A simple and efficient super-short-scan approach of fan-beam reconstruction for multiple circular trajectories: Solution towards the truncated data

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Abstract

The sufficiency condition of the accurate reconstruction of the region of interest (**ROI**) in 2D fanbeam tomography was introduced by F. Noo([1]), and extended by Clackdoyle in presence of the truncated data using virtual fan-beam method (VFB) [2]. In this paper, we propose a simple and efficient super-short scan approach of fan-beam reconstruction on multiple circular trajectories for the whole object reconstruction, in the case of the field of view (FOV) of scanner much smaller than the object.

I. Problem and Solution

Problem:

FOV $<$ whole object.

Objectif:

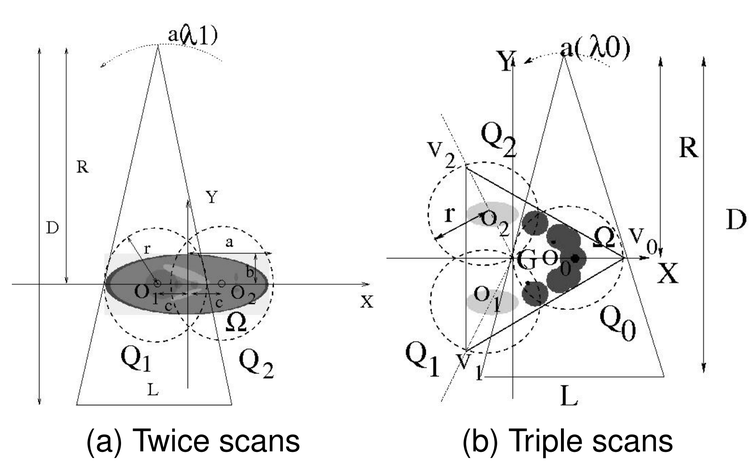
- The whole object reconstruction.
- Dose reduction.

Solution:

- Multiple circular scans (two or three)
- Super-short (SS) scan

II. Twice super-short scans

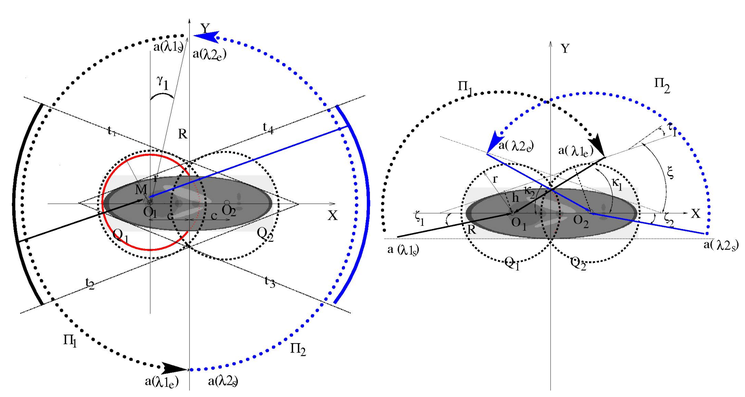
1. System geometry of twice scans (Fig.1.(a)).



(a) Twice scans (b) Triple scans

Fig. 1: Twice scans (Q_1 and Q_2) with ellipse support of the object (a). and triple scans (Q_0, Q_1 and Q_2) for the triangular object support (b).

2. Solutions of the whole object reconstruction
   - 2.1 Reduced scan (VFB [2]) (Fig 2.(a)).
   - 2.2 Proposed SS-scans (Fig 2.(b)).



(a) VFB (b) SS-scan

Fig. 2: The reduced scan by VFB ([2]) (a) and proposed SS-scans (b).

III. Triple super-short scans

1. System geometry of triple scans (Fig.1.(b))
2. Solutions of the whole object reconstruction
   - 2.1 Short-scans [3].
   - 2.2 Super-short scans (Fig 3).

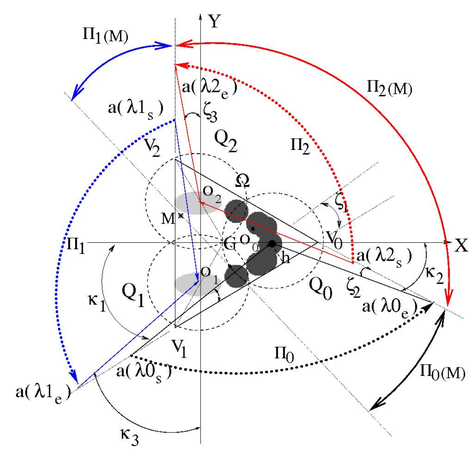


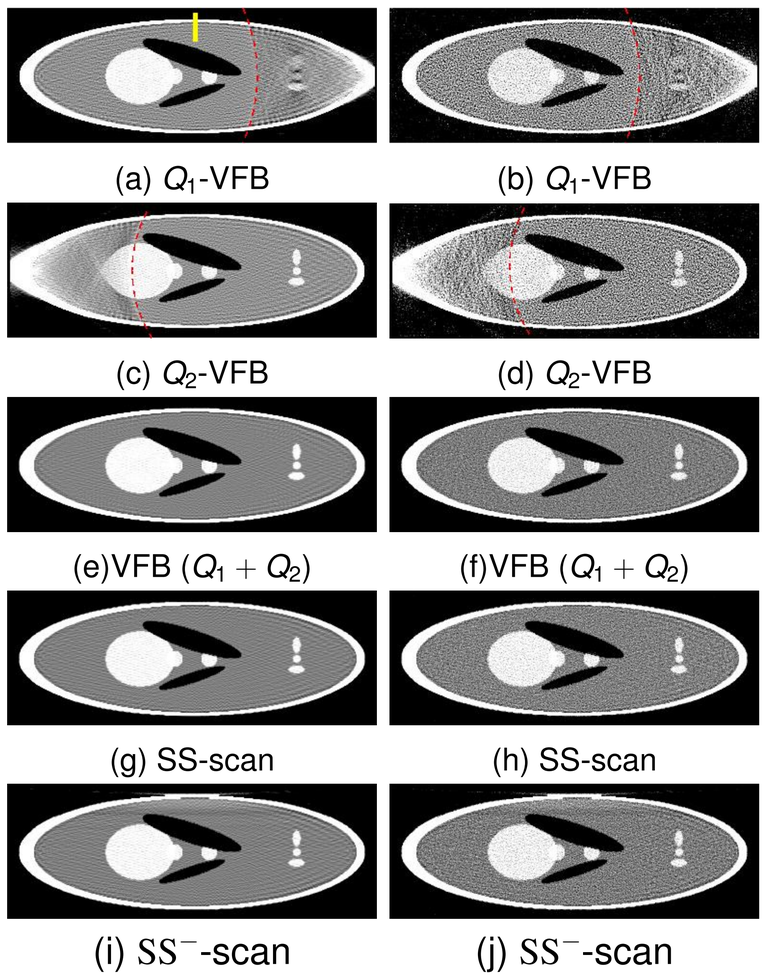
Fig. 3: The triple super-short-scans. M is an arbitrary pixel in the object. The scanning ranges $\Pi_i(M)$, $i = 0, 1, 2$ are sufficient for the exact reconstruction of M.

IV. Image reconstruction

1. Forward model: $\mathbf{g} = \mathbf{H}\mathbf{f} + \epsilon$
   g, mesurement vector,**H**, system matrix,and **f**, unknown object vector.
2. Iterative least-squares method

$$\hat{\mathbf{f}} = \arg\min_{\mathbf{f}} \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2/2$$

V. Simulation: Twice scans(1/2)



(a) Q_1-VFB (b) Q_1-VFB

(c) Q_2-VFB (d) Q_2-VFB

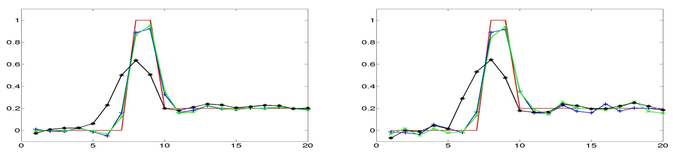
(e)VFB ($Q_1 + Q_2$) (f)VFB ($Q_1 + Q_2$)

(g) SS-scan (h) SS-scan

(i) SS^--scan (j) SS^--scan

Fig. 4: Reconstructed images of the modified Shepp-Logan (MSL) from the nois-free fan-beam data (left column)and the noisy fan-beam data (right column). *SS^- scan* is defined by reducing a bit of projections from SS scan. The display window is [0.1 0.3].

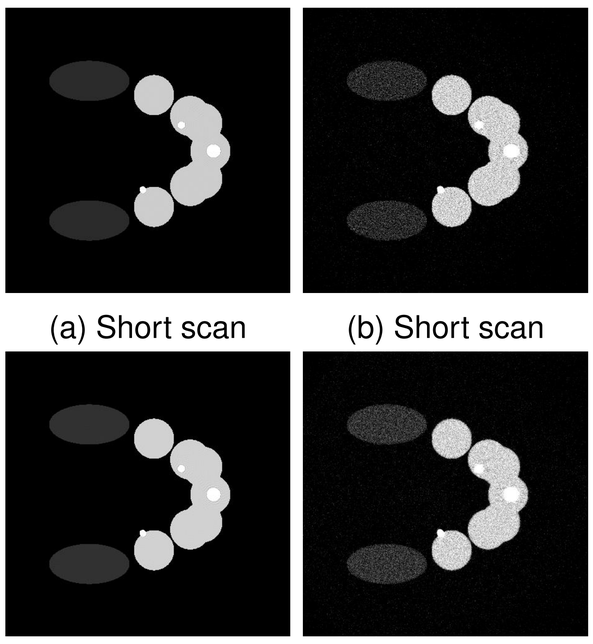
VI. Simulation: Twice scans(2/2)



(a) Noise-free data (b) Noisy data

Fig. 5: Profiles of the reconstructed images of the whole MSL from SS^--scan (a) noise-free data and (b) noisy data. The profiles are drawn along the yellow solid line in 4.(a).

VII. Simulation: triple scans



(a) Short scan (b) Short scan

(c) SS-scan (d) SS-scan

(e) SS^--scan SS^--scan

Fig. 6: Reconstructed images from the noise-free fan-beam data (left column) of the simulated jaw and the noisy data (right column) of the different acquisitions: short scan in first row (a) and (b), SS-scan in second row (c) and (d),SS^--scan in last row (e) and (f). The display window is [0.01 0.07].

VIII. Conclusion

1. A simple and efficient solution of the whole object reconstruction from too small FOV.
2. Reduction of the necessary scanning range with our porposed super-short scans on twice and triple circular trajectories compared to the other feasible scans.

IX. Prospects

1. Extension of our approaches in cone-beam CT tomography.
2. Applications of our proposed super-short scans in dental imaging for a panoramic view with small detector.

Reference

[1] F. Noo, and M. Defrise, and R. Clackdoyle, and H. Kudo, “Image reconstruction from fan-beam projections on less than a short scan“, *Phys. Med. Biol.*,47(2), pp2525–2546, 2002.

[2] R. Clackdoyle, and F. Noo, and J. Guo, and J.A Roberts, ”Quantitative reconstruction from truncated projections in classical tomography”, *IEEE Trans. Nucl. Sci.*,51(5), pp2570–2578, 2004.

[3] D. L. Parker, ”Optimal short scan convolution reconstruction for fanbeam CT”, *Med. Phys.*, 9(2), pp 254–257, 1982.