

How complex is origami design?

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The latest decade saw the emergence of a new approach to the ancient paperfolding art: the computational origami. Indeed, the mechanical behavior of the paper sheet is usually idealized: a zero bending stiffness, but an infinite in-plane stiffness. As a consequence, the problem reduces to a discrete geometry one, with a finite number of folds (lines of strain singularity), a finite mechanism with large rotations during the folding process, and a diffuse unilaterality to prevent the paper self-interpenetration

Several complex planar geometry problems have been studied from their complexity and design efficiency point of view, such as the color changing: starting from a single square of paper with one color on each of its sides, what are the feasible designs to make special patterns appear, without cutting nor gluing? The chessboard (8-per-8 checkered board) is one of the most challenging problems, especially when constrained by searching the most efficient design, i.e. with the less waste of paper. The complexity of a particular solution for a n -per- n checkered board has been estimated with the smallest possible semiperimeter s of the initial square of paper. Bounds are available: a first result in 2000 [3] gave $s = n^2$, and was suspected to be sharp. Nevertheless, a breakthrough in 2009 [2] provided a new bound: $s = \frac{1}{2}n^2 + 8n + 8 - 5(n \bmod 4)$, smaller for $n \geq 16$. This polynomial complexity measures the difficulty to fold the pattern, so to check a solution. It is not the complexity of the design problem, i.e. the algorithmic problem of finding the solutions. Indeed, as soon as 1996, a simpler origami problem was proved to be NP-complete [1].

We propose herein the illustration of complexity on a pixel-matrix origami design. This is a complication of the chessboard: each board square should be able to change its color independently, using as many flapping mechanisms as necessary, but still using a single square sheet of paper with a minimal waste. We also discuss the ability of computer resources to help solving the design problem, together with a more or less deeper analysis by the human paperfolder (for instance using graph theory, with a 2-scale approach for the paper corrugation along the edge) to render the problem tractable, at least up to $n = 8$ [4].

References

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