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Technical Report Associated with the Paper: “Determining the Singularities for the Observation of Three Image Lines”

Sébastien Briot, Philippe Martinet and François Chaumette.

Singularity cases in the visual servoing of three general lines in space

We consider here the case where the three observed lines have a general configuration. We define the frame $\mathcal{F}_b : (Q, x_b, y_b, z_b)$ attached to the observed body $B$ such that $x_b$ is collinear to $U_1$, $y_b$ is lying in the plane $\mathcal{P}$ containing $L_1$ and $L_2$. We parameterize the lines as follows (see Fig. 1)

$$\overrightarrow{OP_1} = [(X - b) (Y - c) (Z - a)]^T, \; U_1 = [1 \; 0 \; 0]^T$$
$$\overrightarrow{OP_2} = [(X - b) (Y - c) (Z + a)]^T, \; U_2 = [d \; e \; 0]^T$$
$$\overrightarrow{OP_3} = [(X + b) (Y + c) Z]^T, \; U_3 = [f \; g \; h]^T$$

where $d$, $e$, $f$, $g$ and $h$ are variables parameterizing the direction of the lines $L_2$ and $L_3$.

Then, we have

$$f_{12} \propto U_1 \times \overrightarrow{OP_2}, \; m_{12} \propto U_1 \times \overrightarrow{OP_3}$$

which, from the singularity conditions which are recalled here for reasons of clarity,

$$f_1 = f_{12}^T (f_{21} \times f_{31}) = 0 \quad \text{or} \quad f_2 = m_{12}^T (m_{22} \times m_{32}) = 0$$

leads to

$$f_1 = 0 \iff a_{22} Z^2 + a_{12} Z + a_{02} = 0$$
$$f_2 = 0 \iff b_{22} Z^2 + b_{12} Z + b_{02} = 0$$

where

$$a_{22} = 2e(cf - bg)$$
$$a_{12} = c_1 y + c_0 y$$
$$a_{02} = d_2 y^2 + d_1 y + d_0 y$$
$$b_{22} = e_1 x + e_1 y + e_0$$

in which

$$c_1 y = 2adg - ac f + 2bch$$
$$c_0 y = abg - 2acdg + ace f - (2ch - acg)eX$$
$$d_{2y} = -2adh$$

and

$$e_{1x} = (ghe + dfh)e$$
$$e_{1y} = (fhe - dgh)e$$
$$e_{0} = -eh(bdf + beg - cdg + cef)$$

$$f_{2xy} = -e(dy^2 - cf g + dh^2)$$
$$f_{1xy} = -2c^2 h^2$$
$$f_{1x} = -e(2ce f^2 + adf h - 2cdf g + aeh g)$$
$$f_{1y} = 2ahd^2 f + 2ced f^2 - 2bde f g + 2cde^2 f^2 + ahdeg + ahe f^2 - 2bce g^2$$
$$f_0 = b^2 d e g^2 + b^2 d e h^2 - b^2 e^2 f g + 2bce^2 f^2 + 2bce^2 h^2 + abde h + a e c^2 g h - 2c^2 d e f^2 - c^2 d e g^2 - c^2 d e h^2 - c^2 e^2 f g - 2acde f h - acdegh - a e c^2 f h$$
$$g_{3y} = deg h$$
$$g_{22xy} = -e^2 f h$$
$$g_{22xy} = dfh e - ghe^2$$
$$g_{2x} = c f e^2 h + adef^2 + adeh^2 - a f e^2 g$$
$$g_{2y} = 2agf^2 f - ade f^2 + bde f h - ade h^2 - cge de f + age f + bh e g^2$$
$$g_{1xy} = -2ad^2 g^2 - 2ad^2 h^2 - 2ced f h + ace^2 f^2$$
$$g_{1y} = c^2 d e f h + c^2 e^2 g h + 2acde f^2 + 2acde h^2$$
$$g_{1x} = c^2 d e f h + c^2 e^2 g h + 2acde f^2 + 2acde h^2$$

Fig. 1. Observation of three general lines in space
\[ g_{1g} = b^2 e^2 f h - 2bcdefh - 2abcd^2 g^2 - 2ab^2 d^2 h^2 \\
+ 2abdefg - abc^2 f^2 - abc^2 g^2 - 2abc^2 h^2 \\
- c^2 degh \\
g_0 = b^2 ce^2 f h - ab^2 de^2 g^2 - ab^2 deh^2 + ab^2 e^2 fg \\
+ bc^2 de f h - bc^2 e^2 gh + 2abcd^2 g^2 + 2abcd^2 h^2 \\
- abce^2 f^2 + abce^2 g^2 + c^3 degh - 2ac^2 d^2 fg \\
+ ac^2 de f^2 + ac^2 deh^2 - ac^2 e^2 fg \]