Technical Report Associated with the Paper: "Revisiting the Determination of the Singularity Cases in the Visual Servoing of Image Points through the Concept of Hidden Robot"

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FULL EXPRESSION OF THE NULLSPACE OF THE WRENCH SYSTEM $\boldsymbol{\xi}$

The expression of the nullspace given in Eq. (19) is recalled here

$$\begin{aligned}\mathbf{t}_{s1} = [0 \quad 0 \quad &- ZYf_{11}(X,Y) \quad - Zf_{12}(X,Y) \\ &- Zf_{13}(X,Y) \quad (X+1)f_{14}(X,Y)]^T\end{aligned}$$

in which $f_{11}(X,Y)$ takes the form

$$\begin{aligned}f_{11} =&\alpha_1((a_{21}a_{31} - a_{22}a_{32})X^2 \\ &+ (a_{21}a_{32} + a_{22}a_{31})XY - a_{21}a_{31})\end{aligned}$$

with

$$\alpha_1 = a_{21}a_{32} - a_{22}a_{31} + a_{22}n_3 - a_{32}n_2$$

The other functions $f_{12}(X,Y)$, $f_{13}(X,Y)$ and $f_{14}(X,Y)$ are given by

$$\begin{aligned}f_{12} =&\gamma_1\alpha_1X^2 - (a_{22}a_{32}\beta_1 - \gamma_2\alpha_1Y)X \\ &- \gamma_1\gamma_3 - a_{21}a_{31}\beta_1 + a_{22}a_{32}\beta_2Y\end{aligned}$$

$$\begin{aligned}f_{13} =&X(a_{22}a_{32}\beta_2 - \gamma_1\alpha_1Y) - \gamma_3\gamma_2 \\ &+ Y(-a_{32}n_3a_{21}^2 + a_{22}n_2a_{31}^2 + \gamma_3n_2n_3) \\ &+ a_{21}a_{32}^2n_2 - a_{22}^2a_{31}n_3 + \gamma_2\alpha_1X^2\end{aligned}$$

$$\begin{aligned}f_{14} =&- a_{32}n_3a_{21}^2 + a_{22}n_2a_{31}^2 + \gamma_1\alpha_1X^2 \\ &+ (\gamma_2\alpha_1Y - \gamma_3(\gamma_1 - \beta_3 + n_2n_3))X \\ &+ a_{22}a_{32}\gamma_3 + \gamma_3n_2n_3 \\ &+ \gamma_3Y(a_{22}n_3 - \gamma_2 + a_{32}n_2)\end{aligned}$$

where

$$\gamma_1 = a_{21}a_{31} - a_{22}a_{32}$$

$$\gamma_2 = a_{21}a_{32} + a_{22}a_{31}$$

$$\gamma_3 = a_{21}a_{32} - a_{22}a_{31}$$

$$\beta_1 = a_{22}n_3 - a_{32}n_2$$

$$\beta_2 = a_{21}n_3 - a_{31}n_2$$

$$\beta_3 = a_{21}n_3 + a_{31}n_2$$

These expressions were obtained thanks to the use of the Matlab Symbolic Toolbox using the following methodology (see the Matlab scripts associated with this technical report: ScriptNullSpace_R2015a.m (or ScriptNullSpace_R2013a.m when using versions of Matlab previous to R2015)):

- Compute the components $\mathbf{f}_{ij}$ and $\mathbf{m}_{ij}$ of the actuation wrenches $\boldsymbol{\xi}_{ij}$ when the origin of the camera frame becomes to the circumcircle of the 3-D points as shown in Eqs. (15)–(18) in the paper (lines 21–46 in ScriptNullSpace_R2015a.m)
- Build the actuation wrench matrix $\boldsymbol{\xi}$ composed of all actuation wrenches $\boldsymbol{\xi}_{ij}$ (line 49 in ScriptNullSpace_R2015a.m)
- Compute the nullspace of the actuation wrench matrix $\boldsymbol{\xi}$ (line 52 in ScriptNullSpace_R2015a.m)
- Simplify the expression of the nullspace in order to obtain the equations shown in this report (line 60–237 in ScriptNullSpace_R2015a.m)