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# Aggregation of belief degrees in graded doxastic logic for conjunction and disjunction

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## Extended abstract

### 1 Introduction

Doxastic modal logic [1] provides a formal basis to model and perform logical inference about beliefs, using a purely syntactic and axiomatic notion of belief established in the literature [4]. However, belief expressions are naturally associated to a notion of belief degrees, e.g. allowing to distinguish whether someone believes 'strongly' or only 'more or less' and in general how much he/she believes. In order to increase the expressiveness of modal logics, graded extensions have been proposed [7, 3, 2]. Nevertheless, most of them are not applied to the doxastic case or are based on a semantic point of view.

This extended abstract aims at discussing an axiomatic approach, based on relevant manipulation rules of the graded modalities in particular to establish rules for distributing and factorising beliefs over the disjunctive and conjunctive connectors. Such rules for instance aim at formally transcribing possible correlation between different beliefs: if an agent strongly believes a piece of information, represented as a formula  $\varphi$ , but not that much another one, represented as  $\psi$ , they make it possible to infer that the agent believes  $\varphi \wedge \psi$  jointly and, more to the point, how much.

To that aim, the paper briefly sketches existing models for graded beliefs, discusses the interpretation of the belief degree and studies the issue of belief degree aggregation.

### 2 Related works

This section presents weighted modal logics using the belief modality. Then, the manipulation of disjunctive and conjunctive formulae is presented in several frameworks which are place in a classical or non-classical logic formalism.

**Modal Logics for Graded Beliefs** Different frameworks have been considered to define a weighted extension of modal logics semantics, from a formal point of view [10, 11] or as an application to multi-agent systems [5]. As detailed in [12], these models can be organised according to three axes: weight position in Kripke semantics, formal paradigm for the interpretation, and weighted modality semantics.

**Manipulation rules for conjunctive and disjunctive formulae** Depending on the considered logical formalism, different types of rules have been proposed for the manipulation of formulae over the disjunction and conjunction connectors. These connectors can be introduced or removed according to the corresponding rules, as in the classical sequent calculus. They also are manipulated in the context of distribution and factorisation rules, especially in non-classical logics for which a formalism is built to reason about non binary informations, as modal logics, in the particular case of the normal ones [4], fuzzy logic [13] and possibilistic description logic [9].

### 3 Formal Paradigms for the Interpretation of the Belief Degree

This section sketches the discussion regarding the interpretation of the belief degrees, considering two formal paradigms in turn, namely fuzzy set theory [13] and possibility theory [14]. Both frameworks are not usually associated to belief degrees but to imprecise measures and uncertain pieces of knowledge respectively. Their adaptation to belief degree interpretation is briefly discussed, regarding the structural consequences of each formalism.

The first reading is based on a fuzzy interpretation of graded beliefs: the belief degree is considered as a degree of membership in the belief set. Thus, the belief base is considered as a fuzzy subset of the crisp set of well-formed formulae. The formulae are not believed at the same level: it depends on their membership.

Another possible reading for belief degrees is based on the notion of uncertainty, which is formalised in possibility theory. It seems natural to measure how much an information can be believed as its truthfulness certainty. However, many examples in life show that an uncertain piece of information can be strongly believed, as can for instance be illustrated by a superstition.

Therefore, in the rest of the paper, we consider a fuzzy set approach. It must be underlined that this choice establishes a justification for the propositions of aggregation relevant degrees, as sketched in the next Section.

### 4 Manipulation of Belief Degrees: Conjunction and Disjunction

The definitions of conjunction and disjunction manipulation rules aim at allowing to computing realistic models for reasoning with beliefs. For each connector, two rules are considered, namely factorisation and distribution: the purpose is to determine a way to aggregate belief degrees in the equivalence rule:

$$B(\varphi, \alpha) * B(\psi, \beta) \Leftrightarrow B(\varphi * \psi, \gamma)$$

where  $B(F, \alpha)$  means "the agent believes  $F$  at a degree  $\alpha$ ",  $*$  can be a conjunctive or disjunctive connector and  $\gamma$  is the aggregation of  $\alpha$  and  $\beta$ . However, more than a gathering of two belief degrees, these rules need to define relation between the separated degrees  $\alpha$  and  $\beta$  and their fusion  $\gamma$ . This relation is called an aggregation. Also, each category of this proposition represents one direction of this equivalence: the left to right direction corresponds to factorisation whereas the right to left one corresponds to distribution.

**Factorisation** The doxastic graded modality can be factorised depending on the logic connector which links the concerned modal formulae. The issue induced by this factorisation concern the way to aggregate the two belief degrees related to the modal formulae. In this factorisation part, which is the left to right direction, the aim is to discuss the principal aggregation properties for both connectors  $\wedge$  and  $\vee$ . For instance, the monotonicity proposes that if  $\alpha$  or  $\beta$  increase then the value of  $\gamma$  should also increase. Thus, this property is read in a doxastic interpretation: the rule  $B(\varphi, \alpha) * B(\psi, \beta) \Rightarrow B(\varphi * \psi, \gamma)$  is compatible with the jointly increasing of  $\alpha$ , or  $\beta$ , and  $\gamma$ . Assuming  $\alpha' \geq \alpha$  and the fact  $\varphi$  is believed at a degree  $\alpha'$  and  $\psi$  at a degree  $\beta$ , then the belief degree value accorded to the formula  $\varphi * \psi$  is higher than  $\gamma$ .

Likewise, it can be argued that the underlying operator must satisfy the properties of commutativity, increasing monotonicity, associativity and neutral element, the discussion being omitted here because of place constraint

**Distribution** The distribution part is proposed as a complementary manipulation of the modality regarding its factorisation. Distributivity is basically related to a phenomenon of belief degree division, whereas factorisation describes a gathering. In other terms, the dissociation of the united believed formulae  $\varphi * \psi$  induces a dissociation between the degree respectively allocated to  $\varphi$  and  $\psi$ . The aggregation operator concerns the degrees associated to  $\varphi$  and  $\psi$  separately. Indeed, the combination of these degrees can be assigned to  $\varphi * \psi$  according to the factorisation process, since distributivity is its reverse operation. Thus, the distribution rule  $B(\varphi * \psi, \gamma) \Rightarrow B(\varphi, \alpha) * B(\psi, \beta)$  is discussed as a constraint for some aggregation properties.

## 5 Selection of Fuzzy Operators

The last part of this work discusses and selects the fuzzy aggregation operators [6] adapted to the selected properties.

For instance, disjunctive and conjunctive operators have to be commutative, increasing monotone and associative, with a neutral element equal to 1 and 0 respectively. In the fuzzy literature, t-norms and t-conorms [6] satisfy the required properties, other families can be considered as well, eg uninorm [8]. These families can require other additional properties which cannot be detailed here because of place constraint.

## 6 Conclusion and Future Works

This paper considered the issue of for belief degrees manipulation. It first compared existing models for graded beliefs and their manipulation and discussed possible interpretations of the meaning of the belief degrees. It then considered the issue of aggregation, so as to determine how much a conjunction or disjunction of graded beliefs is equally believed. It discussed the selection of relevant aggregation using the considered fuzzy tools.

Future works include extending this proposition to other manipulation rules for graded beliefs. For example, the doxastic axioms can be read and discussed according to the proposed fuzzy reading of belief.

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