

The key papers to help explain the theory behind the attached code are:

- Chen *et al.*, *The computation of wavelet-Galerkin approximation on a bounded interval*. International Journal for Numerical Methods in Engineering, **39**, 2921-2944 (1996)
<https://hal.archives-ouvertes.fr/hal-01330583>
- Zhang *et al.*, *Comments on “The computation of wavelet-Galerkin approximation on a bounded interval”*. International Journal for Numerical Methods in Engineering, **72**, 244-251 (2007)
<https://hal.archives-ouvertes.fr/hal-01330597>
- Jones *et al.*, *The computation of wavelet-Galerkin three-term connection coefficients on a bounded domain*, Progress in Applied Mathematics, **7**(1), 1-8 (2014)
<https://hal.archives-ouvertes.fr/hal-01059821>

The Chen *et al.* paper is the foundation of all the code, however the paper is **full** of mistakes. The mistakes do not really become evident until you understand the theory. The Zhang *et al.* paper addresses some of the mistakes in the original paper, and our 2014 paper takes care of the 3-term errors.

In the Wavelet_Galerkin_Code folder are included:

- Two main programs: PG_uDB_vDB_expMDOF_condensed.m and Calculate_3termCC.m
- A number of functions required for the main programs
- One data set: timeStep_150Hz_full.mat

The program PG_uDB_vDB_expMDOF_condensed.m:

This program predicts the tip-displacement of an axially vibrating rod subjected to a tip force. This is the model used for the ASME_DETC2014-34725 conference paper. The parameters in the .m file may be slightly different than those used in the paper.

The code is set up to plot the tip response for a given forcing frequency. If the forcing frequency is 150Hz, the time-stepping solution will plot along with the wavelet-Galerkin solution, using the data set timeStep_150Hz_full.mat.

The level of the scaling function (as explained in the conference paper and the Chen paper) can be selected. Something between 4 and 8 is to be recommended to allow the fsolve algorithm to find a solution: it is currently set to $J = 5$.

The order of the Daubechies wavelet can also be chosen, which must be an even integer greater than 6. This constraint is explained further in the conference paper.

The program Calculate_3termCC.m

This code is used to compile the data for the ASME_DETC2014-34725 conference paper on the 3-term connection coefficients. The algorithm proposed in the Chen paper will work for a small set of parameters, but in general the numerical error in the algorithm results in the equality being infeasible. Our method uses a least-squares approach to get the “best fit” to the over-constrained set of equations.

All the functions

There are numerous functions which are called by the main programs to compute the necessary parameters. The parameter naming convention should follow that laid out in the Chen paper.

To get started on this topic by getting familiar with this code, below is the recommended approach!

- 1) Read the conference paper ASME_DETC2014-34725 to understand the overall goal of the PG_uDB_vDB_expMDOF_condensed.m code. This will hopefully explain how the Galerkin approach has been applied using Daubechies scaling functions for the temporal domain and finite elements for the spatial domain.
- 2) Read the Chen paper to understand the basic algorithms used when doing wG methods with Daubechies scaling functions. The reason for the Chen paper and all of this work is because the inner product of a

Daubechies scaling function with either itself or other functions cannot be accurately computed using standard integration schemes (*i.e.* analytic, Gauss-integration, etc.) because the Daubechies functions are fractal in nature. However, due to their recursive definition, it is possible to compute the exact inner product between the scaling functions and some other functions using the theory presented in these papers.

- 3) Starting from `func_calculate_2termCC.m`, trace the code through all of the associated functions to see how the 2-term connect-coefficients are computed. The 2-term connection coefficients are the inner-product terms for scaling functions. Chen's paper is for a bounded domain, so a bunch of the paper deals with computing the connection coefficients for the special scaling functions that must be constructed near the boundaries of the domain.
- 4) Using the `PG_uDB_vDB_expMDOF_condensed.m` code as a template, solve a vibrating mass on spring/damper problem (*i.e.* a single spatial degree of freedom) without any contact conditions. Once you can solve that problem, you are set! The rest is just using the wG approach in more useful ways.
- 5) If 3-term connection coefficients are required (useful for non-linear equations), then the `Calculate_3termCC.m` code should be relatively easy to follow now that the standard 2-term connection coefficients are understood.