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► **To cite this version:**

Chao Xu, Vineeth Varma, Min S Sheng, Tony Q S Quek. Wireless service provider selection and bandwidth resource allocation in multi-tier HCNs. IEEE Global Communications Conference, GLOBECOM 2016, Dec 2016, Washington, United States. 10.1109/TCOMM.2016.2613083 . hal-01407861

HAL Id: hal-01407861

<https://hal.science/hal-01407861>

Submitted on 2 Dec 2016

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Wireless Service Provider Selection and Bandwidth Resource Allocation in Multi-tier HCNs

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Abstract—In this work, the inter-linked problems of bandwidth allocation by wireless service providers (WSPs) and WSP selection by users in heterogeneous cellular networks (HCNs) are addressed. This work employs a multi-hierarchical game framework. While the interaction between users are modelled using evolutionary game theory, the interactions between competing WSP's are modelled as a non-cooperative spectrum bandwidth allocation game (N-BAG). Finally the interaction between the WSPs and the users is modelled as a multi-leader multi-follower Stackelberg game. Moreover, for the formulated evolutionary game, the existence and uniqueness of the evolutionary equilibrium (EE) was investigated. Conditioned on the obtained EE, the existence of a Nash equilibrium (NE) for the proposed N-BAG was proven and an offline algorithm to achieve the equilibrium state was proposed. Finally, simulation results verify the validity of the analysis and demonstrate that a unique NE would be achieved by the HCNs adopting the proposed scheme.

I. INTRODUCTION

Heterogeneous cellular networks (HCNs) are regarded as a promising approach to cater for the increased demand of high-data-rate services and requirement of ubiquitous access [1]–[4]. In this light, HCNs is a novel network structure for the future. Particularly, in contrast to the traditional pure macro cellular network, HCNs would consist of high power macro base stations (MBSs) as well as various classes of low power nodes, which may include micro BSs (often called eNBs), femto BSs (FBSs), and possibly future radiating elements [5].

Several works on game theory focused on HCN based scenarios can be found in literature [6]–[9]. The major difference between the above mentioned works and ours the type of HCN considered in these works. Specifically, different tiers are deployed by different wireless service providers (WSPs) and each WSP only provides one service to users. Hence, their main concerns are spectrum leasing among different WSPs. This work however, jointly studies the WSP selection by users and spectrum allocation of WSPs.

In this work, the interactions between the users in an environment where HCNs are deployed by several WSPs is studied and the population share for each WSP is predicted using results from evolutionary game theory. The WSPs assume that the user population stabilizes at the EE and competes for a higher revenue. Each WSP determines the bandwidth allocation for each service while the users decide which WSP to subscribe to. Naturally, as every user and WSP are independent decision makers, this results in a situation best modelled through game theory.

II. SYSTEM MODEL

In the following sections, we consider an area of interest where a total of K different service would be provided to users (e.g., the macrocell service, picocell service and femtocell service) by N different WSPs having deployed N different HCNs. Additionally, for each HCN n , there are K_n tiers, each of which has the BSs of a particular class, such as MBSs or FBSs. Note that the BSs in different tiers may be differ among each other due to differences in the transmit power, coverage area and spatial density [10], [11].

A. Network deployment model

For notational simplicity, we denote the set of WSPs as $\mathcal{N} = \{1, 2, \dots, N\}$ and meanwhile, for each WSP n , we denote the set of the K_n tiers of networks as \mathcal{K}_n . Without loss of generality, we consider that $\mathcal{K}_n \subseteq \mathcal{K} = \{1, 2 \dots K\}$, which means different WSPs may like to provide different services to users. For instance, some WSPs may provide macrocell service, picocell service and femtocell service, but some WSPs may only provide macrocell service.

The HCNs are operating on the orthogonal spectrum and each WSP n has to properly allocate the available spectrum bandwidth B_n to different tiers, i.e., the WSP deployment framework is adopted [12]. For the HCN operated by WSP n , we denote the spectrum bandwidth allocated to tier k by $B_{n,k}$ and the spectrum allocation profile by $\mathbf{B}_n = (B_{n,1}, B_{n,2}, \dots, B_{n,K_n})$. Therefore, there is no inter-tier interference, only intra-tier interference. Meanwhile, each user can independently decide to get the corresponding service from any WSP they prefer. For each user, let us denote the price charged by WSP n for service k as $C_{n,k}$ which is the fixed access fee having units \$ per unit of time. For each service k , the WSPs supporting this service are given by \mathcal{N}_k .

Finally, we assume that the user population density is given by λ_k^u . Here, each service is associated to a certain user density. This is explained by the preferences of different kinds of users to different services. For example a highly mobile user population will prefer the macro-cell service in order to avoid regular hand-overs. Of course in practice, the same user can jump from one population set to another, but on an average the population of each set of user types can be assumed to be a constant. Each user in a certain population type, picks one out of the N WSPs to get his service. This choice is determined by the data rates and prices offered by the WSP, and results

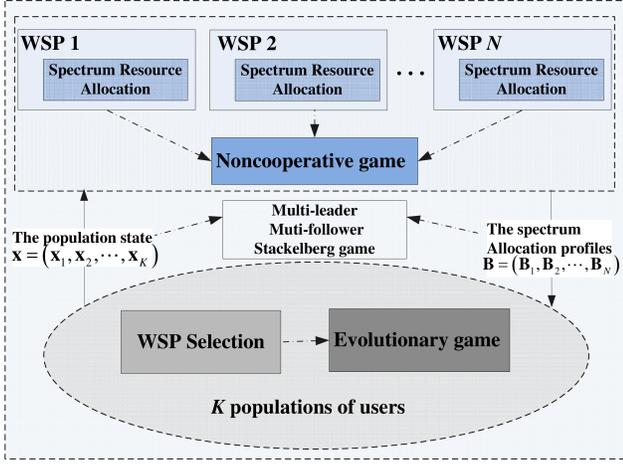


Fig. 1. Illustration of the hierarchical game framework.

in a population share of $x_{n,k}$ associated to each WSP n and service k , where $\sum_{n \in \mathcal{N}_k} x_{n,k} = 1, \forall k$.

B. Hierarchical game framework

For each WSP n , a key problem becomes how to accurately estimate the equilibrium behavior of users and thus make an informed decision. Here, we consider that all the WSPs and users are selfish, i.e., each individual user wants to maximize his payoff by selecting the best WSP and meanwhile, each WSP wants to attract more users and further improve his own revenue by properly allocating the spectrum resource to different tiers. To be more specific, after selecting the service provided by a particular WSP, each user will obtain some payoff determined by both the service price and expected transmission rate. Intuitively, the expected rate of each user will be determined by both the number of users choosing the same network (i.e., the effects of other users' actions) and the available spectrum bandwidth of the network (i.e., the effect of WSPs' actions). Similarly, to attract more users, each WSP also need to make his own decision on the spectrum partition by fully considering the behavior of other WSPs and the WSP selection of users. The scenario under investigation therefore allows two levels of competition and meanwhile one cyclic dependency as described above.¹

These competitions and cyclic dependency can be illustrated with the hierarchical framework shown Fig. 1, where three different game formulation are proposed to investigate the concerned problem. Specifically, in the lower level we formulate the WSP selection of users as an evolutionary game, with which the long-term equilibrium behavior of users can be captured. Meanwhile, in the upper level we have formulated a non-cooperative game to depict the competition among WSPs. In addition, the cyclic dependency interaction between the users and WSPs is modeled by applying the multi-leader multi-follower Stackelberg game theory, where the WSPs are the

¹The definitions for both the user's payoff function and WSP's revenue will be sequentially given in the following section.

leaders and the populations of users are the followers.

III. EQUILIBRIUM ANALYSIS

The formulated Stackelberg game can be solved with backward induction, i.e., the leaders (WSPs) can make their responses (i.e., \mathbf{B}^*) by fully considering the followers' (users') equilibrium reactions (i.e., $\mathbf{x}^*(\mathbf{B}^*)$). The mapping $\mathbf{x}^*(\mathbf{B}^*)$ denotes the equilibrium behaviour of users, when given the WSPs' strategy profile \mathbf{B}^* . Here, $\mathbf{x} = (x_1, x_2, \dots, x_K)$ where $\forall k \in \mathcal{K}, \mathbf{x}_k = (x_{a_k,k}; a_k \in \mathcal{N}_k)$ represents the population states. In this section, the evolutionary game modelling the user reaction to the WSP bandwidth allocation is studied first. Following which, the WSP's compete for a higher revenue assuming the user population to stabilize at the proposed EE based on their bandwidth allocation.

A. Evolutionary equilibrium analysis

For the WSP selection of users, the formulated evolutionary game can be formally depicted as follows:

- **Population:** The users that request for the various services. The density of these users is denoted by λ_k^u .
- **Strategy space:** Each user in the k -th population can decide to get the service from which WSP. Therefore, his strategy space can be depicted with the set

$$\mathcal{N}_k = \{n | n \in \mathcal{N}, k \in \mathcal{K}_n\}. \quad (1)$$

- **Population state:** Given by \mathbf{x}
- **Payoff function:** The payoff is used to quantify the satisfaction level or fitness of a user adopting a strategy when given the population state. Mathematically speaking, the payoff function of an individual player choosing strategy a_k can be considered as a mapping $\pi_{a_k} : \mathbf{x} \rightarrow \mathbb{R}, a_k \in \mathcal{N}_k, \forall k$. We define the payoff function of each user as:

$$\pi_{a_k,k} = \ln \left(1 + \frac{\beta_{a_k,k}}{x_{a_k,k}} \right) \quad (2)$$

where $\beta_{a_k,k} = \frac{\bar{R}_{a_k,k} B_{a_k,k}}{\ln 2 \lambda_k^u C_{a_k,k}}$ and $R_{a_k,k}$ is the expected rate per bandwidth allocated for a user on WSP n accessing the network through tier k ². We note that, from the economic perspective, the logarithmic payoff function $\pi = \ln(1+y)$ can capture both the user's non-satiation, i.e., $\frac{d\pi}{dy} > 0$, and risk aversion, i.e., $\frac{d^2\pi}{dy^2} < 0$, properties. Hence, such a function has been widely used in wireless resource allocation to evaluate satisfaction degree of an agent [13]. It is worth noticing that although only the logarithmic function is considered here, the following analysis also holds when other monotonically increasing functions are adopted.

To depict the dynamically learning behavior of users, the replicator dynamic is adopted in this paper, which can be

² $R_{n,k}$ is determined by the infrastructure, transmission power, noise and many such factors but for the purpose of this work, is treated as a constant which can be calculated using a stochastic geometry approach.

expressed as follows:

$$\begin{aligned} \dot{x}_{a_k} &= \delta x_{a_k} (\pi_{a_k}(\mathbf{x}_k) - \bar{\pi}_k(\mathbf{x}_k)) \\ &= \delta x_{a_k} \left(\pi_{a_k} - \sum_{a'_k \in \mathcal{A}_k} x_{a'_k} \pi_{a'_k}(\mathbf{x}_k) \right), \forall k \in \mathcal{K} \end{aligned} \quad (3)$$

where $\delta > 0$ is the rate of strategy adaptation and $\bar{\pi}(\mathbf{x})$ is the average payoff in the population. Under this dynamic in each population, the percentage growth rate of the population share of each strategy is proportional to the excess of the strategy's payoff over the population's average payoff. This dynamic could be interpreted biologically as a model of natural selection, and economically as a model of imitation [14]. As shown in the above equation, the dynamics of service selection can be described with K first-order differential equations. Hence, in order to investigate the equilibrium behaviour of users, we consider the solution of this evolutionary dynamic as the fixed point of the differential equations, which is termed as the evolutionary equilibrium (EE) represented by \mathbf{x}^* . When the population state of the game is at EE, then no user would like to change his strategy.

Based on the support, the EE can be mainly divided into two classes, i.e., the boundary EE and interior EE. Particularly, let us consider a EE \mathbf{x}^* and denote its support as $\text{supp}(\mathbf{x}^*) = \{(a_k, k) \mid a_k \in \mathcal{A}_k, k \in \mathcal{K}, x_{a_k}^* > 0, x_{a_k}^* \in \mathbf{x}^*\}$. If $\text{supp}(\mathbf{x}^*) = (\mathcal{A}_k, k)_{k \in \mathcal{K}}$ we say \mathbf{x}^* is an interior EE. Otherwise, we term it as a boundary EE. Generally, the class of the achieved EE for each replicator dynamic is determined by the initial state $\mathbf{x}(0)$, i.e., $\text{supp}(\mathbf{x}(0)) = \text{supp}(\mathbf{x}^*)$. In the concerned problem, we note for the applied replicator dynamic the boundary EE is not stable since any small perturbation will make the system deviate from this boundary state. Hence, in the rest of this paper, we just focus on the interior EE. Furthermore, we have the following Theorem.

Theorem 1: For the replicator dynamic in (3), the formulated WSP selection game has an unique interior EE state. Let us denote this EE by \mathbf{x}^* , and each element $x_{a_k}^*$ in \mathbf{x}^* can be expressed as

$$x_{a_k}^* = \frac{\eta_{a_k} B_{a_k, k}}{\sum_{a'_k \in \mathcal{A}_k} \eta_{a'_k} B_{a'_k, k}}, \forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{K} \quad (4)$$

where $\eta_{a_k, k} = \frac{\lambda_{a_k, k} \bar{R}_{a_k, k}^0}{C_{a_k, k}}$. Intuitively, $\eta_{a_k, k}$ can be seen as the cost performance per unit bandwidth of the k -th service provided by WSP a_k .

Proof: Substituting (2) into (3), we can equally transform the replicator dynamics of the formulated evolutionary service selection game as (5). Based on (5), we note that to get the EE \mathbf{x}^* we should make sure that $\dot{x}_{a_k}^* = 0, \forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{K}$. Meanwhile, recalling that $x_{a_k}^* > 0 (\forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{K})$ and $\sum_{a_k \in \mathcal{A}_k} x_{a_k} = 1$, we have

$$x_{a_k}^* = \frac{\beta_{a_k, k} \bar{R}_{a_k, k}^0}{\beta_{a''_k, k} \bar{R}_{a''_k, k}^0} x_{a''_k}^*, \forall a_k, a''_k \in \mathcal{A}_k \quad (6)$$

and

$$\sum_{a'_k \in \mathcal{A}_k} \frac{\beta_{a'_k, k} \bar{R}_{a'_k, k}^0}{\beta_{a''_k, k} \bar{R}_{a''_k, k}^0} x_{a''_k}^* = 1 \quad (7)$$

Further, we can get the expression of the EE as

$$x_{a_k}^* = \frac{\beta_{a_k, k} \bar{R}_{a_k, k}^0}{\sum_{a'_k \in \mathcal{A}_k} \beta_{a'_k, k} \bar{R}_{a'_k, k}^0} = \frac{\frac{\lambda_{a_k, k} B_{a_k, k}}{C_{a_k, k}} \bar{R}_{a_k, k}^0}{\sum_{a'_k \in \mathcal{A}_k} \frac{\lambda_{a'_k, k} B_{a'_k, k}}{C_{a'_k, k}} \bar{R}_{a'_k, k}^0} \quad (8)$$

where $\forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{K}$.

Then, replacing the term $\frac{\lambda_{a_k, k} B_{a_k, k}}{C_{a_k, k}}$ by $\eta_{a_k, k}$ we conclude the proof. ■

Remark 1: From an economic perspective, elements in \mathbf{x}^* denote the expected long-term equilibrium of market shares of different services. For instance, the expected market share of each WSP n can be represented as $\sum_{k \in \mathcal{K}} a_k \in \mathcal{N}_k, a_k = n x_{a_k}^*$. Therefore, with such statistical results, WSPs can get some insights about the effect of bandwidth allocation on users' WSP selection and further, appropriately deploy the HCNs.

B. Spectrum Bandwidth Allocation Game

In order to obtain a higher market share and revenue, a WSP must properly allocate the available spectrum. Since the spectrum allocation profile may not be frequently adjusted, each WSP should make his decision based on the equilibrium state of the service selection of users, i.e., the EE.³ Hence, we define the utility function for the WSP as follows

$$\begin{aligned} U_n(\mathbf{B}_n, \mathbf{B}_{-n}) &= \sum_{k \in \mathcal{K}_n} U_{n, k} = \sum_{k \in \mathcal{K}_n} \lambda_k^u x_{n, k}^* C_{n, k} \\ &= \sum_{k \in \mathcal{K}_n} \frac{\varphi_{n, k} B_{n, k}}{\sum_{m \in \mathcal{N}_k} \eta_{m, k} B_{m, k}} \\ &= \sum_{k \in \mathcal{K}_n} \frac{\varphi_{n, k} B_{n, k}}{\sum_{m \in \mathcal{N}_k, m \neq n} \eta_{m, k} B_{m, k} + \eta_{n, k} B_{n, k}} \end{aligned} \quad (9)$$

where $\mathbf{B}_{-n} = (\mathbf{B}_1, \dots, \mathbf{B}_{n-1}, \mathbf{B}_{n+1}, \dots, \mathbf{B}_N)$ denotes bandwidth allocation profiles of WSPs other than WSP n . Where $\varphi_{n, k} = \lambda_k^u \eta_{n, k} C_{n, k}, \forall k \in \mathcal{K}_n, \forall n \in \mathcal{N}$. We also impose the constraint $B_{n, k} \geq b$ on each WSP's bandwidth allocation strategy, where $b > 0$ is a positive constant.⁴

As shown in (9), the utility of each WSP is determined by both its own bandwidth allocation profile and those of other WSPs. In contrast to the study in the previous section, we assume that each WSP is fully capable of acquiring all the information on HCNs deployed by its competitors (other operators) i.e., the expected average rate and pricing for each service. Conditioned on this, we can formulate the problem as

³This consideration is mainly because in practice, each operator usually makes a long-term decision instead of changing his decision frequently [15].

⁴This b can be interpreted as a minimum bandwidth allocation required in order to deploy a certain service. $b = 0$ leads to a potentially undefined utility function when $B_{n, k} = 0, \forall n$ for any k . So from a mathematical point of view this constraint keeps the function continuous and easier to handle.

$$\dot{x}_{a_k} = \delta x_{a_k} \left(\ln \left(1 + \frac{\beta_{a_k,k} \bar{R}_{a_k,k,u}}{x_{a_k}} \right) - \sum_{a'_k \in \mathcal{A}_k} x_{a'_k} \ln \left(1 + \frac{\beta_{a'_k,k} \bar{R}_{a'_k,k,u}}{x_{a'_k}} \right) \right), \forall a_k \in \mathcal{A}_k, \forall k \in \mathcal{N} \quad (5)$$

a non-cooperative spectrum Bandwidth Allocation Game (N-BAG) and hereafter, the terms WSP and player would be used interchangeably in this section.

Definition 1: N-BAG: This game can be represented by the tuple

$$\mathcal{G} = \Gamma (\mathcal{N}, (\mathbb{B}_n)_{n \in \mathcal{N}}, (U_n)_{n \in \mathcal{N}}). \quad (10)$$

Here, \mathcal{N} denotes the set of players which is identical to the WSP set. For each player n , its strategy space \mathbb{B}_n is defined as the set of available spectrum bandwidth allocation profiles

$$\mathbb{B}_n = \left\{ \mathbf{B}_n \mid \mathbf{B}_n = (B_{n,k})_{k \in \mathcal{K}_n}, B_{n,k} > b, \sum_{k \in \mathcal{K}_n} B_{n,k} = B_n \right\} \quad (11)$$

Based on (9), when given a strategy profile

$$(\mathbf{B}_n)_{n \in \mathcal{N}} = (\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_N) \in (\mathbb{B}_n)_{n \in \mathcal{N}}, \quad (12)$$

the utility function of each player $n, n \in \mathcal{N}$, can be expressed as

$$U_n(\mathbf{B}_n, \mathbf{B}_{-n}) = \sum_{k \in \mathcal{K}_n} \frac{\varphi_{n,k} B_{n,k}}{\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k}} \quad (13)$$

where the term $\chi_{n,k}(\mathbf{B}_{-n}) = \sum_{m \in \mathcal{N}_k, m \neq n} \eta_{m,k} B_{m,k}$ depicts the competition caused to player n from other players.

For a non-cooperative game, NE is the standard solution standing for the equilibrium state, under which no player can unilaterally improve its own utility by choosing a different strategy [16]. Accordingly, we have the following theorem about the existence of NE for N-BAG.

Theorem 2: For our formulated non-cooperative game N-BAG, there always admits at least one NE under which no WSP would like to unilaterally change his own bandwidth allocation strategy. Furthermore, when given other WSPs' strategy profile \mathbf{B}_{-n} , the best response of WSP n , $\mathbf{B}_n^* = (B_{n,k}^*)_{k \in \mathcal{K}_n}$, is unique and can be expressed as

$$B_{n,k}^* = \max \left\{ b, \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n^* \eta_{n,k}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}}} \right\}. \quad (14)$$

where

$$\sum_{k \in \mathcal{K}_n} \max \left\{ b, \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n^* \eta_{n,k}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}}} \right\} = B_n. \quad (15)$$

Proof: First, we will prove the existence of NE for the formulated game N-BAG. After some algebraic computations,

the utility of WSP n is transformed into:

$$U_n(\mathbf{B}_n, \mathbf{B}_{-n}) = \tau_n - \sum_{k \in \mathcal{K}_n} \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k}} \quad (16)$$

where $\tau_n = \sum_{k \in \mathcal{K}_n} \frac{\varphi_{n,k}}{\eta_{n,k}} = \sum_{k \in \mathcal{K}_n} \lambda_k^u C_{n,k}$. It is straightforward to verify that the n -th player's utility U_n is concave over his strategy set \mathbb{B}_n for each fixed \mathbf{B}_{-n} and meanwhile, the strategy space $(\mathbb{B}_n)_{n \in \mathcal{N}}$ is a closed and bounded convex set [17]. Therefore, we can know that \mathcal{G} is a concave game which always admits at least one NE [18]. We note that this proposition can be proved through three steps. First, when given a strategy profile, the best response of each player should be compact and convex. This is true due to the continuity and concavity of the utility function. Then, it can be proved that the best response function $\Omega : (\mathbb{B}_n)_{n \in \mathcal{N}} \rightarrow (\mathbb{B}_n)_{n \in \mathcal{N}}$ is upper semi-continuous. Finally, by applying Kakutani fixed point theorem, it can be proved that there exists a fixed point $\Omega((\mathbf{B}_n^*)_{n \in \mathcal{N}}) = (\mathbf{B}_n^*)_{n \in \mathcal{N}}$. To be more specific, this fixed point $(\mathbf{B}_n^*)_{n \in \mathcal{N}}$ is an NE for the considered concave game.

Next, we will derive the expression of the best response of WSP n when given the strategy profile of other WSPs \mathbf{B}_{-n} . In this case, for each player n , selecting the strategy maximizing his own utility is equivalent to solving the following optimal spectrum allocation problem

$$\mathbf{P}: \min_{\mathbf{B}_n} \sum_{k \in \mathcal{K}_n} \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k}} \quad (17)$$

$$\text{s.t. } B_{n,k} \geq b, \forall k \in \mathcal{K}_n, \quad (18)$$

$$\sum_{k \in \mathcal{K}_n} B_{n,k} = B_n, \quad (19)$$

which is a convex problem. For problem \mathbf{P} , the Lagrangian can be written as

$$L_n = \sum_{k \in \mathcal{K}_n} \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k}} - \sum_{k \in \mathcal{K}_n} \theta_{n,k} (B_{n,k} - b) + v_n \left(\sum_{k \in \mathcal{K}_n} B_{n,k} - B_n \right) \quad (20)$$

where $\theta_{n,k} \geq 0$ and v_n are the Lagrange multipliers [17]. Moreover, the KKT conditions can be expressed as

$$\begin{aligned} \frac{\partial L_n}{\partial B_{n,k}} &= -\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n}) \eta_{n,k}}{(\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k})^2} - \theta_{n,k} + v_n = 0, \\ B_{n,k} &\geq b, \quad \sum_{k \in \mathcal{K}_n} B_{n,k} = B_n, \\ \theta_{n,k} &\geq 0, \quad \theta_{n,k} (B_{n,k} - b) = 0, \quad \forall k \in \mathcal{N}_k. \end{aligned} \quad (21)$$

Since problem \mathbf{P} is a convex problem, the optimal variable $\mathbf{B}_n^* = (B_{n,k}^*)_{k \in \mathcal{K}_n}$ can be achieved by solving the KKT

conditions (21), which could be further transformed as

$$\begin{aligned} v_n &\geq \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n}) \eta_{n,k}}{(\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k})^2}, \\ (B_{n,k} - b) \left(v_n - \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n}) \eta_{n,k}}{(\chi_{n,k}(\mathbf{B}_{-n}) + \eta_{n,k} B_{n,k})^2} \right) &= 0, \\ B_{n,k} - b &\geq 0, \quad \sum_{k \in \mathcal{K}_n} B_{n,k} = B_n, \quad \forall k \in \mathcal{N}_k. \end{aligned} \quad (22)$$

To solve the above equations, two disjoint cases need to be considered. Firstly, if $v_n < \frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n}) \eta_{n,k}}{(\chi_{n,k}(\mathbf{B}_{-n}))^2} = \frac{\lambda_k^u C_{n,k} \eta_{n,k}}{\chi_{n,k}(\mathbf{B}_{-n})}$, the first three conditions in (22) only hold when

$$B_{n,k} = \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n \eta_{n,k}}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}} \geq b. \quad (23)$$

or

$$B_{n,k} = b, \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n \eta_{n,k}}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}} < b. \quad (24)$$

On the other hand, if $v_n \geq \frac{\lambda_k^u C_{n,k} \eta_{n,k}}{\chi_{n,k}(\mathbf{B}_{-n})}$, then $B_{n,k} > b > 0$ and the second condition in (22) can not be satisfied simultaneously, which further means $B_{n,k}$ can only equal to b . To this end, the optimal solution $\mathbf{B}_n^* = (B_{n,k}^*)_{k \in \mathcal{K}_n}$ can be expressed as

$$B_{n,k}^* = \max \left\{ b, \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n^* \eta_{n,k}}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}} \right\}.$$

Meanwhile, recalling the constraint that $\sum_{k \in \mathcal{K}_n} B_{n,k} = B_n$, we can get the optimal Lagrange multiplier v_n^* by solving

$$\sum_{k \in \mathcal{K}_n} \max \left\{ b, \sqrt{\frac{\lambda_k^u C_{n,k} \chi_{n,k}(\mathbf{B}_{-n})}{v_n^* \eta_{n,k}}} - \frac{\chi_{n,k}(\mathbf{B}_{-n})}{\eta_{n,k}} \right\} = B_n.$$

Finally, we will prove the uniqueness of \mathbf{B}_n^* when given \mathbf{B}_{-n} . It should be noted that the left side of the above equation is a piece-wise-linear increasing function of $\frac{1}{v_n^*}$, hence the solution of this equation is unique when given \mathbf{B}_{-n} , which further means that the solution shown in (14) is also unique in the same case, i.e., given \mathbf{B}_{-n} . ■

Based on the above, an iterative algorithm has been developed to obtain the NE of N-BAG, which is shown in Algorithm 1. Particularly, at the beginning of this algorithm, the starting point will be initialized based on the available bandwidth of each WSP, i.e., $\sum_{k \in \mathcal{K}_n} B_{n,k} = B_n, \forall n \in \mathcal{N}$. After that, the algorithm goes into a loop. At each iteration t , each WSP n calculates the optimal Lagrange multiplier v_n^* with (15) and then, updates the best response $\mathbf{B}_n(t)$ by applying (14). Then, the loop will stop when the relative difference of the two optimal solutions obtained after the consecutive iterations (i.e., $(\mathbf{B}_n(t-1))_{n \in \mathcal{N}}$ and $(\mathbf{B}_n(t))_{n \in \mathcal{N}}$) is small enough. Mathematically speaking, the stopping criterion of Algorithm 1 can be expressed as

$$\frac{\|(\mathbf{B}_n(t))_{n \in \mathcal{N}} - (\mathbf{B}_n(t-1))_{n \in \mathcal{N}}\|}{\|(\mathbf{B}_n(t-1))_{n \in \mathcal{N}}\|} < \varepsilon \quad (25)$$

Algorithm 1 : Computation of NE $(\mathbf{B}_n^*)_{n \in \mathcal{N}}$.

- 1: **Initialization:** For $\forall n \in \mathcal{N}$, initialize the starting point $B_{n,k}(0)$ satisfying $\sum_{k \in \mathcal{K}_n} B_{n,k} = B_n$ and set $t = 0$.
 - 2: **repeat**
 - 3: **Set** $t = t + 1$ and $\mathbf{B}_n(t) = \mathbf{B}_n(t - 1)$.
 - 4: **for** $n = 1$ to N users **do**
 - 5: **Calculate** the optimal Lagrange multiplier v_n^* with (15) and then, update the best response $\mathbf{B}_n(t)$ by applying (14).
 - 6: **end for**
 - 7: **until** the stop criterion shown in (25) is satisfied.
 - 8: **For** $B_{n,k}(t)$ being equal to b set its value (i.e., $B_{n,k}(t)$) and the corresponding utility $U_{n,k}(t)$ to 0, $\forall k \in \mathcal{K}_n, \forall n \in \mathcal{N}$.
 - 9: **Set** $\mathbf{B}_n^* = \mathbf{B}_n(t)$ and $U_n^* = \sum_{n \in \mathcal{N}} U_{n,k}(t), \forall n \in \mathcal{N}$.
-

where $\|\cdot\|$ denotes a proper vector norm and ε is a small positive constant.

Remark 2: It should be noted that when the initial point changes, the finally achieved solution may be different unless the NE of N-BAG is unique. However based on our extensive simulation results, we conjecture that the NE is unique and hence, the algorithm will converge to the same point irrespective of the starting allocation chosen.

Remark 3: From a mathematical perspective, note that this game and the corresponding IWF algorithm proposed is indeed quite similar to the well known IWFA for power allocation [19]. However, a significant difference that makes the problem harder to prove uniqueness of the NE is that, the water level in each service k for this work are different unlike the uniform water level over channels seen in [19].

Note that the developed algorithm is an offline algorithm. In addition, since we assume that each WSP can acquire the related information of the co-players, this algorithm can be independently implemented by each WSP and will converge to an NE. Since we observe from simulations that the NE is unique, all WSPs will naturally operate at the unique NE by adopting this algorithm.

IV. SIMULATION RESULTS

We have tested the uniqueness of the NE for an exhaustive set of parameters using Monte-Carlo simulations. In this section, we would like to study the efficiency of the NE. For this purpose, we compare it with:

- **Global Optimum :** In a hypothetical case where all WSPs are given instructions from a centralized agent, the WSPs choose the bandwidth allocation maximizing their sum utilities.
- **Proportion to density of BSs (PDBS):** With this approach each WSP would allocate the available bandwidth to each tier based on the density of BSs, i.e.,

$$B_{n,k} : B_{n,l} = \lambda_{n,k} : \lambda_{n,l}, \forall k, l \in \mathcal{K}_n, \forall n \in \mathcal{N}. \quad (26)$$

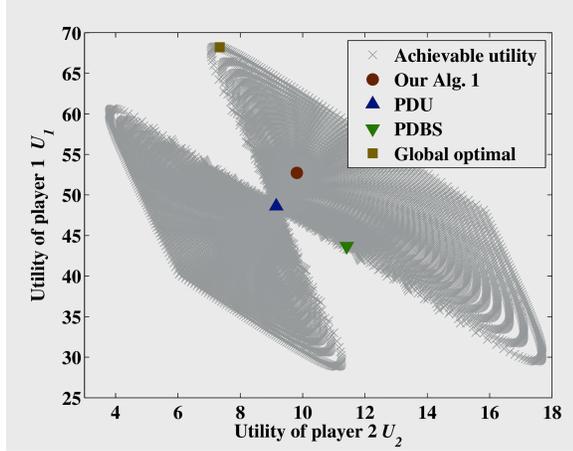


Fig. 2. The utility obtained by WSP 1 and WSP 2 when different schemes are adopted. The point indicated by "Our Alg. 1" converges is the NE.

- Proportion to density of users (PDU): With this approach each WSP would allocate the available bandwidth to each tier based on the density of potential users in each population, i.e.,

$$B_{n,k} : B_{n,l} = \lambda_k^u : \lambda_l^u, \forall k, l \in \mathcal{K}_n, \forall n \in \mathcal{N}. \quad (27)$$

We note that this scheme is similar to the one proposed in [13], which focuses on the single WSP scenario.

Here, we consider there are two WSPs, each of which deploys a two-tier HCN⁵ to provide services to users with the population density as $(\lambda_1^u, \lambda_2^u) = (20, 60)$ per km². In addition, the available bandwidth of the three WSPs are set as $(B_1, B_2) = (10, 5)$ MHz, which means that the bandwidth obtained by each WSP is not the same and may sometimes happen in some practical cases. Based on the different deployment of HCNs, we consider that the price charged by WSPs are also different: $(C_{1,1}, C_{1,2}) = (2, 0.8)$ and $(C_{2,1}, C_{2,2}) = (0.8, 0.1)$. Also, we take $R_{n,k} \approx 1.49$ nats/sec/Hz, $\forall n \in \{1, 2\}, \forall k \in \{1, 2\}$. The performance of these schemes in terms of the utility achieved by WSPs are compared in Fig. 2.

V. CONCLUSIONS

The bandwidth allocation of WSPs in a multi-tier HCN has been formulated as a non-cooperative game and the existence of Nash equilibrium (NE) has been proven. An algorithm used to achieve the NE has also been developed. Simulation results have shown the NE is unique. The resulting NE has also been compared to the global optimum and other strategies. These results can be used by competing WSPs to achieve the NE without having to run an online algorithm which could potentially take a long time to stabilize.

There are many interesting extensions for this work. One of the possible extensions is to investigate infrastructure sharing among WSPs, i.e., the WSPs co-operate to maximize their revenue while minimizing the cost of infrastructure.

⁵Note that considering a three or higher tier HCN with more WSPs is easily possible, however for the numerical part we consider the case of two for better presentation of the results.

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