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# Technical Report Associated with the Paper: “Determining the Singularities for the Observation of Three Image Lines”

Sébastien Briot, Philippe Martinet and François Chaumette.

## SINGULARITY CASES IN THE VISUAL SERVOING OF THREE GENERAL LINES IN SPACE

We consider here the case where the three observed lines have a general configuration. We define the frame  $\mathcal{F}_b$  :  $(Q, \mathbf{x}_b, \mathbf{y}_b, \mathbf{z}_b)$  attached to the observed body  $\mathcal{B}$  such that  $\mathbf{x}_b$  is collinear to  $\mathbf{U}_1$ ,  $\mathbf{y}_b$  is lying in the plane  $\mathcal{P}$  containing  $\mathcal{L}_1$  and  $\mathcal{L}_2$ . We parameterize the lines as follows (see Fig. 1)

$$\begin{aligned}\overrightarrow{OP_1} &= [(X-b)(Y-c)(Z-a)]^T, \mathbf{U}_1 = [1\ 0\ 0]^T \\ \overrightarrow{OP_2} &= [(X-b)(Y-c)(Z+a)]^T, \mathbf{U}_2 = [d\ e\ 0]^T \\ \overrightarrow{OP_3} &= [(X+b)(Y+c)Z]^T, \mathbf{U}_3 = [f\ g\ h]^T\end{aligned}\quad (1)$$

where  $d, e, f, g$  and  $h$  are variables parameterizing the direction of the lines  $\mathcal{L}_2$  and  $\mathcal{L}_3$ .

Then, we have

$$\mathbf{f}_{i1} \propto \mathbf{U}_i \times \overrightarrow{OP_i}, \mathbf{m}_{i2} \propto \mathbf{U}_i \times \overrightarrow{\mathbf{f}_{i1}}\quad (2)$$

which, from the singularity conditions which are recalled here for reasons of clarity,

$$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0 \text{ or } f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0\quad (3)$$

leads to

$$\begin{aligned}f_1 = 0 &\Leftrightarrow a_{2z}Z^2 + a_{1z}Z + a_{0z} = 0 \\ f_2 = 0 &\Leftrightarrow b_{2z}Z^2 + b_{1z}Z + b_{0z} = 0\end{aligned}\quad (4)$$

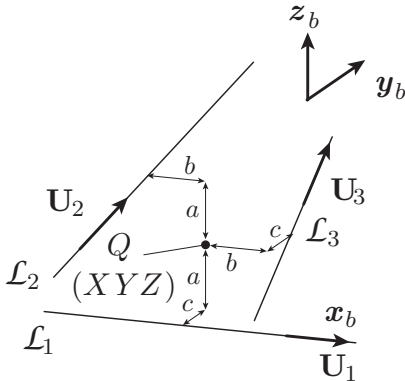


Fig. 1. Observation of three general lines in space

where

$$\begin{aligned}a_{2z} &= 2e(cf - bg) \\ a_{1z} &= c_{1y}Y + c_{0y} \\ a_{0z} &= d_{2y}Y^2 + d_{1y}Y + d_{0y} \\ b_{2z} &= e_{1x}X + e_{1y}Y + e_0 \\ b_{1z} &= f_{2xy}(Y^2 + Y^2) + f_{1xy}XY + f_{1x}X + f_{1y}Y + f_0 \\ b_{0z} &= g_{3y}Y^3 + g_{21xy}X^2Y + g_{22xy}XY^2 + g_{2x}X^2 \\ &\quad + g_{2y}Y^2 + g_{1xy}XY + g_{1x}X + g_{1y}Y + g_0\end{aligned}\quad (5)$$

in which

$$\begin{aligned}c_{1y} &= 2adg - aef + 2beh \\ c_{0y} &= abeg - 2acd + acef - (2ch - ag)eX \\ d_{2y} &= -2adh \\ d_{1y} &= 2ehaX - efa^2 \\ d_{0y} &= a^2beg - a^2cef + 2ac^2dh - 2abceh + a^2egX\end{aligned}\quad (6)$$

and

$$\begin{aligned}e_{1x} &= (ghe + dfh)e \\ e_{1y} &= (fhe - dgh)e \\ e_0 &= -eh(bdf + beg - cdg + cef) \\ f_{2xy} &= -e(dg^2 - efg + dh^2) \\ f_{1xy} &= -2e^2h^2 \\ f_{1x} &= -e(2cef^2 + adfh - 2cdfg + aegh) \\ f_{1y} &= 2ahd^2f + 2cdef^2 - 2bdefg + 2cdeg^2 \\ &\quad + ahdeg + ahe^2f - 2be^2g^2 \\ f_0 &= b^2deg^2 + b^2deh^2 - b^2e^2fg + 2bce^2f^2 \\ &\quad + 2bce^2g^2 + 2bce^2h^2 + abdefh + abe^2gh \\ &\quad - 2c^2def^2 - c^2deg^2 - c^2deh^2 - c^2e^2fg \\ &\quad - 2acd^2fh - acdegh - ace^2fh \\ g_{3y} &= deg \\ g_{21xy} &= -e^2fh \\ g_{22xy} &= dfhe - ghe^2 \\ g_{2x} &= cfe^2h + adeg^2 + adeh^2 - afe^2g \\ g_{2y} &= 2agd^2f - adef^2 + bdefh - adeh^2 \\ &\quad - cgdeh + age^2f + bge^2h \\ g_{1xy} &= -2ad^2g^2 - 2ad^2h^2 - 2cdefh + ae^2f^2 \\ &\quad - ae^2g^2 \\ g_{1x} &= c^2defh + c^2e^2gh + 2acd^2g^2 + 2acd^2h^2 \\ &\quad - 2acdefg + ace^2f^2 + ace^2g^2 + 2ace^2h^2\end{aligned}$$

$$\begin{aligned}
g_{1y} &= b^2 e^2 f h - 2 b c d e f h - 2 a b d^2 g^2 - 2 a b d^2 h^2 \\
&+ 2 a b d e f g - a b e^2 f^2 - a b e^2 g^2 - 2 a b e^2 h^2 \\
&- c^2 d e g h \\
g_0 &= b^2 c e^2 f h - a b^2 d e g^2 - a b^2 d e h^2 + a b^2 e^2 f g \\
&+ b c^2 d e f h - b e^2 e^2 g h + 2 a b c d^2 g^2 + 2 a b c d^2 h^2 \\
&- a b c e^2 f^2 + a b c e^2 g^2 + c^3 d e g h - 2 a c^2 d^2 f g \\
&+ a c^2 d e f^2 + a c^2 d e h^2 - a c^2 e^2 f g
\end{aligned}$$