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"Determining the Singularities for the Observation of
Three Image Lines"**

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Technical Report Associated with the Paper: “Determining the Singularities for the Observation of Three Image Lines”

Sébastien Briot, Philippe Martinet and François Chaumette.

SINGULARITY CASES IN THE VISUAL SERVOING OF THREE GENERAL LINES IN SPACE

We consider here the case where the three observed lines have a general configuration. We define the frame \mathcal{F}_b : $(Q, \mathbf{x}_b, \mathbf{y}_b, \mathbf{z}_b)$ attached to the observed body \mathcal{B} such that \mathbf{x}_b is collinear to \mathbf{U}_1 , \mathbf{y}_b is lying in the plane \mathcal{P} containing \mathcal{L}_1 and \mathcal{L}_2 . We parameterize the lines as follows (see Fig. 1)

$$\begin{aligned}\overrightarrow{OP_1} &= [(X-b)(Y-c)(Z-a)]^T, \mathbf{U}_1 = [1\ 0\ 0]^T \\ \overrightarrow{OP_2} &= [(X-b)(Y-c)(Z+a)]^T, \mathbf{U}_2 = [d\ e\ 0]^T \\ \overrightarrow{OP_3} &= [(X+b)(Y+c)Z]^T, \mathbf{U}_3 = [f\ g\ h]^T\end{aligned}\quad (1)$$

where d, e, f, g and h are variables parameterizing the direction of the lines \mathcal{L}_2 and \mathcal{L}_3 .

Then, we have

$$\mathbf{f}_{i1} \propto \mathbf{U}_i \times \overrightarrow{OP_i}, \mathbf{m}_{i2} \propto \mathbf{U}_i \times \overrightarrow{\mathbf{f}_{i1}}\quad (2)$$

which, from the singularity conditions which are recalled here for reasons of clarity,

$$f_1 = \mathbf{f}_{11}^T(\mathbf{f}_{21} \times \mathbf{f}_{31}) = 0 \text{ or } f_2 = \mathbf{m}_{12}^T(\mathbf{m}_{22} \times \mathbf{m}_{32}) = 0\quad (3)$$

leads to

$$\begin{aligned}f_1 = 0 &\Leftrightarrow a_{2z}Z^2 + a_{1z}Z + a_{0z} = 0 \\ f_2 = 0 &\Leftrightarrow b_{2z}Z^2 + b_{1z}Z + b_{0z} = 0\end{aligned}\quad (4)$$

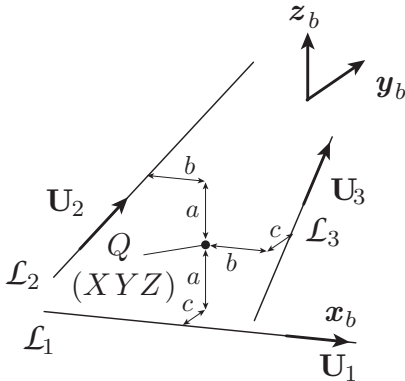


Fig. 1. Observation of three general lines in space

where

$$\begin{aligned}a_{2z} &= 2e(cf - bg) \\ a_{1z} &= c_{1y}Y + c_{0y} \\ a_{0z} &= d_{2y}Y^2 + d_{1y}Y + d_{0y} \\ b_{2z} &= e_{1x}X + e_{1y}Y + e_0 \\ b_{1z} &= f_{2xy}(Y^2 + Y^2) + f_{1xy}XY + f_{1x}X + f_{1y}Y + f_0 \\ b_{0z} &= g_{3y}Y^3 + g_{21xy}X^2Y + g_{22xy}XY^2 + g_{2x}X^2 \\ &\quad + g_{2y}Y^2 + g_{1xy}XY + g_{1x}X + g_{1y}Y + g_0\end{aligned}\quad (5)$$

in which

$$\begin{aligned}c_{1y} &= 2adg - aef + 2beh \\ c_{0y} &= abeg - 2acd + acef - (2ch - ag)eX \\ d_{2y} &= -2adh \\ d_{1y} &= 2ehaX - efa^2 \\ d_{0y} &= a^2beg - a^2cef + 2ac^2dh - 2abceh + a^2egX\end{aligned}\quad (6)$$

and

$$\begin{aligned}e_{1x} &= (ghe + dfh)e \\ e_{1y} &= (fhe - dgh)e \\ e_0 &= -eh(bdf + beg - cdg + cef) \\ f_{2xy} &= -e(dg^2 - efg + dh^2) \\ f_{1xy} &= -2e^2h^2 \\ f_{1x} &= -e(2cef^2 + adfh - 2cdfg + aegh) \\ f_{1y} &= 2ahd^2f + 2cdef^2 - 2bdefg + 2cdeg^2 \\ &\quad + ahdeg + ahe^2f - 2be^2g^2 \\ f_0 &= b^2deg^2 + b^2deh^2 - b^2e^2fg + 2bce^2f^2 \\ &\quad + 2bce^2g^2 + 2bce^2h^2 + abdefh + abe^2gh \\ &\quad - 2c^2def^2 - c^2deg^2 - c^2deh^2 - c^2e^2fg \\ &\quad - 2acd^2fh - acdegh - ace^2fh \\ g_{3y} &= deg h \\ g_{21xy} &= -e^2fh \\ g_{22xy} &= dfhe - ghe^2 \\ g_{2x} &= cfe^2h + adeg^2 + adeh^2 - afe^2g \\ g_{2y} &= 2agd^2f - adef^2 + bdefh - adeh^2 \\ &\quad - cgdeh + age^2f + bge^2h \\ g_{1xy} &= -2ad^2g^2 - 2ad^2h^2 - 2cdefh + ae^2f^2 \\ &\quad - ae^2g^2 \\ g_{1x} &= c^2defh + c^2e^2gh + 2acd^2g^2 + 2acd^2h^2 \\ &\quad - 2acdefg + ace^2f^2 + ace^2g^2 + 2ace^2h^2\end{aligned}$$

$$\begin{aligned}
g_{1y} &= b^2 e^2 f h - 2 b c d e f h - 2 a b d^2 g^2 - 2 a b d^2 h^2 \\
&+ 2 a b d e f g - a b e^2 f^2 - a b e^2 g^2 - 2 a b e^2 h^2 \\
&- c^2 d e g h \\
g_0 &= b^2 c e^2 f h - a b^2 d e g^2 - a b^2 d e h^2 + a b^2 e^2 f g \\
&+ b c^2 d e f h - b e^2 e^2 g h + 2 a b c d^2 g^2 + 2 a b c d^2 h^2 \\
&- a b c e^2 f^2 + a b c e^2 g^2 + c^3 d e g h - 2 a c^2 d^2 f g \\
&+ a c^2 d e f^2 + a c^2 d e h^2 - a c^2 e^2 f g
\end{aligned}$$