The Galactic Dark Matter As Relativistic Necessity

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Introduction

Since the assumption of the existence of galactic dark matter (dark mass) by Vera Rubin to explain the flatness of galactic rotation curves[1,2,3], no convincing explanation about the nature of this dark mass has been made. Attempts to explain this missing mass by an invisible form of ordinary baryonic matter was largely refuted by the programs MACHO[4], EROS[5] and AGAPE[6], it is the same for explanations using ordinary non-baryonic matter. Numerous detection attempts of exotic particle that could explain the missing mass, have also all been unsuccessful. Similarly, the new CERN accelerator appears to confirm that the physics is limited to the standard model and the existence of an exotic particle is less and less probable[7,8,9].

It also seems extremely difficult, even impossible, to explain this phenomenon with the current theory of gravitation either the Newtonian gravity[10,11] (NG) or general relativity (GR). An alternative is to modify gravity so as to adapt it to regime change at galactic level. On the contrary, such a project is exposed to the prodigious adequacy of the GR to the phenomenological reality[12,13] and the physical existence of dark matter[14,15].

The explanation proposed in this article is of an entirely different nature. The dark mass is not some form of actual matter, it is also not an epiphenomenon caused by gravitation. The dark mass is actually a necessary consequence of relativistic mechanics (RM), that is to say, the combination of classical mechanics (CM) and special relativity (SR). It will be demonstrated that this relativistic mass is necessary if a body like a galaxy can physically collapse into a compact ball. The explanation proposed ignores the forces of physics and therefore is a purely mechanical explanation.

The existence of a compact ball state

The theorem of the Schwarzschild radius limit can be simply derived from NG and the postulate of the light speed as a maximum speed. Indeed, simply pose that escape velocity $V = \sqrt{(2GM/Rs)}$ is equal to c which gives $Rs = 2GM/c^2$. It is also known that the same equation can be derived with GR.

The fundamental axiom of our demonstration is that a galaxy can contract in a compact ball with a radius proportional to the mass thus $R_\alpha = \alpha M$. The only constraint is that R_α radius is much smaller than that of the compacted galaxy, which limits the choice of α. To simplify calculations, we'll set $\beta = \alpha\, c^2/2$ and so $R_\alpha = 2\beta M/c^2$ **[D1]**.

We consider two models for the dynamics of this compact ball: 1) the rigid ball in relativistic rotation at constant angular velocity $v(r) = \omega r$ and 2) the homogeneous energy ball at constant linear velocity $\omega(r) = v/r$. The rigid ball requires the existence of a bonding force, non existent in RM only, while a simple force of friction would produce a homogenization of speeds.

In the case of the rigid ball, the spin is defined as $a = \omega/\omega_{max}$ and in the case of the homogeneous energy ball $a = \omega(r)/\omega_{max}(r)$. The spin value is a constant and by posing $\omega = \omega(R)$ and $\omega_{max} = \omega_{max}(R)$ then in both cases $a = \omega(r)/\omega_{max}(r) = \omega(R)/\omega_{max}(R) = \omega/\omega_{max}$.

The energy and angular momentum of the ball in relativistic rotation

First calculate the mass M of a rigid ball of radius R in relativistic rotation around a central axis. $\rho(r)$ is the relativistic density at the distance r from the axis of rotation and ρ_0 the inert density on the axis of rotation (no relativistic expansion of the mass). The relativistic density is $\rho(r) = \rho_0/\sqrt{[1-v(r)^2/c^2]}$. The height of the basic cylinder at the distance r from the axis of rotation is $h(r) = 2\sqrt{[R^2-r^2]}$ and its volume is given by $V(r) = 2\pi rh(r)\, dr$.

Therefore, taking into account the relativistic mass change by integrating from 0 to R: $M = \int \rho(r)\, V(r) = \int \rho(r)\, 2\pi rh(r)\, dr = 2\pi\rho_0R^2 \int 2\sqrt{[R^2-r^2]}/\sqrt{[1-v(r)^2/c^2]}\, d[r^2/2R^2] = 2\pi\rho_0R^3 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-v(r)^2/c^2]}\, d[r^2/R^2]$.

1) In the case where the ball is rigid and rotates at constant angular speed $\omega_{max} = c/R$ (maximum speed at the equator) we have $M = 2\pi\rho_0R^3 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-(r\omega_{max})^2/c^2]}\, d[r^2/R^2] = 2\pi\rho_0R^3 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-c^2r^2/c^2R^2]}\, d[r^2/R^2] = 2\pi\rho_0R^3 \int d[r^2/R^2] = 2\pi\rho_0R^3$. So $M = 2\pi\rho_0R^3$ **[L1]**. Because $M_0 = 4\pi\rho_0R^3/3$ (density by the volume of the ball) therefore $M = 3M_0/2$. The energy gain $M_r = M-M_0$ when $a = 1$ is maximum $M_r = M_0/2$ **[T1]**.

2) In the case where the ball has a homogeneous energy and rotates at the constant linear velocity $v = r\omega(r)$ we have $M = 2\pi\rho_0R^3 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-(r\omega(r))^2/(r\omega_{max}(r))^2]}\, d[r^2/R^2] = 2\pi\rho_0R^3/\sqrt{(1-a^2)} \int \sqrt{(1-r^2/R^2)}\, d[r^2/R^2] = 4\pi\rho_0R^3/\sqrt{(1-a^2)} \int r\sqrt{(1-r^2/R^2)}/R^2\, dr = 4\pi\rho_0R^3/3\sqrt{(1-a^2)}$ **[L2]**. Since that $M_0 = 4\pi\rho_0R^3/3$ (density by the volume of the ball) then $M = M_0/\sqrt{(1-a^2)}$. The energy gain $M_h = M-M_0 = M_0(1/\sqrt{(1-a^2)}-1)$ **[T2]** tends to infinity with the spin of the ball ($a = 1$) and is zero for a nul spin ($a = 0$).

It is now possible to calculate the relativistic moment of inertia of the ball : $I = \int r^2\, dm = \int r^2\rho(r)\, V(r) = \int r^2\rho(r)\, 2\pi rh(r)\, dr = 2\pi\rho_0R^4 \int 2\sqrt{[R^2-r^2]}/[1-v(r)^2/c^2]\, d[r^4/4R^4] = \pi\rho_0R^5 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-v(r)^2/c^2]}\, d[r^4/R^4]$.

1) In the case where the ball is rigid and rotates at constant angular speed $\omega_{max} = c/R$ (maximum speed at the equator) we have $I = \pi\rho_0R^5 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-(r\omega_{max})^2/c^2]}\, d[r^4/R^4] = \pi\rho_0R^5 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-c^2r^2/c^2R^2]}\, d[r^4/R^4] = \pi\rho_0R^5$. Thus $I = \pi\rho_0R^5$ and by [L1] $I_r = ½MR^2$.

2) In the case where the ball has a homogeneous energy and rotates at the constant linear velocity $v = r\omega(r)$ we have $I = \pi\rho_0R^5 \int \sqrt{[1-r^2/R^2]}/\sqrt{[1-(r\omega(r))^2/(r\omega_{max}(r))^2]}\, d[r^4/R^4] = \pi\rho_0R^5/\sqrt{(1-a^2)} \int \sqrt{(1-r^2/R^2)}\, d[r^4/R^4] = \pi\rho_0R^5/\sqrt{(1-a^2)} \int 4r^3\sqrt{(1-r^2/R^2)}/R^4 = 8\pi\rho_0R^5/15\sqrt{(1-a^2)}$. So $I = 8\pi\rho_0R^5/15\sqrt{(1-a^2)}$ and by [L2] $I_r = 2MR^2/5$.

The relativistic angular momentum of the maximum speed rotating rigid ball is given by $J_r = I_r\omega = ½MR^2\omega$ **[T3]**, which is not very different from the non-relativistic angular momentum $J = 3MR^2\omega/5$. This calculation is not exact because J_r should vary from $3MR^2\omega/5$ for a spin of $a = 0$ to $½MR^2\omega$ for a spin of $a = 1$. The relativistic angular momentum of the rigid ball of homogenous energy is $J_h = I_h\omega = 2MR^2\omega/5$ **[T4]**, this calculation is, on the other hand, exact.

The dark mass production

Consider a rigid ball in relativistic rotation with a radius R_α, thus by [T3] $J = I\omega = ½MR_\alpha{}^2\omega$ **[L3]**. Since $a = \omega/\omega_{max}$ and $\omega_{max} = c/R_\alpha$ then $\omega = ac/R_\alpha$ and by [L3] $J = ½acMR_\alpha$ **[L4]**. By [D1] and [L4] we obtain $J_{br} = a\beta M^2/c$ **[T5]**, which is exactly the calculated angular momentum[16] by Kerr if $\beta = G$. Thus, the moment of inertia of the Kerr black hole, regardless of its spin, is that of a rigid ball of which the surface rotates at the speed of light. Using the same calculation with the ball of homogeneous energy is obtained by [T4] and [D1] $J_{bh} = 4a\beta M^2/5c = 4J_{br}/5$ **[T6]**.

For virtually any rigid body $I = \varphi MR^2$ such as φ is a constant characterizing the shape of the body : φ = 1 for the ring, φ = 1/2 for the disc, φ = 2/3 for the sphere, and φ = 3/5 for the ball. Since $\omega = V/R$, it follows that $J_\varphi = I\omega = (\varphi MR^2)(V/R) = \varphi MRV$ **[T7]**.

If such a body is compacted into a ball with a radius R_α then by [T5] $J_f = a\beta M^2/c$ and by [T7] $J_i = \varphi MRV$. Therefore, by the conservation of angular momentum $J_i = J_f$ and so $a = \varphi RVc/\beta M$. Yet, if the values of the Milky Way are used[17,18] $R = 3.3\times10^{20}$, $V = 2.2\times10^5$, $M = 2\times10^{42}$ and $\varphi = 1$ and $\beta = G$ we get a spin $a = 262$, which is not an admissible value because $a \in [0,1]$. In fact, we must modify of one order of magnitude at least three parameters to get a possible value. Worse, the mass M should be multiplied by a factor to approach a valid value but it is already the total mass including the dark mass.

Our calculation is incorrect because the final ball and the initial Galaxy are not in the same Galilean reference frame. Indeed, the mass of J_i is actually only the baryonic mass whereas the mass of J_f is the total mass expanded by relativistic velocity produced by the rotation. Therefore, to maintain the same mass-energy of the initial to the final state, it is necessary to contract the mass-energy of the final state. This is exactly a change of relativistic reference frame requiring the introduction of a Lorentz factor. Since the unit of angular momentum is kg×m²/s then $\gamma^2 = J_i/J_f$ because $\gamma^2 = (1/\gamma)\text{kg}\times(\gamma^2)\text{m}^2/(1/\gamma)\text{s}$ This allows to write $\gamma^2 = J_i/J_f = \varphi RVc/a\beta M$ **[T8]** and get by setting $\varphi = a = 1$ and $\beta = G$ $\gamma = M/M_0 \approx 16$. What is much more than the maximum dilatation of $M = 3M_0/2$ by [T1], we must conclude that the compact ball is not rigid.

Using the same calculation for the homogeneous energy ball it is obtained by [T6] and [T7] $\gamma^2 = J_i/J_f = 5\varphi RVc/4a\beta M$ **[T9]** and posing $\varphi = a = 1$ and $\beta = G$ for the Milky Way one obtains $\gamma = M/M_0 \approx 18$. That which is, here, an allowed energy value by [T2].

To get a better approximation, it is possible to model a spiral galaxy by a disk of homogeneous surface density σ_0 of mass $M = \sigma_0\pi R^2$. It is subsequently possible to divide the disk in rings of masses $m(r) = \sigma_0 2\pi r\, dr$, of moments of inertia $I(r) = m(r)\, r^2$ and angular momenta $J(\mathrm{r}) = I(r)\,\omega(r)$ with $\omega(r) = V/r$ so a constant speed of rotation for the entire disc. Integrating $J(r)$ from 0 to R we obtain $J = 2MRV/3$, thus by [T7] $\varphi = 2/3$ which gives for the values of the Milky Way applied to [T8] $\gamma = M/M_0 \approx 13$ or $\gamma = M/M_0 \approx 15$ by using [T9].

If we compare the equation of the rigid ball $a\gamma^2 = \varphi RVc/\beta M$ with that of the homogeneous energy ball $a\gamma^2 = 5\varphi RVc/4\beta M$ we realize that it is practically the same equation with a small multiplicative factor difference. Since the equation of the angular momentum of the rigid ball is identical to that obtained using the Kerr black hole, it will be favored. However, to not contradict the value of energies by the boundary of the rigid ball, we will use the energy of the homogeneous ball, $a\gamma^2 = \varphi RVc/\beta M$ and $\gamma = M/M_0 = 1/\sqrt{(1-a^2)}$ **[T10]**.

The baryonic Tully-Fisher relation

Using [T10] and $\beta = G$ then $a\gamma^2 = \varphi RVc/GM$, the radius R being the maximum galactic radius, V the linear speed at the galactic circumference and M the total galactic mass, it is possible to apply the virial theorem[19] $M = 2V^2R/G$. By posing $a \approx 1$, it is thus possible to obtain $\gamma^2 = (M/M_0)^2 \approx \varphi RVc/GM$ then $8V^5R^2/M_0^2 \approx \varphi cG^2$ by applying the virial and thus $V^5 \approx \varphi cG^2(M_0^2/8R^2)$. By simply modeling the galaxy as a disk with a thickness e and a homogeneous density σ_0, we obtain $M_0 = \pi\sigma_0 eR^2$ so $R^2 = M_0/\pi\sigma_0 e$ therefore $V^5 \approx M_0\,(\pi\sigma_0 e)(\varphi cG^2/8)$ which implies a relation of the type $M_0 \propto V^5$. However, this calculation ignores the coefficient $(\pi\sigma_0 e)$ which lacks only the multiplication by R^2 to get M_0. Let

$e = R/b$ thus $M_0 = \pi\sigma_0(R^3/b)$ **[L5]** and so $\ln(M_0) = \ln(\pi\sigma_0/b)+3\ln(R)$ **[L6]**, therefore, by posing $M_0^{\kappa} = \pi\sigma_0(R/b)$ then $\kappa = [\ln(\pi\sigma_0/b) + \ln(R)]/\ln(M_0)$ and consequently $\kappa = [\ln(\pi\sigma_0/b) + \ln(R)]/[\ln(\pi\sigma_0/b) + 3\ln(R)]$ by [L5] and [L6]. Since $\pi\sigma_0/b$ is on the order of 10^{-2} and R on the order of 10^{20} we can approximate $\kappa \approx 1/3$ and therefore $V^5 \approx M_0^{4/3}(\varphi c G^2/8)$ which implies a relation of the type $M_0 \propto V^{3.75}$. This relationship is in perfect agreement with the baryonic Tully-Fisher law[20] $V^{3.5} \propto M_0 \propto V^4$ and it is therefore possible to derive this law without changing Newton's gravitation[21] or the general relativity.

Discussion on the mechanical determinism

Since a galaxy can contract into compact ball with a great amount of kinetic energy and this energy can be found, by relativity, as mass-energy, one must ask where this energy comes from. By the conservation of energy, if that energy does not come from a source outside the system, it must pre-exist in the galaxy before the compactification. So it must be a tremendous amount of energy in the form of mass-energy in the galaxy. If the cause of the compactification is gravitation, because it is an internal force to the system, this energy must lie somewhere in the gravitational energy field.

The problem in our explanation is that the mass energy being found in a system depends on a potential destiny. If the destiny of a system is not to collapse into a black hole because it does not have the critical mass[22] of Chandrachekar, it should produce much less dark mass. How the other forces interact with gravity to control this mass production is, for now, a mystery.

Theoretical prediction

Since a galaxy is found to be a Galilean reference frame, comparable to a body at a constant speed. Thus, the classical Lorentz transformation of the velocities composition must consequently be used. This consequently causes a shift of the radiation frequency from a transmitting galaxy to a receiving galaxy of this radiation. This intrinsic frequency shift is distinct from the gravitational shift.

Thus, from a radiation emitting galaxy $1/\gamma_e = M_{0e}/M_e$ to a galaxy receiving the radiation $1/\gamma_r = M_{0r}/M_r$, it is possible to get the velocities equivalents $v_e = c\sqrt{(1-1/\gamma_e^2)}$ and $v_r = c\sqrt{(1-1/\gamma_r^2)}$ which may be composed as $v = (v_r - v_e)/[1+(v_r v_e/c^2)]$ which gives a shift $z = \sqrt{[(1+v/c)/(1-v/c)]} - 1$. This can lead, for the Milky Way, to intrinsic shifts between $z \approx -1\%$ and $z \approx 8\%$. This shift should affect the redshift but also the measurement of regular phenomena as pulsars (expansion and contraction of the time).

The Tully-Fisher relation is the most important secondary measure of the distance measurements of a broad set of spiral galaxies, it has a significant influence on the conventional calculation (through the creation of distances scales) of the Hubble constant, yet the errors persist inexplicably[23]. Russell (2015)[24] after a comprehensive analysis concluded at the existence of an intrinsic redshift which can exceed 5000 km/s and a clear tendency for the intrinsic redshifts to be more important than the intrinsic blueshifts. The result of this analysis is in perfect agreement with our theory.

Conclusion

The results obtained are mere consequences of the application of relativistic mechanics (RM) to the problem of contraction of a galactic sized body into a ball with the Schwarzschild radius, no other hypothesis on the nature of gravitation was used. A first surprising result is that the relativistic angular momentum of a rigid ball, with the Schwarzschild radius, is strictly that calculated by Kerr using general relativity (GR). We must therefore conclude that, from a strictly mechanical point of view, the moment of inertia of a black hole is identical to that of a rigid ball rotating at maximum speed and its energy is probably that of a homogeneous energy ball.

The second result is that the kinetic energy of rotation of a ball resulting from the compactification of a galactic sized system is enormous, more than thirteen times its own mass in the case of the Milky Way. If this compactification can indeed occur by the mere internal force of gravity, without any external energy input to the system then, by the law of the conservation of energy, this energy does exist somewhere in the system before the compactification. Since this surplus energy can't be stored mechanically in the galaxy, it is necessarily stored in the gravitational energy field. This energy surplus enables the derivation of the Tully-Fisher relation and correlates well with the amounts of galactic dark matter (mass). All this evidence suggest that this necessary mechanical consequence is the cause of the galactic dark matter.

The third result is a consequence of the fact that without an external energy input, the system is in the same inertial reference frame before and after the compactification. Therefore, an intrinsic redshift phenomenon should occurs between galaxies transmitting and receiving radiation. This new experimental prediction, which permits to accurately calculate the intrinsic redshift from the galactic dark mass, offers a possibility of experimental refutation of this theory. Note that if this theory is proved false, this will strongly question the consistency of relativistic mechanics.

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