, it is possible to observe that the DPF model strongly underestimates the PX
 402 purity and recovery when compared to the 2MENCSTR model that is used as a reference. The

403 deviation of residual impurities in the extract line is over 80%. This signifies that using the DPF model
404 to design a SMB process may lead to a loss of productivity and/or yield.

405 In order to further investigate the importance of a detailed hydrodynamic description in SMB modelling,
406 the same study was conducted for a 12 column SMB process. The concentration profiles obtained are
407 presented in figure 11.

408 Even though in this case, the concentration profiles provided by both models appear to provide a
409 better agreement than in the case of the SMB with 6 adsorption columns, a major profile deviation is
410 found near the extract (bed 2) stream at both linear and logarithmic scales. The resulting performance
411 parameters are shown in table 4.

412 The DPF model underestimates the extract purity, as for the 6 column SMB, with an absolute deviation
413 of 0.21% and a deviation of residual impurities of 210.00% , while the extract recovery deviations are
414 of 0.43% (in absolute) and 70.49% (relative loss of PX in the raffinate line). When these values are
415 compared to those obtained for a 6 bed SMB, the absolute deviations decrease, as the SMB leans
416 towards a “perfect operation” and both extract purity and recovery approach 100%. However the
417 deviations calculated by Eqs. 23 and 24 are more important for the 12 bed SMB, meaning that a
418 detailed hydrodynamic description is more important as the SMB approaches high purities and
419 recoveries.

420 **4 CONCLUSION**

421 Gomes et al. (2015) proposed a method to develop 1D models based on the transport of the moments
422 of the fluid age distribution by CFD (Liu and Tilton, 2010 and Liu, 2012). In this work, this method is
423 used and the 1D model deduced can simulate an adsorption process more accurately than the
424 traditionally used DPF model, when coupled with adsorption. The accuracy gain when using the 1D
425 model proposed, the 2MENCSTR model, instead of the DPF model is more evident when the mass
426 transfer rates are fast. The 2MENCSTR can converge in few minutes, contrasting with the CFD
427 simulations of multicomponent adsorption coupled with hydrodynamics that can take several days to
428 converge at present. This facilitates the parametric study and optimization of the SMB or other
429 industrial applications involving adsorbers.

430 The adsorber geometry reveals itself to be a minor issue for slow mass transfer rates, yet not
431 negligible. However, if the development of an adsorbent with better mass transfer performances is
432 considered the adsorber geometry becomes preponderant. The addition of free flow volumes and
433 obstacles to the packed bed can severely decrease the separation efficiency of the adsorber.

434 Then, the 2MENCSTR model was used as a reference in a cyclic solver and compared to the DPF
435 model to investigate the importance of a good hydrodynamic description when studying SMB
436 processes. The DPF model is able to provide a reasonable agreement regarding the 2MENCSTR
437 SMB concentration profiles but the performance parameters are visibly different from the ones
438 obtained with the 2MENCSTR model, reaching important extract purity and recovery deviations when
439 residual impurity concentrations are compared.

440 In this work 1D models were deduced for two-dimensional adsorber geometries, although the same
441 method can be used for 3D geometries and for diverse applications of separation processes or
442 reactors.

443

444 **NOTATIONS**

445

b_i adsorbent coefficient of affinity for the component i ($\text{m}^{-3} \text{kg}^{-1}$)
 C_i concentration of the component i in the bulk phase (kg m^{-3})

$C_{m,i}$	concentration of the component i in the macro-porous liquid phase (kg m^{-3})
D_M	molecular diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
D_P	mechanical dispersion coefficient ($\text{m}^2 \text{s}^{-1}$)
d_p	particle diameter (m)
D_T	turbulent diffusivity coefficient ($\text{m}^2 \text{s}^{-1}$)
J	degree of mixing (dimensionless)
K	bed permeability (m^2)
k_1	fluid film mass transfer coefficient (s^{-1})
k_2	internal mass transfer coefficient (s^{-1})
m_n	raw moment of order n (s^n)
N	number of equivalent CSTR (dimensionless)
N_i^ε	mass flux between the bulk phase and the macro-porous phase ($\text{mol m}^3 \text{s}^{-1}$)
p	pressure (Pa)
$Pe_{ax/tr}$	Peclet number relative to the particle diameter ($= v d_p / D_P$)
Q	volumetric flow rate ($\text{m}^3 \text{s}^{-1}$)
q_i	average concentration of the component i in the adsorbed phase (dimensionless)
q_i^{**}	adsorbed concentration of i at equilibrium with the macro-porous liquid phase
$q_{m,i}$	adsorbent mass capacity for the component i (dimensionless)
Sc_T	turbulent Schmidt number ($= \mu / \rho D_T$)
u	superficial liquid velocity (m s^{-1})

446

447 Greek letters

β	non-Darcy coefficient (m^{-1})
ε	bed interstitial porosity (dimensionless)
ε_p	macroporosity (dimensionless)
μ	dynamic viscosity (Pa s)
μ_1	first moment of the concentration curve (s)
μ_e	effective dynamic viscosity (Pa s)
μ_T	turbulent dynamic viscosity (Pa s)
ρ	mass density (kg m^{-3})
ρ_p	apparent adsorbent mass density (kg m^{-3})
σ^2	variance of the concentration curve (s^2)

448

449 Abbreviations

2MENCSTR	Multiple Entrances and Multiple Exits NCSTR models
CFD	Computational Fluid Dynamics
CSTR	Continuous Stirred-tank Reactor
DPF	Dispersed Plug Flow
EB	ethylbenzene
LDF	Linear Driving Force
MX	<i>m</i> -xylene
NCSTR	Cascade of N CSTR
NTP	Number of Theoretical Plates
OX	<i>o</i> -xylene
PDEB	<i>p</i> -diethylbenzene
PET	polyethylene terephthalate
PFR	Plug Flow Reactor
PSA	Pressure Swing Adsorption
PUR	Purity
PX	<i>p</i> -xylene
RANS	Reynolds-averaged Navier-Stokes
REC	Recovery
RTD	Residence Time Distribution
SMB	Simulated Moving Bed

450

451 **REFERENCES**

- 452 Augier, F., Hotier, G., 2011. U.S. Patent Application 13/154,874.
- 453 Augier, F., Laroche, C., Brehon, E., 2008. Application of computational fluid dynamics to fixed
454 bed adsorption calculations: effect of hydrodynamics at laboratory and industrial scale. *Separation and*
455 *Purification Technology*, 63(2), 466-474.
- 456 Bart, H. J., Germerdonk, R., Ning, P., 1996. Two-dimensional non-isothermal model for toluene
457 adsorption in a fixed-bed adsorber. *Chemical Engineering and Processing: Process Intensification*,
458 35(1), 57-64.
- 459 Benyahia, F., 2004. On the modeling of flow in packed bed systems. *Particulate science and*
460 *technology*, 22(4), 367-378.
- 461 Bergeot, G., Leinekugel-Le-Cocq, D., Wolff, L., Muhr, L., Bailly, M., 2010. Intensification of
462 paraxylene production using a simulated moving bed reactor. *Oil & Gas Science and Technology–*
463 *Revue d'IFP Energies nouvelles*, 65(5), 721-733.
- 464 Chan, E. C., Lien, F. S., Yovanovich, M. M., 2000. Numerical study of forced flow in a back-step
465 channel through porous layer. In 2000 National Heat Transfer Conference, Pittsburgh, PA.
- 466 Combest, D. P., Ramachandran, P. A., Dudukovic, M. P., 2011. On the gradient diffusion
467 hypothesis and passive scalar transport in turbulent flows. *Industrial & Engineering Chemistry*
468 *Research*, 50(15), 8817-8823.
- 469 Danckwerts, P. V., 1952. The definition and measurement of some characteristics of mixtures.
470 *Applied Scientific Research, Section A*, 1952, vol. 3, no 4, p. 279-296.
- 471 Danckwerts, P. V., 1958. The effect of incomplete mixing on homogeneous reactions. *Chemical*
472 *Engineering Science*, 8(1), 93-102.
- 473 Delgado, J. M. P. Q., 2006. A critical review of dispersion in packed beds. *Heat and mass*
474 *transfer*, 42(4), 279-310.
- 475 Ergun, S., 1952. Fluid flow through packed columns. *Chem. Eng. Prog.*, 48, 89-94.
- 476 Gomes, L. F., Augier, F., Leinekugel-le-Cocq, D., Vinkovic, I., Simoëns, S., 2015. Hydrodynamic
477 modelling of complex fixed bed geometries in simulated moving bed adsorption processes. *Chemical*
478 *Engineering Science*, 132, 46-58.
- 479 Flesch, T. K., Prueger, J. H., Hatfield, J. L., 2002. Turbulent Schmidt number from a tracer
480 experiment. *Agricultural and Forest Meteorology*, 111(4), 299-307.
- 481 Giese, M., Rottschäfer, K., Vortmeyer, D., 1998. Measured and modeled superficial flow profiles
482 in packed beds with liquid flow. *AIChE Journal*, 44(2), 484-490.
- 483 Jones, W. P., Launder, B., 1972. The prediction of laminarization with a two-equation model of
484 turbulence. *International journal of heat and mass transfer*, 15(2), 301-314.
- 485 Koeltzsch, K., 2000. The height dependence of the turbulent Schmidt number within the boundary
486 layer. *Atmospheric Environment*, 34(7), 1147-1151.
- 487 Kwapinski, W., Salem, K., Mewes, D., & Tsotsas, E., 2010. Thermal and flow effects during
488 adsorption in conventional, diluted and annular packed beds. *Chemical Engineering Science*, 65(14),
489 4250-4260.
- 490 Liu, M., Tilton, J. N., 2010. Spatial distributions of mean age and higher moments in steady
491 continuous flows. *AIChE journal*, 56(10), 2561-2572.
- 492 Liu, M., 2012. Age distribution and the degree of mixing in continuous flow stirred tank reactors.
493 *Chemical Engineering Science*, 69(1), 382-393.

494 Liu, Y., Zheng, X., Dai, R., 2014. Numerical study of flow maldistribution and depressurization
495 strategies in a small-scale axial adsorber. *Adsorption*, 20(5-6), 757-768.

496 Mazzotti, M., Storti, G., Morbidelli, M., 1997a. Optimal operation of simulated moving bed units
497 for nonlinear chromatographic separations. *Journal of Chromatography A*, 769(1), 3-24.

498 Mazzotti, M., Storti, G., Morbidelli, M., 1997b. Robust design of countercurrent adsorption
499 separation processes: 4. Desorbent in the feed. *AIChE journal*, 43(1), 64-72.

500 Mendes, A. M., Costa, C. A., Rodrigues, A. E., 1996. Extension of the linear driving force-dusty
501 gas model approximation to include surface or micropore diffusion. *Gas separation & purification*,
502 10(3), 141-148.

503 Migliorini, C., Mazzotti, M., Morbidelli, M., 1999. Simulated moving-bed units with extra-column
504 dead volume. *American Institute of Chemical Engineers. AIChE Journal*, 45 (7), 1411.

505 Minceva, M., Rodrigues, A. E., 2002. Modeling and simulation of a simulated moving bed for the
506 separation of p-xylene. *Industrial & engineering chemistry research*, 41(14), 3454-3461.

507 Minceva, M., Rodrigues, A. E., 2003. Influence of the transfer line dead volume on the
508 performance of an industrial scale simulated moving bed for p-xylene separation. *Separation science
509 and technology*, 38(7), 1463-1497.

510 Minceva, M., Rodrigues, A. E., 2007. Understanding and revamping of industrial scale SMB units
511 for p-xylene separation. *AIChE journal*, 53(1), 138-149.

512 Nield, D. A., 1991. The limitations of the Brinkman-Forchheimer equation in modeling flow in a
513 saturated porous medium and at an interface. *International Journal of Heat and Fluid Flow*, 12(3), 269-
514 272.

515 Pais, L. S., Loureiro, J. M., Rodrigues, A. E., 1997a. Modeling, simulation and operation of a
516 simulated moving bed for continuous chromatographic separation of 1, 1'-bi-2-naphthol enantiomers.
517 *Journal of Chromatography A*, 769(1), 25-35.

518 Pais, L. S., Loureiro, J., Rodrigues, A. E., 1997b. Separation of 1, 1'-bi-2-naphthol enantiomers
519 by continuous chromatography in simulated moving bed. *Chemical engineering science*, 52(2), 245-
520 257.

521 Pavone, D., Hotier, G., 2000. System approach modelling applied to the eluxyl process. *Oil & Gas
522 Science and Technology*, 55(4), 437-446.

523 Ruthven, D. M., 1984. *Principles of adsorption and adsorption processes*. John Wiley & Sons.

524 Silva, M.S.P., Rodrigues, A. E., Mota, J. P. B., 2016. Effect of dead volumes on the
525 performance of an industrial-scale simulated moving-bed Parex unit for p-xylene purification. *AIChE
526 Journal*, 62(1), 241.

527 Storti, G., Baciocchi, R., Mazzotti, M., Morbidelli, M., 1995. Design of optimal operating
528 conditions of simulated moving bed adsorptive separation units. *Industrial & engineering chemistry
529 research*, 34(1), 288-301.

530 Vafai, K., Tien, C. L., 1982. Boundary and inertia effects on convective mass transfer in porous
531 media. *International Journal of Heat and Mass Transfer*, 25(8), 1183-1190.

532 Vafai, K., Kim, S. J., 1995. On the limitations of the Brinkman-Forchheimer-extended Darcy
533 equation. *International Journal of Heat and Fluid Flow*, 16(1), 11-15.

534 Wu, Y. X., Yu, H. W., Ching, C. B., 2004. A computational fluid dynamics study of binary
535 adsorption separation in chromatography. *Chemical engineering & technology*, 27(9), 955-961.

536 Zheng, X., Liu, Y., & Liu, W., 2010. Two-dimensional modeling of the transport phenomena in the
537 adsorber during pressure swing adsorption process. *Industrial & Engineering Chemistry Research*,
538 49(22), 11814-11824.

539 Zwietering, T. N., 1959. The degree of mixing in continuous flow systems. *Chemical Engineering*
540 *Science*, 11(1), 1-15.

541

542

543 **Table Captions**

544 Table 1 – Simulation parameters.

545 Table 2 – SMB operating conditions.

546 Table 3 – PX purity and recovery obtained with the DPF and 2MENCSTR models for a SMB unit with
547 6 adsorption columns, for k_1 and k_2 equal to 5 and 1.53 s, respectively.

548 Table 4 – PX purity and recovery obtained with the DPF and 2MENCSTR models for a SMB unit with
549 12 adsorption columns, for k_1 and k_2 equal to 5 and 1.53 s, respectively.

550

551

552 **Figure Captions**

553 Figure 1 – Schematic view of an example of SMB process (Gomes et al., 2015).

554 Figure 2 – Geometry and dimensions of the adsorption bed.

555 Figure 3 – Scheme of the modelling approach proposed (a), resulting system of parallel NCSTR
556 models with the distinction the dissociation of the 3 zones (b) and resulting 2MENCSTR model (c).

557 Figure 4 – (a) Schematic diagram of a SMB process for the separation of a binary mixture, (b)
558 2MENCSTR model and (c) DPF model, both used to model each adsorption column.

559 Figure 5 – Matrix with the fractions of flow exchanged between the two CSTR cascades of the
560 2MENCSTR model, number (N) and volume (V) of the CSTR.

561 Figure 6 – RTD comparison between the different models of similar variances σ^2 .

562 Figure 7 – PX and MX concentration profiles obtained with the three studied models; CFD, DPF and
563 2MENCSTR.

564 Figure 8 – PX and MX concentration profiles obtained with the three studied models; CFD, DPF and
565 2MENCSTR, for k_1 and k_2 equal to 5 and 1.53 s, respectively.

566 Figure 9– Deviations between the concentration profiles obtained by CFD and obtained with the 1D
567 models, DPF and 2MENCSTR for different mass transfer scenarii.

568 Figure 10 – Mass fraction profiles obtained with the DPF and 2MENCSTR models for a SMB with 6
569 adsorption columns, for k_1 and k_2 equal to 5 and 1.53 s, respectively.

570 Figure 11 – Mass fraction profiles obtained with the DPF and 2MENCSTR models for a SMB with 12
571 adsorption columns, for k_1 and k_2 equal to 5 and 1.53 s, respectively.

572

573

574

575

Table 1

Parameter	Value	Unit
μ	2E-4	Pa.s
ρ	725	kg.m ⁻³
d_p	6E-4	m
ε	0.32	
Sc_T	0.7	
Pe_{ax}	2	
Pe_{tra}	11	
k_1	0.5	s ⁻¹
k_2	0.153	s ⁻¹
ε_p	0.37	
ρ_p	1480	kg.m ⁻³
q_{PX}	0.1024	kg _{PX} /kg _{adsorbent}
q_{MX}	0.0917	kg _{MX} /kg _{adsorbent}
q_{PDEB}	0.1045	kg _{PDEB} /kg _{adsorbent}
b_{PX}	1.9409	m ³ .kg ⁻¹
b_{MX}	0.8884	m ³ .kg ⁻¹
b_{PDEB}	1.9028	m ³ .kg ⁻¹

576

577

578

579

Table 2

Parameter/Number of columns	6	12
Configuration	1-2-2-1	2-5-3-2
Q_F ($\text{m}^3 \cdot \text{h}^{-1}$)	45.30	100.60
Q_D ($\text{m}^3 \cdot \text{h}^{-1}$)	73.95	147.90
Q_X ($\text{m}^3 \cdot \text{h}^{-1}$)	43.35	86.70
Q_R ($\text{m}^3 \cdot \text{h}^{-1}$)	75.90	161.80
Q_{PA} ($\text{m}^3 \cdot \text{h}^{-1}$)	464.10	928.20
Δt_{sw} (s)	90	45
$C_{PX,F}$ ($\text{kg} \cdot \text{m}^{-3}$)	1986.5	
$C_{MX,F}$ ($\text{kg} \cdot \text{m}^{-3}$)	5263.5	

580

581

582

583

Table 3

Parameter	Values	Deviations on residual impurities
PUR_X^{DPF}	96.24%	82.52%
$PUR_X^{2MENCSTR}$	97.94%	
REC_X^{DPF}	97.88%	19.77%
$REC_X^{2MENCSTR}$	98.23%	

584

585

586

587

Table 4

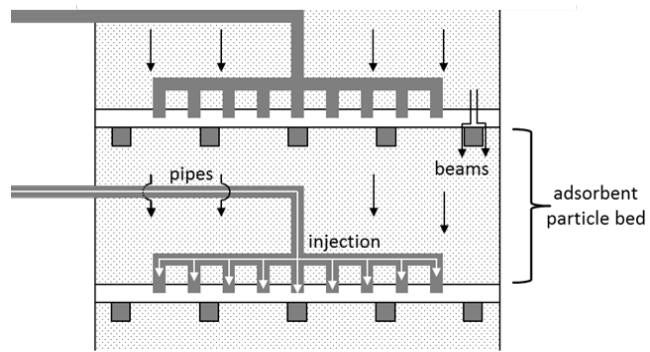
Parameter	Values	Deviations on residual impurities
PUR_X^{DPF}	99.69%	210.00%
$PUR_X^{2MENCSTR}$	99.90%	
REC_X^{DPF}	98.96%	70.49%
$REC_X^{2MENCSTR}$	99.39%	

588

589

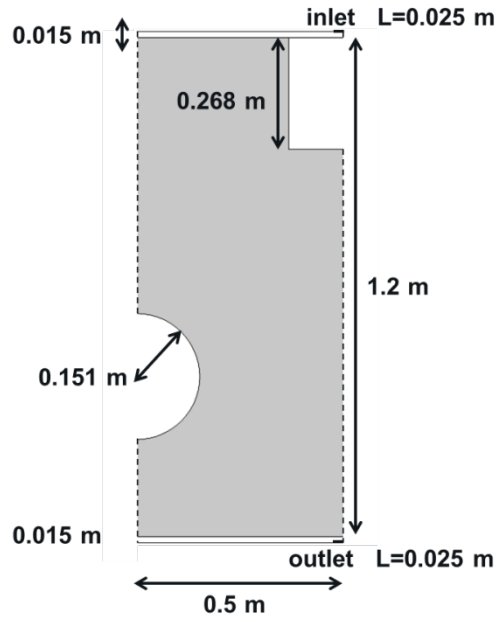
590
591
592
593
594
595
596
597
598
599
600
601
602

Figure 1



603
604
605
606
607
608
609
610
611
612
613
614
615
616
617

Figure 2



618
 619
 620
 621
 622
 623
 624
 625
 626
 627
 628
 629
 630

Figure 3

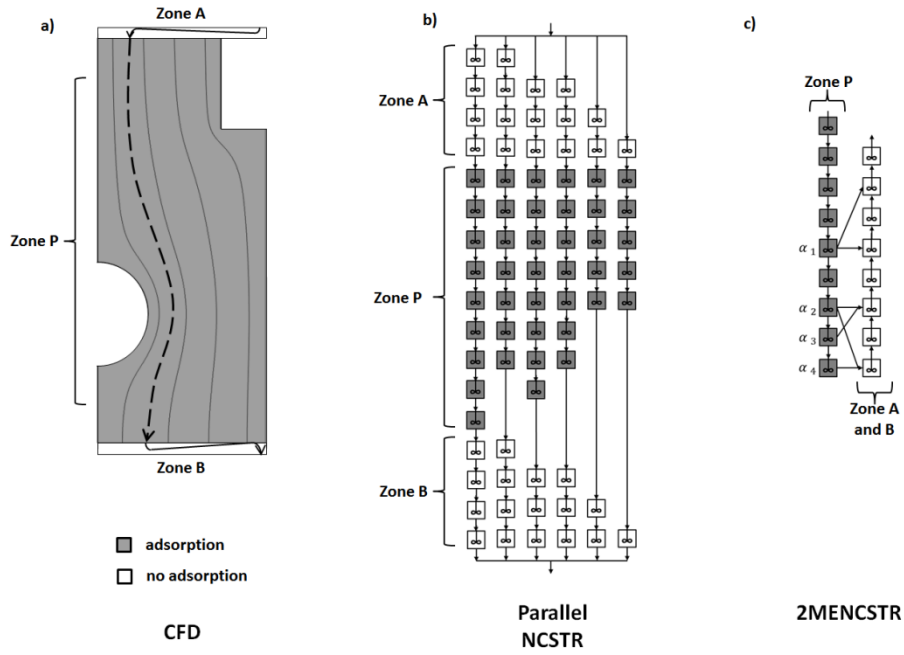


Figure 4

631
632
633
634
635
636
637
638
639
640
641
642
643
644

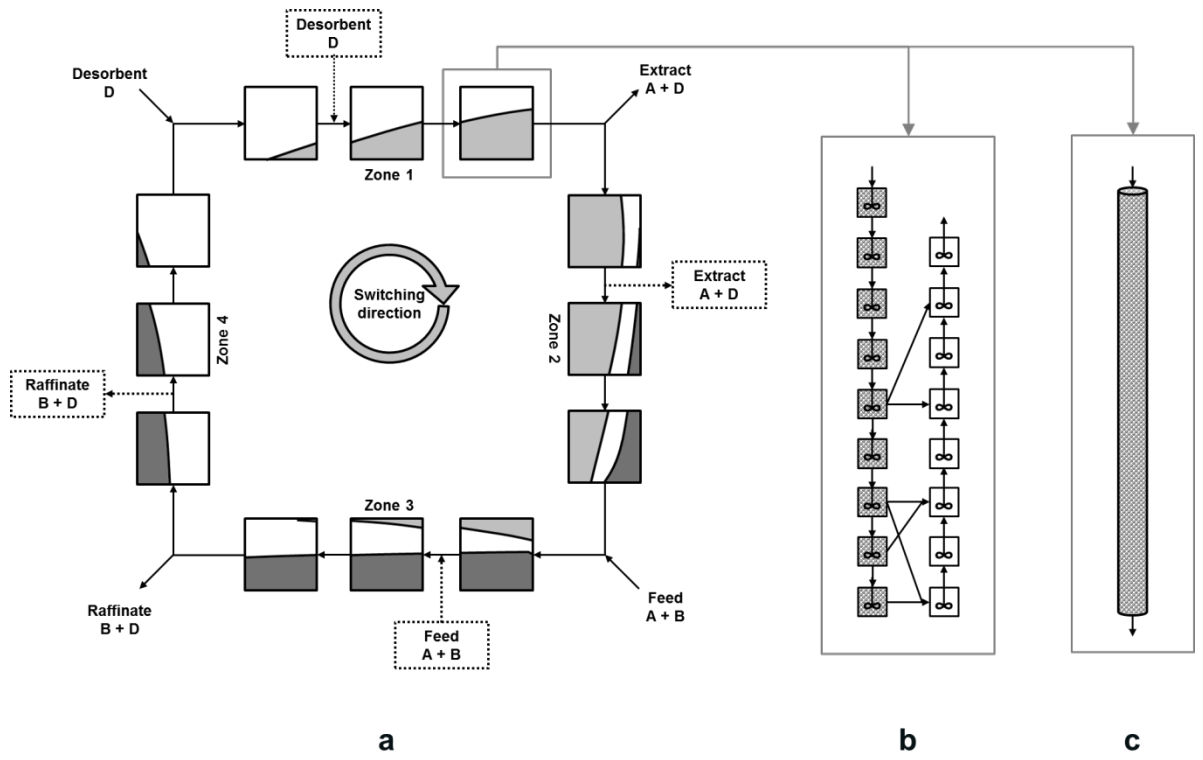


Figure 5

645
646
647
648
649
650
651
652
653
654
655
656
657
658
659

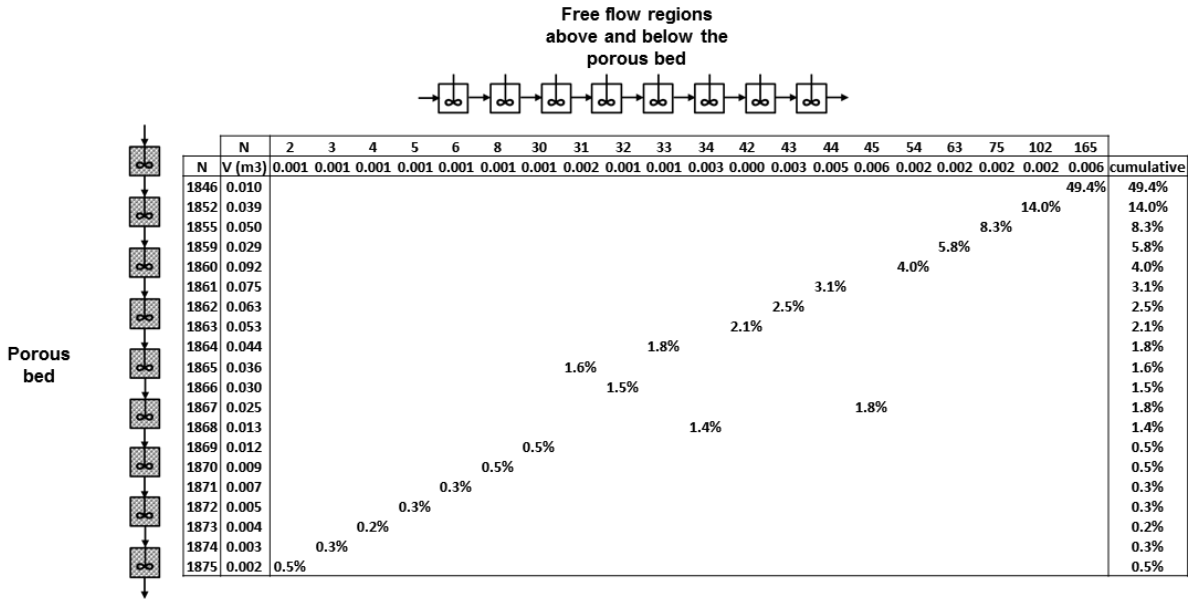
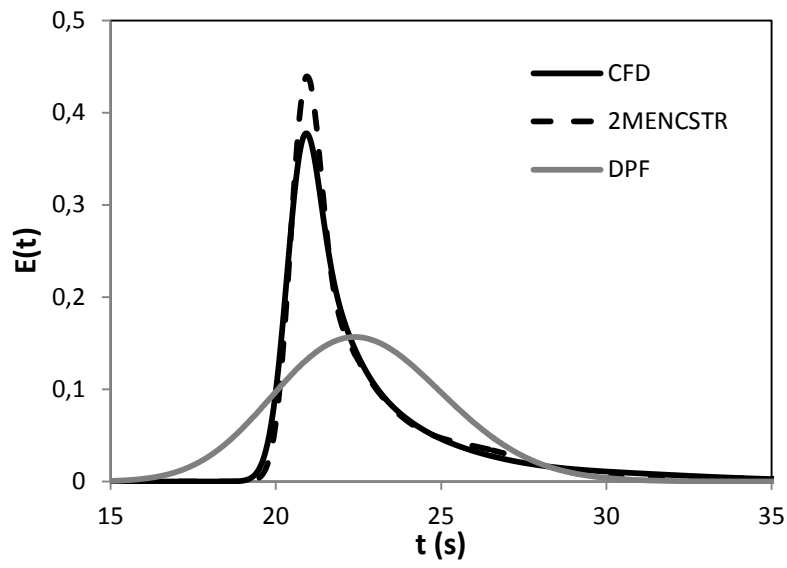


Figure 6

660

661



662

663

664

665

666

667

668

669

670

671

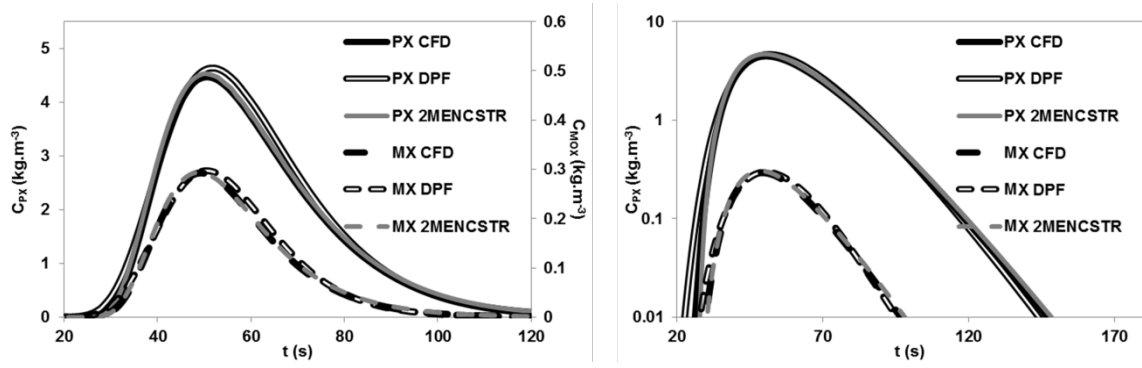
672

673

674

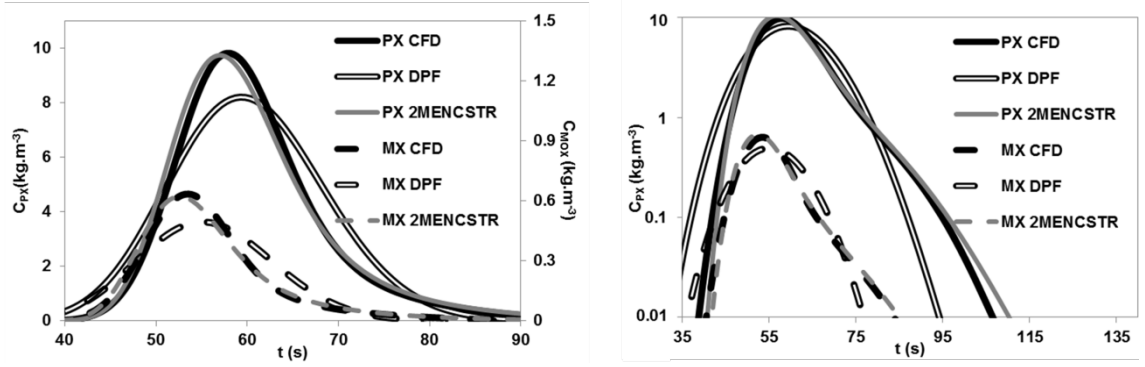
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689

Figure 7



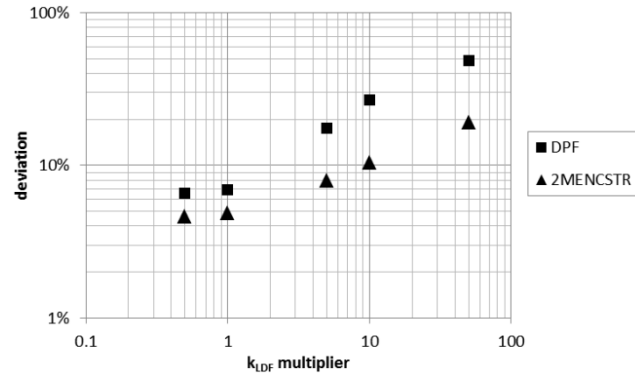
690
691
692
693
694
695
696
697
698
699
700
701
702

Figure 8



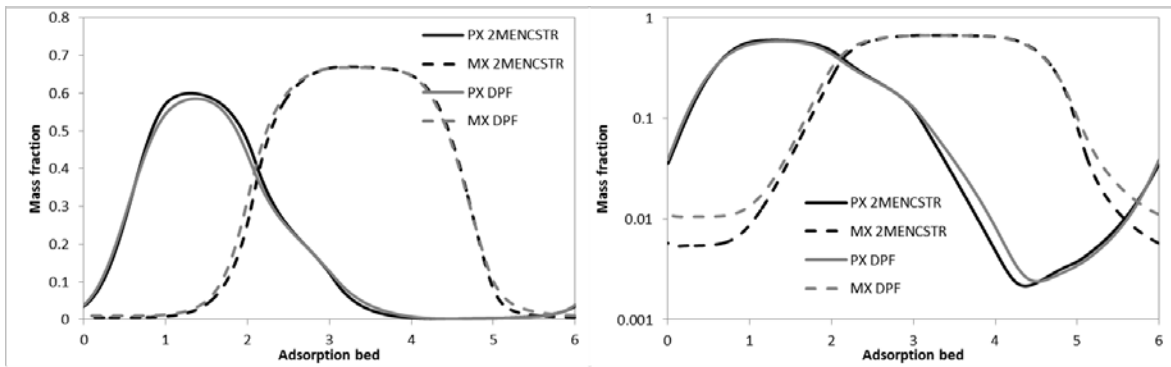
703
704
705
706
707
708
709
710
711
712
713
714
715

Figure 9



716
717
718
719
720
721
722
723
724
725
726
727

Figure 10



728

729

Figure 11

730

731

732

733

734

735

736

737

