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Liquid menisci equilibrium in a wet granular medium in pendular regime

– Cohesive strength estimation for random smooth spheres packings



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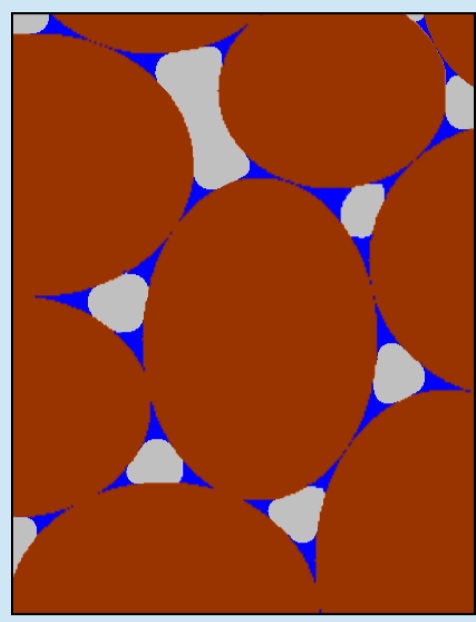
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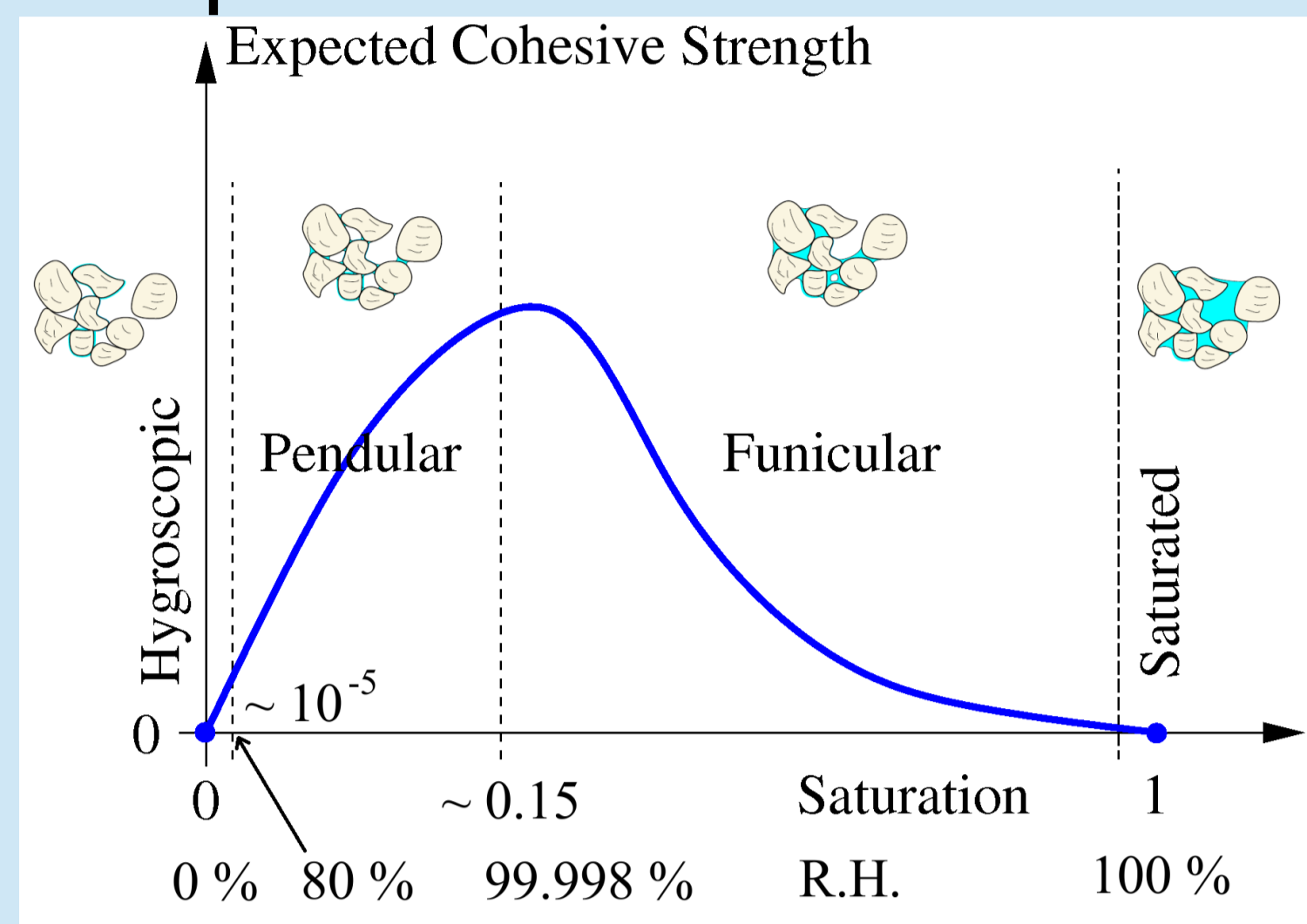
1 – Can a simple model well represent the cohesion?



Sandcastle: a small water content gives strong capillary forces!



In pendular regime, capillary forces come from surface tension



Maximum of this cohesive strength should occur at the end of the pendular regime... (see Richefeu *et al.*, 2016, for a realistic 2D simulation)

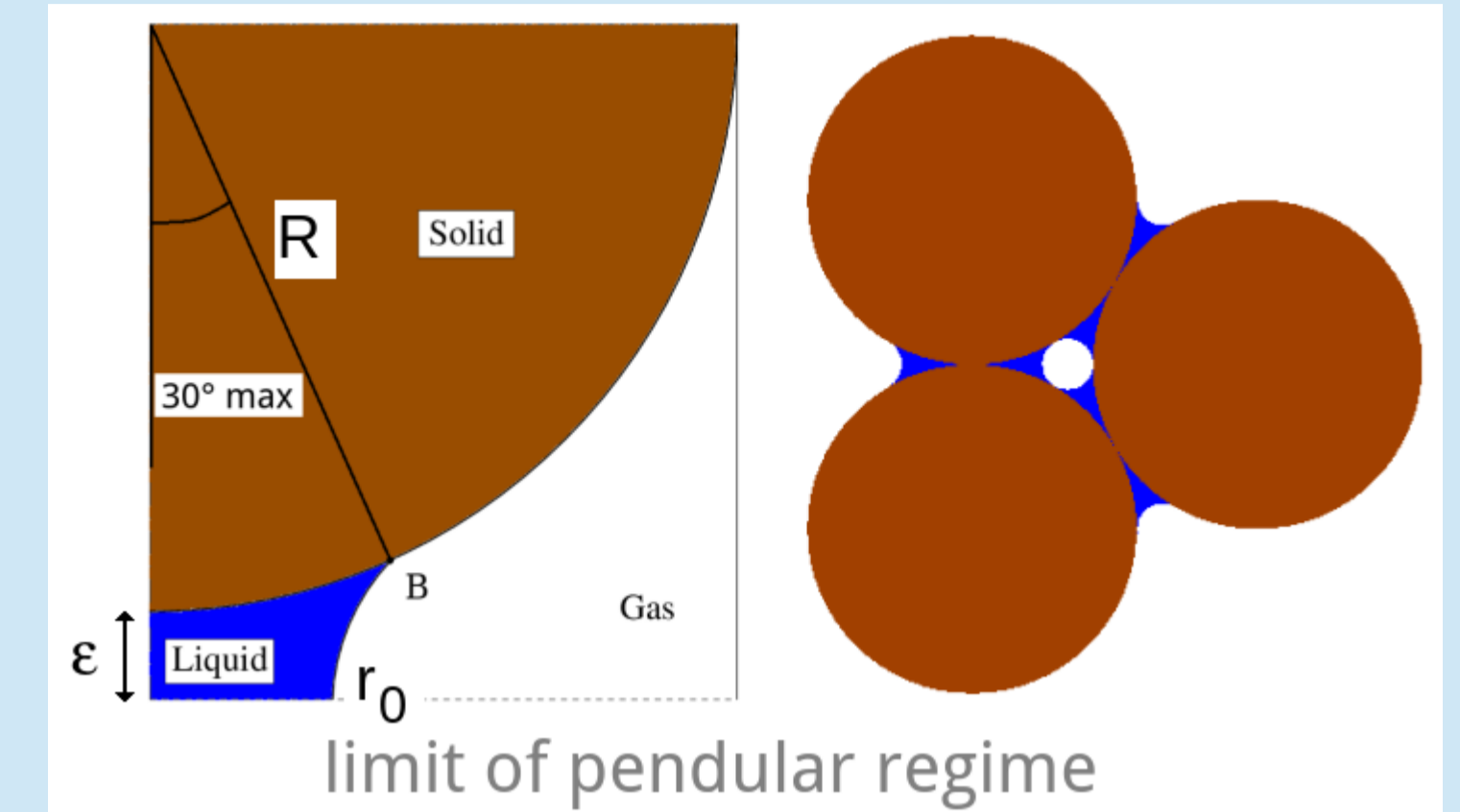
2 – Two smooth spherical grains' system

Control parameters:

- R radius of the spherical grains
- ϵ half of the gap between them
- κ total curvature of the meniscus surface
- σ surface tension
- θ contact angle liquid-solid

Variables obtained after computation:

- r_0 radius of the neck
- r_B radius of the wet surface of the solid part
- V volume of the liquid meniscus
- S surface area of the liquid meniscus
- F attractive force between the two grains



limit of pendular regime

$$F = \sigma (2 \pi r_0 + \kappa \pi r_0^2)$$

Meniscus shape computed numerically: integration of an ODE system, in an axisymmetric coord. system, specifying a constant total curvature κ
Numerical results for r_0 , V, S, F: stored in a database

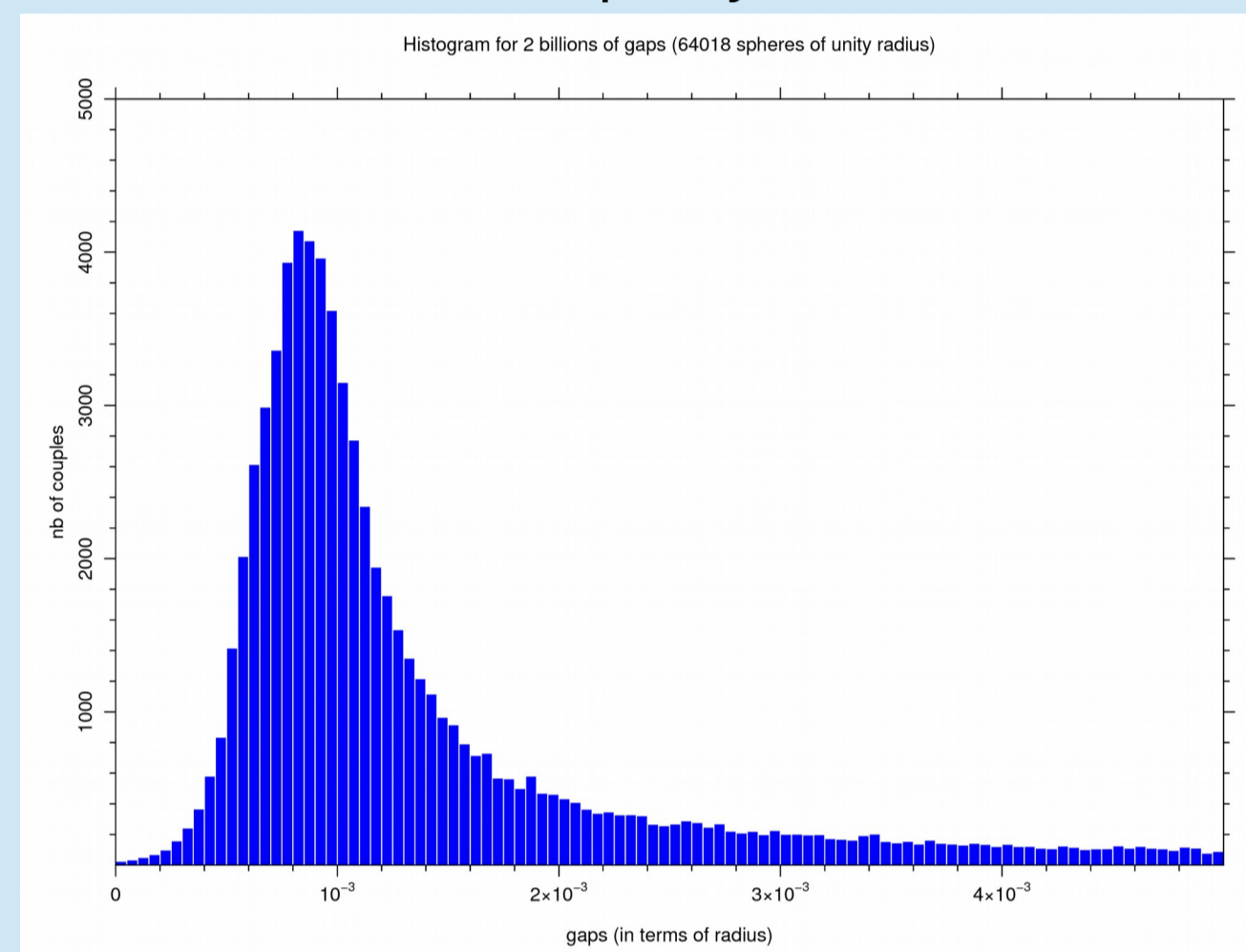
3 – Statistics for random spheres' packings

→ numerically obtained via DEM (L. Oger, IPR)

Distribution of the gaps: $D(\epsilon)$

isotropic case

increasing radius (no gravity) + monodisperse
→ compacity: 0.64



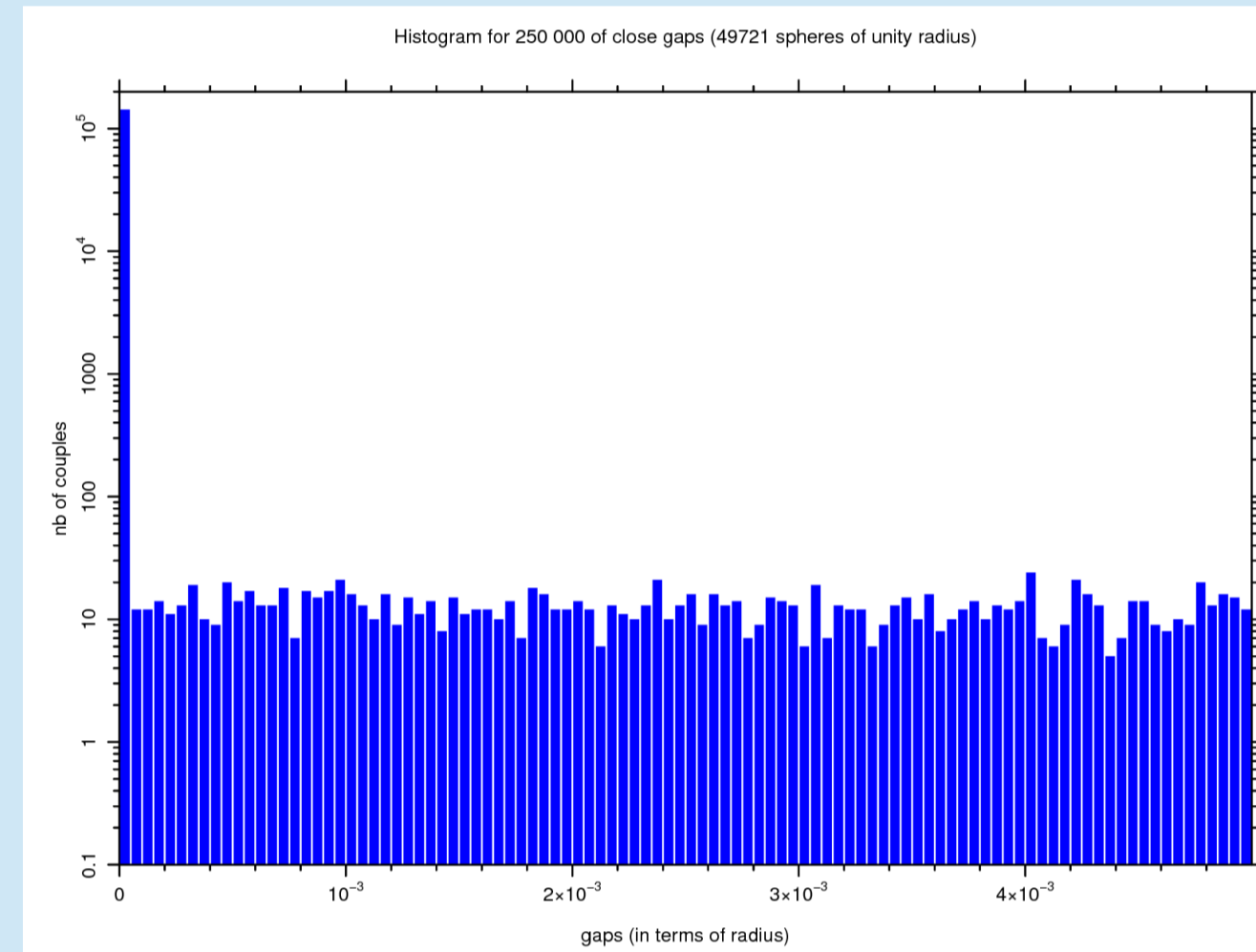
gaps: **Log-Cauchy distribution**
no true contact between spheres

density of gaps (per R^3): $N_g \sim 1.03$

- Easy to parameterize
- isotropic case: median of log(gap) + sharpness
- anisotropic case: % of true contacts + distribution width

anisotropic case

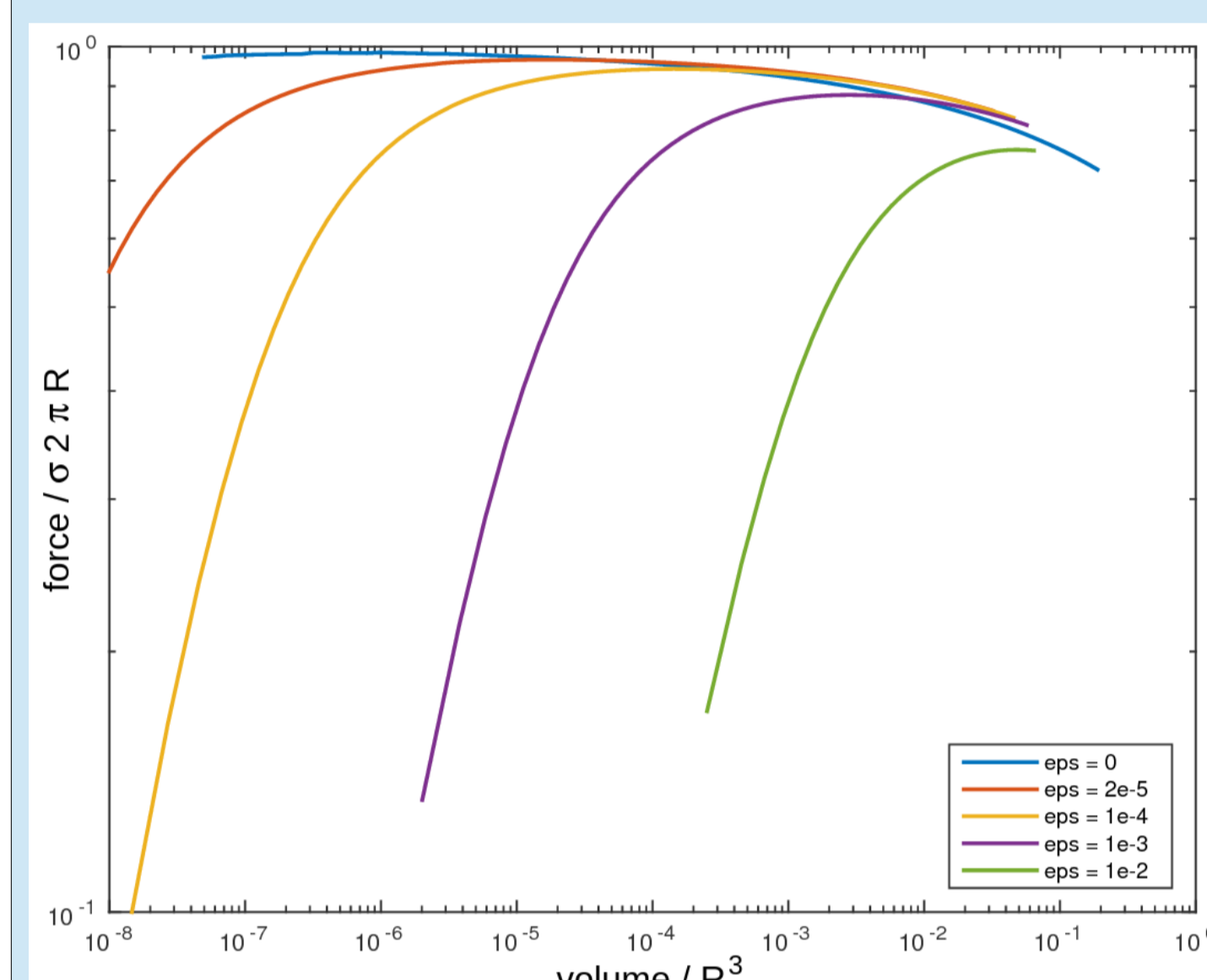
falling balls (gravity) + quasi-monodisperse
→ compacity: 0.61 – low radius STD: 0.005



gaps: **Dirac + Uniform distribution**
46% of true contacts between spheres

density of gaps (per R^3): $N_g \sim 0.82$

Main tendencies for the behaviour of the capillary force $F(\epsilon, V)$



With respect to the volume V:
→ force \sim Cst for contact ($\epsilon=0$)
(even for infinitesimal volume!)

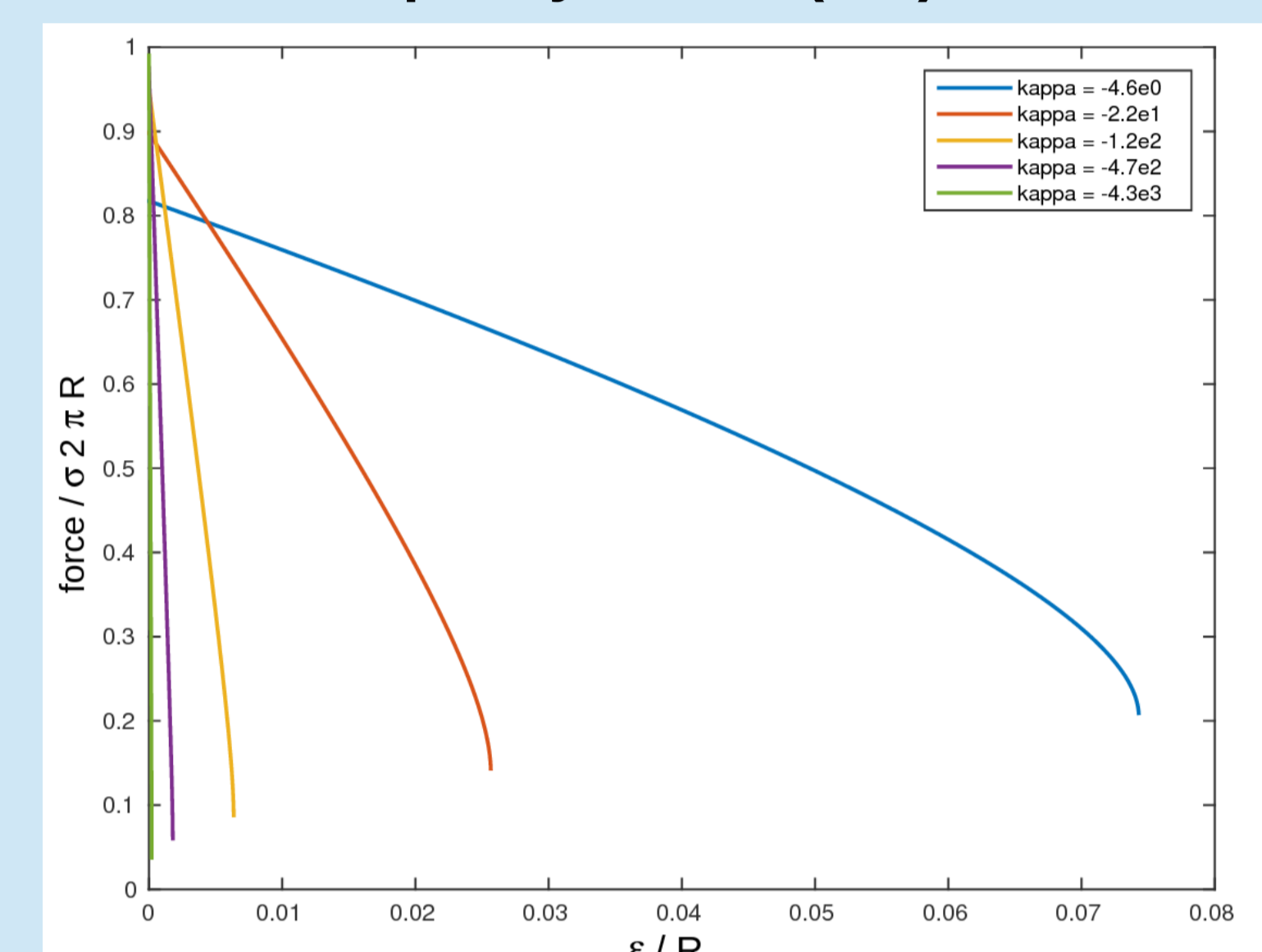
Energy approach: Gibbs free potential

Defining the Gibbs free energy as:
(P. Attard, 2000; Elliott & Voitcu, 2007)

$$G(\epsilon, \kappa) = \sigma (A_{LV} - A_{SL} \cos \theta - 2 V \kappa)$$

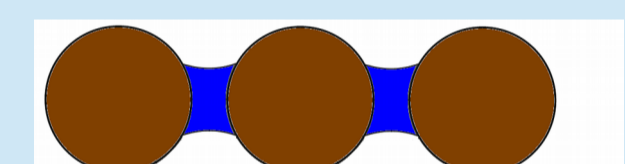
where A_{LV} and A_{SL} are the liquid-vapor and solid-liquid surface area

then $F = \frac{\partial G}{\partial \epsilon}$ with κ fixed (not V!)



With respect to the gap ϵ :
→ force is always decreasing

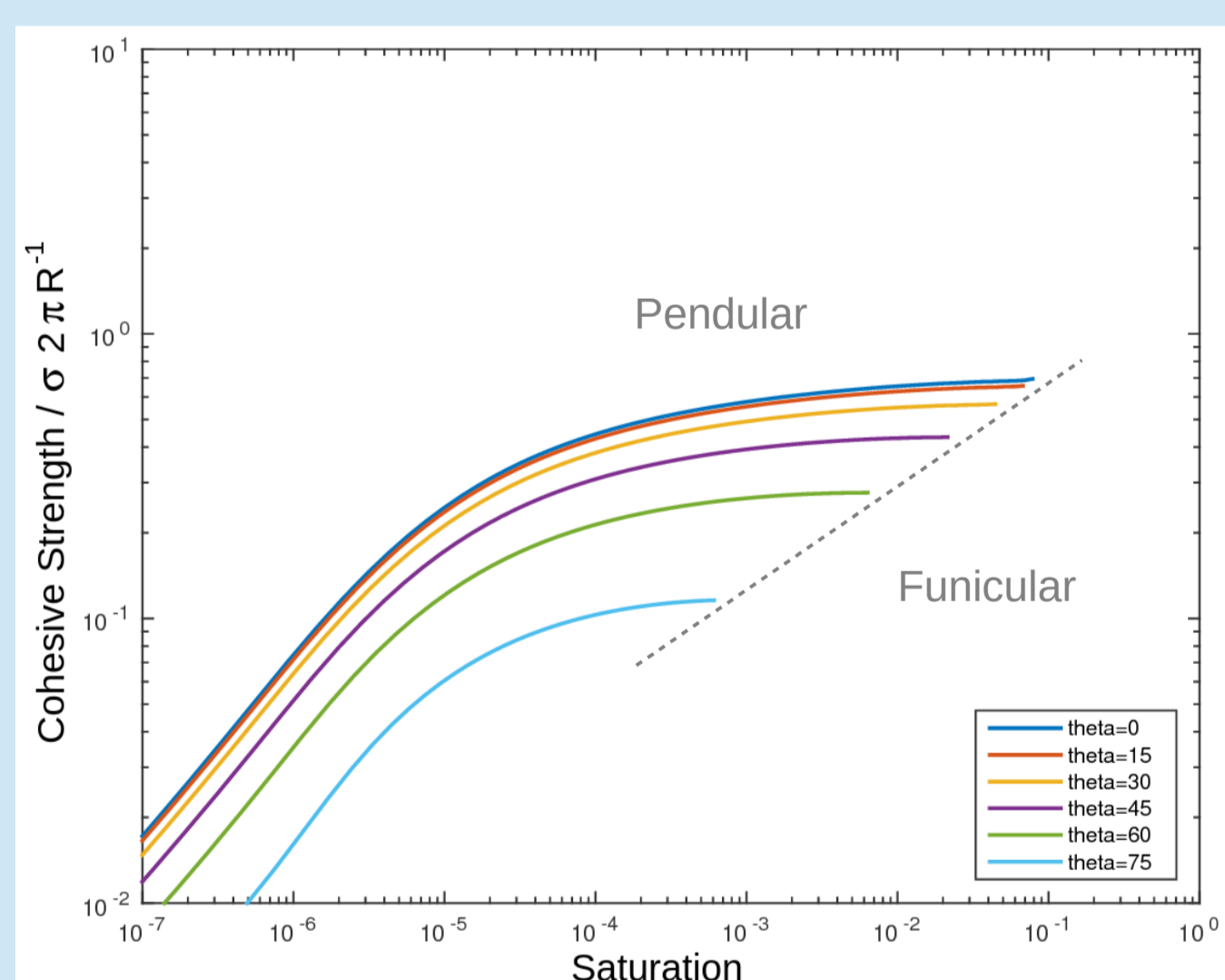
Leads to *unstable situation* as the following:



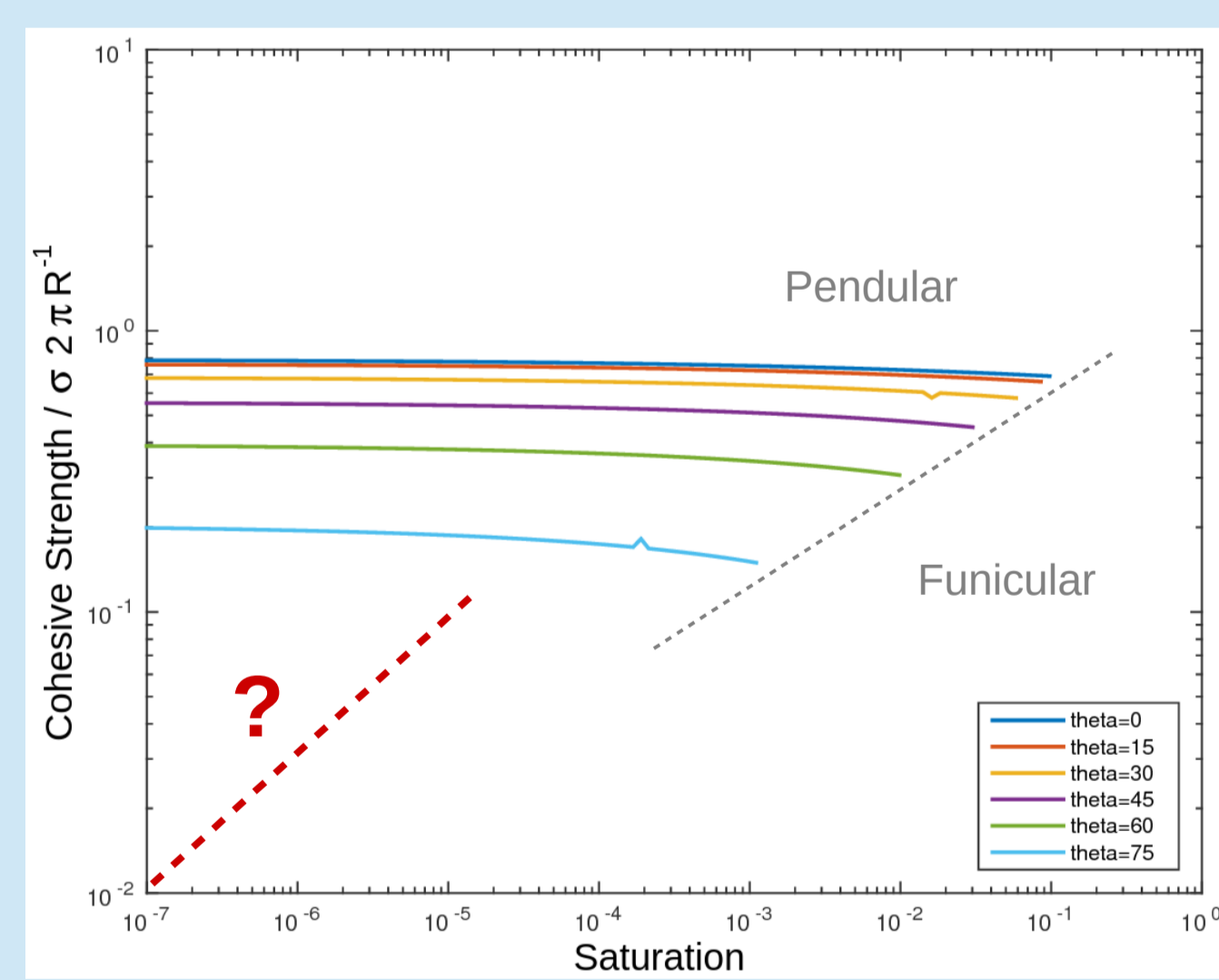
→ humidity aggregates grains as clusters:
(Staron *et al.*, 2005)

5 – Results and Conclusion

- Strong dependency with the contact angle, as expected
- Plots w.r.t. saturation (strongly linked to Relative Humidity and curvature κ via the Kelvin equation)



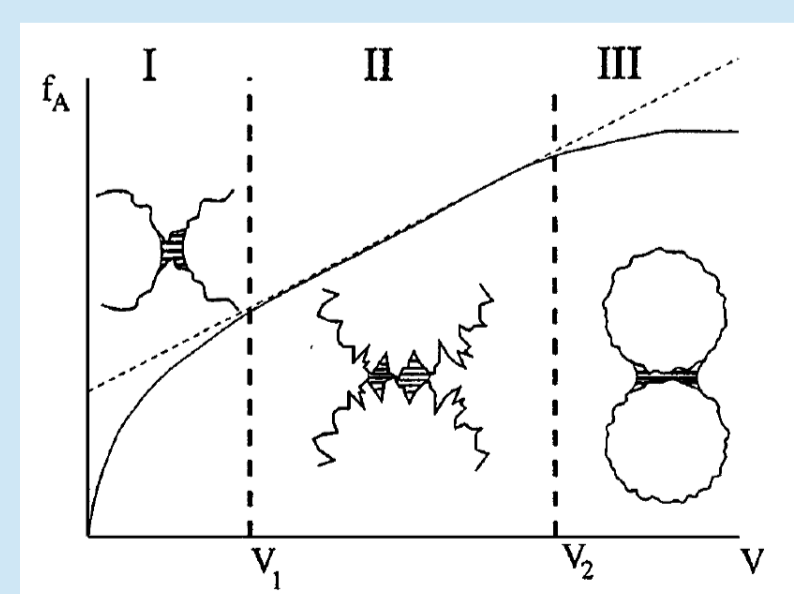
→ Good behavior but not realistic packing (because lot of true contacts must occur!)



Behaviour not expected!
Smooth spheres model not appropriate!
→ Needs introduction of roughness

Advantages of roughness introduction:

- already emphasized by Bocquet *et al.*, 2002
- avoids true contacts between smooth spheres
- allows the decreasing of the cohesive strength w.r.t. liquid volume (or saturation):
see the regime II of Halsey & Levine, 1998
- another possibility is the cone-plane contact model of Albert *et al.*, 1997



Three-regime model of Halsey & Levine, 1998
→ capillary force decrease as liquid volume tends to zero

4 – Computing the cohesive strength

Density of Gibbs free energy: $\bar{G}(\kappa) = N_g \int_0^{\epsilon_{max}} D(\epsilon) G(\epsilon, \kappa) d\epsilon$

The Cohesive Strength χ :

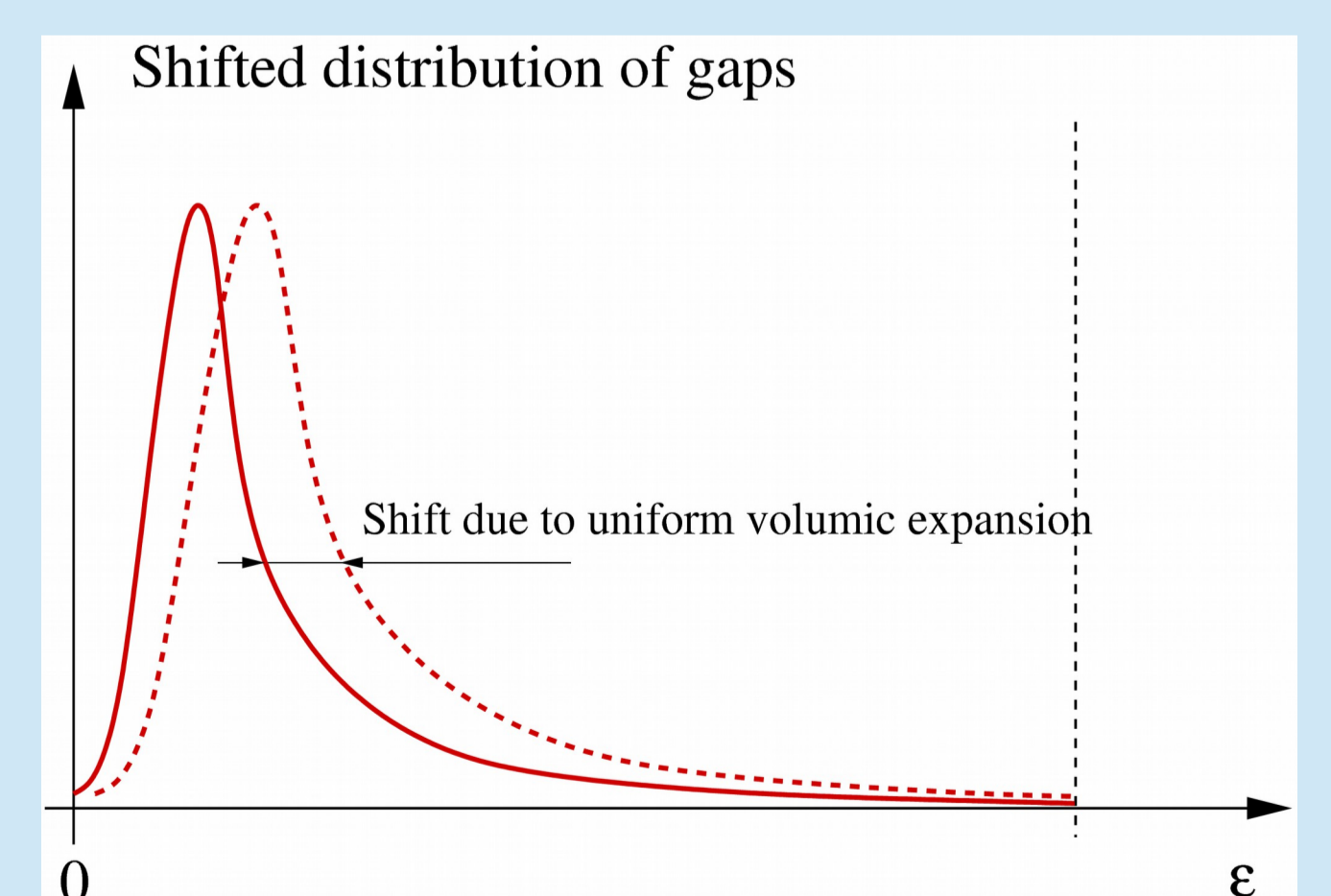
- average of the normal components of the tensile tensor (Richefeu *et al.*, 2016); (avoid the full tensile tensor, as in Scholtès *et al.*, 2009)
- computed via a Uniform Volumetric Expansion $\chi(\kappa) = \frac{\partial \bar{G}}{\partial \beta}$ where β is the dilatation
- doesn't need the direction of the contacts, uses only statistical information (see below)

Equilibrium → all liquid menisci have the same curvature κ

κ related to Relative Humidity via Kelvin equation

Recast in terms of $\frac{\partial D(\epsilon)}{\partial \epsilon}$ instead of $\frac{\partial G(\epsilon)}{\partial \epsilon}$ because $D(\epsilon)$ is well approximated by a mathematical function: → needs to work with a shifted distribution of gaps

The cohesive strength $\chi(\kappa)$ has the dimension of a tension N/m^2



$$\chi(\kappa) = -\frac{1}{3} N_g \left[\int_0^{\epsilon_{max}} (1+\epsilon) \frac{\partial D}{\partial \epsilon} G(\epsilon, \kappa) d\epsilon + D(0) \bar{G}(0) \right]$$

Non constant shift due to the volumetric dilatation of the grains' packing

Boundary effect at the origin, comes from the integration by parts

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