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# A preliminary study on hybrid dynamical systems to control a N-paralleled boost converter

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**Abstract:** This paper addresses the control problem of driving N-paralleled boost converters to a functioning mode by using hybrid dynamical theory, that accounts for the continuous and discrete nature of these particular systems. The continuous dynamics correspond to the evolution of the electrical signals and, the discrete dynamics correspond to the switch signals that decide the converter configuration in every switching cycle. The hybrid control manages the switches, with the advantage of reducing switching frequency as well as guaranteeing any optimality level. The efficiency of the method is evaluated in simulation.

*Keywords:* Hybrid dynamical systems, affine, switching variables, converters, optimality.

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## 1. INTRODUCTION

The advances of recent years in the field of materials and components lead to converter topologies or structures with many possibilities while improving performance. These opportunities allow us to consider complex converter topologies, such as the N-paralleled boost converter [Cid-Pastor et al. 2011], which is generally used in telecommunication power supplies, microprocessors and renewable sources. This class of converters works in interleaving, it means that the converters operate at an operation point at the same constant switching frequency, with their switching waveforms shifted in phase with respect to another by  $2\pi/N$  over a switching period. Consequently, this class of switched systems presents a main control difficulty, which is to drive the converter to its functioning point. At a second problem addresses to guarantee the interleaved operation.

This N-paralleled boost converter has been addressed by using averaging control methods such as neural networks [Veerachary et al. 2003] or model predictive control [Babu and Veerachary 2005]. This means that the authors have used an averaged model for the switching dynamics. Some efforts have been done considering both continuous dynamics of the electrical signals and, discrete dynamics of the switches that decide the converter configuration in every switching cycle. For example, in the work presented in [Cid-Pastor et al. 2011], the authors control the paralleled boost converter by using sliding modes. Likewise, the interleaving is guaranteed applying a method already developed, as for example using a low-pass filter [Cid-Pastor et al. 2011] or employing a coupled inductor [Gu and Zhang 2013], among others.

In this work, we consider the continuous and discrete dynamics, proposing a hybrid controller that follows the hybrid framework presented in [Goebel et al. 2012]. With this formalism, we get a relevant advantage, which is to

manage the switching frequency during transient state and even during steady-state, while a prescribed optimality level is achieved. This controller is then tested in simulation and compared with the controller proposed in [Cid-Pastor et al. 2011].

The rest of this paper is organized as follows: in Section II, the problem statement is established. A hybrid controller is presented in Section III. Next, Section IV deal with to guarantee some optimality level. Some simulations are presented in Section V. The paper ends with conclusions and a discussion on the future works.

**Notation:** Through out the paper  $\mathbb{R}$  denotes the set of real numbers,  $\mathbb{R}^n$  the n-dimensional euclidean space and  $\mathbb{R}^{n \times m}$  the set of all real  $m \times n$  matrices. Likewise, we consider  $\bar{N} = \{v \in \mathbb{R}^n \text{ st } \forall i = 1, \dots, n, v_i \in \{0, 1\}\}$ , being  $\text{card}(\bar{N}) = 2^n$ . The set  $\mathbb{S}^s$  denotes the set of symmetric positive definite matrices of matrices  $\mathbb{R}^{n \times n}$ .

## 2. N-PARALLELED BOOST CONVERTER MODEL AND PROBLEM FORMULATION

A N-paralleled boost converter is composed of a constant voltage source  $V_{in}$ , a capacitance  $C$  and a purely resistive load  $R$ . Moreover, it is composed of  $N$  inductances  $L$  in series with a switch  $u_i \in \{0, 1\}$  and with a parasite resistance  $R_{LS}$  that models the energy dissipation in the inductance and in the switch. The  $N$  switches (see Fig. 1) makes that the converter presents  $2^N$  operating modes. Assuming that all  $L$  and  $R_{LS}$  are the same and that  $R_{LS} \ll R$ , the converter can be modelled as follows

$$\begin{aligned} \dot{x} &= A_u x + B, \\ z &= Fx, \end{aligned} \tag{1}$$

where  $x \in \mathbb{R}^{N+1}$  represents the state of the system and is given by

$$x^T = [i_{L,1} \ i_{L,2} \ \dots \ i_{L,N} \ v_c]^T \in \mathbb{R}^{N+1},$$

$z \in \mathbb{R}$  denotes the output measurement and,  $u \in \bar{N}$  is the control input. The matrices  $A, B$  and  $F$  are given by

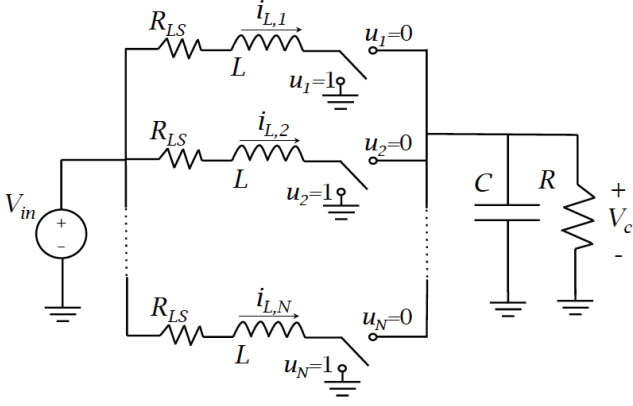


Fig. 1. Schematic representation of a N-paralleled boost converter.

$$A_u = \begin{bmatrix} -\frac{R_{LS}}{L} I_N & -\frac{u^T}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \in \mathbb{R}^{(N+1) \times (N+1)}, \quad (2)$$

$$B^T = \begin{bmatrix} \frac{V_{in}}{L} \mathbf{1} & 0 \end{bmatrix}^T \in \mathbb{R}^{N+1}$$

$$F = \begin{bmatrix} \mathbf{0} & 1 \end{bmatrix} \in \mathbb{R}^{N+1}.$$

Note that System (1) is a switched affine system, that can be controlled following the hybrid formalism given in [Albea et al. 2015] with the deal to guarantee suitable convergence of  $x$  to a desired equilibrium from arbitrary switching. A necessary and sufficient condition characterizing this desired equilibrium is represented by the following assumption (see [Deaecto et al. 2010, Liberzon and Morse 1999]).

*Assumption 1.* There exists an equilibrium  $x_e$  such that

$$x_e = \begin{bmatrix} i_e \cdot \mathbf{1}^N \\ v_e \end{bmatrix} \in \{(i_e, v_e) : \frac{V_{in}}{NR + R_{LS}} \leq i_e \leq \frac{V_{in}}{R_{LS}}, v_e^2 + RR_{LS}i_e^2 - NV_{in}Ri_e = 0\}.$$

There also exists  $\lambda_e = [\lambda_{e,1}, \lambda_{e,2}, \dots, \lambda_{e,2N}]$  satisfying  $\sum_{i \in \bar{N}} \lambda_{e,i} = 1$ , such that the following convex combination holds:

$$\sum_{i \in \bar{N}} \lambda_{e,i} A_i x_e + B = 0. \quad (3)$$

Note that this Assumption 1 is necessary and sufficient in order to guarantee that  $x_e$ , achieved by a suitable switching signal, is forward invariant, understanding solutions in the generalized sense of Krasovskii or Filippov. Indeed, under (3), this equilibrium is a periodic sequence of arbitrary period  $T$ , with a time equal to  $\lambda_{e,i}T$  in mode  $i$ . Conversely, if Assumption 1 does not hold, this signal does not exist because there is not an arbitrary switching signal corresponding to a convex combination of every mode  $i$  in (3) that can only generate an equivalent action on  $\dot{x}$ .

*Problem 1.* The objectives of this paper are two folds:

- To propose a suitable hybrid dynamic formalism to account for the continuous and discrete dynamics of the converter (1), in order to generate a switching signal  $u$  that stabilizes an equilibrium  $x_e$  satisfying Assumption 1.

- To introduce a degree of freedom in the control law to tune the switching frequency.

In the next section, a hybrid formalism is proposed to provide a solution to Problem 1. To this end, we invoke the property already given in [Albea et al. 2015, Deaecto et al. 2010] to assume the existence of a set of matrices  $P$  and  $Q$  as well as  $u \in \bar{N}$ .

*Property 1.* There exists a matrix  $P, Q \in \mathbb{S}^{N+1}$ , satisfying

$$A_u^T P + P A_u + 2Q < 0, \quad (4)$$

for all  $u \in \bar{N}$ , where  $A_u$  is given in (2).

Note that this property is reasonable from a converter modelled with (1) and (2).

In Section 4, we will discuss of the selection of matrices  $P$  and  $Q$  as well as  $u \in \bar{N}$  satisfying Property 1, following an optimization-based procedure that guarantees stability and optimality.

### 3. HYBRID MODEL AND CONTROL DESIGN

We deal here with the Problem 1 through the hybrid dynamical system formalism proposed in [Goebel et al. 2012], wherein continuous-time behavior is similar to the evolution in (1), and the discrete-time behavior aims at representing the rapid change in the functioning mode of the converter. We represent the global dynamics as a hybrid dynamical system given by

$$\mathcal{H} : \begin{cases} \begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = f(x, u), & (x, u) \in \mathcal{C} \\ \begin{bmatrix} x^+ \\ u^+ \end{bmatrix} \in G(x), & (x, u) \in \mathcal{D}, \end{cases} \quad (5)$$

where the set valued map  $G$  and  $f$  are

$$f(x, u) := \begin{bmatrix} A_u x + B \\ 0 \end{bmatrix}$$

$$G(x) := \left[ \underset{i \in \bar{N}}{\operatorname{argmin}} (x - x_e)^T P (A_i x + B) \right] \quad (6)$$

and where the “flow” and “jump” sets  $\mathcal{C}$  and  $\mathcal{D}$  are given by

$$\mathcal{C} := \{(x, u) : \tilde{x}^T P (A_u x + B) \leq -\eta \tilde{x}^T Q \tilde{x}\} \quad (7)$$

$$\mathcal{D} := \{(x, u) : \tilde{x}^T P (A_u x + B) \geq -\eta \tilde{x}^T Q \tilde{x}\} \quad (8)$$

where  $\tilde{x} = x - x_e$  and scalar  $\eta \in (0, 1)$  is a design parameter that will be introduced to provide a trade-off between reducing the switching and optimality level in the transient-state.

The next lemma is fundamental to prove uniform global asymptotic stability (UGAS) of the converter in Theorem 1 below.

*Proposition 1.* The hybrid dynamical system (5)–(8) satisfies the basic hybrid conditions [Goebel et al. 2012, Assumption 6.5], which guarantees the well-possessedness of the system.

*Proof.* To prove the hybrid basic conditions we see that the sets  $\mathcal{C}$  and  $\mathcal{D}$  are closed. Moreover  $f$  is a continuous function, thus trivially satisfying outer semicontinuity and convexity properties. The map  $G$  is closed, therefore it also is outer semicontinuous [Goebel et al. 2012, Lemma

5.1]. In addition,  $f$  and  $G$  are locally bounded. Finally, the second conclusion of the proposition comes from [Goebel et al. 2012, Theorem 6.30].  $\square$

*Lemma 1.* Consider matrices  $P \in \mathbb{S}^{N+1}$  and  $Q \in \mathbb{S}^{N+1}$  satisfying Property 1, a point  $x_e^T \in \mathbb{R}^{N+1}$  satisfying Assumption 1. Then, for any  $x^T \in \mathbb{R}^{N+1}$ ,

$$\min_{i \in \bar{N}} \tilde{x}^T P(A_i x + B) \leq \min_{i \in \bar{N}} -\tilde{x}^T Q \tilde{x}. \quad (9)$$

$\square$

*Proof.* First notice that the left hand side of (3) is linear in the components of  $\lambda_e$ . Moreover,  $\lambda_e$  belongs to the compact set  $\Lambda = \left\{ \lambda_n \in [0, 1]^N \mid \sum_{i=1}^N \lambda_{n,i} = 1 \right\}$ . Then, the following minimum is obtained at the extreme points:

$$\begin{aligned} & \min_{u \in \bar{N}} \tilde{x}^T P(A_u x + B) \\ &= \min_{\lambda_n \in \Lambda} \tilde{x}^T P \left( \sum_{u \in \bar{N}} \lambda_{n,u} A_u x + B \right) \\ &= \min_{\lambda_n \in \Lambda} \left( \tilde{x}^T P \sum_{i \in \bar{N}} \lambda_{n,i} A_i \tilde{x} + \tilde{x}^T P \left( \sum_{i \in \bar{N}} \lambda_{n,i} A_i x_e + B \right) \right) \\ &\leq \tilde{x}^T P \sum_{i \in \bar{N}} \lambda_{e,i} A_i \tilde{x} + \tilde{x}^T P \left( \sum_{i \in \bar{N}} \lambda_{e,i} A_i x_e + B \right) \\ &\leq -\tilde{x}^T Q \tilde{x}, \end{aligned}$$

where in the last step we used relations (3) and (4).  $\square$

The switching signals  $u$  generated by our solution depend on Lemma 1. Indeed, Property 1 with (7) shows that unless  $x = x^+ = 0$  (which means that the system at the equilibrium  $x = x_e$ ), the solution always jumps to the interior of the flow set  $\mathcal{C}$  because

$$-\tilde{x}^T Q \tilde{x} < -\eta \tilde{x}^T Q \tilde{x},$$

and  $\eta < 1$ . This fact, together with stability (ensuring boundedness of solutions) and the sector growth condition

$$|\dot{\tilde{x}}| = |\dot{x}| \leq |A_u(x - x_e)| + |A_u x_e + B| \leq \kappa_1 |\tilde{x}| + \kappa_2$$

with constants  $\kappa_1, \kappa_2 > 0$ , coming from the flow dynamics (1), implies that between each pair of consecutive resets before solutions approach  $x = x_e$ , there exists a uniform lower bound on the dwell time. Note that, as in other cases, (see [Deaecto et al. 2010], for instance) the equilibrium  $x = x_e$  is achieved with an arbitrarily fast switching. The notably difference in our approach is that during transient time the solution may be characterized by a relative reduction the switching adjusted by  $\eta$ . In [Theunisse et al. 2015] it is shown the same paradigm using very similar techniques, but applied to different class of systems.

Following up, we will establish uniform stability and convergence properties of the hybrid system (5)–(8) to the compact attractor

$$\mathcal{A} := \{(x, u) : x = x_e, u \in \bar{N}\}. \quad (10)$$

*Theorem 1.* Under Assumptions 1 and Property 1 the attractor (10) is uniformly globally asymptotically stable (UGAS) for hybrid system (5)–(8).

*Proof.* Let us take the candidate Lyapunov function

$$V(\tilde{x}) = \frac{1}{2} \tilde{x}^T P \tilde{x}.$$

Note, that in the flow set,  $\mathcal{C}$ , using its definition in (7), we

$$\langle \nabla V(\tilde{x}, z), f(x, u) \rangle = \tilde{x}^T P(A_u x + B) \leq -\eta \tilde{x}^T Q \tilde{x} \quad (11)$$

Across jumps we trivially get:

$$V(\tilde{x}^+) - V(\tilde{x}) = \frac{1}{2} \{ \tilde{x}^T P \tilde{x} - \tilde{x}^T P \tilde{x} \} = 0. \quad (12)$$

because  $\tilde{x}^+ = x^+ - x_e = x - x_e = \tilde{x}$ .

UGAS is then shown applying [Priour et al. 2014, Theorem 1]. Indeed, since the distance of  $x$  to the attractor (10) is defined by  $|x|_{\mathcal{A}} = |\tilde{x}|$ , we have that [Priour et al. 2014, eq. (6)] holds from the structure of  $V$  and from (11) and (12). [Priour et al. 2014, Theorem 1] also requires building a restricted hybrid system  $\mathcal{H}_{\delta, \Delta}$  by intersecting  $\mathcal{C}$  and  $\mathcal{D}$  with the set

$$S_{\delta, \Delta} = \{(\tilde{x}, u) : |\tilde{x}| \geq \delta \text{ and } |\tilde{x}| \leq \Delta\}$$

and then (semi-global) practical persistence flow for  $\mathcal{H}_{\delta, \Delta}$  is provided, for each fixed pair values of  $(\delta, \Delta)$ . More precisely, practical persistent flow is got showing that there exists  $\gamma \in \mathcal{K}_{\infty}$  and  $M \geq 0$ , such that, all solutions to  $\mathcal{H}_{\delta, \Delta}$  satisfy

$$t \geq \gamma(j) - M, \quad \forall t \in \bigcup_{j \in \text{dom}_j \xi} I^j \times \{j\} \quad (13)$$

(see [Priour et al. 2014] for details). To establish (13), notice that after each jump, from the definition of  $G$  in (6) and from property (9) (in Lemma 1), we have

$$\tilde{x}^T(A_{u^+}x + B) \leq -\tilde{x}^T Q \tilde{x} < -\eta \tilde{x}^T Q \tilde{x}, \quad (14)$$

where we used the fact that  $\eta < 1$  and that  $(0, u) \notin S_{\delta, \Delta}$ . Therefore, if any solution to  $\mathcal{H}_{\delta, \Delta}$  performs a jump from  $S_{\delta, \Delta}$ , it will remain in  $S_{\delta, \Delta}$  (because  $\tilde{x}$  remains unchanged) and then, from (8) and (14), it jumps to the interior of the flow set  $\mathcal{C} \cap S_{\delta, \Delta}$ . Moreover, from the strict inequality in (14), then all non-terminating solutions must flow for some time and since  $\mathcal{C} \cap S_{\delta, \Delta}$  is bounded, there is a uniform dwell-time  $\rho(\delta, \Delta)$  between each pair of consecutive jumps. This dwell-time  $(\delta, \Delta)$  clearly implies [Priour et al. 2014, eq. (4)] with the class  $\mathcal{K}_{\infty}$  function  $\gamma(j) = \rho(\delta, \Delta)j$  and  $M = 1$ . Then, all the assumptions of [Priour et al. 2014, Theorem 1] hold and UGAS of  $\mathcal{A}$  is concluded.  $\square$

*Corollary 1.* The hybrid dynamical system (5)–(8) is UGAS and is robust with respect to the presence of small noise in the state, unmodeled dynamics, and spatial regularization to relax the rate of switching, because the attractor (10) is compact.

*Proof.* From Theorem 1, we prove that the hybrid dynamic system is UGAS, and from Proposition 1 we see that it is well-posed. Moreover, as the attractor (10) is compact then it is robustly  $\mathcal{KL}$  asymptotically stable in a basin of attraction.  $\square$

*Remark 1.* Note that, according to Theorem 1, system (5)–(8) may exhibit a Zeno behaviour when  $z \in \mathbb{R} \rightarrow 0$ , and consequently an infinitely fast switching may be expected, which is not acceptable in practice. This will be practically avoided later introducing an additional dwell-time logic to obtain a temporal-regularisation of the dynamics, thereby weakening asymptotic will convergence into practical convergence.

#### 4. OPTIMALITY AND PARAMETERS TUNING

Once that UGAS of the attractor is established for our solution, we focus on providing a suitable performance guarantee as reducing the energy cost, current peaks and response time, for instance. For this goal we apply the same paradigm shown in Albea et al. [2015] for a hybrid context applied to affine switched systems.

Following the hybrid theory given in [Goebel et al. 2012], we consider that the domain of a solution  $\xi$  corresponds to a finite or infinite union of intervals defined as

$$\text{dom } \xi = \bigcup_{j \in \text{dom}_j \xi} I^j \times \{j\}, \quad (15)$$

with  $I^j = [t_j, t_{j+1}]$  being a bounded time interval of the ordinary continuous time  $t$  and having the discrete-time (or “jump times”)  $t_j$  as extremes, where  $j$  represents the number of switches; or  $I^j = [t_j, +\infty)$  being a last unbounded interval.

Let adapt the notation  $\text{dom}_j \xi := \{j \in \mathbb{Z} : (t, j) \in \text{dom } \xi, \text{ for some } t \in \mathbb{R}\}$ . Let us also represent an LQ performance metric focusing on the flows of the inverter, using the expression

$$J(\xi) := \sum_{k \in \text{dom}_j \xi} \int_{t_k}^{t_{k+1}} x^T F^T F x d\tau, \quad (16)$$

where  $\xi = (x, u) : \text{dom } \xi \rightarrow \mathbb{R}^{N+1} \times \bar{N}$  is a solution to hybrid system (5)–(8),  $\tilde{z}(t, j) = F_{u(t, j)} \tilde{x}(t, j)$  for all  $(t, j) \in \text{dom } \xi$ .

The following theorem guarantees the performance cost (16) for our hybrid system.

*Theorem 2.* Consider hybrid system (5)–(8) satisfying Assumption 1 and Property 1. If

$$F^T F \leq Q, \quad (17)$$

then the following bound holds along any solution  $\xi = (x, u)$  of (5)–(8):

$$J(\xi) \leq \eta^{-1} \tilde{x}(0, 0)^T P \tilde{x}(0, 0), \quad (18)$$

where  $\tilde{x}(t, j) = x(t, j) - x_e$ , for all  $(t, j) \in \text{dom}(\xi)$ .

*Proof.* To prove the optimality property in (18), consider any solution  $\xi = (x, u)$  to  $\mathcal{H}$ . Then for each  $(t, j) \in \text{dom } \xi$  and denoting  $t = t_{j+1}$  to simplify notation, we have from (11)

$$\begin{aligned} & V(\tilde{x}(t, j)) - V(\tilde{x}(0, 0)) \\ &= \sum_{k=0}^j V(\tilde{x}(t_{k+1}, k)) - V(\tilde{x}(t_k, k)) \\ &= \sum_{k=0}^j \int_{t_k}^{t_{k+1}} \langle \nabla V(\tilde{x}(\tau, k)), f(x(\tau, k), u(\tau, k)) \rangle d\tau \\ &\leq \sum_{k=0}^j \int_{t_k}^{t_{k+1}} -\eta \tilde{x}^T(\tau, k) Q \tilde{x}(\tau, k) d\tau \\ &\leq -\eta \sum_{k=0}^j \int_{t_k}^{t_{k+1}} \tilde{x}^T(\tau, k) F^T F \tilde{x}(\tau, k) d\tau, \end{aligned} \quad (19)$$

where the last inequality comes from applying (17). Now, considering  $\tilde{y}(\tau, k) = F \tilde{x}(\tau, k)$ , taking the limit as  $t \rightarrow +\infty$

and using the fact that UGAS established in Theorem 1 implies  $\lim_{t \rightarrow +\infty} V(\tilde{x}(t, j)) = 0$ , we get from (19)

$$\eta J(\xi) \leq V(\tilde{x}(0, 0)) = \tilde{x}(0, 0)^T P \tilde{x}(0, 0),$$

as to be proven.  $\square$

*Remark 2.* Note that for a given  $P$  and  $Q$  matrices that satisfies (17), the guaranteed performance level is proportional to the inverse of  $\eta \in (0, 1)$ . That means that large values of  $\eta$  (as close as possible to 1) in transient time are expected to drive to improved LQ performance along solutions.

On the other hand, note from the flow and jump sets in (7) and (8), that larger values of  $\eta$  (close to 1) correspond to strictly larger jump sets (and smaller flow sets), which reveals that solutions with larger values of  $\eta$  exhibit a larger switching frequency. In other words, through parameter  $\eta$  we can find a trade off between switching frequency and performance along solutions in transient time, affecting the level of guaranteed optimality given in (18).

##### 4.1 Computation of $P$ and $Q$

Now, we address the problem of the computation of parameters  $P$ ,  $Q$ , following any class of optimization that reduces as much as possible the right hand side in bound (18). To this end, we select

$$Q = F^T F + \nu I, \quad (20)$$

where  $\nu > 0$  is a positive constant small enough, which must be selected different to zero if  $F^T F \geq 0$  (as happens in our case in (20)), ensuring  $Q > 0$  as well as restriction (17).

Once  $Q$  is selected, and noting that matrix  $A_u$  is Hurwitz, the following convex optimization expressed by the following linear matrix inequality always leads to a feasible solution:

$$\min_{P=P^T > 0} \text{Trace } P, \text{ subject to:} \quad (21)$$

$$A_u^T P + P A_u^T \leq -2Q,$$

and this optimal solution clearly satisfies (4).

#### 5. SIMULATIONS

The hybrid control scheme for a ring converter is tested in a circuit composed of  $N=3$  boost converters. We select the cost function  $J$  in (16) as follows:

$$\min_u \sum_{k \in \text{dom}_j(\xi)} \int_{t_k}^{t_{k+1}} \frac{\rho}{R} (v_c(\tau, k) - v_e)^2 + R_{LS} (i_L(\tau, k) - i_e)^2 d\tau$$

where  $\rho$  is a positive scalar. Note that the constant parameters of each term express the weighted sum of the energy of the error signal of each state variable.

Following (20), we take

$$Q = \text{diag}\{R_{LS}, R_{LS}, R_{LS}, \frac{\rho}{R}\}.$$

We take the parameters given in [Cid-Pastor et al. 2011] for comparison with the switched control algorithm presented therein. The considered nominal values are:  $V_{in} = 20V$ ,

$R = 20\Omega$ ,  $L = 70\mu H$ ,  $C = 220\mu F$  and  $R_{LS} = 0.1\Omega$ . Moreover, we take a sampling time  $T_s = 1\mu s$  and  $\rho = 1000$ . Note that, we give more weight to the voltage than to the current, expecting to obtain a voltage convergence faster at the expense of a larger current peak. Simulations are performed in MATLAB/Simulink by using the HyEQ Toolbox [Sanfelice et al. 2013].

The simulation parameters chosen are:

$$x_e = [2.7 \quad 2.7 \quad 2.7 \quad 40]^T,$$

$$P = \begin{bmatrix} 6.8 & 0.06 & 0.06 & 0.1 \\ 0.06 & 6.8 & 0.06 & 0.1 \\ 0.06 & 0.06 & 6.8 & 0.1 \\ 0.06 & 0.06 & 0.06 & 22 \end{bmatrix} \cdot 10^{-2}.$$

Note that Assumption 1 and Property 1 are satisfied. Figure 2 shows the voltage and current evolutions of the 3-paralleled boost converter controlled by our proposed hybrid dynamic control scheme and compared by the switched control proposed in [Cid-Pastor et al. 2011], for different values of  $\eta$ . Note that the voltage response-time are similar, being in less than  $5ms$  with a maximum current peak near to  $3.2A$  of magnitude.

On the other hand, the voltage evolutions are essentially the same for different values of  $\eta$ . However, we insist that choosing a smaller value of  $\eta$ , the switching frequency can be reduced. In particular, Fig. 3 shows the switching frequency in a time slot of the transient for different values of  $\eta$ . From the figure it appears that using  $\eta = 0.99$  we are closer of a similar frequency as the one obtained with the switched control proposed in [Cid-Pastor et al. 2011]. Nevertheless, according to Remark 2, as  $\eta$  is reduced, a reduced switching frequency is expected, which is consistent with the upper two plots of Fig. 3. This trend of the switching frequency with respect to  $\eta$  can be well appreciated in Fig. 4 that shows the normalized switching frequency as a function of  $\eta$  (different curves correspond to different initial conditions) and the normalized cost function  $J$ . The curves are normalized to the switching frequency experienced with the method of [Cid-Pastor et al. 2011]. As expected, for large values of  $\eta$ , we recover a closer frequency as the one in [Cid-Pastor et al. 2011], but reducing  $\eta$  we may give up a little on optimality level and suitably adjust the switching frequency. In particular, for  $\eta = 0.07$  we see that can obtain a trade-off between switching frequency and optimality level in the simulated time slot.

In Remark 1, we note that there is infinitely fast switching in the steady-state. In practice, this is not desired in terms of energy efficiency and reliability, since every switch dissipates energy and reduces the switch lifespan. This is not appreciated in Fig.2 because we used a sampling time. If we want to avoid this infinitely fast switching without the need to introduce a sampling time, we propose a space-regularisation through a dwell-time that separates the regions (7) and (8). For this aim, the proposed flow and jump sets are

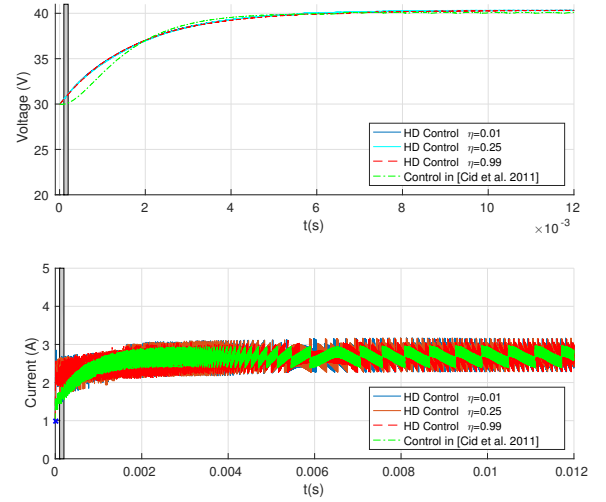


Fig. 2. 3-paralleled boost converter with  $\eta_2 = 0$ .

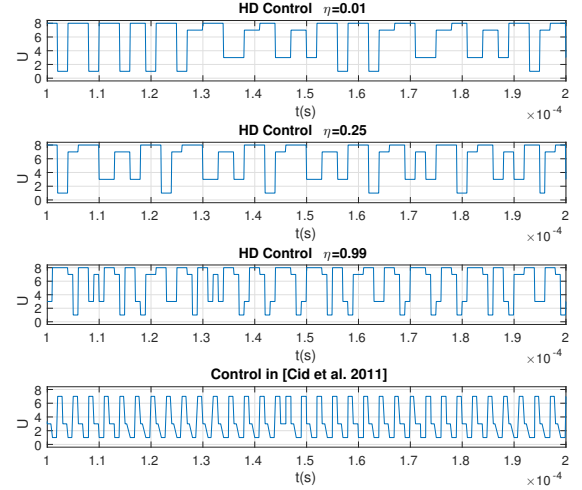


Fig. 3. Zoom of  $U = [1 \ 2 \ 4]u^T + 1$  in the 3-paralleled boost converter with  $\eta_2 = 0$ .

$$\mathcal{C} := \{(\tilde{x}, u) : \tilde{x}^T P(A_u x + B) \leq -\eta \tilde{x}^T Q \tilde{x} \text{ or } V(\tilde{x}) \leq \eta_2\} \quad (22)$$

$$\mathcal{D} := \{(\tilde{x}, u) : \tilde{x}^T P(A_u x + B) \geq -\eta \tilde{x}^T Q \tilde{x} \ \& \ V(\tilde{x}) \geq \eta_2\}, \quad (23)$$

being  $\eta_2 > 0$  small enough and  $\eta \in (0, 1)$ . With these flow and jump maps a reduction of switching in the steady-state is expected as  $\eta_2$  increases.

Some simulations are given in Fig. 5 with the practice Hybrid system (5)–(6) with (22)–(23) and compared with the main Hybrid system (5)–(8), which is equivalent to  $\eta_2 = 0$ . Note that the voltage evolution is essentially the same for different values of  $\eta_2$ . On the other hand, as was mentioned above, note as  $\eta_2$  is increased as reduced switching frequency is expected, which is consistent with the upper two plots of Fig. 3. For these simulations we chose the value of  $\eta = 0.07$ .

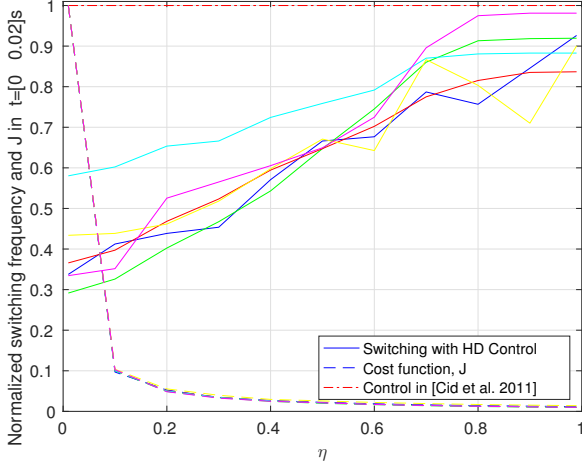


Fig. 4. Evolution of the normalized switching frequency w.r.t.  $\eta$  for different initial conditions and  $\eta_2 = 0$ .

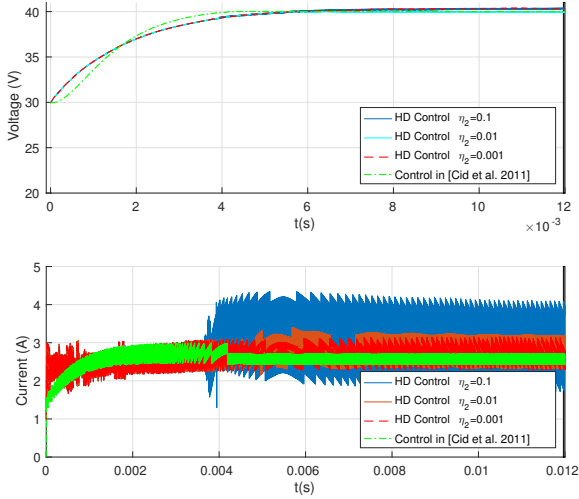


Fig. 5. 3-Ring converter with  $\eta = 0.07$ .

## 6. CONCLUSIONS AND FUTURE WORK

In this article, a paralleled boost converter is controlled by using an hybrid control scheme, which is based on a hybrid modelling. In comparison with the work presented in [Cid-Pastor et al. 2011], we got to take into account a dissipative resistance of the inductances and, to manage the states and of an optimality level. The potential of considering a hybrid representation is to obtain two design parameters:  $\eta$ , that adjusts a trade-off between performance and switching frequency in the transient-state and  $\eta_2$ , that can reduce the switching in the steady-state. In future work, we will establish that the compact attractor

$$\mathcal{A}_{\eta_2} = \{V(\tilde{x}) \leq \eta_2, u \in \bar{N}\}, \quad (24)$$

is UGAS for Hybrid system (5)–(6) with (22)–(23), and global practical asymptotic stability of the attractor (10) with respect to  $\eta_2$ . Other future work is to consider the interleaved operation in the hybrid dynamical schema.

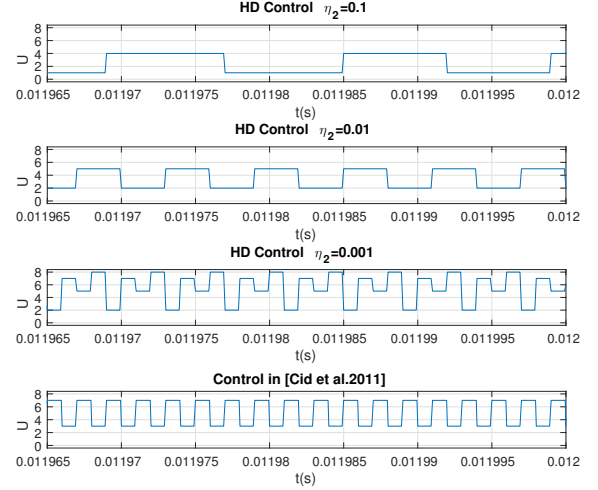


Fig. 6. Zoom of  $U = [1 \ 2 \ 4]u^T + 1$  in the 3-ring converter with  $\eta = 0.07$ .

This constitutes the main an important direction for future researches.

## ACKNOWLEDGMENTS

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