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# A simple and efficient super-short-scan algorithm of Fan-beam reconstruction for multiple circular trajectories: solution towards the truncated data

Long Chen, Thomas Rodet, and Nicolas Gac

**Abstract**—The sufficiency condition of the accurate reconstruction of the region of interest (ROI) in fan-beam tomography with non-truncated data was introduced by F. Noo([1]). When the detector does not cover the whole objet, R. Clackdoyle extended this condition of the accurate reconstruction of ROI in presence of the truncated data using virtual fan-beam method (VFB) [2]. In this paper, we are interested in the image reconstruction of the whole object from truncated data, since the whole object reconstruction is always preferred even with small detector, instead of the accurate reconstruction of only a region. We propose a simple and efficient super-short-scan algorithm for multiple circular trajectories to reconstruct the whole object from truncated fan-beam projections. This algorithm is validated by simulation studies.

## I. INTRODUCTION

In fan-beam tomography, the short-scan ( $180^\circ$  plus the fan angle) is a minimal complet data set to reconstruct the whole object [3]. This had been also hold even for the reconstruction of only a ROI [4]. A new sufficient condition of the accurate reconstruction of ROI was introduced in [1]. The ROI can be exactly reconstructed only and if only all the lines through the ROI are known. All the fan-beam projections should be non-truncated as the short scan.

When the detector is too small to cover all the object, the projection data is inevitably truncated. The accurate reconstruction of ROI is still able to be achieved in some cases with VFB in [2]. An optimal virtual fan-beam circular trajectory is found by rebinning the truncated projections to non-truncated projections. Additionally, the whole object reconstruction from the truncated data has been interesting, as in dental imaging, the panoramic view of the whole jaw from a medium detector. In this paper, we propose a simple and efficient super-short-scan (SS-scan) algorithm on twice and triple circular trajectories to achiev the reconstruction of the whole object. 'Simple' means the movement of the scanner is not complexe, and 'efficient' stands for as less fan-beam projections as

possible to reconstruct the whole object. The super-short-scan projections here are not necessarily non-truncated, which are different from those in [5].

The rest of this paper is organized as follows: the proposed super-short-scans on twice and triple circular trajectories are described in section 2, we use a standard iterative least-squares method for image reconstruction in section 3. the proposed super-short-scan algorithm is evaluated by the simulation studis in section 4, and we draw the conclusion in the final section.

## II. SUPER-SHORT-SCANS ON MULTIPLE CIRCLE TRAJECTORIES

In CT tomography, how to reconstruct a whole object from the truncated data is a challenge, when the detector is too small to cover all the object. Obviously, a single circular scan is able to reconstruct only a region, not the whole object. More scans are needed to achieve the whole object reconstruction. We present the proposed super-short-scans on two and three circular trajectories below.

### A. Twice super-short-scans

Usually, the object support is known *a priori*. Without loss of generality, let the object support  $\Omega$  be an ellipse depicted by tow semi-axis parametres ( $a$  and  $b$ ,  $a > b$ ), which is centered at origine in figure 1.(a). Two iso-centers  $O_1$  and  $O_2$  of scans are located symetrically to the axis Y, with distance  $c$  to the origine. The system geometry is decribed in the figure 1.(a) with distance from the X-ray source to the iso-center R, and from the source to the detector D. The length of detector is L. We have  $r = L * R / D$ , where  $r$  is the radius of the field of view (FOV). Obviously,  $r$  should be greater than  $c$  for the whole object reconstruction.

Intuitively, the first choice of the whole object reconstruction is to reconstruct a half ellipse from each scan. In presence of the truncated data, an optimum virtual trajectory of the X-ray is found in red solid circle using VFB ([2]), in the figure 2.(a). According to the sufficient condition in [1], two reasonable trajectories are drawn in the figure 2.(a), the scanning angle of each scan  $Q_1$  or  $Q_2$  is  $\pi + 2 * \gamma_1$ , where  $\gamma_1 = \arcsin(c/R)$ . As  $c < r$ , the angle is smaller than half of the fan angle  $\gamma_m$  with  $\gamma_m = \arcsin(r/R)$ . These tracjectories are refered to the reduced scan (R-scan). When we look at the two reduced

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scans, a random pixel M in the left half of object is measured twice in one line in the figure 2.(a). The fan-beam projections of the whole object are double from the scanning range between the lines  $t_1$  and  $t_2$  and between  $t_3$  and  $t_4$ , which are the shared tangents of the object support  $\Omega$  and two fields of view (FOV) of the twice scans.

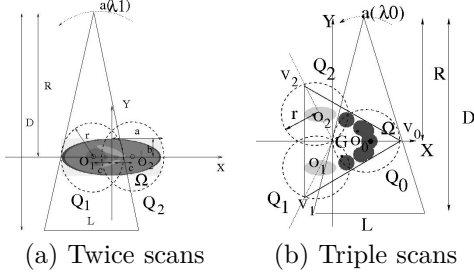


Fig. 1: System geometries of twice scans ( $Q_1$  and  $Q_2$ ) with ellipse support of the object (a). and triple scans ( $Q_0$ ,  $Q_1$  and  $Q_2$ ) for the triangular object support (b).  $O_i$  denotes the iso-center,  $i = 0, 1, 2$ , In (b),  $V_0$ ,  $V_1$ , and  $V_2$  are the three vertexes, their three iso-centers,  $O_0$ ,  $O_1$  and  $O_2$ , are located on the middle of the segments between the triangle gravity G and each vertex, respectively. The phantoms of Shepp-Logan (a) and simulated jaw (b) inside  $\Omega$  are given here only as example.

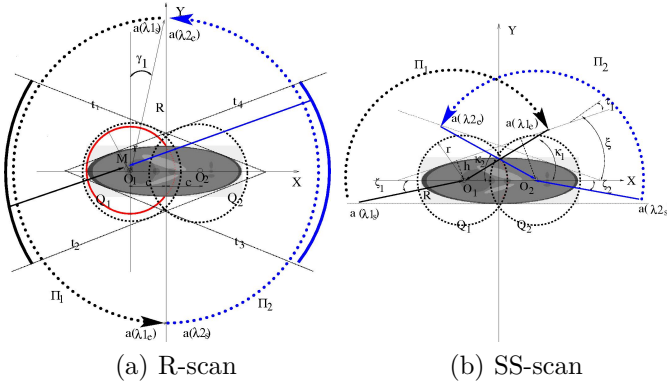


Fig. 2: Illustration of the reduced scan (a) and super-short-scan (b).

In order to reduce the data redundancy above, we propose two shorter scans referred to super-short-scans (SS-scan) in the figure 2.(b). The scanning ranges of the two super-short-scans  $\Pi_1$  et  $\Pi_2$  are  $[\lambda_{1s} \lambda_{1e}]$  and  $[\lambda_{2s} \lambda_{2e}]$  respectively. The start positions of the X-ray source  $a(\lambda_{1s})$  and  $a(\lambda_{2s})$  are on the tangent to the object support. In the figure 2.(b), the angle is defined positive in counterclockwise direction.  $\zeta_1 = \zeta_2 = \arcsin(b/R)$ ,  $\tau_1 = \arcsin(h/R) = \arcsin(\frac{r-2*c*\sin\xi}{R})$ , and  $\kappa_1 = \kappa_2$ ,

$$\lambda_{1s} = \pi + \zeta_1, \lambda_{1e} = \kappa_1 = \xi + \tau_1 \quad (1)$$

$$\lambda_{2s} = -\zeta_2, \lambda_{2e} = \pi - \kappa_2 = \pi - (\xi + \tau_1) \quad (2)$$

where the tangent angle  $\xi$  is given by

$$\tan\xi = 1/\sqrt{\left(\frac{r*c + \sqrt{(a^2 - b^2)*(r^2 - b^2) + b^2*c^2}}{r^2 - b^2}\right)^2 - 1},$$

which is derived from the solution of the tangent to the FOV of  $Q_1$  or  $Q_2$  and the object support function. Compared to the reduced scan, the SS scan requires less fan-beam projections. Quantitatively, the reduced projection angle is given below :

$$\Delta_{\Pi} = \pi + 2*\gamma_1 - |\lambda_{1e} - \lambda_{1s}| = 2*\gamma_1 + \xi + \tau_1 - \zeta_1 \quad (3)$$

A numerical result will be given in the simulation studies,  $\Delta_{\Pi} = 30^\circ$ .

### B. Triple super-short-scans

When the object is much larger than the field of view of the scanner, triple scans are required for the whole object reconstruction from the truncated data. For simplification, let the object support  $\Omega$  be an equilateral triangle, as in the figure 1.(b). The other parameters are defined as above.

As the object takes a great part of the FOV of each scan, no less than short-scan is able to reconstruct the part of the object inside the FOV, separately, according to Noo's sufficient condition. A good choice is short-scan. The scanning angle of short-scan is  $\pi$  plus the fan angle  $2*\gamma_m$ . The data redundancy here is larger than in case of twice scans. We propose a triple super-short-scans to decrease

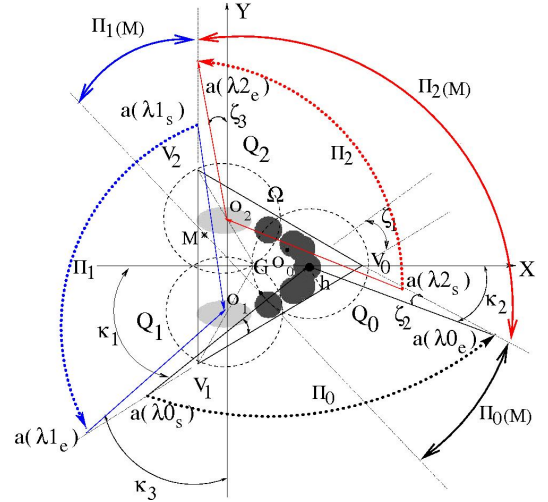


Fig. 3: Illustration of the triple super-short-scans. M is an arbitrary pixel in the object. The scanning ranges  $\Pi_0(M)$ ,  $\Pi_1(M)$  and  $\Pi_2(M)$  are sufficient for the accurate reconstruction of M.

the data redundancy in the figure 3. The scanning ranges of the triple super-short-scans are  $\Pi_0$ ,  $\Pi_1$ , and  $\Pi_2$ , where  $\Pi_0 = [\lambda_{0s} \lambda_{0e}]$ ,  $\Pi_1 = [\lambda_{1s} \lambda_{1e}]$ , and  $\Pi_2 = [\lambda_{2s} \lambda_{2e}]$ . In the figure 3, we have the angles  $\angle O_0V_0a(\lambda_{0s}) = \angle V_2V_0O_0 = \pi/6$  in the equilateral triangle  $\Omega$ , the start and end angular

positions of  $\Pi_0$ ,  $\Pi_1$ , and  $\Pi_2$  are obtained as follows:

$$\lambda 0_s = \pi + \kappa_1, \lambda 0_e = 2 * \pi - \kappa_2 \quad (4)$$

$$\lambda 1_s = \frac{\pi}{2} + \angle O_2 O_1 a(\lambda 1_s), \lambda 1_e = \frac{3\pi}{2} - \kappa_3 \quad (5)$$

$$\lambda 2_s = -\frac{\pi}{2} + \angle O_1 O_2 a(\lambda 2_s), \lambda 2_e = \frac{\pi}{2} + \zeta_3 \quad (6)$$

where  $h = r * \sin(\angle O_0 V_0 a(\lambda 0_s)) = 1/2 * r$ ,  $\zeta_1 = \zeta_2 = \zeta_3 = \angle O_2 O_1 a(\lambda 1_s) = \arcsin(h/R) = \arcsin(\frac{r}{2*R})$ ,  $\kappa_1 = \angle O_0 V_0 a(\lambda 0_s) + \zeta_1$ ,  $\kappa_2 = \angle V_2 V_0 O_0 - \zeta_2$ ,  $\kappa_3 = \pi/3 - \arcsin(\frac{r}{2*R})$ , and  $\angle O_1 O_2 a(\lambda 2_s) = \pi/3 + \arcsin(\frac{r}{2*R})$ . All the three scanning angles  $|\lambda i_e - \lambda i_s|$  of  $\Pi_i$  are  $2/3\pi$ , with  $i=0,1,2$ . For the whole object reconstruction, our triple super-short scans only needs  $2\pi$  fan-beam projections, which are  $\pi + 6 * \gamma_m$  less than those of the triple short-scans, since each short-scan needs  $\pi + 2 * \gamma_m$  fan-beam projections.

We verify the sufficient condition where every lines through the object support should be known in the figure 3: Let M be an arbitrary pixel inside the object. All the lines passing M are intersected with the three colorful solid arcs (black, red, and blue). Therefore, M is able to be reconstructed exactly. This will be validated in the results of image reconstruction from the simulated data.

### III. IMAGE RECONSTRUCTION

In this paper, we consider the image reconstruction as an optimisation problem. A standard iterative least-squares (ILS) method is used for image reconstruction. The analytic methods suitable for the multiple scans are beyond this topic.

In fan-beam tomography, a forward model of data acquisition is given as follows:

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \epsilon \quad (7)$$

where  $\mathbf{g}$  is the measurement vector,  $\mathbf{H}$  represents the system matrix, whose element  $h_{ij}$  means the contribution of  $j^{th}$  pixel of the object on the  $i^{th}$  measurement unit, the vector  $\mathbf{f}$  describes the unknown object, and  $\epsilon$  contains the detector noise and modelling errors.  $\mathbf{f}$  is estimated by minimizing the criteria function  $J(\mathbf{f})$  defined below:

$$J(\mathbf{f}) = 1/2 \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 \quad (8)$$

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f}} J(\mathbf{f}) \quad (9)$$

where  $\|\cdot\|^2$  is  $\mathcal{L}_2$  norm. In the case of the twice scans or triple scans,  $\mathbf{H}$  is rewritten as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}, \text{ or } \mathbf{H} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix}$$

respectively, where  $\mathbf{H}_0$ ,  $\mathbf{H}_1$ , and  $\mathbf{H}_2$  are the system matrices of the scans  $Q_0$ ,  $Q_1$ , and  $Q_2$ , separately. In case of the twice, the gradient  $\nabla J(\mathbf{f})$  and the optimal step  $\hat{\alpha}$  of ILS are deducted as follows:

$$\nabla J(\mathbf{f}) = \mathbf{H}_1^T (\mathbf{H}_1 \mathbf{f} - \mathbf{g}_1) + \mathbf{H}_2^T (\mathbf{H}_2 \mathbf{f} - \mathbf{g}_2) \quad (10)$$

$$\hat{\alpha} = \frac{\|\nabla J(\mathbf{f})\|^2}{\nabla J(\mathbf{f})^T (\mathbf{H}_1^T \mathbf{H}_1 + \mathbf{H}_2^T \mathbf{H}_2) \nabla J(\mathbf{f})} \quad (11)$$

where  $\mathbf{H}_i^T$  are the backprojections,  $i = 1, 2$ ,  $\mathbf{g}_1$  and  $\mathbf{g}_2$  are the measurements of the scans  $Q_1$  and  $Q_2$ .

### IV. SIMULATION STUDIES

The proposed methods of image reconstruction of the whole object from the truncated data are evaluated by the noise-free and noisy simulated data. The geometry configuration of our CT scanner fits for dental imaging, R and D are 440 and 690 mms, there are 680 units of the detector with size of  $120 \mu\text{m}$ , as a result, the achievable image size by a single circular scan is  $52 \times 52 \text{ mm}^2$ . However, in our simulation, the support size of the standard modified Shepp-Logan (MSL) phantom (Figure 5) is  $72 \times 24 \text{ mm}^2$ , whose attenuation coefficients are defined in matlab, and a larger simulated jaw (Figure 6) is included in an equilateral triangle support with side length 90 mm. Twice or triple circular scans are required to reconstruct the whole object. Our jaw contains 3 materials to simulate the tissue, teeth and high attenuated implants with attenuation coefficient of 0.02, 0.06 and  $0.3 \text{ mm}^{-1}$ . The fan-beam data was acquired using our ray-driven projector. To verify a stable reconstruction with the proposed method, the noisy data was generated with addition of a gaussian noise in the noise-free fan-beam projections.

The images are reconstructed in  $384 \times 384$  with pixel size  $200 \mu\text{m}$ . The reconstructed images from the noise-free fan-beam data of MSL are presented in the left column of Figure 5. Each half of MSL is accurately reconstructed, even more the pixels inside the FOV (red dashed arc in 5.(a) and .(c)) from the reduced scan. The ILS method is able to reconstruct the MSL inside the FOV not only half of MSL from the reduced scan, as OSEM in [2]. Both twice reduced scans and twice super-short scans allow to reconstruct the whole object exactly, in the figure 5.(e) and .(g). Moreover, the scanning angle of the proposed super-short-scan decreases by 16.3%, from  $184^\circ$  of the reduced scan ( $\pi + 2 * \gamma_1$ ) to  $154^\circ$  (equations (1) and (2)). To assure the efficacy of our proposed super-short-scan, we reduce a bit of its scanning ranges, which is referred to the SS<sup>-</sup>-scan,  $[\lambda 1_s - \epsilon \lambda 1_e + \epsilon]$  (clockwise) and  $[\lambda 1_s + \epsilon \lambda 1_e - \epsilon]$ , with a small angle  $\epsilon$  (one or two angular steps), in the figure 2.(b). The reconstruction result is given in the figure 5.(i). The upper and centre edge of the MSL is contaminated, clearly in the profil curves of the figure 4.(a). The reconstructed images from the noisy fan-beam data are demonstrated in the right column of Figure 5. We achieve the stable reconstructions from all the noisy data. The image of SS<sup>-</sup>-scan (Figure 5.(j)) and the profile (Figure 4.(b)) show again that the upper and center edge of MSL is not able to be reconstructed accurately decreasing the scanning range of our SS-scans.

The reconstructed images from the noise-free data and the noisy data of the simulated jaw are given in the left column of the figure 6.(a)-(c) and in the right column 6.(d)-(f). The accurate reconstructions of the whole jaw are achieved both on the triple short-scans (Figure 6.(a) and .(c)) and the proposed SS-scans (Figure 6.(b) and

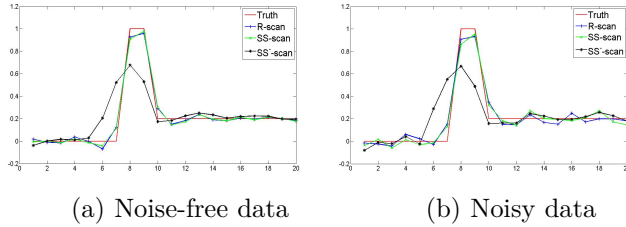


Fig. 4: Profiles of the reconstructed images of the whole MSL from  $SS^-$ -scan (a) noise-free data and (b) noisy data. The profiles are drawn along the yellow solid line in 5.(a).

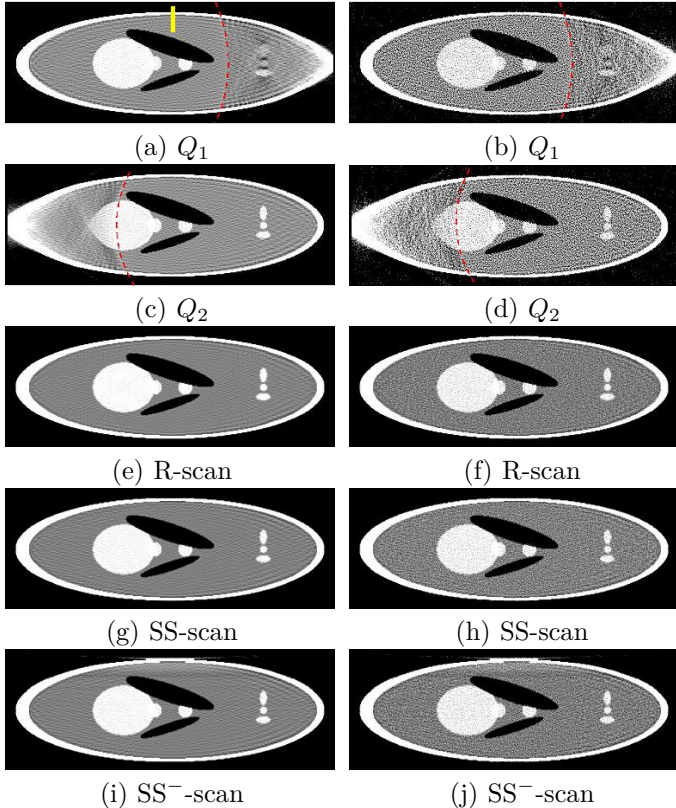


Fig. 5: Reconstructed images from the noise-free fan-beam data (left column) and the noisy fan-beam data of MSL (right column). The display window is  $[0.1 \ 0.3]$ .

(e). However the proposed SS-scan requires only the fan-beam projections of  $120^\circ$ , which is much less than  $187^\circ$  ( $\pi + \text{the fan-beam angle} \cdot 2 \cdot \gamma_m$ ) of the short-scan. The scanning range using SS-scan decreases by 35.8% compared to the short-scan. Moreover, the proposed SS-scan is the minimal scan for the whole jaw reconstruction. If we reduce a few of fan-beam projections ( $SS^-$ -scan) with the scanning ranges  $[\lambda_{i_s} + \varepsilon \ \lambda_{i_e} - \varepsilon]$ ,  $i = 0, 1, 2$  (Figure 3), some artifacts appear in the figure 6.(c) and .(f).

## V. CONCLUSION

We propose a simple and efficient super-short-scan algorithm on twice and triple circular trajectories for the whole object image reconstruction from the truncated data. The scanning angle decreases by 16.3% in the case

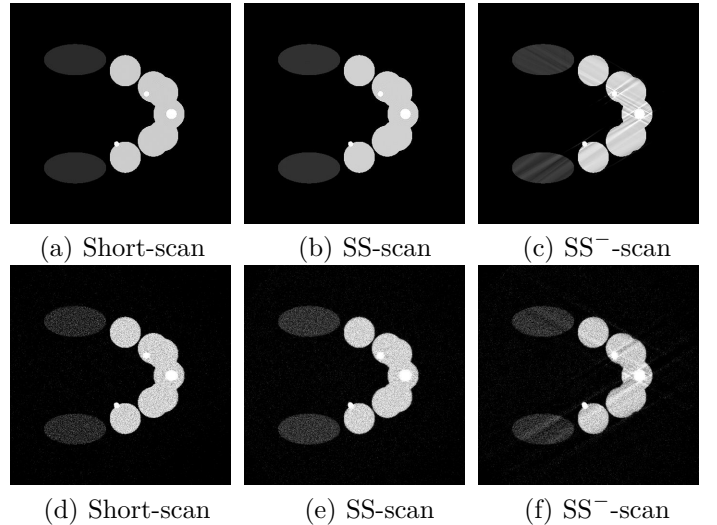


Fig. 6: Reconstructed images from the noise-free fan-beam data of the simulated jaw of the different acquisitions (a) short-scan, (b) SS-scan and (c)  $SS^-$ -scan. and from the noisy data (d) short-scan, (e) SS-scan and (f)  $SS^-$ -scan. The display window is  $[0.01 \ 0.07]$ .

of twice super-short scans compared to the reduced scans, and by 35.8% in the case of the proposed triple super-short scans compared to the short-scans. Our proposed super-short scans algorithms could be applied in dental imaging for a panoramic view of the whole jaw with a small detector, as well as for a fast scan and reduction of dose in other applications of CT tomography. Furthermore, these super-short scans will be easily extended in cone-beam CT tomography.

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